CHAPTER 9

Transportation, Assignment, and Network Models

To accompany
Quantitative Analysis for Management, Twelfth Edition,
by Render, Stair, Hanna and Hale
Power Point slides created by Jeff Heyl

LEARNING OBJECTIVES

After completing this chapter, students will be able to:

1. Structure LP problems for the transportation, transshipment, and assignment models.
2. Solve facility location and other application problems with transportation models.
3. Use LP to model shortest-route and maximal-flow problems.
4. Solve minimal-spanning tree problems.

CHAPTER OUTLINE

Introduction

- LP problems modeled as networks
  – Helps visualize and understand problems
    - Transportation problem
    - Transshipment problem
    - Assignment problem
    - Maximal-flow problem
    - Shortest-route problem
    - Minimal-spanning tree problem
  – Specialized algorithms available

Introduction

- Common terminology for network models
  - Points on the network are referred to as **nodes**
    - Typically circles
  - Lines on the network that connect nodes are called **arcs**

The Transportation Problem

- Deals with the distribution of goods from several points of supply (**sources**) to a number of points of demand (**destinations**)
  – Usually given the capacity of goods at each source and the requirements at each destination
  – Typically objective is to minimize total transportation and production costs

Linear Program for Transportation

- Executive Furniture Corporation transportation problem
  - Minimize transportation cost
  - Meet demand
  - Not exceed supply

Linear Program for Transportation

Let X_{ij} = number of units shipped from source i to destination j

Where

i = 1, 2, 3, with 1 = Des Moines, 2 = Evansville, and 3 = Fort Lauderdale

j = 1, 2, 3, with 1 = Albuquerque, 2 = Boston, and 3 = Cleveland

Linear Program for Transportation

Minimize total cost = $5X_{11} + 4X_{12} + 3X_{13} + 8X_{21} + 4X_{22}$
$+ 3X_{23} + 9X_{31} + 7X_{32} + 5X_{33}$

Subject to:

$X_{11} + X_{12} + X_{13} \leq 100$ (Des Moines supply)

$X_{21} + X_{22} + X_{23} \leq 300$ (Evansville supply)

$X_{31} + X_{32} + X_{33} \leq 300$ (Fort Lauderdale supply)

$X_{11} + X_{21} + X_{31} = 300$ (Albuquerque demand)

$X_{12} + X_{22} + X_{32} = 200$ (Boston demand)

$X_{13} + X_{23} + X_{33} = 200$ (Cleveland demand)

$X_{ij} \geq 0$ for all i and j

Linear Program for Transportation

- Optimal solution

  100 units from Des Moines to Albuquerque

  200 units from Evansville to Boston

  100 units from Evansville to Cleveland

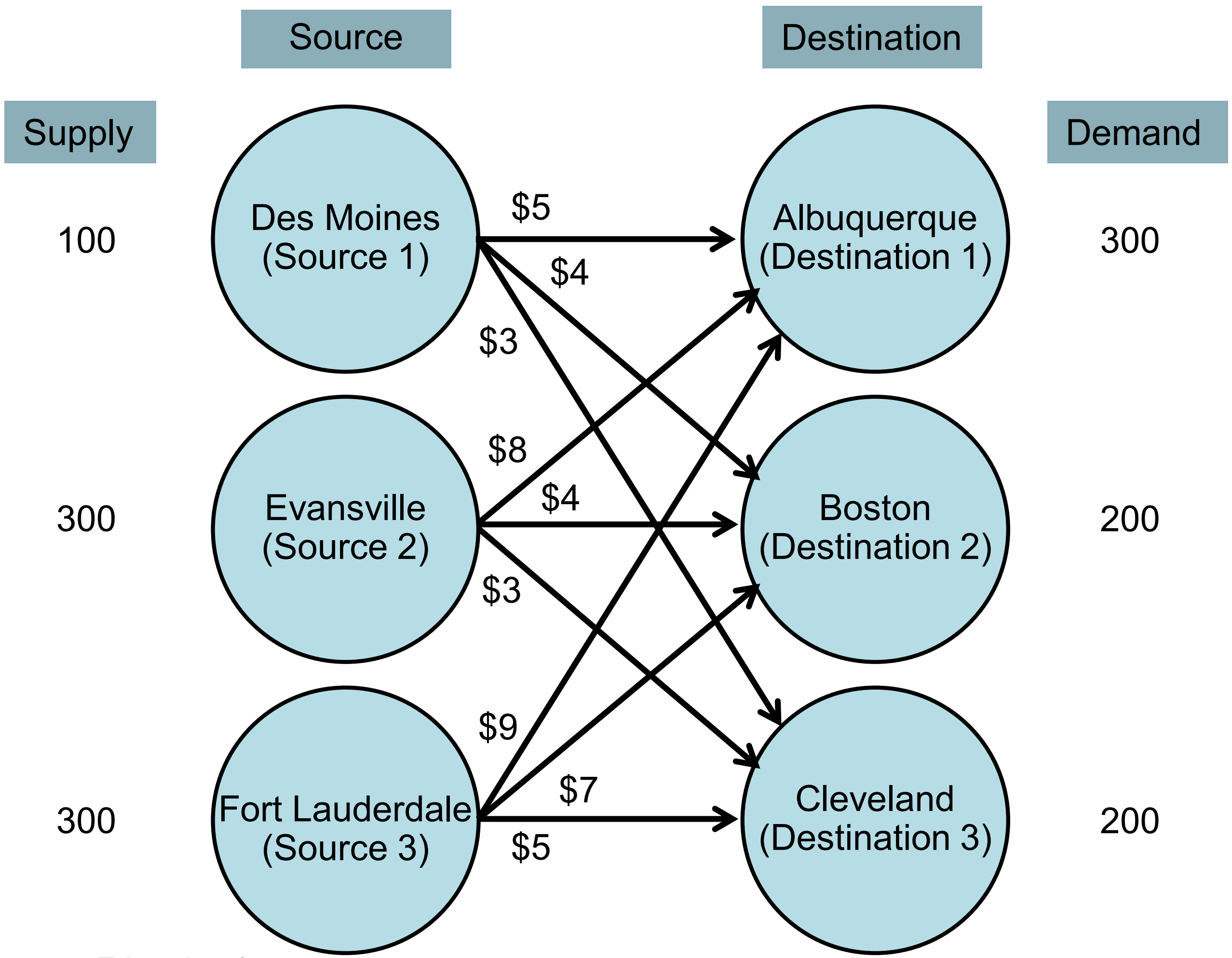
  200 units from Ft. Lauderdale to Albuquerque

  100 units from Ft. Lauderdale to Cleveland

  Total cost = $3,900

Linear Program for Transportation

FIGURE 9.1 – Network Representation of a Transportation Problem



Using Excel QM

PROGRAM 9.1 – Executive Furniture Corporation Solution in Excel 2013 Using Excel QM

From Excel QM ribbon, select Menu (Alphabetical or By Chapter). Select Transportation from the drop-down menu, and then input 3 Origins (sources) and 3 Destination.

From the Data tab, select Solver and then click Solve.

Data

COSTS	Albuquerque	Boston	Cleveland	Supply
Des Moines	5	4	3	100
Evansville	8	4	3	300
Ft. Lauderdale	9	7	5	300
Demand	300	200	200	700 \ 700

Fill in the table with the costs, supplies, and demands.

Shipments

Shipments	Albuquerque	Boston	Cleveland	
Des Moines	100			100
Evansville		200	100	300
Ft. Lauderdale	200		100	300
Column Total	300	200	200	700 \ 700

Total Cost	3900

The solution is shown here.

A General LP Model for Transportation Problems

Let

X_{ij} = number of units shipped from source i to destination j

c_{ij} = cost of one unit from source i to destination j

s_i = supply at source i

d_j = demand at destination j

A General LP Model for Transportation Problems

$$\text{Minimize cost} = \sum_{j=1}^{n} \sum_{i-1}^{m} c_{ij} x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \le s_i \qquad i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1,2\ldots,n$$

$$x_{ij} \ge 0 \qquad \text{for all } i \text{ and } j$$

Facility Location Analysis

- Transportation method especially useful
- New location is major financial importance
- Several alternative locations evaluated
- Subjective factors are considered
- Final decision also involves minimizing total shipping and production costs
- Alternative facility locations analyzed within the framework of one *overall* distribution system

Facility Location Analysis

- Hardgrave Machine Company produces computer components in Cincinnati, Salt Lake City, and Pittsburgh
- Four warehouses in Detroit, Dallas, New York, and Los Angeles
- Two new plant sites being considered – Seattle and Birmingham
- Which of the new locations will yield the lowest cost for the firm in combination with the existing plants and warehouses?

