CHAPTER 10

Integer Programming, Goal Programming, and Nonlinear Programming

To accompany
***Quantitative Analysis for Management*, *Twelfth Edition*,**
by Render, Stair, Hanna and Hale
Power Point slides created by Jeff Heyl

LEARNING OBJECTIVES

After completing this chapter, students will be able to:

1. Understand the difference between LP and integer programming.
2. Understand and solve the three types of integer programming problems.
3. Formulate and solve goal programming problems using Excel and QM for Windows.
4. Formulate nonlinear programming problems and solve using Excel.

CHAPTER OUTLINE

Introduction

- There are other mathematical programming models that can be used when the assumptions of LP are not met
- A large number of business problems require variables have integer values
- Many business problems have multiple objectives
- Goal programming permits multiple objectives
- Nonlinear programming allows objectives and constraints to be nonlinear

Integer Programming

- An integer programming model is one where one or more of the decision variables has to take on an integer value in the final solution
- Three types of integer programming problems
  1. Pure integer programming – all variables have integer values
  2. Mixed-integer programming – some but not all of the variables will have integer values
  3. Zero-one integer programming – special cases in which all the decision variables must have integer solution values of 0 or 1

Integer Programming

- An int
  one o
  take c
- Three
  1. Pu
     integer values
  2. Mixed-integer programming – some but not all of the variables will have integer values
  3. Zero-one integer programming – special cases in which all the decision variables must have integer solution values of 0 or 1

- Solving an integer programming problem is much more difficult than solving an LP problem
- Solution time required may be excessive

Harrison Electric Company Example of Integer Programming

- Company produces two products, old-fashioned chandeliers and ceiling fans
- Both require a two-step production process involving wiring and assembly
  - It takes about 2 hours to wire each chandelier and 3 hours to wire a ceiling fan
  - Final assembly of the chandeliers and fans requires 6 and 5 hours, respectively
  - Only 12 hours of wiring time and 30 hours of assembly time are available
  - Each chandelier produced nets the firm $7 and each fan $6

Harrison Electric Company Example of Integer Programming

Production mix LP formulation

$$\begin{aligned}
&\text{Maximize profit} = \$7X_1 + \$6X_2 \\
&\text{subject to} \quad 2X_1 + 3X_2 \le 12 \quad \text{(wiring hours)} \\
&\qquad\qquad\quad\;\; 6X_1 + 5X_2 \le 30 \quad \text{(assembly hours)} \\
&\qquad\qquad\qquad\quad X_1, X_2 \ge 0
\end{aligned}$$

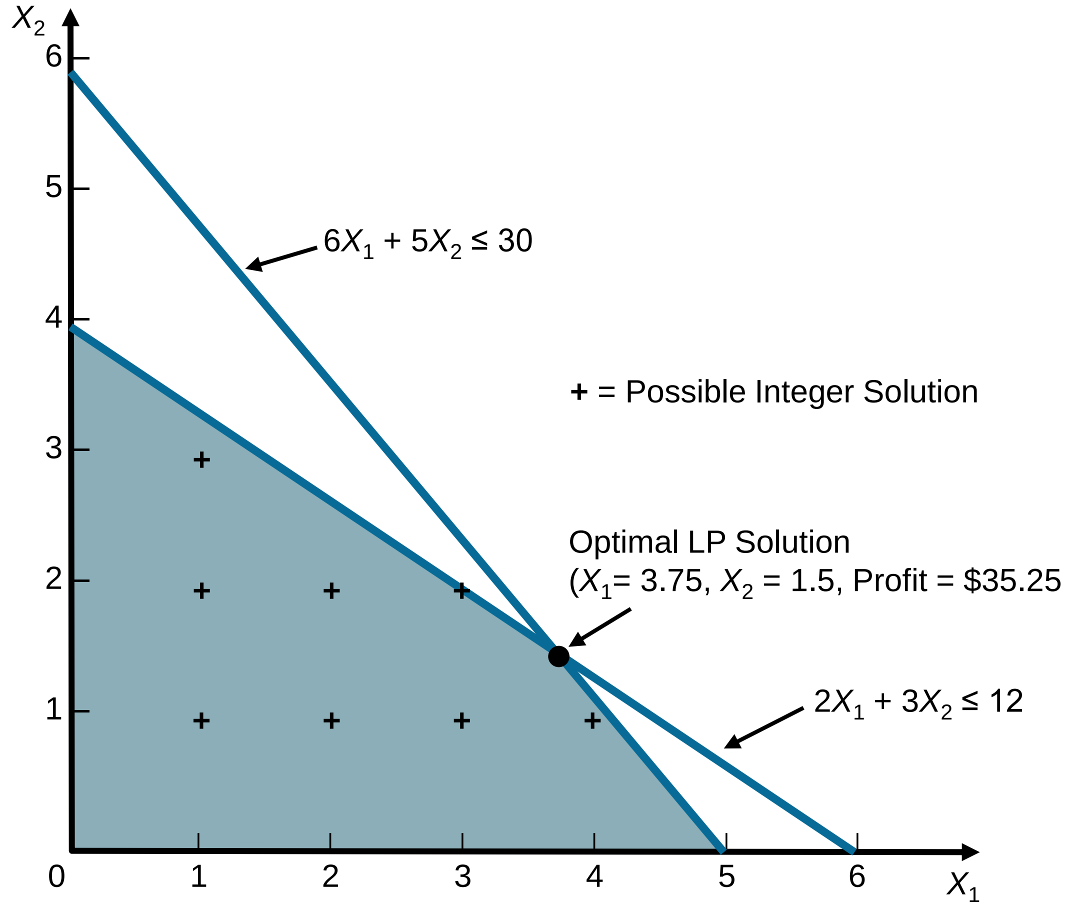
where

X_1 = number of chandeliers produced

X_2 = number of ceiling fans produced

Harrison Electric Company Example of Integer Programming

FIGURE 10.1 – Harrison Electric Problem



Harrison Electric Company Example of Integer Programming

- Production planner recognizes this is an integer problem
  - First attempt at solving it is to round the values to $X_1 = 4$ and $X_2 = 2$
  - However, this is not feasible
  - Rounding X_2 down to 1 gives a feasible solution, but it may not be optimal
- This could be solved using the enumeration method
  - Generally not possible for large problems

Harrison Electric Company Example of Integer Programming

TABLE 10.1 – Integer Solutions to the Harrison Electric Company Problem

CHANDELIERS (X_1)	CEILING FANS (X_2)	PROFIT ($\$7X_1 + \$6X_2$)	
0	0	$0	
1	0	7	
2	0	14	
3	0	21	
4	0	28	
5	0	35	← Optimal solution to integer programming problem
0	1	6	
1	1	13	
2	1	20	
3	1	27	
4	1	34	← Solution if rounding is used
0	2	12	
1	2	19	
2	2	26	
3	2	33	
0	3	18	
1	3	25	
0	4	24	

