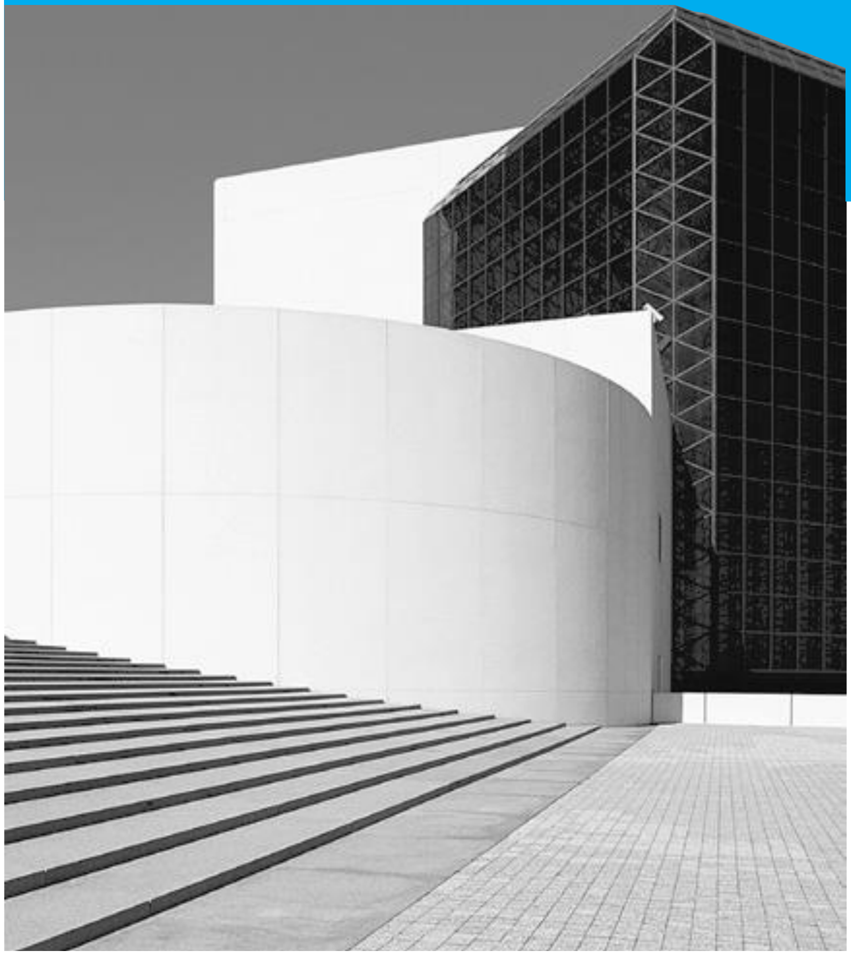
CHAPTER 8



Linear Programming Applications

To accompany
Quantitative Analysis for Management, *Twelfth Edition*,
by Render, Stair, Hanna and Hale
Power Point slides created by Jeff Heyl

LEARNING OBJECTIVES

After completing this chapter, students will be able to:

1. Model a wide variety of medium to large LP problems.
2. Understand major application areas, including marketing, production, labor scheduling, fuel blending, transportation, and finance.
3. Gain experience in solving LP problems with Excel Solver software.

CHAPTER OUTLINE

Introduction

- The graphical method of LP is useful for understanding how to formulate and solve small LP problems
- Many types of problems can be solved using LP
- Principles developed here are applicable to larger problems

Marketing Applications

- Linear programming models have been used in the advertising field as a decision aid in selecting an effective media mix
- Media selection LP problems can be approached from two perspectives
  - Maximize audience exposure
  - Minimize advertising costs

Win Big Gambling Club

- Club promotes gambling junkets to the Bahamas
  - $8,000 per week to spend on advertising
  - Goal is to reach the largest possible high-potential audience
  - Media types and audience figures shown below
  - Place at least five radio spots per week
  - No more than $1,800 can be spent on radio advertising each week

Win Big Gambling Club

- Advertising options

MEDIUM	AUDIENCE REACHED PER AD	COST PER AD ($)	MAXIMUM ADS PER WEEK
TV spot (1 minute)	5,000	800	12
Daily newspaper (full-page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

Win Big Gambling Club

- Problem formulation

X_1 = number of 1-minute TV spots taken each week
X_2 = number of daily newspaper ads taken each week
X_3 = number of 30-second prime-time radio spots taken each week
X_4 = number of 1-minute afternoon radio spots taken each week

Objective:

Maximize audience coverage = $5{,}000X_1 + 8{,}500X_2 + 2{,}400X_3 + 2{,}800X_4$

Subject to

X_1	≤ 12	(max TV spots/wk)
X_2	≤ 5	(max newspaper ads/wk)
X_3	≤ 25	(max 30-sec radio spots/wk)
X_4	≤ 20	(max 1-min radio spots/wk)
$800X_1 + 925X_2 + 290X_3 + 380X_4$	$\leq \$8{,}000$	(weekly advertising budget)
$X_3 + X_4$	≥ 5	(min radio spots contracted)
$290X_3 + 380X_4$	$\leq \$1{,}800$	(max dollars spent on radio)
X_1, X_2, X_3, X_4	≥ 0	

Win Big

Solution

$X_1 = 1.97$ TV spots
$X_2 = 5$ newspaper ads
$X_3 = 6.2$ 30-second radio spots
$X_4 = 0$ 1-minute radio spots

- Problem formula

X_1 = number of 1-minute T
X_2 = number of daily news
X_3 = number of 30-second
X_4 = number of 1-minute afternoon radio spots taken each week

Objective:

Maximize audience coverage = $5{,}000X_1 + 8{,}500X_2 + 2{,}400X_3 + 2{,}800X_4$

Subject to

X_1	≤ 12	(max TV spots/wk)
X_2	≤ 5	(max newspaper ads/wk)
X_3	≤ 25	(max 30-sec radio spots/wk)
X_4	≤ 20	(max 1-min radio spots/wk)
$800X_1 + 925X_2 + 290X_3 + 380X_4$	$\leq \$8{,}000$	(weekly advertising budget)
$X_3 + X_4$	≥ 5	(min radio spots contracted)
$290X_3 + 380X_4$	$\leq \$1{,}800$	(max dollars spent on radio)
X_1, X_2, X_3, X_4	≥ 0	

Solution in Excel 2013

PROGRAM 8.1 – Win Big Solution

	A	B	C	D	E	F	G	H
1	**Win Big Gambling Club**							
2				**Radio**	**Radio**			
3		**TV**	**Newspaper**	**30 sec.**	**1 min.**			
4	**Variables**	**X1**	**X2**	**X3**	**X4**			
5	**Solution**	1.9688	5	6.2069	0	**Total Audience**		
6	**Audience per ad**	5000	8500	2400	2800	67240.3017		
7								
8	**Constraints**					**LHS**		**RHS**
9	**Max. TV**	1				1.9688	≤	12
10	**Max. Newspaper**		1			5	≤	5
11	**Max. 30-sec. radio**			1		6.2069	≤	25
12	**Max. 1 min. radio**				1	0	≤	20
13	**Cost**	800	925	290	380	8000	≤	8000
14	**Radio dollars**			290	380	1800	≤	1800
15	**Radio spots**			1	1	6.2069	≥	5

Solution in Excel 2013

PROGRAM 8.1 – Win Big Solution

Solver Parameter Inputs and Selections

Set Objective: F6

By Changing cells: B5:E5

To: Max

Subject to the Constraints:

F9:F14 <= H9:H14

F15 >= H15

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	F
5	**Total Audience**
6	=SUMPRODUCT($B$5:$E$5,B6:E6)

Copy F6 to F9:F15

Management Sciences Association

- MSA is a marketing research firm
- Several requirements for a statistical validity
  1. Survey at least 2,300 U.S. households
  2. Survey at least 1,000 households whose heads are ≤ 30 years old
  3. Survey at least 600 households whose heads are between 31 and 50
  4. Ensure that at least 15% of those surveyed live in a state that borders Mexico
  5. Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a state that borders Mexico

Management Sciences Association

- MSA decides to conduct all surveys in person
- Estimates of the costs of reaching people in each age and region category
- Goal is to meet the sampling requirements at the least possible cost

	COST PER PERSON SURVEYED ($)		
REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	$7.50	$6.80	$5.50
State not bordering Mexico	$6.90	$7.25	$6.10

Management Sciences Association

- Decision variables

X_1 = number of 30 or younger and in a border state
X_2 = number of 31-50 and in a border state
X_3 = number 51 or older and in a border state
X_4 = number 30 or younger and not in a border state
X_5 = number of 31-50 and not in a border state
X_6 = number 51 or older and not in a border state

Management Sciences Association

Objective function

$$\text{Minimize total interview costs} = \$7.50X_1 + \$6.80X_2 + \$5.50X_3 + \$6.90X_4 + \$7.25X_5 + \$6.10X_6$$

subject to

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 2{,}300 \quad \text{(total households)}$$

$$X_1 + X_4 \geq 1{,}000 \quad \text{(households 30 or younger)}$$

$$X_2 + X_5 \geq 600 \quad \text{(households 31-50)}$$

$$X_1 + X_2 + X_3 \geq 0.15(X_1 + X_2 + X_3 + X_4 + X_5 + X_6) \quad \text{(border states)}$$

$$X_3 \leq 0.20(X_3 + X_6) \quad \text{51+ who can live in border state)}$$

$$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$$

Management Sciences Association

- Optimal solution will cost $15,166

REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	0	600	140
State not bordering Mexico	1,000	0	560

Solution in Excel 2013

PROGRAM 8.2 – MSA Solution

	A	B	C	D	E	F	G	H	I	J
1	**Management Science Associates**									
2										
3	**Variable**	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**			
4	**Solution**	**0**	**600**	**140**	**1000**	**0**	**560**	**Total Cost**		
5	**Min. Cost**	7.5	6.8	5.5	6.9	7.25	6.1	15166		
6										
7	**Constraints**							**LHS**		**RHS**
8	**Total Households**	1	1	1	1	1	1	2300	≥	2,300
9	**30 and Younger**	1	0	0	1	0	0	1000	≥	1,000
10	**31-50**	0	1	0	0	1	0	600	≥	600
11	**Border States**	0.85	0.85	0.85	-0.15	-0.15	-0.15	395	≥	0
12	**51+ Border States**	0	0	0.8	0	0	-0.2	0	≤	0

Solution in Excel 2013

PROGRAM 8.2 – MSA Solution

Solver Parameter Inputs and Selections

Set Objective: H5

By Changing cells: B4:G4

To: Min

Subject to the Constraints:

H8:H11 <= J8:J11

H12 >= J12

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	H
4	**Total Cost**
5	=SUMPRODUCT($B$4:$G$4,B5:G5)

Copy H5 to H8:H12

Manufacturing Applications

- Production Mix
  - LP can be used to plan the optimal mix of products to manufacture
  - Company must meet a myriad of constraints
    - Financial concerns
    - Sales demand
    - Material contracts
    - Union labor demands
  - Primary goal is to generate the largest profit possible

Fifth Avenue Industries

- Produces four varieties of ties
  - Expensive all-silk
  - All-polyester
  - Two are polyester-cotton or silk-cotton blends
- Cost and availability of the three materials used in the production process

MATERIAL	COST PER YARD ($)	MATERIAL AVAILABLE PER MONTH (YARDS)
Silk	24	1,200
Polyester	6	3,000
Cotton	9	1,600

Fifth Avenue Industries

- The firm has contracts with several major department store chains
  - Contracts require a minimum number of ties
  - May be increased if demand increases
- Goal is to maximize monthly profit
- Decision variables