Facility Location Analysis

TABLE 9.1 – Hardgrave's Demand and Supply Data

WAREHOUSE	MONTHLY DEMAND (UNITS)	PRODUCTION PLANT	MONTHLY SUPPLY	COST TO PRODUCE ONE UNIT ($)
Detroit	10,000	Cincinnati	15,000	48
Dallas	12,000	Salt Lake City	6,000	50
New York	15,000	Pittsburgh	14,000	52
Los Angeles	9,000		35,000	
	46,000			

Supply needed from a new plant = 46,000 – 35,000 = 11,000 units per month

ESTIMATED PRODUCTION COST PER UNIT AT PROPOSED PLANTS	
Seattle	$53
Birmingham	$49

Facility Location Analysis

TABLE 9.2 – Hardgrave's Shipping Costs

TO / FROM	DETROIT	DALLAS	NEW YORK	LOS ANGELES
CINCINNATI	$25	$55	$40	$60
SALT LAKE CITY	35	30	50	40
PITTSBURGH	36	45	26	66
SEATTLE	60	38	65	27
BIRMINGHAM	35	30	41	50

Solve two transportation problems – one for each combination

Facility Location Analysis

X_{ij} = number of units shipped from source i to destination j

Where

i = 1, 2, 3, 4 with 1 = Cincinnati, 2 = Salt Lake City, 3 = Pittsburgh, and 4 = Seattle

j = 1, 2, 3, 4 with 1 = Detroit, 2 = Dallas, 3 = New York, and 4 = Los Angeles

Facility Location Analysis

Minimize total cost = $73X_{11} + 103X_{12} + 88X_{13} + 108X_{14} + 85X_{21} + 80X_{22} + 100X_{23} + 90X_{24} + 88X_{31} + 97X_{32} + 78X_{33} + 118X_{34} + 113X_{41} + 91X_{42} + 118X_{43} + 80X_{44}$

Subject to:

$X_{11} + X_{21} + X_{31} + X_{41} = 10{,}000$ Detroit demand

$X_{12} + X_{22} + X_{32} + X_{42} = 12{,}000$ Dallas demand

$X_{13} + X_{23} + X_{33} + X_{43} = 15{,}000$ New York demand

$X_{14} + X_{24} + X_{34} + X_{44} = 9{,}000$ Los Angeles demand

$X_{11} + X_{12} + X_{13} + X_{14} \le 15{,}000$ Cincinnati supply

$X_{21} + X_{22} + X_{23} + X_{24} \le 6{,}000$ Salt Lake City supply

$X_{31} + X_{32} + X_{33} + X_{34} \le 14{,}000$ Pittsburgh supply

$X_{41} + X_{42} + X_{43} + X_{44} \le 11{,}000$ Seattle supply

All variables $X_{ij} \ge 0$

Facility Lo[illegible]

The total cost for the Seattle alternative = $3,704,000

Minimize total cost = $73X_{11}$ + [illegible] X_{12} [illegible] X_{13} [illegible] X_{14} [illegible] X_{21} [illegible] X_{22}
$+ 100X_{23} + 90X_{24} + 88X_{31} + 97X_{32} + 78X_{33}$
$+ 118X_{34} + 113X_{41} + 91X_{42} + 118X_{43} + 80X_{44}$

Subject to:

$X_{11} + X_{21} + X_{31} + X_{41} = 10{,}000$ Detroit demand

$X_{12} + X_{22} + X_{32} + X_{42} = 12{,}000$ Dallas demand

$X_{13} + X_{23} + X_{33} + X_{43} = 15{,}000$ New York demand

$X_{14} + X_{24} + X_{34} + X_{44} = 9{,}000$ Los Angeles demand

$X_{11} + X_{12} + X_{13} + X_{14} \leq 15{,}000$ Cincinnati supply

$X_{21} + X_{22} + X_{23} + X_{24} \leq 6{,}000$ Salt Lake City supply

$X_{31} + X_{32} + X_{33} + X_{34} \leq 14{,}000$ Pittsburgh supply

$X_{41} + X_{42} + X_{43} + X_{44} \leq 11{,}000$ Seattle supply

All variables $X_{ij} \geq 0$

Facility Lo

The total cost for the Seattle alternative = $3,704,000

Minimize total cost = $73X_{11}$ + $103X_{12}$ + $88X_{13}$ + $108X_{14}$ + $85X_{21}$ + $80X_{22}$
+ $100X$ + $90X$ + $88X$ + $97X$ + $78X$
+ $118X$

Subject to:

Reformulating the problem for the Birmingham alternative and solving, the total cost = $3,741,000

$X_{11} + X_{21} + X_{31} + X_{41} =$ 10

$X_{12} + X_{22} + X_{32} + X_{42}$ =

$X_{13} + X_{23} + X_{33} + X_{43}$ =

$X_{14} + X_{24} + X_{34} + X_{44} = 9{,}000$ Los Angeles demand

$X_{11} + X_{12} + X_{13} + X_{14} \leq 15{,}000$ Cincinnati supply

$X_{21} + X_{22} + X_{23} + X_{24} \leq 6{,}000$ Salt Lake City supply

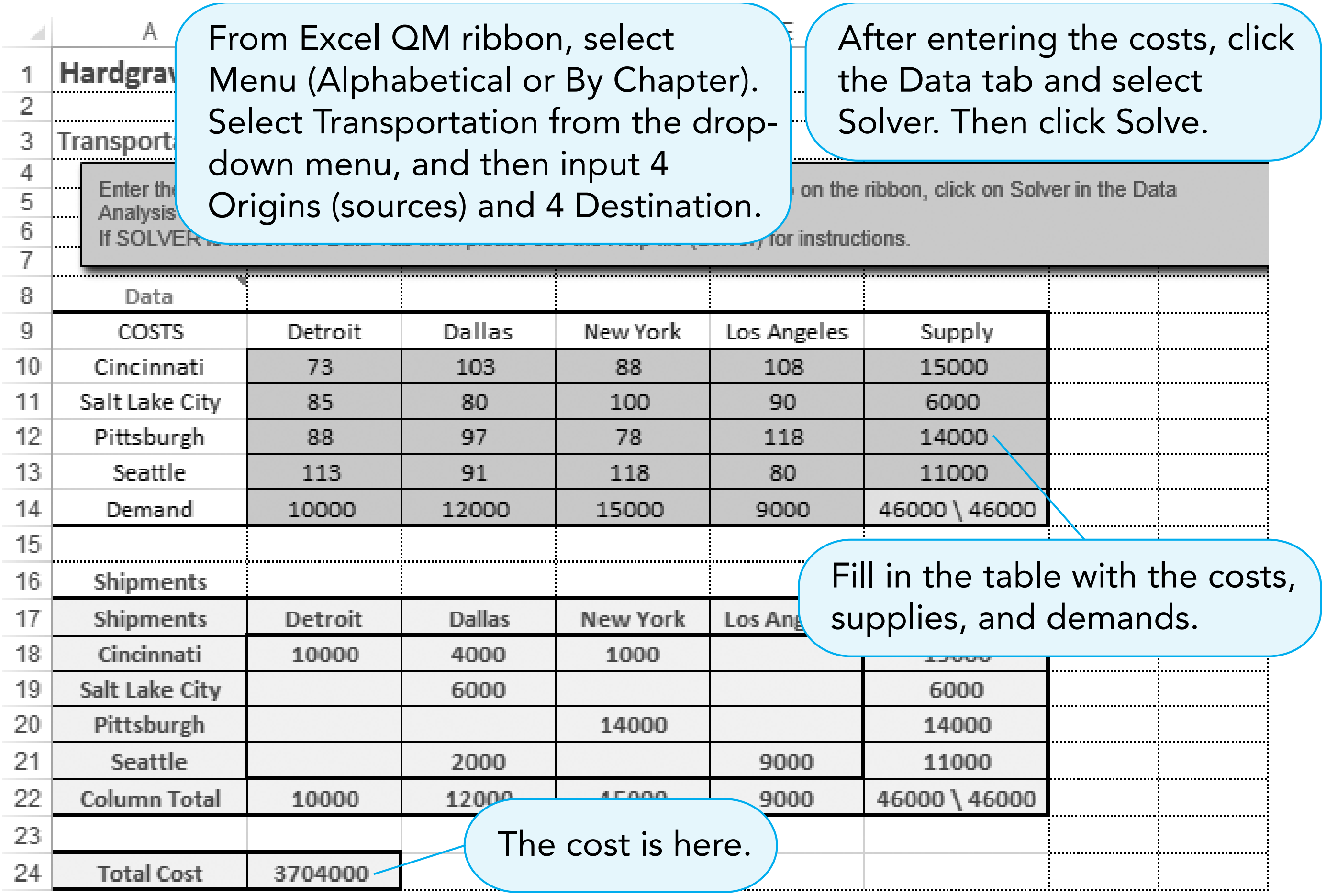
$X_{31} + X_{32} + X_{33} + X_{34} \leq 14{,}000$ Pittsburgh supply

$X_{41} + X_{42} + X_{43} + X_{44} \leq 11{,}000$ Seattle supply

All variables $X_{ij} \geq 0$

Using Excel QM

PROGRAM 9.2 – Facility Location (Seattle) Solution in Excel 2013 Using Excel QM



COSTS	Detroit	Dallas	New York	Los Angeles	Supply
Cincinnati	73	103	88	108	15000
Salt Lake City	85	80	100	90	6000
Pittsburgh	88	97	78	118	14000
Seattle	113	91	118	80	11000
Demand	10000	12000	15000	9000	46000 \ 46000

Shipments

Shipments	Detroit	Dallas	New York	Los Ang	
Cincinnati	10000	4000	1000		[illegible]
Salt Lake City		6000			6000
Pittsburgh			14000		14000
Seattle		2000		9000	11000
Column Total	10000	12000	[illegible]	9000	46000 \ 46000

Total Cost	3704000

Using Excel QM

PROGRAM 9.3 – Facility Location (Birmingham) Solution in Excel 2013 Using Excel QM

Hardgrave Machine

Transportation

Enter the transportation data in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver) for instructions.