Harrison Electric Company Example of Integer Programming

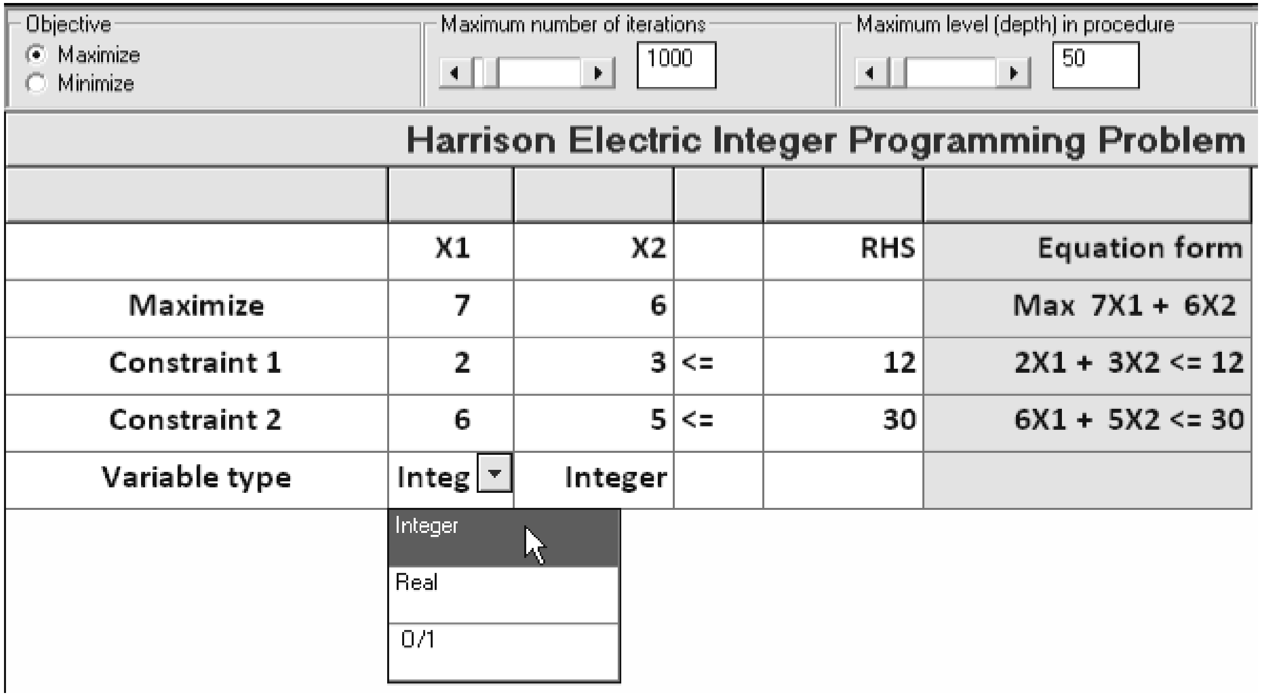
TABLE 10.1 – Integer Solutions to the Harrison Electric Company Problem

CHANDELIERS (X_1)	CEILING FANS (X_2)	PROFIT ($\$7X_1 + \$6X_2$)	
0	0	$0	
1	0	7	
2	0	14	
3	0	21	
4	0	28	
5	0	35	← Optimal solution to integer programming problem
		6	
		13	
		20	
		27	
		34	← Solution if rounding is used
		12	
		19	
2	2	26	
3	2	33	
0	3	18	
1	3	25	
0	4	24	

- The optimal integer solution is less than the optimal LP solution of $35.25
- An integer solution can *never* be better than the LP solution and is *usually* a lesser value

Using Software

PROGRAM 10.1A – QM for Windows Input Screen for Harrison Electric Problem

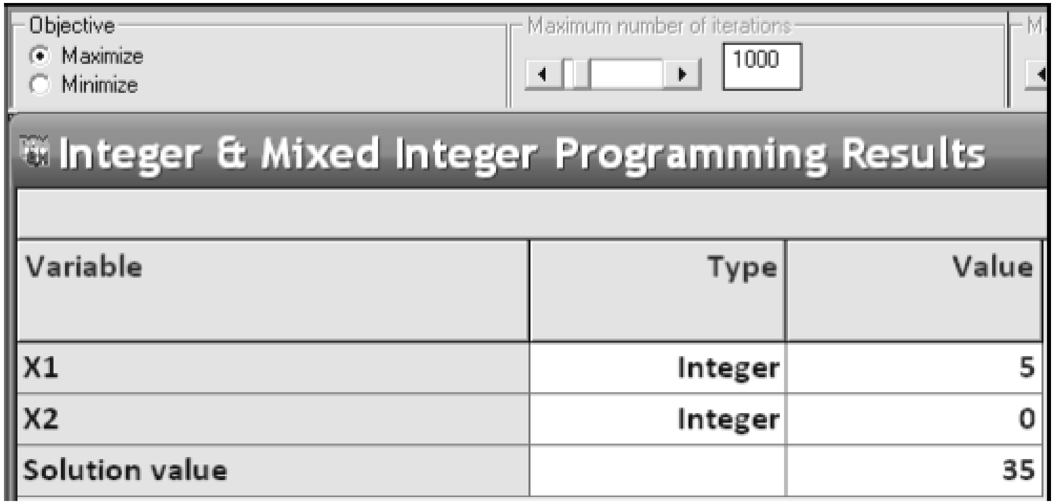


Harrison Electric Integer Programming Problem

	X1	X2		RHS	Equation form
Maximize	7	6			Max 7X1 + 6X2
Constraint 1	2	3	<=	12	2X1 + 3X2 <= 12
Constraint 2	6	5	<=	30	6X1 + 5X2 <= 30
Variable type	Integ	Integer			