X_1 = number of all-silk ties produced per month
X_2 = number all-polyester ties
X_3 = number of blend 1 polyester-cotton ties
X_4 = number of blend 2 silk-cotton ties

Fifth Avenue Industries

TABLE 8.1 – Data for Fifth Avenue

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MONTHLY CONTRACT MINIMUM	MONTHLY DEMAND	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL REQUIREMENTS
All silk	19.24	5,000	7,000	0.125	100% silk
All polyester	8.70	10,000	14,000	0.08	100% polyester
Poly-cotton blend 1	9.52	13,000	16,000	0.10	50% polyester – 50% cotton
Silk-cotton blend 2	10.64	5,000	8,500	0.11	60% silk – 40% cotton

Fifth Avenue Industries

- Establish profit per tie

SILK REQ'D	COST	POLY-ESTER REQ'D	COST	COTTON REQ'D	COST	MATERIAL COST	SELLING PRICE	PROFIT
All-silk X_1								
0.125	$24.00					$3.00	$19.24	$16.24
All-polyester X_2								
		0.08	$6			$0.48	$8.70	$8.22
Poly-cotton blend X_3								
		0.05	$6	0.05	$9	$0.75	$9.52	$8.77
Silk-cotton blend X_4								
0.066	$24.00			0.044	$9	$1.98	$10.64	$8.66

Fifth Avenue Industries

Objective function

Maximize profit = $\$16.24X_1 + \$8.22X_2 + \$8.77X_3 + \$8.66X_4$

Subject to	$0.125X_1 + 0.066X_4 \leq$	1200	(yds of silk)
	$0.08X_2 + 0.05X_3 \leq$	3,000	(yds of polyester)
	$0.05X_3 + 0.44X_4 \leq$	1,600	(yds of cotton)
	$X_1 \geq$	5,000	(contract min for silk)
	$X_1 \leq$	7,000	(contract min)
	$X_2 \geq$	10,000	(contract min for all polyester)
	$X_2 \leq$	14,000	(contract max)
	$X_3 \geq$	13,000	(contract min for blend 1)
	$X_3 \leq$	16,000	(contract max)
	$X_4 \geq$	5,000	(contract min for blend 2)
	$X_4 \leq$	8,500	(contract max)
	$X_1, X_2, X_3, X_4 \geq$	0	

Fifth Avenue Industries

- Optimal solution will result in a profit of $412,028 per month

TIE	QUANTITY PER MONTH
All-Silk	5,112
All-Polyester	14,000
Poly-Silk	16,000
Silk-Cotton	8,500

Solution in Excel 2013

PROGRAM 8.3 – Fifth Avenue Solution

	A	B	C	D	E	F	G	H
1	**Fifth Avenue Industries**							
2								
3		**All silk**	**All poly.**	**Blend 1**	**Blend 2**			
4	**Variables**	**X1**	**X2**	**X3**	**X4**			
5	**Values**	5112	14000	16000	8500	**Total Profit**		
6	**Profit**	16.24	8.22	8.77	8.66	412028.88		
7								
8	**Constraints**					**LHS**		**RHS**
9	**Silk available**	0.125			0.066	1200	≤	1200
10	**Polyester available**		0.08	0.05		1920	≤	3000
11	**Cotton available**			0.05	0.044	1174	≤	1600
12	**Maximum silk**	1				5112	≤	7000
13	**Maximum polyester**		1			14000	≤	14000
14	**Maximum blend 1**			1		16000	≤	16000
15	**Maximum blend 2**				1	8500	≤	8500
16	**Minimum silk**	1				5112	≥	5000
17	**Minimum polyester**		1			14000	≥	10000
18	**Minimum blend 1**			1		16000	≥	13000
19	**Minimum blend 2**				1	8500	≥	5000

Solution in Excel 2013

PROGRAM 8.3 – Fifth Avenue Solution

Solver Parameter Inputs and Selections

Set Objective: F6

By Changing cells: B5:E5

To: Max

Subject to the Constraints:

F9:F15 <= H9:H15

F16:F19 >= H16:H19

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	F
5	**Total Profit**
6	=SUMPRODUCT($B$5:$E$5,B6:E6)

Copy F6 to F9:F19

Manufacturing Applications

- Production Scheduling
  - Low-cost production schedule
    - Period of weeks or months
  - Important factors include
    - Labor capacity
    - Inventory and storage costs
    - Space limitations
    - Product demand
    - Labor relations
  - With more than one product, the scheduling process can be quite complex
  - The problem resembles the product mix model for each time period in the future

Greenberg Motors

- Manufactures two different electric motors for sale under contract to Drexel Corp
  - Orders placed three times a year for four months at a time
  - Demand varies month to month
  - Develop a production plan for the next four months

TABLE 8.2 – Four-Month Order Schedule

MODEL	JANUARY	FEBRUARY	MARCH	APRIL
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

Greenberg Motors

- Production planning must consider four factors
  1. Produce the required number of motors each month and ensure the desired ending inventory
  2. Desire to keep inventory carrying costs down
  3. No-lay-off policy, minimize fluctuations in production levels
  4. Warehouse limitations

Greenberg Motors

- Basic data

MOTOR	ENDING INV	CARRYING COST	LABOR HRS REQ'D	PRODUCTION COST PER UNIT
GM3A	450	$0.36	1.3	$20
GM3B	300	$0.26	0.9	$15

2,240 ≤ Desired labor hrs per month ≤ 2,560

Maximum total inventory space available = 3,300 units

Labor cost increases 10% March 1

Greenberg Motors

- Model formulation

Objective

Minimize total cost (production plus inventory carrying cost)

Constraints

4 demand constraints (1 constraint for each of 4 months) for GM3A

4 demand constraints (1 constraint for each of 4 months) for GM3B

2 constraints (1 for GM3A and 1 for GM3B) for the inventory at the end of April

4 constraints for minimum labor hours (1 constraint for each month)