Data

COSTS	Detroit	Dallas	New York	Los Angeles	Supply
Cincinnati	73	103	88	108	15000
Salt Lake City	85	80	100	90	6000
Pittsburgh	88	97	78	118	14000
Birmingham	84	79	90	99	11000
Demand	10000	12000	15000	9000	46000 \ 46000

Shipments

Shipments	Detroit	Dallas	New York	Los Angeles	Row Total
Cincinnati	10000		1000	4000	15000
Salt Lake City		1000		5000	6000
Pittsburgh			14000		14000
Birmingham		11000			11000
Column Total	10000	12000	15000	9000	46000 \ 46000

Total Cost	3741000

The cost is here.

The Assignment Problem

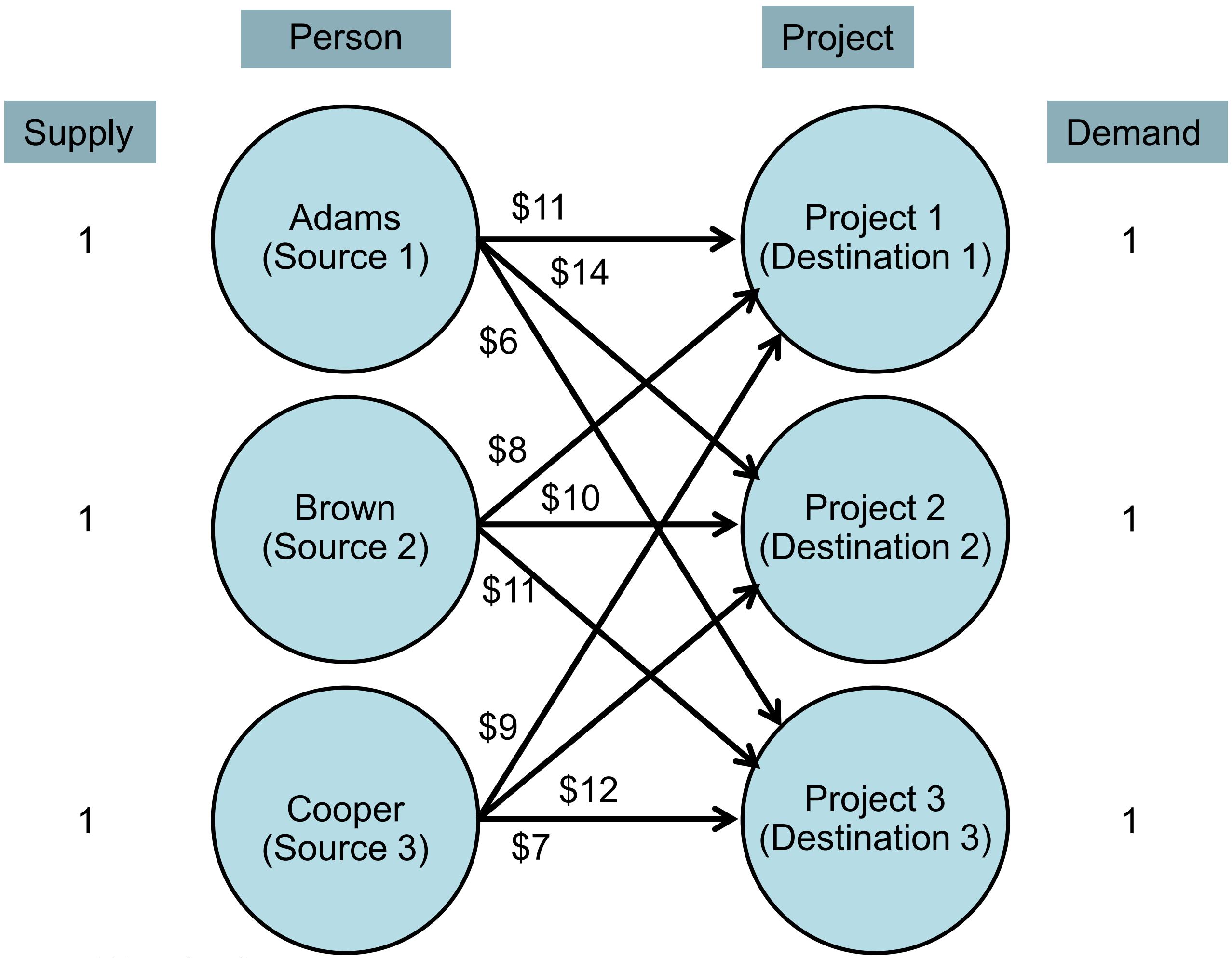
- This class of problem determines the most efficient assignment of people or equipment to particular tasks
- Objective is typically to minimize total cost or total task time

Linear Program for Assignment Example

- The Fix-it Shop has just received three new repair projects that must be repaired quickly
  1. A radio
  2. A toaster oven
  3. A coffee table
- Three workers with different talents are able to do the jobs
- Owner estimates wage cost for workers on projects
- Objective – minimize total cost

Linear Program for Assignment Example

FIGURE 9.2 – Assignment Problem in a Transportation Network Format



Linear Program for Assignment Example

Let

$$X_{ij} = \begin{cases} 1 \text{ if person } i \text{ is assigned to project } j \\ 0 \text{ otherwise} \end{cases}$$

where

i = 1, 2, 3, with 1 = Adams, 2 = Brown, and 3 = Cooper

j = 1, 2, 3, with 1 = Project 1, 2 = Project 2, and 3 = Project 3

Linear Program for Assignment Example

$$\text{Minimize total cost} = 11X_{11} + 14X_{12} + 6X_{13} + 8X_{21} + 10X_{22} + 11X_{23} + 9X_{31} + 12X_{32} + 7X_{33}$$

subject to

$$\begin{aligned}
X_{11} + X_{12} + X_{13} &= 1 \\
X_{21} + X_{22} + X_{23} &= 1 \\
X_{31} + X_{32} + X_{33} &= 1 \\
X_{11} + X_{21} + X_{31} &= 1 \\
X_{12} + X_{22} + X_{32} &= 1 \\
X_{13} + X_{23} + X_{33} &= 1 \\
X_{ij} &= 0 \text{ or } 1 \text{ for all } i \text{ and } j
\end{aligned}$$

Linear Program for Assignment Example

Minimize total cost = $11X_{11} + 14X_{12} + 6X_{13} + 8X_{21}$
$+ 10X_{22} + 11X_{23} + 9X_{31}$
$+ 12X_{32} + 7X_{33}$

subject to

Solution

$X_{13} = 1$, Adams assigned to Project 3
$X_{22} = 1$, Brown assigned to Project 2
$X_{31} = 1$, Cooper is assigned to Project 1

Total cost = $25

Using Excel QM

PROGRAM 9.4 – Mr. Fix-It Shop Assignment Solution in Excel 2013 Using Excel QM

From Excel QM ribbon, select Menu (Alphabetical or By Chapter). Select Assignment from the drop-down menu, and then input the number of assignments (3).

After entering the costs, click the Data tab and select Solver. Then click Solve.

go to
Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver)

Data

COSTS	Project 1	Project 2	Project 3
Adams	11	14	6
Brown	8	10	11
Cooper	9	12	7

Fill in the table with the costs, supplies, and demands.

Assignments

Shipments	Project 1	Project 2	Project 3	Row Total
Adams			1	1
Brown		1		1
Cooper	1			1
Column Total	1			

Total Cost	25

The cost is here.