Using Software

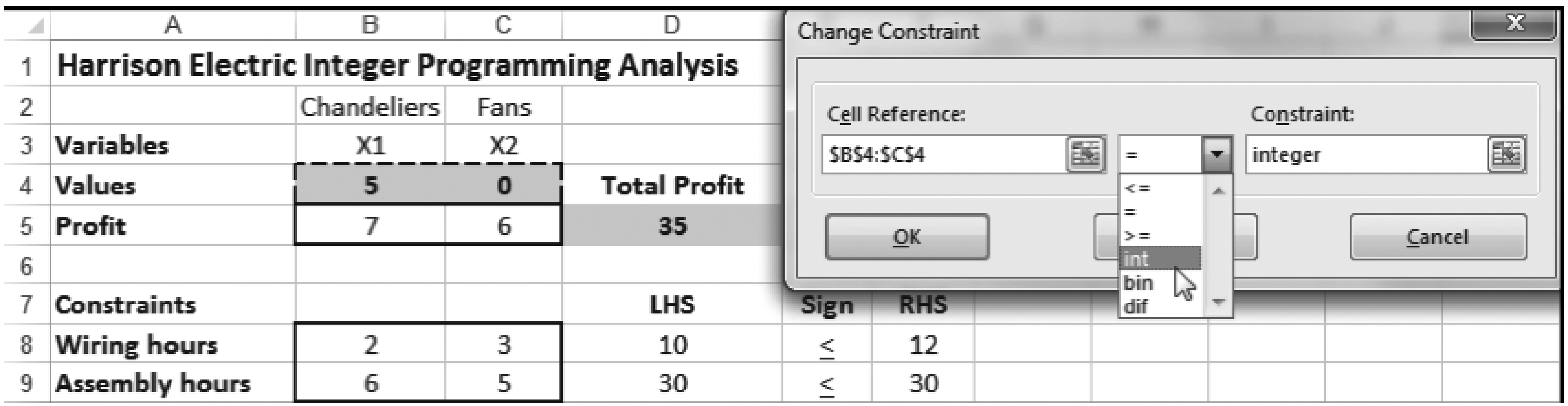
PROGRAM 10.1B – QM for Windows Solution Screen for Harrison Electric Problem



Variable	Type	Value
X1	Integer	5
X2	Integer	0
Solution value		35

Using Software

PROGRAM 10.2 – Excel 2013 Solver Solution for Harrison Electric Problem



	A	B	C	D	E	F
1	**Harrison Electric Integer Programming Analysis**					
2		Chandeliers	Fans			
3	**Variables**	X1	X2			
4	**Values**	**5**	**0**	**Total Profit**		
5	**Profit**	7	6	**35**		
6						
7	**Constraints**			**LHS**	**Sign**	**RHS**
8	**Wiring hours**	2	3	10	≤	12
9	**Assembly hours**	6	5	30	≤	30

Using Software

PROGRAM 10.2 – Excel 2013 Solver Solution for Harrison Electric Problem

Solver Parameter Inputs and Selections

Set Objective: D5

By Changing cells: B4:C4

To: Max

Subject to the Constraints:

D8:D9 >= F8:F9

B4:C4 = integer

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	D
5	=SUMPRODUCT($B$4:$C$4,B5:C5)

Copy D5 to D8:D9

Mixed-Integer Programming Problem Example

- Many situations in which only some of the variables are restricted to integers
  - Bagwell Chemical Company produces two industrial chemicals
    - Xyline must be produced in 50-pound bags
    - Hexall is sold by the pound and can be produced in any quantity
    - Both xyline and hexall are composed of three ingredients – *A*, *B*, and *C*
    - Bagwell sells xyline for $85 a bag and hexall for $1.50 per pound

Mixed-Integer Programming Problem Example

AMOUNT PER 50-POUND BAG OF XYLINE (LB)	AMOUNT PER POUND OF HEXALL (LB)	AMOUNT OF INGREDIENTS AVAILABLE
30	0.5	2,000 lb–ingredient *A*
18	0.4	800 lb–ingredient *B*
2	0.1	200 lb–ingredient *C*

- Objective is to maximize profit

Mixed-Integer Programming Problem Example

Let X = number of 50-pound bags of xyline

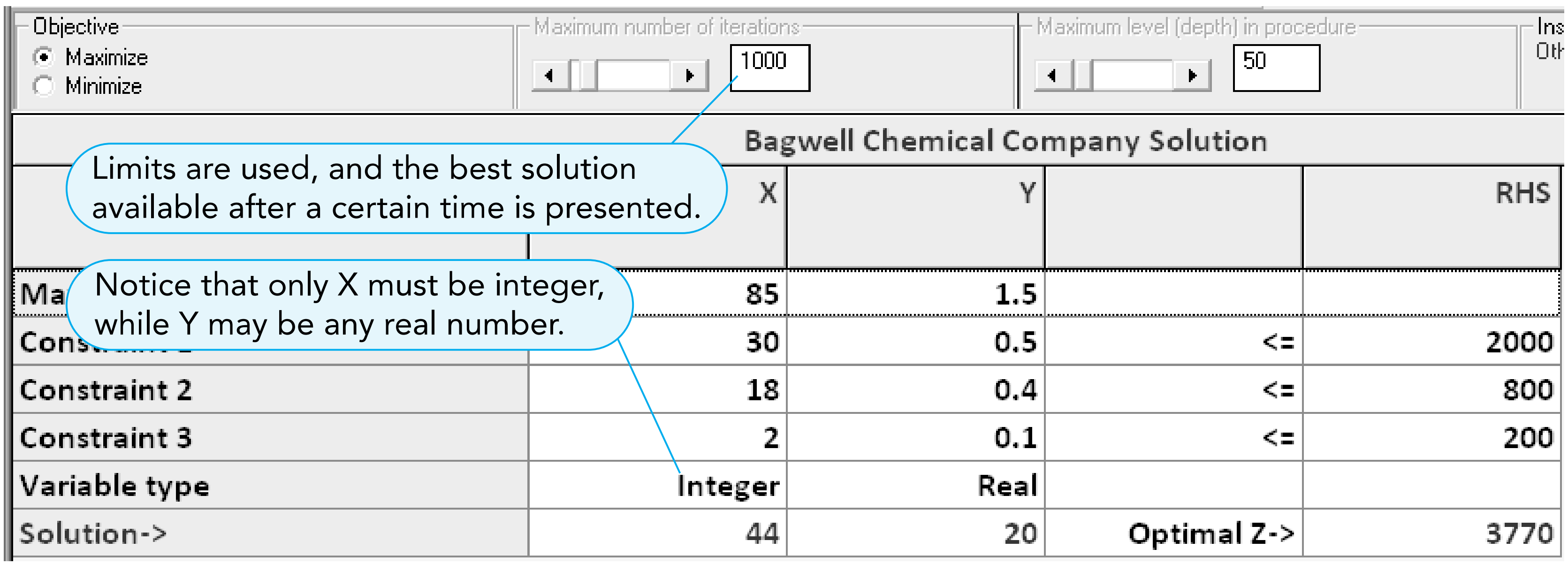
Let Y = number of pounds of hexall

A mixed-integer programming problem as Y is not required to be an integer

$$
\begin{array}{lrrcl}
\text{Maximize profit} = & \$85X + & \$1.50Y & & \\
\text{subject to} & 30X + & 0.5Y & \leq & 2{,}000 \\
 & 18X + & 0.4Y & \leq & 800 \\
 & 2X + & 0.1Y & \leq & 200 \\
 & & X,\ Y & \geq & 0 \text{ and } X \text{ integer}
\end{array}
$$