4 constraints for maximum labor hours (1 constraint for each month)

4 constraints for inventory storage capacity each month

Greenberg Motors

- Objective function – production costs

A_i = Number of model GM3A motors produced in month i (i = 1, 2, 3, 4 for January – April)

B_i = Number of model GM3B motors produced in month i

$$\text{Cost of production} = \$20A_1 + \$20A_2 + \$22A_3 + \$22A_4 + \$15B_1 + \$15B_2 + \$16.50B_3 + \$16.50B_4$$

Greenberg Motors

- Objective function – inventory carrying costs

$IA_i =$ Units of GM3A left in inventory at the end of month i (i = 1, 2, 3, 4 for January – April)

$IB_i =$ Units of GM3B left in inventory at the end of month i (i = 1, 2, 3, 4 for January – April)

$$\text{Cost of carrying inventory} = \$0.36IA_1 + \$0.36IA_2 + \$0.36IA_3 + 0.36IA_4 + \$0.26IB_1 + \$0.26IB_2 + \$0.26IB_3 + \$0.26IB_4$$

Greenberg Motors

- Complete objective function

$$\begin{aligned} \text{Minimize costs} = {} & \$20A_1 + \$20A_2 + \$22A_3 + \$22A_4 \\ & + \$15B_1 + \$15B_2 + \$16.50B_3 + \$16.50B_4 \\ & + \$0.36IA_1 + \$0.36IA_2 + \$0.36IA_3 + 0.36IA_4 \\ & + \$0.26IB_1 + \$0.26IB_2 + \$0.26IB_3 + \$0.26IB_4 \end{aligned}$$

Greenberg Motors

- End of month inventory is calculated using

$$\begin{bmatrix}\text{Inventory}\\ \text{at the}\\ \text{end of}\\ \text{this month}\end{bmatrix} = \begin{bmatrix}\text{Inventory}\\ \text{at the}\\ \text{end of}\\ \text{last month}\end{bmatrix} + \begin{bmatrix}\text{Current}\\ \text{month's}\\ \text{production}\end{bmatrix} - \begin{bmatrix}\text{Sales to}\\ \text{Drexel this}\\ \text{month}\end{bmatrix}$$

- Rearranged to create a standard format for a constraint equation

$$\begin{bmatrix}\text{Inventory}\\ \text{at the}\\ \text{end of}\\ \text{last month}\end{bmatrix} + \begin{bmatrix}\text{Current}\\ \text{month's}\\ \text{production}\end{bmatrix} - \begin{bmatrix}\text{Inventory}\\ \text{at the}\\ \text{end of}\\ \text{this month}\end{bmatrix} = \begin{bmatrix}\text{Sales to}\\ \text{Drexel this}\\ \text{month}\end{bmatrix}$$

Greenberg Motors

- The demand constraints

$$
\begin{aligned}
A_1 - IA_1 &= 800 && \text{(demand for GM3A in Jan)}\\
IA_1 + A_2 - IA_2 &= 700 && \text{(demand for GM3A in Feb)}\\
IA_2 + A_3 - IA_3 &= 1{,}000 && \text{(demand for GM3A in Mar)}\\
IA_3 + A_4 - IA_4 &= 1{,}100 && \text{(demand for GM3A in Apr)}\\
B_1 - IB_1 &= 1{,}000 && \text{(demand for GM3B in Jan)}\\
IB_1 + B_2 - IB_2 &= 1{,}200 && \text{(demand for GM3B in Feb)}\\
IB_2 + B_3 - IB_3 &= 1{,}400 && \text{(demand for GM3B in Mar)}\\
IB_3 + B_4 - IB_4 &= 1{,}400 && \text{(demand for GM3B in Apr)}\\
IA_4 &= 450 && \text{(inventory of GM3A at end of Apr)}\\
IB_4 &= 300 && \text{(inventory of GM3B at end of Apr)}
\end{aligned}
$$

Greenberg Motors

- The labor hour constraints

$1.3A_1 + 0.9B_1 \geq 2{,}240$	(min labor hrs in Jan)
$1.3A_2 + 0.9B_2 \geq 2{,}240$	(min labor hrs in Feb)
$1.3A_3 + 0.9B_3 \geq 2{,}240$	(min labor hrs in Mar)
$1.3A_4 + 0.9B_4 \geq 2{,}240$	(min labor hrs in Apr)
$1.3A_1 + 0.9B_1 \leq 2{,}560$	(max labor hrs in Jan)
$1.3A_2 + 0.9B_2 \leq 2{,}560$	(max labor hrs in Feb)
$1.3A_3 + 0.9B_3 \leq 2{,}560$	(max labor hrs in Mar)
$1.3A_4 + 0.9B_4 \leq 2{,}560$	(max labor hrs in Apr)

Greenberg Motors

- The storage constraints

$IA_1 + IB_1$	$\leq$	3,300	(storage capacity in Jan)
$IA_2 + IB_2$	$\leq$	3,300	(storage capacity in Feb)
$IA_3 + IB_3$	$\leq$	3,300	(storage capacity in Mar)
$IA_4 + IB_4$	$\leq$	3,300	(storage capacity in Apr)
All variables	$\geq$	0	(nonnegativity constraints)