The Transshipment Problem

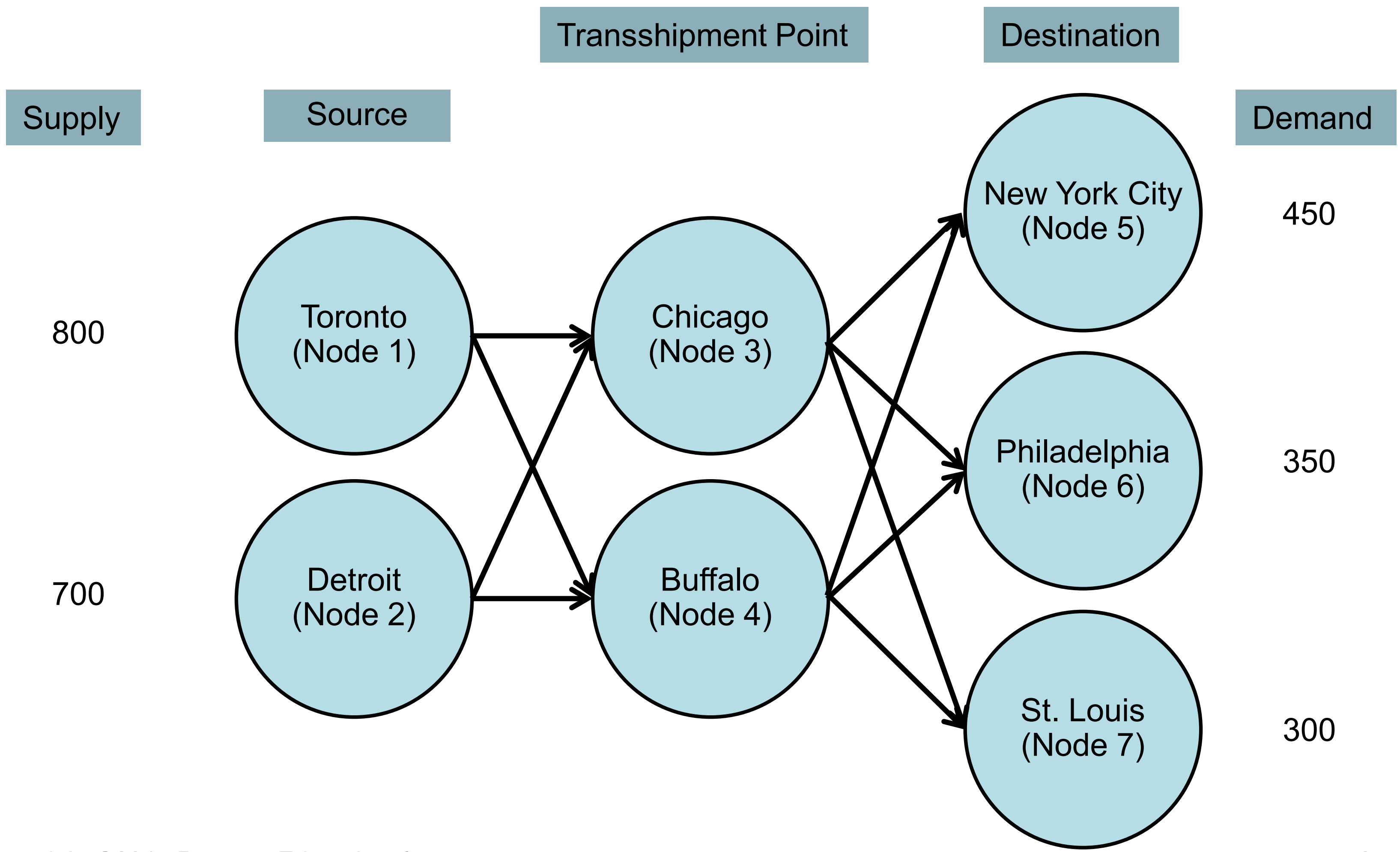
- Items are being moved from a source to a destination through an intermediate point (a *transshipment point*)
- *Transshipment problem*

The Transshipment Problem

- Frosty Machines manufactures snow blowers in Toronto and Detroit
- Shipped to regional distribution centers in Chicago and Buffalo
- Then shipped to supply houses in New York, Philadelphia, and St. Louis
- Shipping costs vary by location and destination
- Snow blowers cannot be shipped directly from the factories to the supply houses

The Transshipment Problem

FIGURE 9.3 – Network Representation of Transshipment Example



The Transshipment Problem

TABLE 9.3 – Frosty Machine Transshipment Data

	TO					
FROM	**CHICAGO**	**BUFFALO**	**NEW YORK CITY**	**PHILADELPHIA**	**ST. LOUIS**	**SUPPLY**
Toronto	$4	$7	—	—	—	800
Detroit	$5	$7	—	—	—	700
Chicago	—	—	$6	$4	$5	—
Buffalo	—	—	$2	$3	$4	—
Demand	—	—	450	350	300	

Minimize transportation costs associated with shipping snow blowers subject to demands and supplies

The Transshipment Problem

- Minimize cost subject to
  1. The number of units shipped from Toronto is not more than 800
  2. The number of units shipped from Detroit is not more than 700
  3. The number of units shipped to New York is 450
  4. The number of units shipped to Philadelphia is 350
  5. The number of units shipped to St. Louis is 300
  6. The number of units shipped out of Chicago is equal to the number of units shipped into Chicago
  7. The number of units shipped out of Buffalo is equal to the number of units shipped into Buffalo

The Transshipment Problem

Decision variables

X_{ij} = number of units shipped from location (node) i to location (node) j

where

$i = 1, 2, 3, 4$

$j = 3, 4, 5, 6, 7$

The Transshipment Problem

$$\text{Minimize cost} = 4X_{13} + 7X_{14} + 5X_{23} + 7X_{24} + 6X_{35} + 4X_{36} + 5X_{37} + 2X_{45} + 3X_{46} + 4X_{47}$$

subject to

$X_{13} + X_{14}$	≤ 800	(Supply at Toronto [node 1])
$X_{23} + X_{24}$	≤ 700	(Supply at Detroit [node 2])
$X_{35} + X_{45}$	$= 450$	(Demand at New York [node 5])
$X_{36} + X_{46}$	$= 350$	(Demand at Philadelphia [node 6])
$X_{37} + X_{47}$	$= 300$	(Demand at St. Louis [node 7])
$X_{13} + X_{23}$	$= X_{35} + X_{36} + X_{37}$	(Shipping through Chicago [node 3])
$X_{14} + X_{24}$	$= X_{45} + X_{46} + X_{47}$	(Shipping through Buffalo [node 4])
X_{ij}	≥ 0 for all i and j	(nonnegativity)

The Transshipment Problem

Minimize cost = $4X_{13} + 7X_{14} + 5X_{23} + 7X_{24} + 6X_{35} + 4X_{36} + 5X_{37} + 2X_{45} + 3X_{46} + 4X_{47}$

subject to

$X_{13} + X_{14} \leq 800$ (Supply at Toronto [node 1])

$X_{23} + X_{24} \leq 700$ (Supply at Detroit [node 2])