Using Software

PROGRAM 10.3 – QM for Windows Solution for Bagwell Chemical Problem



	X	Y		RHS
Ma	85	1.5		
Cons	30	0.5	<=	2000
Constraint 2	18	0.4	<=	800
Constraint 3	2	0.1	<=	200
Variable type	Integer	Real		
Solution->	44	20	Optimal Z->	3770

Using Software

PROGRAM 10.4 – Excel 2013 Solver Solution for Bagwell Chemical Problem

	A	B	C	D	E	F
1	**Bagwell Chemical Company**					
2		**Xyline (bags)**	**Hexall (lbs)**			
3	**Variables**	**X**	**Y**			
4	**Values**	**44**	**20**	**Total Profit**		
5	**Profit**	85	1.5	**3770**		
6						
7	**Constraints**			**LHS**	**sign**	**RHS**
8	**Ingredient A**	30	0.5	1330	≤	2000
9	**Ingredient B**	18	0.4	800	≤	800
10	**Ingredient C**	2	0.1	90	≤	200

Using Software

PROGRAM 10.4 – Excel 2013 Solver Solution for Bagwell Chemical Problem

Solver Parameter Inputs and Selections

Set Objective: D5

By Changing cells: B4:C4

To: Max

Subject to the Constraints:

D8:D10 <= F8:F10

B4 = integer

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	D
5	=SUMPRODUCT($B$4:$C$4,B5:C5)

Copy D5 to D8:D10

Modeling With 0-1 (Binary) Variables

- Demonstrate how 0-1 variables can be used to model several diverse situations
- Typically a 0-1 variable is assigned a value of 0 if a certain condition is not met and a 1 if the condition is met
- This is also called a *binary variable*

Capital Budgeting Example

- Common capital budgeting problem – select from a set of possible projects when budget limitations make it impossible to select them all
  - A 0-1 variable is defined for each project
- Quemo Chemical Company is considering three possible improvement projects for its plant
  - A new catalytic converter
  - A new software program for controlling operations
  - Expanding the storage warehouse
- It cannot do them all

Capital Budgeting Example

- Objective is to maximize net present value of projects undertaken

  subject to Total funds used in year 1 ≤ $20,000

  Total funds used in year 2 ≤ $16,000

TABLE 10.2 – Quemo Chemical Company Information

PROJECT	NET PRESENT VALUE	YEAR 1	YEAR 2
Catalytic Converter	$25,000	$8,000	$7,000
Software	$18,000	$6,000	$4,000
Warehouse expansion	$32,000	$12,000	$8,000
Available funds		$20,000	$16,000

Capital Budgeting Example

Decision variables

$$X_1 = \begin{cases} 1 \text{ if catalytic converter project is funded} \\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ if software project is funded} \\ 0 \text{ otherwise} \end{cases}$$

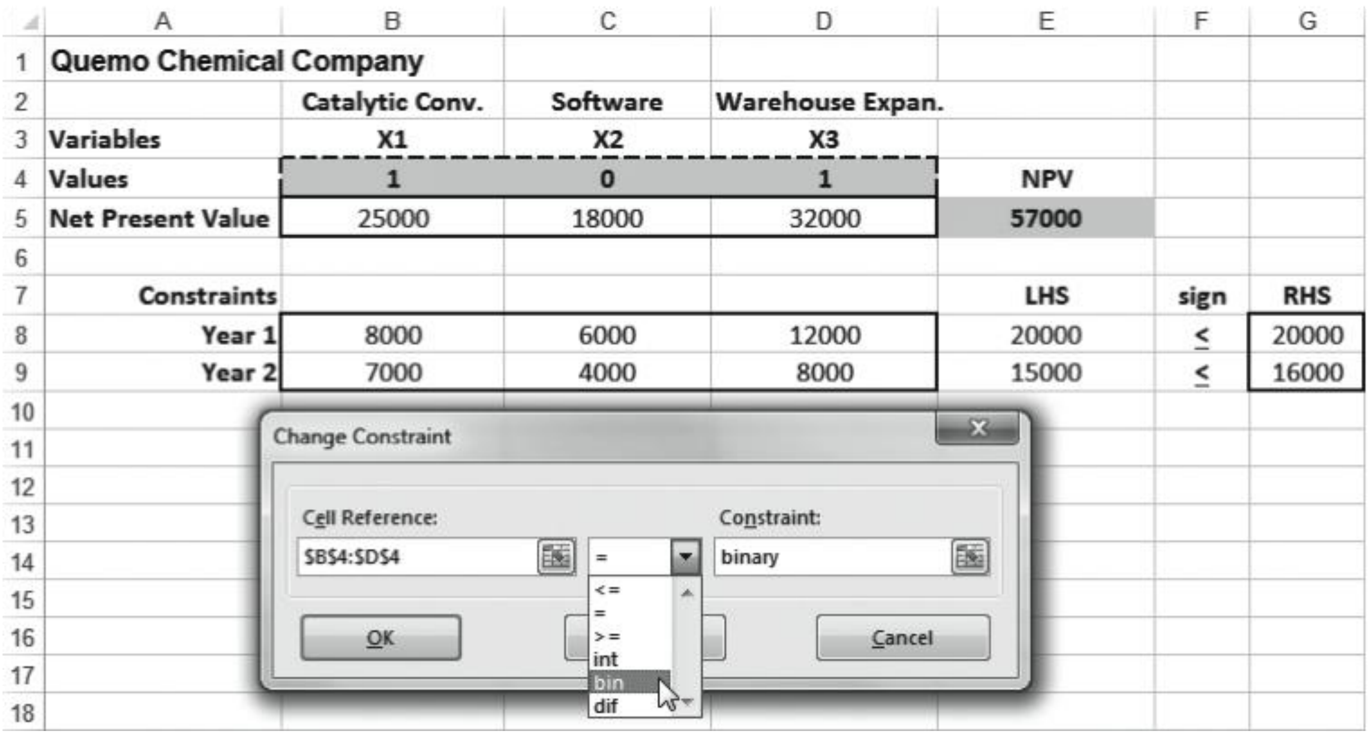
$$X_3 = \begin{cases} 1 \text{ if warehouse expansion project is funded} \\ 0 \text{ otherwise} \end{cases}$$

Formulation

$$\begin{array}{lrrrcl}
\text{Maximize NPV} = & 25{,}000X_1 + & 18{,}000X_2 + & 32{,}000X_3 & & \\
\text{subject to} & 8{,}000X_1 + & 6{,}000X_2 + & 12{,}000X_3 & \leq & 20{,}000 \\
 & 7{,}000X_1 + & 4{,}000X_2 + & 8{,}000X_3 & \leq & 16{,}000 \\
 & & & X_1, X_2, X_3 & = & 0 \text{ or } 1
\end{array}$$