Solution in Excel 2013

PROGRAM 8.4 – Greenberg Motors Solution

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	**Greenberg Motors**																			
2																				
3	**Variable**	A1	A2	A3	A4	B1	B2	B3	B4	IA1	IA2	IA3	IA4	IB1	IB2	IB3	IB4			
4	**Solution**	1276.9	223.1	1757.7	792.3	1000	2522.2	77.8	1700	476.9	0	757.7	450	0	1322.2	0	300	**Total Cost**		
5	**Min. Cost**	20	20	22	22	15	15	16.5	16.5	0.36	0.36	0.36	0.36	0.26	0.26	0.26	0.26	169294.9		
6																				
7	**Demand Constraints**																	**LHS**	**Sign**	**RHS**
8	Jan. GM3A	1								-1								800	=	800
9	Feb. GM3A		1							1	-1							700	=	700
10	Mar. GM3A			1							1	-1						1000	=	1000
11	Apr. GM3A				1							1	-1					1100	=	1100
12	Jan. GM3B					1								-1				1000	=	1000
13	Feb. GM3B						1							1	-1			1200	=	1200
14	Mar. GM3B							1							1	-1		1400	=	1400
15	Apr. GM3B								1							1	-1	1400	=	1400
16	Inv.GM3A Apr.												1					450	=	450
17	Inv.GM3B Apr.																1	300	=	300
18	**Labor Hour Constraints**																			
19	Hrs Min. Jan.	1.3				0.9												2560	≥	2240
20	Hrs Min. Feb.		1.3				0.9											2560	≥	2240
21	Hrs Min. Mar.			1.3				0.9										2355	≥	2240
22	Hrs Min. Apr.				1.3				0.9									2560	≥	2240
23	Hrs Max. Jan.	1.3				0.9												2560	≤	2560
24	Hrs Max. Feb.		1.3				0.9											2560	≤	2560
25	Hrs Max.Mar.			1.3				0.9										2355	≤	2560
26	Hrs Max. Apr.				1.3				0.9									2560	≤	2560
27	**Storage Constraints**																			
28	Jan. Inv. Limit									1				1				476.92	≤	3300
29	Feb. Inv. Limit										1				1			1322.22	≤	3300
30	Mar. Inv. Limit											1				1		757.69	≤	3300
31	Apr. Inv. Limit												1				1	750	≤	3300

Solution in Excel 2013

PROGRAM 8.4 – Greenberg Motors Solution

Solver Parameter Inputs and Selections

Set Objective: F5

By Changing cells: B4:Q4

To: Min

Subject to the Constraints:

R19:R22 >= T19:T22

R23:R26 <= T23:T31

R28:R31 <= T28:T31

R8:R17 = T8:T17

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	R
4	**Total Cost**
5	=SUMPRODUCT($B$4:$Q$4,B5:Q5)

Copy formula in R5 to R8:R17
Copy formula in R5 to R19:R26
Copy formula in R5 to R28:R31

Greenberg Motors

TABLE 8.3 – Solution to Greenberg Motors Problem

PRODUCTION SCHEDULE	JANUARY	FEBRUARY	MARCH	APRIL
Units GM3A produced	1,277	223	1,758	792
Units GM3B produced	1,000	2,522	78	1,700
Inventory GM3A carried	477	0	758	450
Inventory GM3B carried	0	1,322	0	300
Labor hours required	2,560	2,560	2,355	2,560

- Total cost for this four month period is about $169,295
- Complete model has 16 variables and 22 constraints

Employee Scheduling Applications

- Labor Planning
  - Address staffing needs over a particular time
  - Especially useful when there is some flexibility in assigning workers that require overlapping or interchangeable talents

Hong Kong Bank

- Hong Kong Bank of Commerce and Industry requires between 10 and 18 tellers depending on the time of day
- The bank wants a schedule that will minimize total personnel costs
  - Lunch time from noon to 2 pm is generally the busiest
  - Bank employs 12 full-time tellers, many part-time workers

Hong Kong Bank

- Part-time workers must put in exactly four hours per day, can start anytime between 9 am and 1 pm, and are inexpensive
- Full-time workers work from 9 am to 3 pm and have 1 hour for lunch
- Part-time hours are limited to a maximum of 50% of the day's total requirements
- Part-timers earn $8 per hour on average
- Full-timers earn $100 per day on average
- It will release one or more of its full-time tellers if it is profitable to do so

Hong Kong Bank

- Labor requirements

TABLE 8.4

TIME PERIOD	NUMBER OF TELLERS REQUIRED
9 am – 10 am	10
10 am – 11 am	12
11 am – Noon	14
Noon – 1 pm	16
1 pm – 2 pm	18
2 pm – 3 pm	17
3 pm – 4 pm	15
4 pm – 5 pm	10

Hong Kong Bank

- Variables

F = full-time tellers
P_1 = part-timers starting at 9 am (leaving at 1 pm)
P_2 = part-timers starting at 10 am (leaving at 2 pm)
P_3 = part-timers starting at 11 am (leaving at 3 pm)
P_4 = part-timers starting at noon (leaving at 4 pm)
P_5 = part-timers starting at 1 pm (leaving at 5 pm)

- Objective

Minimize total daily personnel cost $= \$100F + \$32(P_1 + P_2 + P_3 + P_4 + P_5)$

Hong Kong Bank

- Constraints

F	$+ P_1$					$\geq$	10	(9 am – 10 am needs)
F	$+ P_1$	$+ P_2$				$\geq$	12	(11 am – noon needs)
$0.5F$	$+ P_1$	$+ P_2$	$+ P_3$			$\geq$	14	(1 pm – 2 pm needs)
$0.5F$	$+ P_1$	$+ P_2$	$+ P_3$	$+ P_4$		$\geq$	16	(noon – 1 pm needs)
F		$+ P_2$	$+ P_3$	$+ P_4$	$+ P_5$	$\geq$	18	(1 pm – 2 pm needs)
F			$+ P_3$	$+ P_4$	$+ P_5$	$\geq$	17	(2 pm – 3 pm needs)
F				$+ P_4$	$+ P_5$	$\geq$	15	(3 pm – 4 pm needs)
F					$+ P_5$	$\geq$	10	(4 pm – 5 pm needs)
F						$\leq$	12	(12 full-time tellers)
	$4P_1$	$+ 4P_2$	$+ 4P_3$	$+ 4P_4$	$+ 4P_5$	$\leq$	0.50(112)	(max 50% part-timers)
					$F, P_1, P_2, P_3, P_4, P_5$	$\geq$	0	(nonnegativity)