$X_{35} + X_{45} = 450$

$X_{36} + X_{46} = 350$

$X_{37} + X_{47} = 300$

$X_{13} + X_{23} = X_{35}$ +

$X_{14} + X_{24} = X_{45}$ +

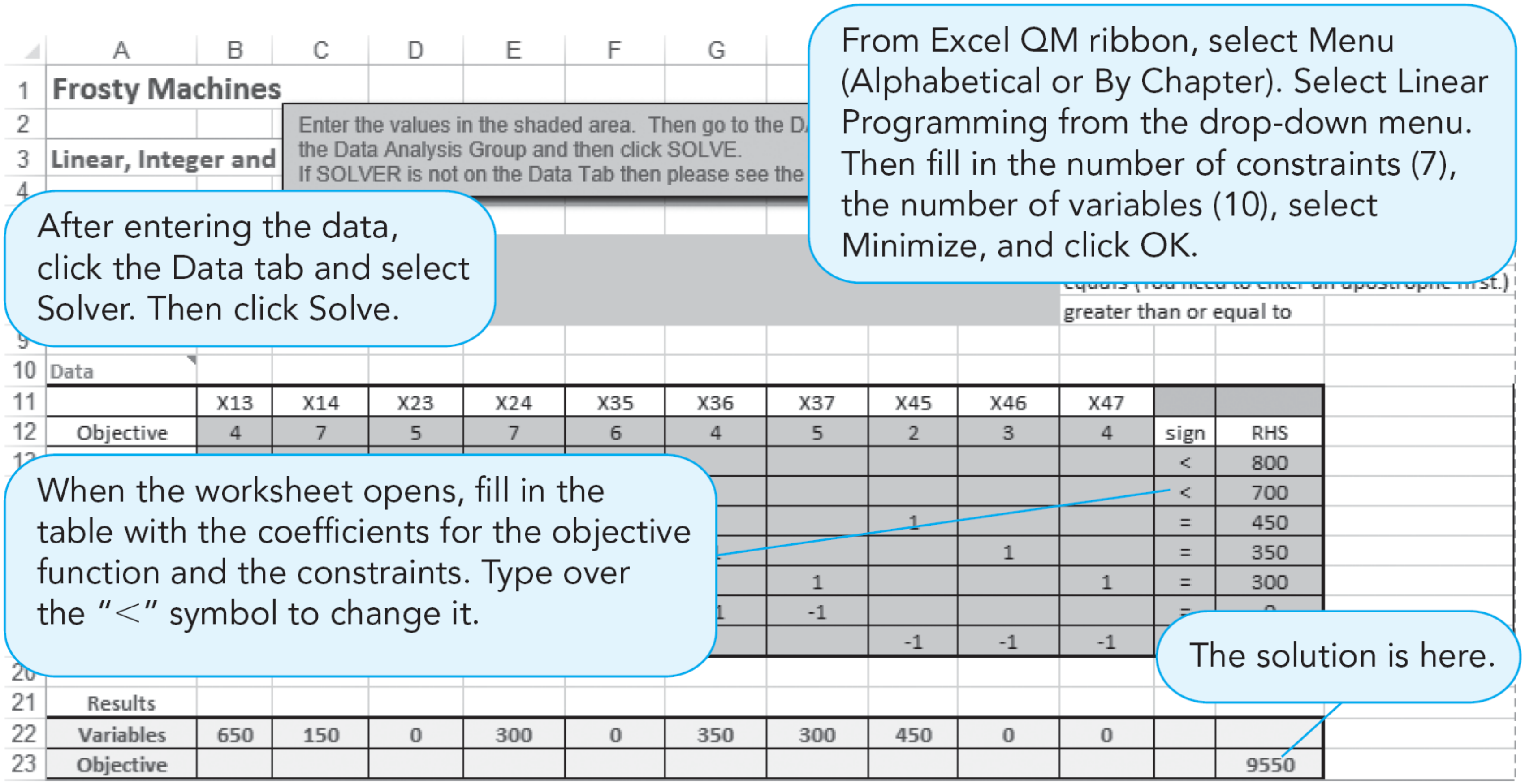
$X_{ij} \geq 0$ for

Ship 650 units from Toronto to Chicago
Ship 150 units from Toronto to Buffalo
Ship 300 units from Detroit to Buffalo
Ship 350 units from Chicago to Philadelphia
Ship 300 units form Chicago to St. Louis
Ship 450 units from Buffalo to New York

Total cost = $9,550

Using Excel QM

PROGRAM 9.5 – Excel QM Solution to Frosty Machine Transshipment Problem



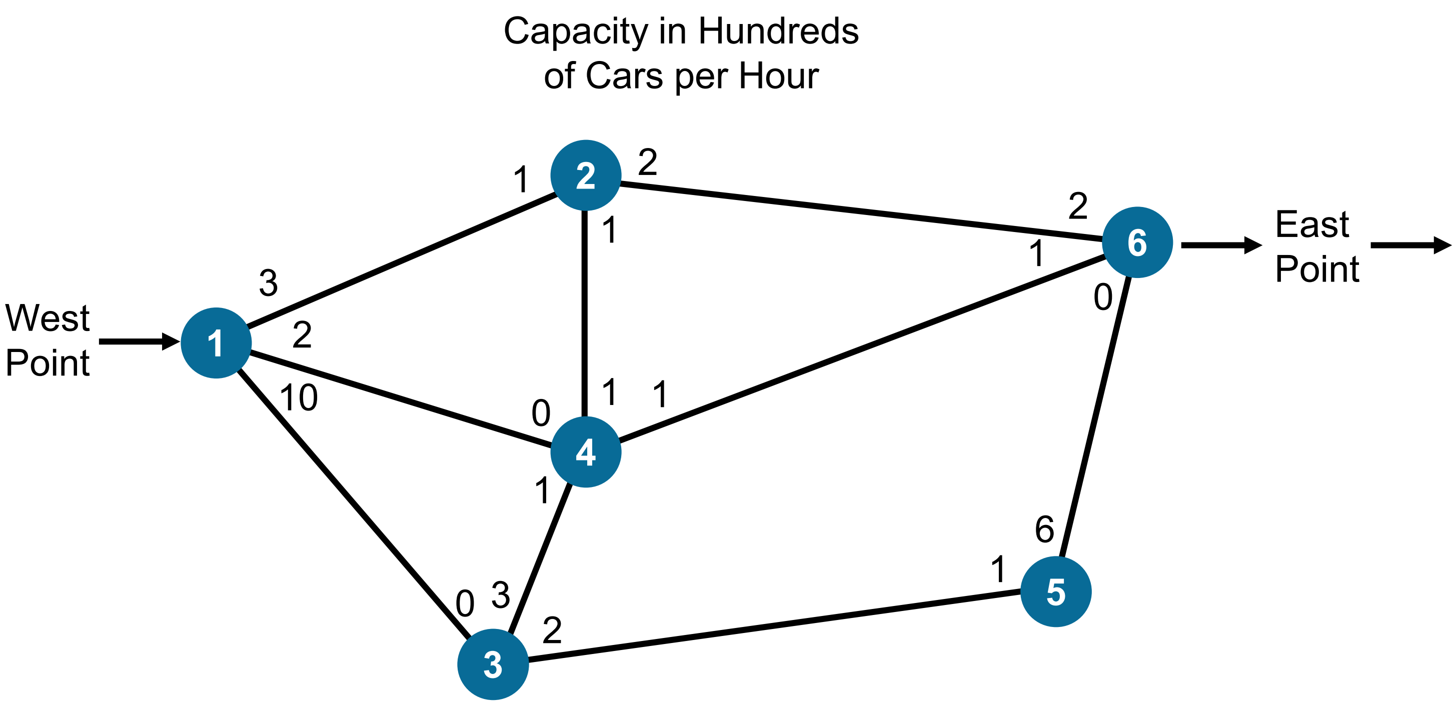
Maximal-Flow Problem

- Determining the maximum amount of material that can flow from one point (the **source**) to another (the **sink**) in a network
- Two common methods
  - Linear programming
  - Maximal-flow technique

Maximal-Flow Problem

- Determine maximum number of cars from east to west for Waukesha WI road system

FIGURE 9.4 – Road Network for Waukesha Maximal-Flow Example



Maximal-Flow Problem

Variables

X_{ij} = flow from node i to node j

where

$i = 1, 2, 3, 4, 5, 6$

$j = 1, 2, 3, 4, 5, 6$

Constraints necessary for

– Capacity of each arc
– Equal flows into and out of each arc

Maximal-Flow Problem

Maximize flow = X_{61}

subject to

X_{12}	$\leq$	3	X_{13}	$\leq$	10	X_{14}	$\leq$	2	Capacities for arcs from node 1
X_{21}	$\leq$	1	X_{24}	$\leq$	1	X_{26}	$\leq$	2	Capacities for arcs from node 2
X_{34}	$\leq$	3	X_{35}	$\leq$	2				Capacities for arcs from node 3
X_{42}	$\leq$	1	X_{43}	$\leq$	1	X_{46}	$\leq$	1	Capacities for arcs from node 4
X_{53}	$\leq$	1	X_{56}	$\leq$	1				Capacities for arcs from node 5
X_{62}	$\leq$	2	X_{64}	$\leq$	1				Capacities for arcs from node 6

$(X_{21} + X_{61}) - (X_{12} + X_{13} + X_{14})$	$=$	0	Flows into = flows out of node 1
$(X_{12} + X_{42} + X_{62}) - (X_{21} + X_{24} + X_{26})$	$=$	0	Flows into = flows out of node 2
$(X_{13} + X_{43} + X_{53}) - (X_{34} + X_{35})$	$=$	0	Flows into = flows out of node 3
$(X_{14} + X_{24} + X_{34} + X_{64}) - (X_{42} + X_{43} + X_{46})$	$=$	0	Flows into = flows out of node 4
$(X_{35}) - (X_{56} + X_{53})$	$=$	0	Flows into = flows out of node 5
$(X_{26} + X_{46} + X_{56}) - (X_{61} + X_{62} + X_{64})$	$=$	0	Flows into = flows out of node 6
X_{ij}	$\geq$	0	

Using Excel QM

PROGRAM 9.6 – Waukesha Maximal-Flow Solution

Waukesha

Maximal Flow

Enter the flow capac
the sink must be set
Data Analysis Group

From Excel QM ribbon, select Menu (Alphabetical or By Chapter). Select Network Analysis as LP and Maximal Flow from the drop-down menu. Then fill in the number of branches or arcs (6). Click OK.

Data - Arc capacities

From\to	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Node 1	0	3	10	2	0	0
Node 2	1	0	0	1	0	2
Node 3	0	0	0	3	2	0
Node 4	0	1	1	0	0	1
Node 5	0	0	1	0	0	6
Node 6	999999	2	0	1	0	0

When the worksheet opens, enter the flows in the table.

This is entered already and is used by Excel QM.

From\to	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Outflow
Node 1			2				2
Node 2						2	2
Node 3					2		2
Node 4						1	1
Node 5						2	2
Node 6	2	2		1			5
Inflow	2	2	2	1	2	5	
Outflow	2	2					
Max Flow	5						

After entering the data, click the Data tab and select Solver. Then click Solve.