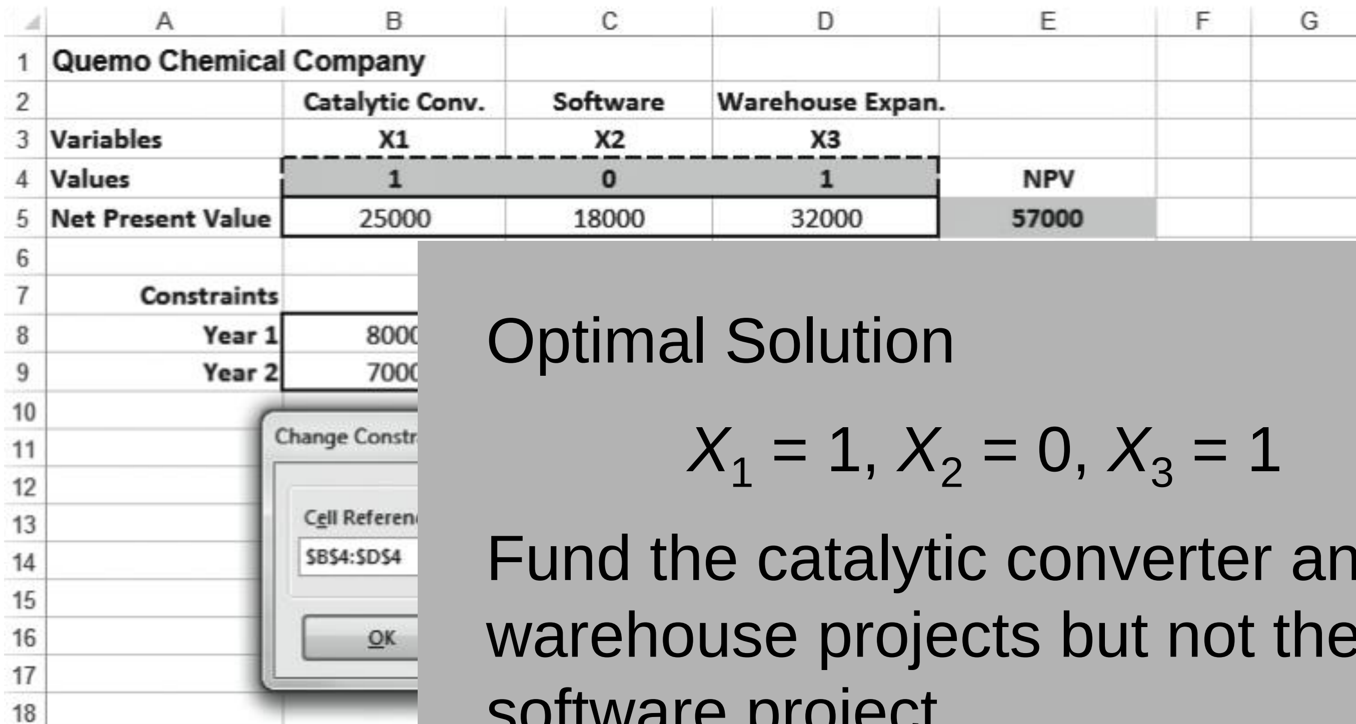
Using Software

PROGRAM 10.5 – Excel 2013 Solver Solution for Quemo Chemical Problem



Using Software

PROGRAM 10.5 – Excel 2013 Solver Solution for Quemo Chemical Problem



Optimal Solution

$$X_1 = 1, X_2 = 0, X_3 = 1$$

Fund the catalytic converter and warehouse projects but not the software project

NPV = $57,000

Using Software

PROGRAM 10.5 – Excel 2013 Solver Solution for Quemo Chemical Problem

Solver Parameter Inputs and Selections

Set Objective: E5

By Changing cells: B4:D4

To: Max

Subject to the Constraints:

E8:E9 <= G8:G9

B4:D4 = binary

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	E
5	=SUMPRODUCT($B$4:$D$4,B5:D5)

Copy E5 to E8:E9

Limiting the Number of Alternatives Selected

- One common use of 0-1 variables involves limiting the number of projects or items that are selected from a group
  - Suppose Quemo Chemical is required to select no more than two of the three projects regardless of the funds available
  - This would require adding a constraint

$$X_1 + X_2 + X_3 \leq 2$$

  - If they had to fund exactly two projects the constraint would be

$$X_1 + X_2 + X_3 = 2$$

Dependent Selections

- At times the selection of one project depends on the selection of another project
  - Suppose Quemo's catalytic converter could only be purchased if the software was purchased
  - The following constraint would force this to occur

$$X_1 \le X_2 \quad \text{or} \quad X_1 - X_2 \le 0$$

  - If we wished for the catalytic converter and software projects to either both be selected or both not be selected, the constraint would be

$$X_1 = X_2 \quad \text{or} \quad X_1 - X_2 = 0$$

Fixed-Charge Problem Example

- Often businesses are faced with decisions involving a fixed charge that will affect the cost of future operations
  - Sitka Manufacturing is planning to build at least one new plant and three cities are being considered
    - Baytown, Texas
    - Lake Charles, Louisiana
    - Mobile, Alabama

Fixed-Charge Problem Example

Constraints

1. Total production capacity at least 38,000 units each year
2. Number of units produced at the Baytown plant is 0 if the plant is not built and no more than 21,000 if the plant is built
3. Number of units produced at the Lake Charles plant is 0 if the plant is not built and no more than 20,000 if the plant is built
4. Number of units produced at the Mobile plant is 0 if the plant is not built and no more than 19,000 if the plant is built

Fixed-Charge Problem Example

TABLE 10.3 – Fixed and Variable Costs for Sitka Manufacturing

SITE	ANNUAL FIXED COST	VARIABLE COST PER UNIT	ANNUAL CAPACITY
Baytown, TX	$340,000	$32	21,000
Lake Charles, LA	$270,000	$33	20,000
Mobile, AL	$290,000	$30	19,000

Fixed-Charge Problem Example

Decision variables

$$X_1 = \begin{cases} 1 \text{ if factory is built in Baytown} \\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ factory is built in Lake Charles} \\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if factory is built in Mobile} \\ 0 \text{ otherwise} \end{cases}$$