Solution in Excel 2013

PROGRAM 8.5 – Labor Planning Solution

	A	B	C	D	E	F	G	H	I	J
1	**Labor Planning Example**									
2										
3										
4	**Variables**	**F**	**P1**	**P2**	**P3**	**P4**	**P5**			
5	**Values**	10	0	7	2	5	0	**Total Cost**		
6	**Cost**	100	32	32	32	32	32	1448		
7										
8	**Constraints**							**LHS**	**Sign**	**RHS**
9	**9 a.m. - 10 a.m.**	1	1					10	≥	10
10	**10 a.m. - 11 a.m.**	1	1	1				17	≥	12
11	**11 a.m. - noon**	0.5	1	1	1			14	≥	14
12	**noon - 1 p.m.**	0.5	1	1	1	1		19	≥	16
13	**1 p.m. - 2 p.m.**	1		1	1	1	1	24	≥	18
14	**2 p.m. - 3 p.m.**	1			1	1	1	17	≥	17
15	**3 p.m. - 4 p.m.**	1				1	1	15	≥	15
16	**4 p.m. - 5 p.m.**	1					1	10	≥	10
17	**Max. Full time**	1						10	≤	12
18	**Total PT hours**		4	4	4	4	4	56	≤	56

Solution in Excel 2013

PROGRAM 8.5 – Labor Planning Solution

Solver Parameter Inputs and Selections

Set Objective: H6

By Changing cells: B5:G5

To: Min

Subject to the Constraints:

H9:H16 >= J9:J16

H17:H18 <= J17:J18

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	H
5	**Total Cost**
6	=SUMPRODUCT($B$5:$G$5,B6:G6)

Copy H6 to H9:H18

Hong Kong Bank

- Alternate solutions are possible for this problem
- Each has the same total cost – $1,448/day

	SOLUTION 1	SOLUTION 2
Full-Time Tellers	10	10
P_1 Tellers	0	6
P_2 Tellers	7	1
P_3 Tellers	2	2
P_4 Tellers	5	5
P_5 Tellers	0	0

Financial Applications

- Portfolio Selection
  - Bank, investment funds, and insurance companies often have to select specific investments from a variety of alternatives
  - Overall objective is generally to maximize the potential return on the investment given a set of legal, policy, or risk restraints

International City Trust

- International City Trust (ICT) invests in short-term trade credits, corporate bonds, gold stocks, and construction loans
- The board of directors has placed limits on how much can be invested in each area

INVESTMENT	INTEREST EARNED (%)	MAXIMUM INVESTMENT ($ MILLIONs)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stocks	19	1.5
Construction loans	15	1.8

International City Trust

- ICT has $5 million to invest and wants to accomplish two things
  - Maximize the return on investment over the next six months
  - Satisfy the diversification requirements set by the board
- The board has also decided that at least 55% of the funds must be invested in gold stocks and construction loans and no less than 15% be invested in trade credit

International City Trust

- Investment possibilities

INVESTMENT	INTEREST RETURN	MAXIMUM INVESTMENT ($ MILLIONs)
Trade credit	7%	1.0
Corporate bonds	11%	2.5
Gold stocks	19%	1.5
Construction loans	15%	1.8

International City Trust

- Variables

X_1 = dollars invested in trade credit
X_2 = dollars invested in corporate bonds
X_3 = dollars invested in gold stocks
X_4 = dollars invested in construction loans

International City Trust

- Formulation

$$\text{Maximize dollars of interest earned} = 0.07X_1 + 0.11X_2 + 0.19X_3 + 0.15X_4$$

subject to:

$$\begin{aligned}
X_1 &\le 1{,}000{,}000 \\
X_2 &\le 2{,}500{,}000 \\
X_3 &\le 1{,}500{,}000 \\
X_4 &\le 1{,}800{,}000 \\
X_3 + X_4 &\ge 0.55(X_1 + X_2 + X_3 + X_4) \\
X_1 &\ge 0.15(X_1 + X_2 + X_3 + X_4) \\
X_1 + X_2 + X_3 + X_4 &\le 5{,}000{,}000 \\
X_1, X_2, X_3, X_4 &\ge 0
\end{aligned}$$

Solution in Excel 2013

PROGRAM 8.6 – ICT Portfolio Solution

	A	B	C	D	E	F	G	H
1	ICT Portfolio Selection							
2								
3	Variable	X1	X2	X3	X4			
4	Solution	750000	950000	1500000	1800000	Total Return		
5	Max. Return	0.07	0.11	0.19	0.15	712000		
6								
7						LHS		RHS
8	Trade	1				750000	≤	1,000,000
9	Bonds		1			950000	≤	2,500,000
10	Gold			1		1500000	≤	1,500,000
11	Construction				1	1800000	≤	1,800,000
12	Min. Gold+Const	-0.55	-0.55	0.45	0.45	550000	≥	0
13	Min. Trade	0.85	-0.15	-0.15	-0.15	0	≥	0
14	Total Invested	1	1	1	1	5000000	≤	5000000

Solution in Excel 2013

PROGRAM 8.6 – ICT Portfolio Solution

Solver Parameter Inputs and Selections

Set Objective: F5

By Changing cells: B4:E4

To: Min

Subject to the Constraints:

F8:F11 <= H8:H11

F12:F13 >= H12:H13

F14 <= H14

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	F
4	**Total Return**
5	=SUMPRODUCT($B$4:$E$4,B5:E5)

Copy F5 to F8:F14

International City Trust

- Optimal solution
  - Make the following investments

$$X_1 = \$750{,}000$$
$$X_2 = \$950{,}000$$
$$X_3 = \$1{,}500{,}000$$
$$X_4 = \$1{,}800{,}000$$

  - Total interest earned = $712,000

Truck Loading Problem

- Truck Loading Problem
  - Deciding which items to load on a truck so as to maximize the value of a load shipped
  - Goodman Shipping has to ship the following six items