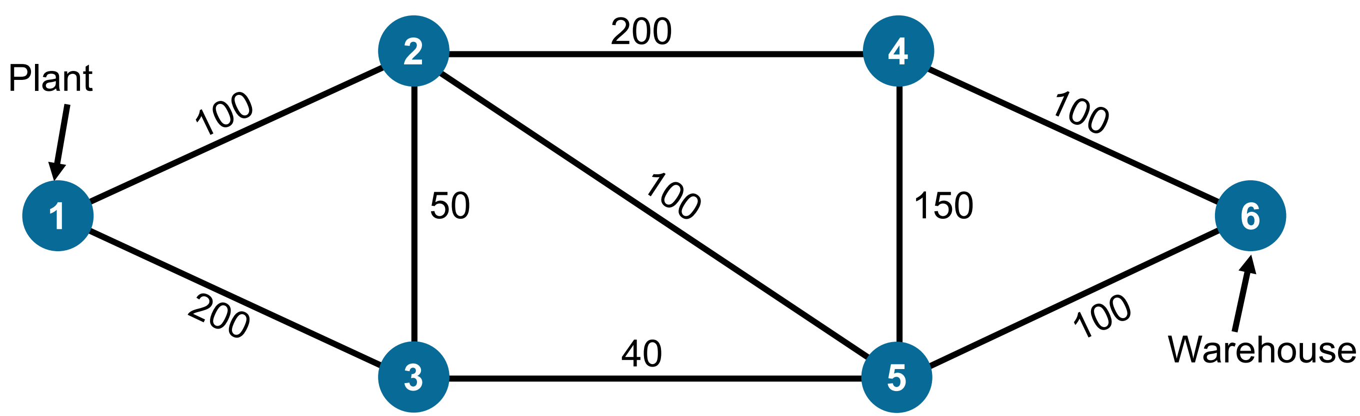
Shortest-Route Problem

- Find the shortest distance from one location to another
- Can be modeled as
  - A linear programming problem with 0-1 variables
    - A special type of transshipment problem
  - Using specialized algorithm

Shortest-Route Problem

- Ray Design transports beds, chairs, and other furniture items from the factory to the warehouse
  - Travel through several cities
  - No direct interstate highways
- Find the route with the shortest distance

FIGURE 9.5 – Roads from Ray's Plant to Warehouse



Shortest-Route Problem

Variables

X_{ij} = 1 if arc from node *i* to node *j* is selected and
X_{ij} = 0 otherwise

where

$i = 1, 2, 3, 4, 5$

$j = 2, 3, 4, 5, 6$

Constraints specify the number of units going into a node must equal the number of units going out of the node

Shortest-Route Problem

Origin point must ship one unit

$$X_{12} + X_{13} = 1$$

Final destination must have one unit shipped into it

$$X_{46} + X_{56} = 1$$

Intermediate nodes must have same amounts entering and leaving

$$X_{12} + X_{32} = X_{23} + X_{24} + X_{25}$$

or

$$X_{12} + X_{32} - X_{23} - X_{24} - X_{25} = 0$$

Shortest-Route Problem

$$\text{Minimize distance} = 100X_{12} + 200X_{13} + 50X_{23} + 50X_{32} + 200X_{24} + 200X_{42} + 100X_{25} + 100X_{52} + 40X_{35} + 40X_{53} + 150X_{45} + 150X_{54} + 100X_{46} + 100X_{56}$$

subject to

$X_{12} + X_{13}$	=	1	Node 1
$X_{12} + X_{32} - X_{23} - X_{24} - X_{25}$	=	0	Node 2
$X_{13} + X_{23} - X_{32} - X_{35}$	=	0	Node 3
$X_{24} + X_{54} - X_{42} - X_{45} - X_{46}$	=	0	Node 4
$X_{25} + X_{35} + X_{45} - X_{52} - X_{53} - X_{54} - X_{56}$	=	0	Node 5
$X_{46} + X_{56}$	=	1	Node 6
All variables	=	0	or 1

Using Excel QM

PROGRAM 9.7 – Ray Designs, Inc. Solution

Ray Design

Shortest Path

Enter the distances from nod
ribbon, click on Solver in the
If SOLVER is not on the Data

From Excel QM ribbon, select Menu (Alphabetical or By Chapter). Select Network Analysis as LP and Shortest Route from the drop-down menu. Then fill in the number of branches or arcs (6). Click OK.

Data - Distance Table

From\to	City 1	City 2	City 3	City 4	City 5	City 6
City 1	0	100	200	0	0	0
City 2	100	0	50	200	100	0
City 3	200	50	0	0	40	0
City 4	0	200	0	0	150	100
City 5	0	100	40	150	0	100
City 6	0	0	0	100	100	0
	1	0	0	0	0	-1

Start=1,Finish=-1

When the worksheet opens, enter the distances in the table.

Flows

From\to	City 1	City 2	City 3	City 4	City 5	City 6	Outflow
City 1		1					1
City 2			1				1
City 3					1		1
City 4							
City 5						1	1
City 6							
Inflow		1	1		1	1	
Outflow	1						
Net Outflow	1						
Minimum distance	290						

After entering the data, click the Data tab and select Solver. Then click Solve.

Using Excel QM

PROGRAM 9.7 – Ray Designs, Inc. Solution

Solution

$X_{12} = X_{23} = X_{35} = X_{56} = 1$

Route is City 1 to City 2 to City 3 to City 5 to City 6

Total distance traveled = 290 miles

18								
19	**Flows**							
20	From\to	City 1	City 2	City 3	City 4	City 5	City 6	Outflow
21	City 1		1					1
22	City 2			1				1
23	City 3					1		1
24	City 4							
25	City 5						1	1
26	City 6							
27	Inflow		1	1		1	1	
28	Outflow	1						
29	Net Outflow	1						
30								
31	Minimum distance	290						

After entering the data, click the Data tab and select Solver. Then click Solve.

Minimal-Spanning Tree Problem

- Connecting all points of a network together while minimizing the total distance of the connections
- Linear programming can be used but is complex
- Minimal-spanning tree technique is quite easy

Minimal-Spanning Tree Problem

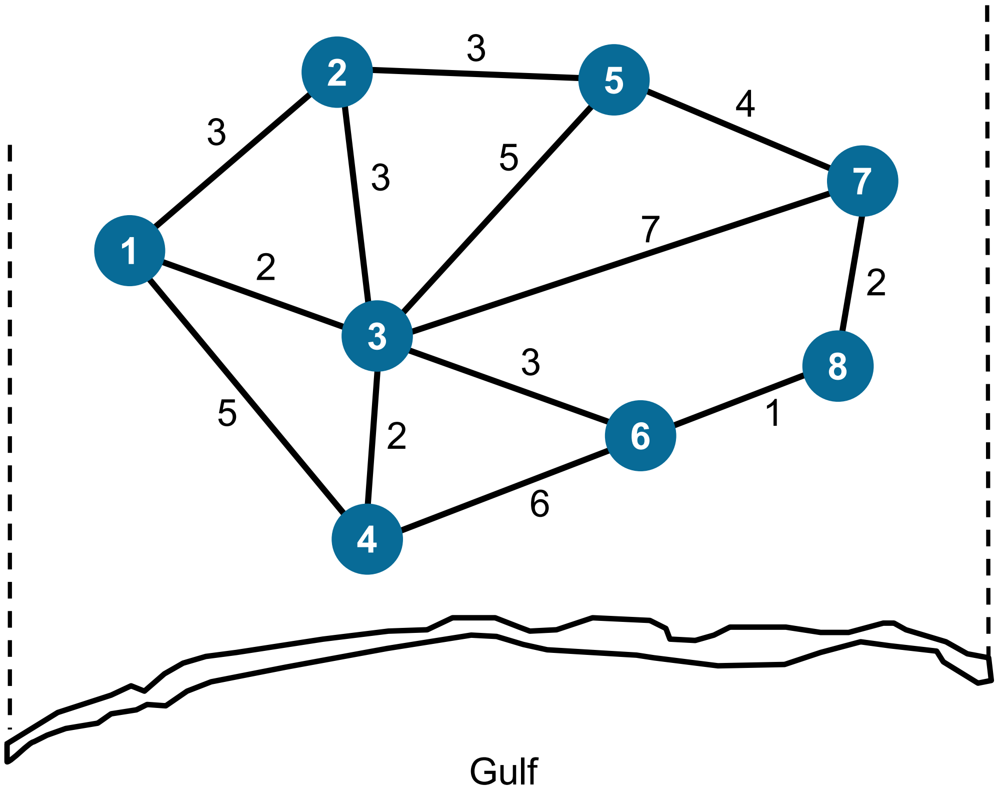
Steps for the Minimal-Spanning Tree Technique

1. Select any node in the network.
2. Connect this node to the nearest node that minimizes the total distance.
3. Considering all of the nodes that are now connected, find and connect the nearest node that is not connected. If there is a tie for the nearest node, select one arbitrarily. A tie suggests there may be more than one optimal solution.
4. Repeat the third step until all nodes are connected.