X_4 = number of units produced at Baytown plant

X_5 = number of units produced at Lake Charles plant

X_6 = number of units produced at Mobile plant

Fixed-Charge Problem Example

Formulation

Minimize cost = $340{,}000X_1 + 270{,}000X_2 + 290{,}000X_3 + 32X_4 + 33X_5 + 30X_6$

subject to

$$
\begin{aligned}
X_4 + X_5 + X_6 &\geq 38{,}000 \\
X_4 &\leq 21{,}000X_1 \\
X_5 &\leq 20{,}000X_2 \\
X_6 &\leq 19{,}000X_3 \\
X_1, X_2, X_3 &= 0 \text{ or } 1 \\
X_4, X_5, X_6 &\geq 0 \text{ and integer}
\end{aligned}
$$

Using Software

PROGRAM 10.6 – Excel 2013 Solver Solution for Sitka Manufacturing Problem

	A	B	C	D	E	F	G	H	I	J
1	Sitka Manufacturing Company									
2		Baytown	Lake Charles	Mobile	Baytown units	L. Charles units	Mobile units			
3	Variables	X1	X2	X3	X4	X5	X6			
4	Values	0	1	1	0	19000	19000	Cost		
5	Cost	340000	270000	290000	32	33	30	1757000		
6										
7	Constraints							LHS	Sign	RHS
8	Minimum capacity				1	1	1	38000	≥	38000
9	Maximum in Baytown	-21000			1			0	≤	0
10	Maximum in L. C.		-20000			1		-1000	≤	0
11	Maximum in Mobile			-19000			1	0	≤	0

Using Software

PROGRAM 10.6 – Excel 2013 Solver Solution for Sitka Manufacturing Problem

	A	B	C	D	E	F	G	H	I	J
1	**Sitka Manufacturing Company**									
2		**Baytown**	**Lake Charles**	**Mobile**	**Baytown units**	**L. Charles units**	**Mobile units**			
3	**Variables**	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**			
4	**Values**	**0**	**1**	**1**	**0**	**19000**	**19000**	**Cost**		
5	**Cost**	340000	270000	290000	32	33	30	**1757000**		
6										
7	**Constraints**							**LHS**	**Sign**	**RHS**

Optimal solution

$X_1 = 0$, $X_2 = 1$, $X_3 = 1$, $X_4 = 0$, $X_5 = 19{,}000$, $X_6 = 19{,}000$

Objective function value = $1,757,000

Using Software

PROGRAM 10.6 – Excel 2013 Solver Solution for Sitka Manufacturing Problem

Solver Parameter Inputs and Selections

Set Objective: H5

By Changing cells: B4:G4

To: Min

Subject to the Constraints:

- H8 >= J8
- H9:H11 <= J9:J11
- B4:D4 = binary
- E4:G4 = integer

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	H
5	=SUMPRODUCT($B$4:$G$4,B5:G5)

Copy H5 to H8:H11

Financial Investment Example

- Simkin, Simkin, and Steinberg specialize in recommending oil stock portfolios
  - One client has the following specifications
    1. At least two Texas firms must be in the portfolio
    2. No more than one investment can be made in a foreign oil company
    3. One of the two California oil stocks must be purchased
  - The client has $3 million to invest and wants to buy large blocks of shares

Financial Investment Example

TABLE 10.4 – Oil Investment Opportunities

STOCK	COMPANY NAME	EXPECTED ANNUAL RETURN ($1,000s)	COST FOR BLOCK OF SHARES ($1,000s)
1	Trans-Texas Oil	50	480
2	British Petroleum	80	540
3	Dutch Shell	90	680
4	Houston Drilling	120	1,000
5	Texas Petroleum	110	700
6	San Diego Oil	40	510
7	California Petro	75	900

Financial Investment Example

Formulation

Maximize return $= 50X_1 + 80X_2 + 90X_3 + 120X_4 + 110X_5 + 40X_6 + 75X_7$

subject to

$X_1 + X_4 + X_5 \geq 2$ (Texas constraint)

$X_2 + X_3 \leq 1$ (foreign oil constraint)

$X_6 + X_7 = 1$ (California constraint)

$480X_1 + 540X_2 + 680X_3 + 1{,}000X_4 + 700X_5 + 510X_6 + 900X_7 \leq 3{,}000$
($3 million limit)

$X_i = 0$ or 1 for all i

Using Software

PROGRAM 10.7 – Excel 2013 Solver Solution for Financial Investment Problem

	A	B	C	D	E	F	G	H	I	J	K
1	**Simkin, Simkin and Steinberg**										
2											
3	**Variables**	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**			
4	**Values**	**0**	**0**	**1**	**1**	**1**	**1**	**0**	**Return**		
5	**Return ($1,000s)**	50	80	90	120	110	40	75	**360**		
6	**Constraints**								**LHS**	**Sign**	**RHS**
7	**Texas**	1			1	1			2	≥	2
8	**Foreign Oil**		1	1					1	≤	1
9	**California**						1	1	1	=	1
10	**$3 Million**	480	540	680	1000	700	510	900	2890	≤	3000

Using Software

PROGRAM 10.7 – Excel 2013 Solver Solution for Financial Investment Problem

Solver Parameter Inputs and Selections

Set Objective: I5

By Changing cells: B4:H4

To: Max

Subject to the Constraints:

I7 >= K7

I8 <= K8

I9 = K9

I10 <= K10

B4:H4 = binary

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	I
5	=SUMPRODUCT($B$4:$H$4,B5:H5)

Copy I5 to I7:I10

Goal Programming

- Firms often have more than one goal
- In linear and integer programming methods the objective function is measured in one dimension only
- It is not possible for LP to have multiple goals unless they are all measured in the same units
  - Highly unusual situation
- **Goal programming** developed to supplement LP

Goal Programming

- Typically goals set by management can be achieved only at the expense of other goals
- Establish a hierarchy of importance so that higher-priority goals are satisfied before lower-priority goals
- Not always possible to satisfy every goal
- Goal programming attempts to reach a satisfactory level of multiple objectives
- May not optimize but have to **satisfice**

Goal Programming

- Main difference is in the objective function
- Goal programming tries to minimize the *deviations* between goals and what can be achieved given the constraints
- Objective is to minimize **deviational variables**

Harrison Electric Company Revisited

Production mix LP formulation

$$
\begin{aligned}
&\text{Maximize profit} = \$7X_1 + \$6X_2 \\
&\text{subject to} \quad 2X_1 + 3X_2 \le 12 \quad \text{(wiring hours)} \\
&\qquad\qquad\quad\; 6X_1 + 5X_2 \le 30 \quad \text{(assembly hours)} \\
&\qquad\qquad\qquad\quad X_1, X_2 \ge 0
\end{aligned}
$$

where

X_1 = number of chandeliers produced

X_2 = number of ceiling fans produced

Harrison Electric Company Revisited

- Moving to a new location and maximizing profit is not a realistic objective
- A profit level of $30 would be satisfactory during this period
- The goal programming problem is to find the production mix that achieves this goal as closely as possible given the production time constraints
- Define two deviational variables

  d_1^- = underachievement of the profit target

  d_1^+ = overachievement of the profit target

Harrison Electric Company Revisited

Single-goal programming formulation

Minimize under or overachievement of profit target $= d_1^- + d_1^+$

subject to

$\$7X_1 + \$6X_2 + d_1^- - d_1^+ = \$30$ (profit goal constraint)