ITEM	VALUE ($)	WEIGHT (POUNDS)
1	22,500	7,500
2	24,000	7,500
3	8,000	3,000
4	9,500	3,500
5	11,500	4,000
6	9,750	3,500

Goodman Shipping

- The objective is to maximize the value of items loaded into the truck
- The truck has a capacity of 10,000 pounds
- Decision variable

  X_i = proportion of each item i loaded on the truck

Goodman Shipping

- Formulation

$$\text{Maximize load value} = \$22{,}500X_1 + \$24{,}000X_2 + \$8{,}000X_3 + \$9{,}500X_4 + \$11{,}500X_5 + \$9{,}750X_6$$

subject to

$$7{,}500X_1 + 7{,}500X_2 + 3{,}000X_3 + 3{,}500X_4 + 4{,}000X_5 + 3{,}500X_6 \leq 10{,}000 \text{ lb capacity}$$

$$X_1 \leq 1$$

$$X_2 \leq 1$$

$$X_3 \leq 1$$

$$X_4 \leq 1$$

$$X_5 \leq 1$$

$$X_6 \leq 1$$

$$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$$

Solution in Excel 2013

PROGRAM 8.7 – Goodman Truck Loading Solution

	A	B	C	D	E	F	G	H	I	J
1	**Goodman Shipping**									
2										
3	**Variables**	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**			
4	**Values**	0.3333	1	0	0	0	0	**Total Value**		
5	**Load Value $**	22500	24000	8000	9500	11500	9750	31500		
6										
7	**Constraints**							**LHS**	**Sign**	**RHS**
8	**Total weight**	7500	7500	3000	3500	4000	3500	10000	≤	10000
9	**% Item 1**	1						0.3333333	≤	1
10	**% Item 2**		1					1	≤	1
11	**% Item 3**			1				0	≤	1
12	**% Item 4**				1			0	≤	1
13	**% Item 5**					1		0	≤	1
14	**% Item 6**						1	0	≤	1

Solution in Excel 2013

PROGRAM 8.7 – Goodman Truck Loading Solution

Solver Parameter Inputs and Selections

Set Objective: H5

By Changing cells: B4:G4

To: Min

Subject to the Constraints:

H8:H14 <= H8:H11

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	H
4	**Total Value**
5	=SUMPRODUCT($B$4:$G$4,B5:G5)

Copy H5 to H8:H14

Goodman Shipping

- Goodman Shipping raises an interesting issue
  - The solution calls for one third of Item 1 to be loaded on the truck
  - What if Item 1 cannot be divided into smaller pieces?
- Rounding down leaves unused capacity on the truck and results in a value of $24,000
- Rounding up is not possible since this would exceed the capacity of the truck
- Using *integer programming*, the solution is to load one unit of Items 3, 4, and 6 for a value of $27,250

Ingredient Blending Applications

- Diet Problems
  - One of the earliest LP applications
  - Used to determine the most economical diet for hospital patients
  - This is also known as the feed mix problem

Whole Food Nutrition Center

- Uses three bulk grains to blend a natural cereal
- Advertises that the cereal meets the U.S. Recommended Daily Allowance (USRDA) for four key nutrients
- Select the blend that will meet the requirements at the minimum cost

NUTRIENT	USRDA
Protein	3 units
Riboflavin	2 units
Phosphorus	1 unit
Magnesium	0.425 unit

Whole Food Nutrition Center

- Variables

X_A = pounds of grain *A* in one 2-ounce serving of cereal

X_B = pounds of grain *B* in one 2-ounce serving of cereal

X_C = pounds of grain *C* in one 2-ounce serving of cereal

TABLE 8.5 – Whole Food's Natural Cereal requirements:

GRAIN	COST PER POUND (CENTS)	PROTEIN (UNITS/LB)	RIBOFLAVIN (UNITS/LB)	PHOSPHOROUS (UNITS/LB)	MAGNESIUM (UNITS/LB)
A	33	22	16	8	5
B	47	28	14	7	0
C	38	21	25	9	6

Whole Food Nutrition Center

- Formulation

Minimize total cost of mixing a 2-ounce serving $= \$0.33X_A + \$0.47X_B + \$0.38X_C$

subject to

$$
\begin{array}{rcl l}
22X_A + 28X_B + 21X_C & \geq & 3 & \text{(protein units)} \\
16X_A + 14X_B + 25X_C & \geq & 2 & \text{(riboflavin units)} \\
8X_A + 7X_B + 9X_C & \geq & 1 & \text{(phosphorous units)} \\
5X_A + 0X_B + 6X_C & \geq & 0.425 & \text{(magnesium units)} \\
X_A + X_B + X_C & = & 0.125 & \text{(total mix)} \\
X_A, X_B, X_C & \geq & 0 &
\end{array}
$$

Solution in Excel 2013

PROGRAM 8.8 – Whole Food Diet Solution

	A	B	C	D	E	F	G
1	Whole Foods Nutrition Problem						
2							
3		**Grain A**	**Grain B**	**Grain C**			
4	**Variable**	**Xa**	**Xb**	**Xc**			
5	**Solution**	0.025	0.05	0.05	**Total Cost**		
6	**Minimize**	0.33	0.47	0.38	0.05075		
7							
8	**Constraints**				**LHS**	**Sign**	**RHS**
9	**Protein**	22	28	21	3	≥	3
10	**Riboflavin**	16	14	25	2.35	≥	2
11	**Phosphorus**	8	7	9	1	≥	1
12	**Magnesium**	5	0	6	0.425	≥	0.425
13	**Total Weight**	1	1	1	0.125	=	0.125