Minimal-Spanning Tree Problem

- Lauderdale Construction
  - Housing project in Panama City Beach
  - Determine the least expensive way to provide water and power to each house

FIGURE 9.6 – Network for Lauderdale Construction

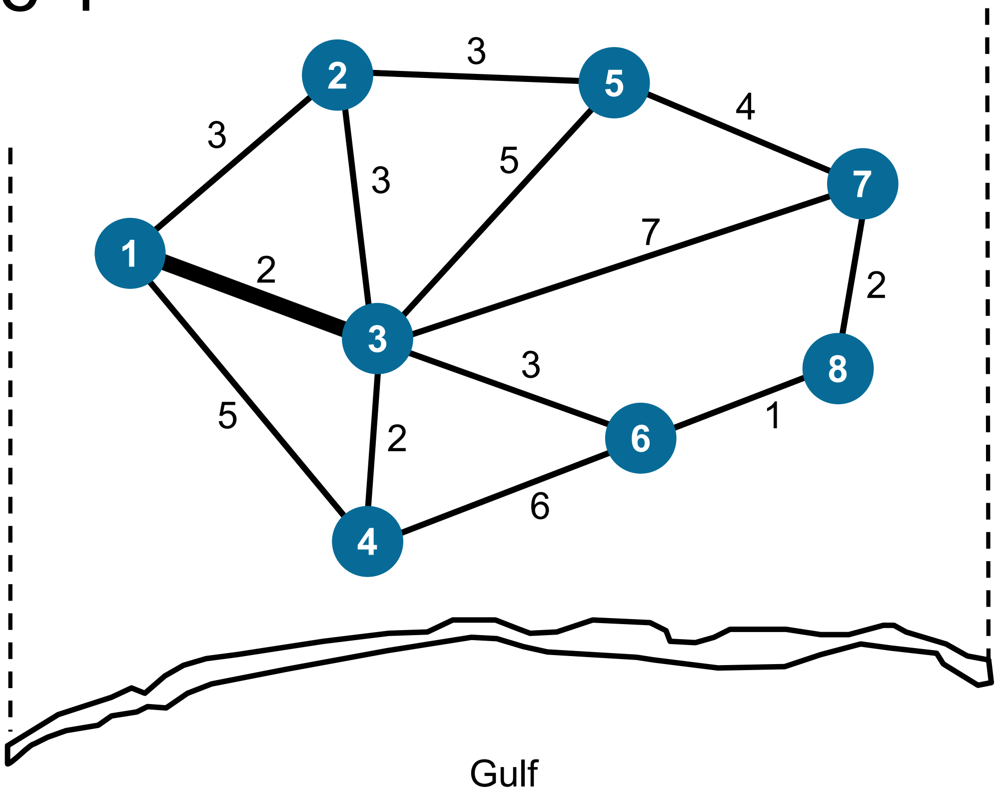


Minimal-Spanning Tree Problem

Step 1 – Arbitrarily select node 1

Step 2 – Connect node 1 to node 3 (nearest)

FIGURE 9.7 – First Iteration

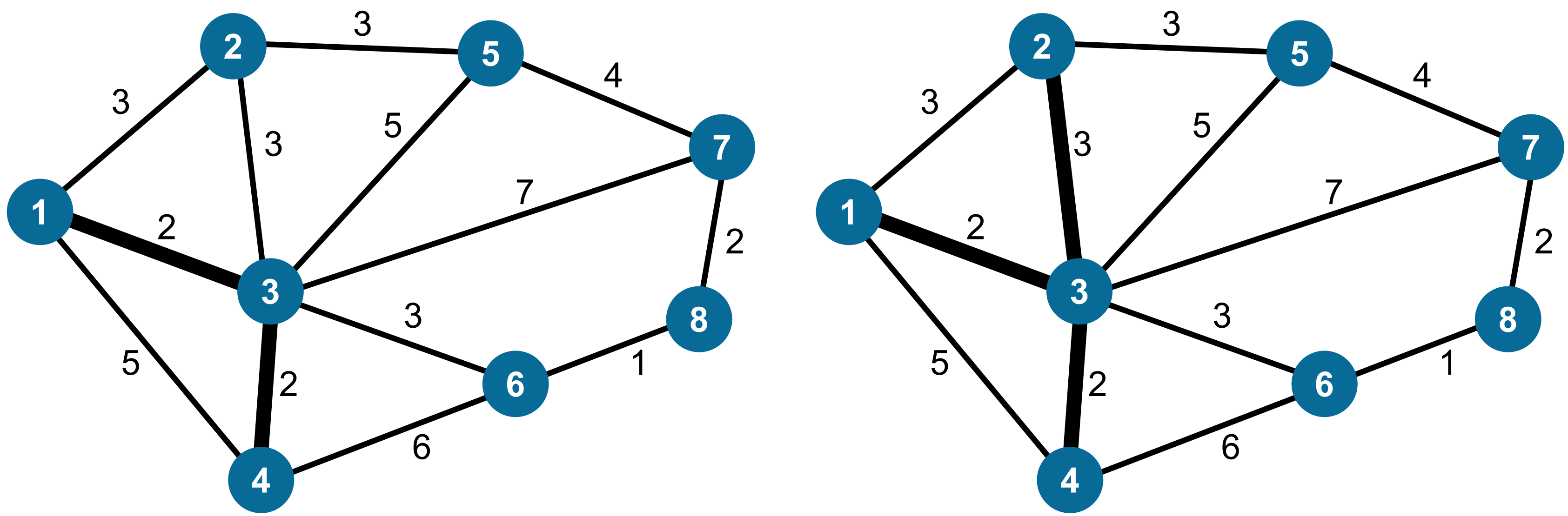


Minimal-Spanning Tree Problem

Step 3 – Connect next nearest unconnected node, node 4

Continue for other unconnected nodes

FIGURE 9.8 – Second and Third Iterations



(a) Second Iteration

(b) Third Iteration

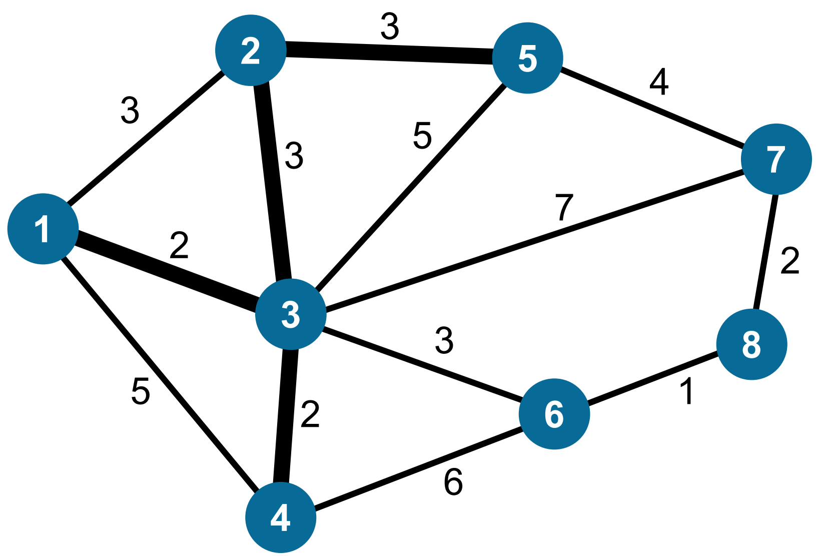
Minimal-Spanning Tree Problem

- Here the algorithm is not complete since we only go to the next node with minimum weight that does not form a cycle. Check the compete algorithm and its implementation at the following link
- https://www.geeksforgeeks.org/kruskals-minimum-spanning-tree-algorithm-greedy-algo-2/