$2X_1 + 3X_2 \leq 12$ (wiring hours)

$6X_1 + 5X_2 \leq 30$ (assembly hours)

$X_1, X_2, d_1^-, d_1^+ \geq 0$

Harrison Electric Company Revisited

Single-goal programming formulation

Minimize under or overachievement of profit target $= d_1^- + d_1^+$

subj

$

2

6

- Analyze each goal to see if underachievement or overachievement of that goal is acceptable
- If overachievement is acceptable, eliminate the appropriate d^+ variable from the objective function
- If underachievement is okay, the d^- variable should be dropped
- If a goal must be attained exactly, both d^- and d^+ must appear in the objective function

Extension to Equally Important Multiple Goals

Achieve several goals that are equal in priority

Goal 1: to produce a profit of $30 if possible during the production period

Goal 2: to fully utilize the available wiring department hours

Goal 3: to avoid overtime in the assembly department

Goal 4: to meet a contract requirement to produce at least seven ceiling fans

Extension to Equally Important Multiple Goals

The deviational variables can be defined as

d_1^- = underachievement of the profit target
d_1^+ = overachievement of the profit target
d_2^- = idle time in the wiring department (underutilization)
d_2^+ = overtime in the wiring department (overutilization)
d_3^- = idle time in the assembly department (underutilization)
d_3^+ = overtime in the assembly department (overutilization)
d_4^- = underachievement of the ceiling fan goal
d_4^+ = overachievement of the ceiling fan goal

Extension to Equally Important Multiple Goals

Management is unconcerned about d_1^+, d_2^+, d_3^-, and d_4^+ so these may be omitted from the objective function

New objective function and constraints

Minimize total deviation $= d_1^- + d_2^- + d_3^+ + d_4^-$

subject to

$\$7X_1 + \$6X_2 + d_1^- - d_1^+ = \$30$ (profit constraint)

$2X_1 + 3X_2 + d_2^- - d_2^+ = 12$ (wiring hours constraint)

$6X_1 + 5X_2 + d_3^- - d_3^+ = 30$ (assembly hours constraint)

$X_2 + d_4^- - d_4^+ = 7$ (ceiling fan constraint)

Ranking Goals with Priority Levels

- In most goal programming problems, one goal will be more important than another
- Lower-order goals considered only after higher-order goals are met
- Priorities (P_is) are assigned to each deviational variable
  - P_1 is the most important goal
  - P_2 the next most important
  - P_3 the third, and so on

Ranking Goals with Priority Levels

Harrison Electric has set the following priorities for their four goals

GOAL	PRIORITY
Reach a profit as much above $30 as possible	P_1
Fully use wiring department hours available	P_2
Avoid assembly department overtime	P_3
Produce at least seven ceiling fans	P_4

Priority 1 is infinitely more important than Priority 2, which is infinitely more important than the next goal, and so on

Ranking Goals with Priority Levels

Harrison Electric has set the following priorities for their four goals

GOAL	PRIORITY
Reach a profit as much above $30 as possible	P_1
Fully use wiring department hours available	P_2
Avoid assembly department overtime	P_3
Produce at least seven ceiling fans	P_4

With ranking of goals considered, the new objective function is

Minimize total deviation = $P_1 d_1^- + P_2 d_2^- + P_3 d_3^+ + P_4 d_4^-$

Goal Programming with Weighted Goals

- Priority levels assume that each level is infinitely more important than the level below it
- However a goal may be only two or three times more important than another
- Instead of placing these goals on different levels, they are placed on the same level but with different weights
- The coefficients of the deviation variables in the objective function include both the priority level and the weight

Goal Programming with Weighted Goals

- Suppose Harrison decides to add another goal of producing at least two chandeliers
- The goal of producing seven ceiling fans is considered twice as important as this goal
- The goal of two chandeliers is assigned a weight of 1 and the goal of seven ceiling fans is assigned a weight of 2 and both of these will be priority level 4
- The new constraint and objective function are

$$X_1 + d_5^- - d_5^+ = 2 \quad \text{(chandeliers)}$$

$$\text{Minimize total deviation} = P_1d_1^- + P_2d_2^- + P_3d_3^+ + P_4(2d_4^-) + P_4d_5^-$$

Using Software

PROGRAM 10.8A – Harrison Electric's Goal Programming Analysis Using QM for Windows: Inputs

Harrison Electric Company								
	Wt(d+)	Prty(d+)	Wt(d-)	Prty(d-)	X1	X2		RHS
Constraint 1	0	0	1	1	7	6	=	30
Constraint 2	0	0	1	2	2	3	=	12
Constraint 3	1	3	0	0	6	5	=	30
Constraint 4	0	0	1	4	0	1	=	7

Using Software

PROGRAM 10.8B – Summary Solution Screen for Harrison Electric's Goal Programming Problem Using QM for Windows

Summary

Harrison Electric Company Solution

Item			
Decision variable analysis	Value		
X1	0.		
X2	6.		
Priority analysis	Nonachievement		
Priority 1	0.		
Priority 2	0.		
Priority 3	0.		
Priority 4	1.		
Constraint Analysis	RHS	d+ (row i)	d- (row i)
Constraint 1	30.	6.	0.
Constraint 2	12.	6.	0.
Constraint 3	30.	0.	0.
Constraint 4	7.	0.	1.

Nonlinear Programming

- The methods seen so far have assumed that the objective function and constraints are linear
- However, there are many nonlinear relationships in the real world that would require the objective function and/or constraint equations to be nonlinear
- Computational procedures for nonlinear programming (NLP) may only provide a **local optimum** solution rather than a **global optimum**

Nonlinear Objective Function and Linear Constraints

- The Great Western Appliance Company sells two models of toaster ovens, the Microtoaster (X_1) and the Self-Clean Toaster Oven (X_2)
- They earn a profit of $28 for each Microtoaster no matter the number of units sold
- For the Self-Clean oven, profits increase as more units are sold due to a fixed overhead
  - The profit function for the Self-Clean oven

$$21X_2 + 0.25X_2^2$$

Nonlinear Objective Function and Linear Constraints

- The objective function is nonlinear and there are two linear constraints on production capacity and sales time available

Maximize profit = $28X_1 + 21X_2 + 0.25X_2^2$

subject to

$$
\begin{aligned}
X_1 + X_2 &\leq 1{,}000 && \text{(units of production capacity)} \\
0.5X_1 + 0.4X_2 &\leq 500 && \text{(hours of sales time available)} \\
X_1, X_2 &\geq 0
\end{aligned}
$$