Solution i

PROGRAM 8.8 – Whole Food Diet Soluti

	A	B	C				
1	Whole Foods Nutrition Pro						
2							
3		Grain A	Grain B	Grain C			
4	**Variable**	Xa	Xb	Xc			
5	**Solution**	0.025	0.05	0.05	Total Cost		
6	**Minimize**	0.33	0.47	0.38	0.05075		
7							
8	**Constraints**				LHS	Sign	RHS
9	**Protein**	22	28	21	3	≥	3
10	**Riboflavin**	16	14	25	2.35	≥	2
11	**Phosphorus**	8	7	9	1	≥	1
12	**Magnesium**	5	0	6	0.425	≥	0.425
13	**Total Weight**	1	1	1	0.125	=	0.125

This solution is in pounds of grain

Expressed as ounces/serving, the optimal mix is:

0.4 oz Grain A

0.8 oz Grain B

0.8 oz Grain C

Solution in Excel 2013

PROGRAM 8.8 – Whole Food Diet Solution

Solver Parameter Inputs and Selections

Set Objective: E6

By Changing cells: B5:D5

To: Min

Subject to the Constraints:

E9:E12 >= G9:G12

E13 = G13

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	E
5	**Total Cost**
6	=SUMPRODUCT($B$5:$D$5,B6:D6)

Copy E6 to E9:E13

Ingredient Blending Applications

- Ingredient Mix and Blending Problems
  - Diet and feed mix problems are special cases of a more general class of problems known as ingredient or blending problems
  - Blending problems arise when decisions must be made regarding the blending of two or more resources to produce one or more product
  - Resources may contain essential ingredients that must be blended so that a specified percentage is in the final mix

Low Knock Oil Company

- Company produces two grades of cut-rate gasoline for industrial distribution
- Regular and economy grades created by blending two different types of crude oil
- The crude oil differs in cost and in its content of crucial ingredients

CRUDE OIL TYPE	INGREDIENT A (%)	INGREDIENT B (%)	COST/BARREL ($)
X100	35	55	30.00
X220	60	25	34.80

Low Knock Oil Company

- Variables

X_1 = barrels of crude X100 blended to produce the refined regular

X_2 = barrels of crude X100 blended to produce the refined economy

X_3 = barrels of crude X220 blended to produce the refined regular

X_4 = barrels of crude X220 blended to produce the refined economy

Low Knock Oil Company

- Formulation

$$\text{Minimize cost} = \$30X_1 + \$30X_2 + \$34.80X_3 + \$34.80X_4$$

subject to

$$
\begin{array}{lllll}
X_1 & + X_3 & & \geq 25{,}000 & \text{(demand for regular)} \\
X_2 & & + X_4 & \geq 32{,}000 & \text{(demand for economy)}
\end{array}
$$

Low Knock Oil Company

45% of each barrel of regular must be ingredient A

$(X_1 + X_3)$ = total amount of crude blended to produce regular

Thus,

$0.45(X_1 + X_3)$ = minimum amount of ingredient A required

But

$0.35X_1 + 0.60X_3$ = amount of ingredient A in regular

So

$$0.35X_1 + 0.60X_3 \geq 0.45X_1 + 0.45X_3$$

or

$$-0.10X_1 + 0.15X_3 \geq 0 \quad \text{(A in regular constraint format)}$$

Low Knock Oil Company

- Formulation

Minimize cost = $\$30X_1 + \$30X_2 + \$34.80X_3 + \$34.80X_4$

subject to

X_1		$+ X_3$		$\geq$	25,000	(demand for regular)
	X_2		$+ X_4$	$\geq$	32,000	(demand for economy)
$-0.10X_1$		$+ 0.15X_3$		$\geq$	0	(ingredient A in regular)
	$0.05X_2$		$-0.25X_4$	$\leq$	0	(ingredient B in economy)
			X_1, X_2, X_3, X_4	$\geq$	0	(nonnegativity)

Solution in Excel 2013

PROGRAM 8.9 – Low Knock Oil Solution

	A	B	C	D	E	F	G	H
1	**Low Knock Oil Company**							
2								
3		**X100 Reg**	**X100 Econ**	**X220 Reg**	**X220 Econ**			
4	**Variable**	**X1**	**X2**	**X3**	**X4**			
5	**Solution**	15000	26666.67	10000	5333.33	**Total Cost**		
6	**Cost**	30	30	34.8	34.8	1783600		
7								
8	**Constraints**					**LHS**	**Sign**	**RHS**
9	**Demand Regular**	1		1		25000	$\geq$	25000
10	**Demand Economy**		1		1	32000	$\geq$	32000
11	**Ing. A in Regular**	-0.1		0.15		0	$\geq$	0
12	**Ing. B in Economy**		0.05		-0.25	0	$\leq$	0

Solution in Excel 2013

PROGRAM 8.9 – Low Knock Oil Solution

Solver Parameter Inputs and Selections

Set Objective: F6

By Changing cells: B5:E5

To: Min

Subject to the Constraints:

F9:F11 >= H9:H11

F12 <= H12

Solving Method: Simplex LP

☑ Make Variables Non-Negative

Key Formulas

	F
5	**Total Cost**
6	=SUMPRODUCT($B$5:$E$5,B6:E6)

Copy F6 to F9:F12

Other LP Applications

- Revenue Management
  - Developed by American Airlines
  - Differential pricing of seats to generate additional revenue
  - How many seats to make available to each type of passenger
  - Adopted by hotel industry

Other LP Applications

- Data Envelopment Analysis (DEA)
  - Measure efficiency of similar operating units
  - Can be used when there is no single objective to be optimized
    - Identify inputs and outputs
    - Develop constraints for each unit in the system
  - Objective is to minimize the resources required to generate specific levels of output
  - Identify areas where improvement might be possible

Other LP Applications

- Transportation, Transshipment, Assignment Problems
  - Very widely used in business
  - Special purpose algorithms have been developed
  - Allow more rapid solution of these types of problems
  - Presented in Chapter 9

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