Nonlinear Objective Function and

- The objectiv
linear constr
time availabl

When an objective function contains a squared term and the problem constraints are linear, it is called a quadratic programming problem

Maximize profit = $28X_1 + 21X_2 + 0.25X_2^2$

subject to

$$
\begin{array}{rrll}
X_1 + & X_2 \leq & 1,000 & \text{(units of production capacity)} \\
0.5X_1 + & 0.4X_2 \leq & 500 & \text{(hours of sales time available)} \\
 & X_1, X_2 \geq & 0 &
\end{array}
$$

Using Software

PROGRAM 10.9 – Excel 2013 Solver Solution for Great Western Appliance NLP Problem

	A	B	C	D	E	F	G
1	Great Western Appliance						
2		Micro	Self-Clean				
3	Variables	X1	X2				
4	Values	0	1000				
5							
6	Terms	X1	X2	$X2^2$			
7	Calculated Values	0	1000	1000000	Profit		
8	Profit	28	21	0.25	271000		
9							
10	Constraints				LHS	Sign	RHS
11	Capacity	1	1		1000	≤	1000
12	Hours Available	0.5	0.4		400	≤	500

Using Software

PROGRAM 10.9 – Excel 2013 Solver Solution for Great Western Appliance NLP Problem

Solver Parameter Inputs and Selections

Set Objective: E8

By Changing cells: B4:C4

To: Max

Subject to the Constraints:

E11:E12 <= G11:G12

Solving Method: GRG Nonlinear

☑ Make Variables Non-Negative

Key Formulas

	E
8	**=SUMPRODUCT($B$7:$D$7,B8:D8)**
9	
10	**LHS**
11	=SUMPRODUCT($B$4:$C$4,B11:C11)
12	=SUMPRODUCT($B$4:$C$4,B12:C12)

	B	C	D
7	**=B4**	**=C4**	**=C4^2**

Both Nonlinear Objective Function and Nonlinear Constraints

- The annual profit at a medium-sized (200-400 beds) Hospicare Corporation hospital depends on
  - The number of medical patients admitted (X_1)
  - The number of surgical patients admitted (X_2)
- The objective function for the hospital is nonlinear
- There are three constraints, two of which are nonlinear
  - Nursing capacity - nonlinear
  - X-ray capacity - nonlinear
  - Marketing budget required

Both Nonlinear Objective Function and Nonlinear Constraints

- Objective function and constraint equations

Maximize profit = $\$13X_1 + \$6X_1X_2 + \$5X_2 + \$1/X_2$

subject to

$$
\begin{aligned}
2X_1^2 + 4X_2 &\leq 90 \quad \text{(nursing capacity in thousands of labor-days)}\\
X_1 + X_2^3 &\leq 75 \quad \text{(x-ray capacity in thousands)}\\
8X_1 - 2X_2 &\leq 61 \quad \text{(marketing budget required in thousands of \$)}
\end{aligned}
$$

Using Software

PROGRAM 10.10 – Excel 2013 Solution to the Hospicare NLP Problem

	A	B	C	D	E	F	G	H	I	J
1	**Hospicare Corp**									
2										
3	**Variables**	**X1**	**X2**							
4	**Values**	**6.0663**	**4.1003**							
5										
6	**Terms**	**X1**	**$X1^2$**	**X1*X2**	**X2**	**$X2^3$**	**1/X2**			
7	**Calculated Values**	6.0663	36.7995	24.8732	4.1003	68.9337	0.2439	**Total Profit**		
8	**Profit**	13	0	6	5		1	**248.8457**		
9										
10	**Constraints**							**LHS**	**Sign**	**RHS**
11	**Nursing**		2		4			90.00	$\leq$	90
12	**X-Ray**	1				1		75.00	$\leq$	75
13	**Budget**	8			-2			40.33	$\leq$	61

Using Software

PROGRAM 10.10 – Excel 2013 Solution to the Hospicare NLP Problem

Solver Parameter Inputs and Selections

Set Objective: H8

By Changing cells: B4:C4

To: Max

Subject to the Constraints:

H11:H13 <= J11:J13

Solving Method: GRG Nonlinear

☑ Make Variables Non-Negative

Key Formulas

	H
8	=SUMPRODUCT($B$7:$G$7,B8:G8)

Copy H8 to H11:H13

	B	C	D	E	F	G
7	=B4	=B4^2	=B4*C4	=C4	=C4^3	=1/C4

Linear Objective Function and Nonlinear Constraints

- Thermlock Corp. produces massive rubber washers and gaskets like the type used to seal joints on the NASA Space Shuttles
  - It combines two ingredients, rubber (X_1) and oil (X_2)
  - The cost of the industrial quality rubber is $5 per pound and the cost of high viscosity oil is $7 per pound
  - Two of the three constraints are nonlinear

Linear Objective Function and Nonlinear Constraints

- Objective function and constraints

Minimize costs = $\$5X_1 + \$7X_2$

subject to

$$3X_1 + 0.25X_1^2 + 4X_2 + 0.3X_2^2 \geq 125 \quad \text{(hardness constraint)}$$
$$13X_1 + X_1^3 \geq 80 \quad \text{(tensile strength)}$$
$$0.7X_1 + X_2 \geq 17 \quad \text{(elasticity)}$$

Using Software

PROGRAM 10.11 – Excel 2013 Solution to the Thermlock NLP Problem

	A	B	C	D	E	F	G	H	I
1	Thermlock Gaskets								
2									
3	Variables	X1	X2						
4	Values	3.325	14.672	Total Cost					
5	Cost	5	7	119.333					
6									
7		X1	$X1^2$	$X1^3$	X2	$X2^2$			
8	Value	3.325	11.058	36.771	14.672	215.276			
9	Constraints						LHS	Sign	RHS
10	Hardness	3	0.25		4	0.3	136.012	≥	125
11	Tensile Strengt	13		1			80	≥	80
12	Elasticity	0.7			1		17	≥	17

Using Software

PROGRAM 10.11 – Excel 2013 Solution to the Thermlock NLP Problem

Solver Parameter Inputs and Selections

Set Objective: D5

By Changing cells: B4:C4

To: Min

Subject to the Constraints:

G10:G12 >= I10:I12

Solving Method: GRG Nonlinear

☑ Make Variables Non-Negative

Key Formulas

	D
5	=SUMPRODUCT($B$4:$C$4,B5:C5)

	G
10	=SUMPRODUCT($B$8:$F$8,B10:F10)

Copy G10 to G11:G12

	B	C	D	E	F
8	=B4	=B4^2	=B4^3	=C4	=C4^2

Copyright