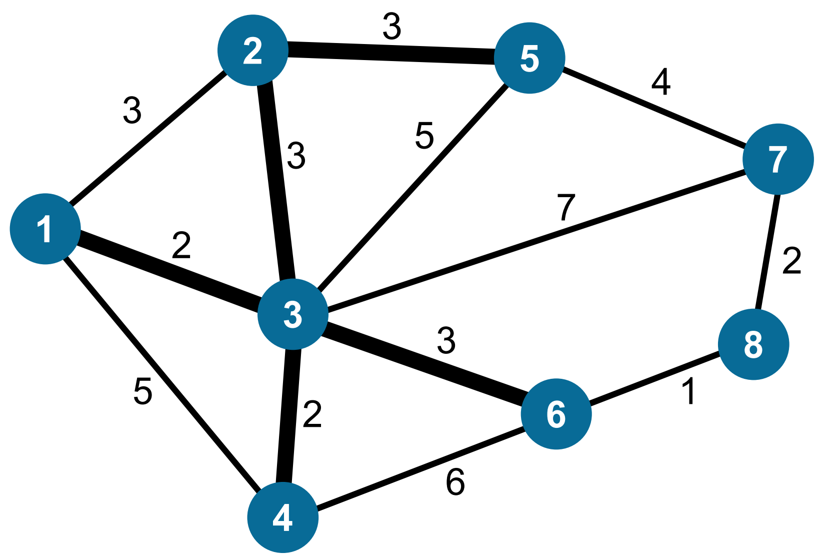
Minimal-Spanning Tree Problem

Step 4 – Repeat the process

FIGURE 9.9 – Last Four Iterations



(a) Fourth Iteration

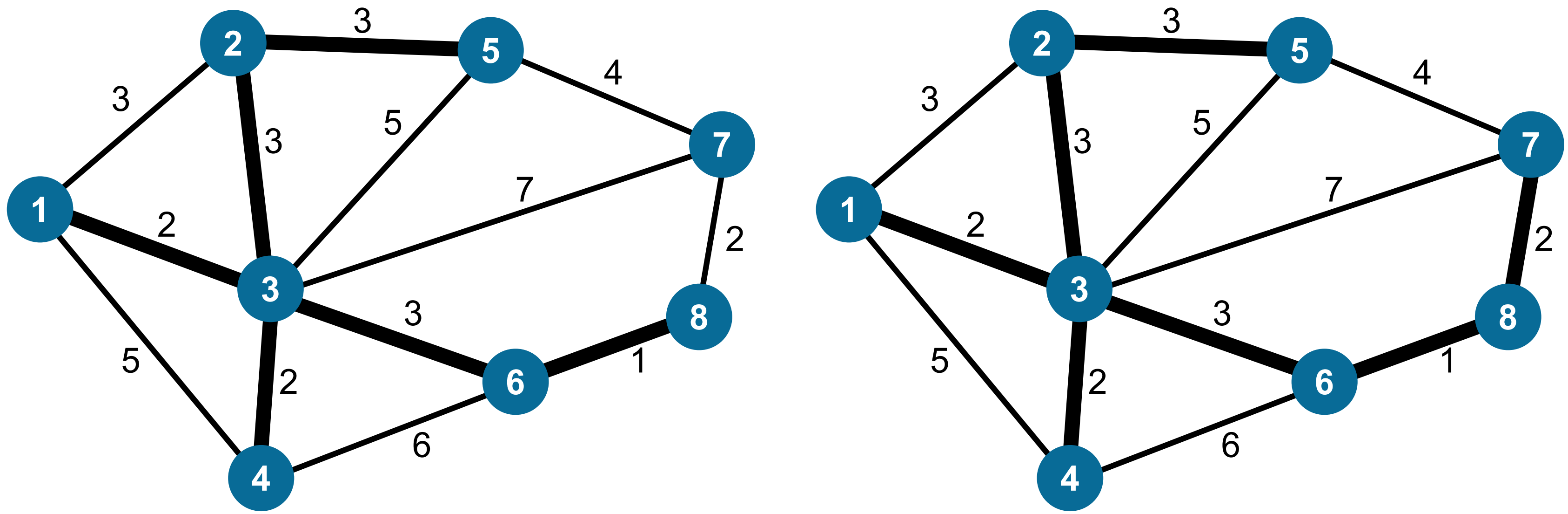


(b) Fifth Iteration

Minimal-Spanning Tree Problem

Step 4 – Repeat the process

FIGURE 9.9 – Last Four Iterations



(c) Sixth Iteration

(d) Seventh Iteration

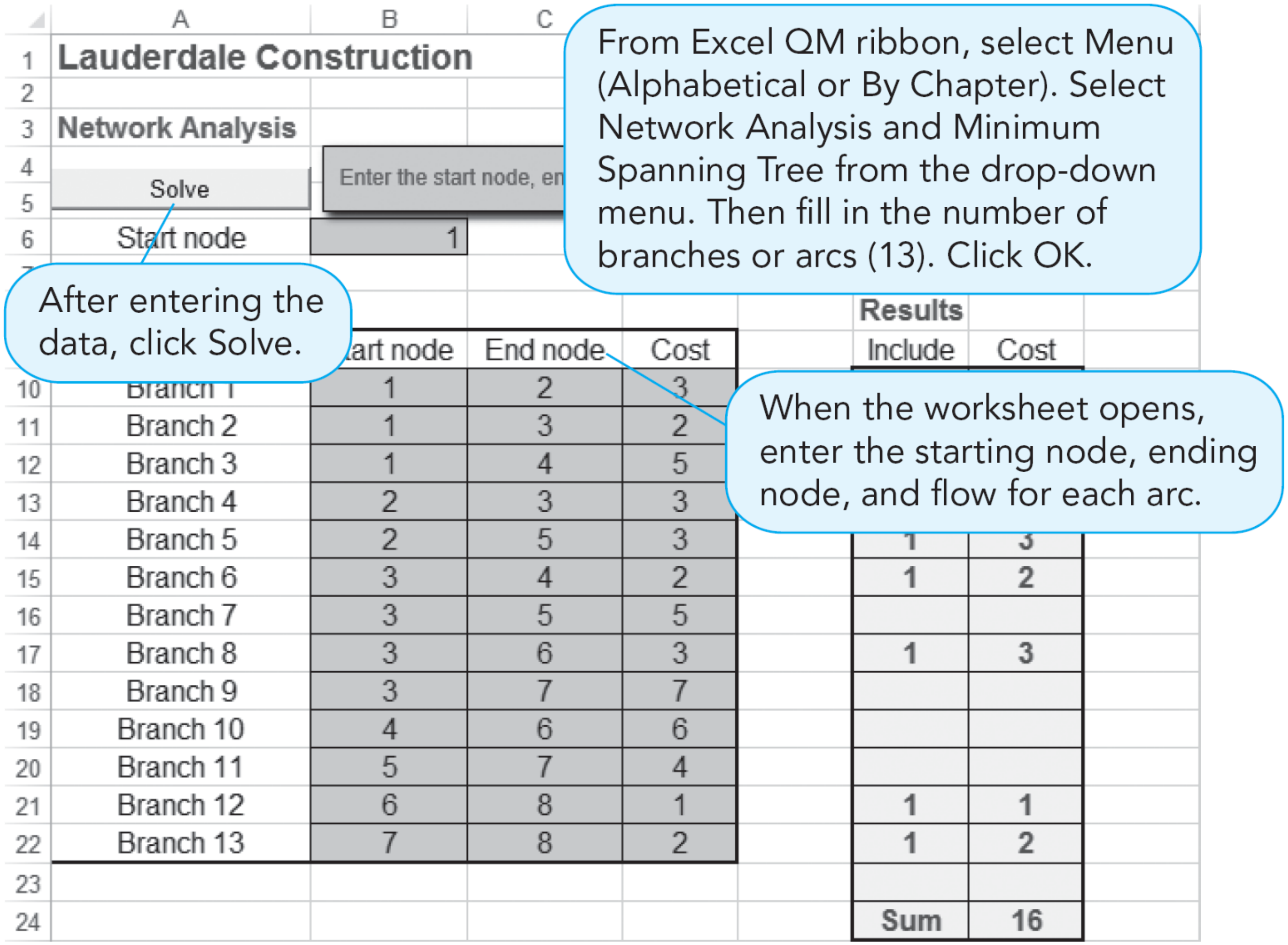
Minimal-Spanning Tree Problem

TABLE 9.4 – Summary of Steps in Lauderdale Construction Minimal-Spanning Tree Problem

STEP	CONNECTED NODES	UNCONNECTED NODES	CLOSE UNCONNECTED NODES	ARC SELECTED	ARC LENGTH	TOTAL DISTANCE
1	1	2, 3, 4, 5, 6, 7, 8	3	1–3	2	2
2	1, 3	2, 4, 5, 6, 7, 8	4	3–4	2	4
3	1, 3, 4	2, 5, 6, 7, 8	2 or 6	3–2	3	7
4	1, 2, 3, 4	5, 6, 7, 8	5 or 6	2–5	3	10
5	1, 2, 3, 4, 5	6, 7, 8	6	3–6	3	13
6	1, 2, 3, 4, 5, 6	7, 8	8	6–8	1	14
7	1, 2, 3, 4, 5, 6, 8	7	7	8–7	2	16

Using Excel QM

PROGRAM 9.8 – Lauderdale Construction Minimal-Spanning Tree Example



	tart node	End node	Cost	Include	Cost
Branch 1	1	2	3		
Branch 2	1	3	2		
Branch 3	1	4	5		
Branch 4	2	3	3		
Branch 5	2	5	3	1	3
Branch 6	3	4	2	1	2
Branch 7	3	5	5		
Branch 8	3	6	3	1	3
Branch 9	3	7	7		
Branch 10	4	6	6		
Branch 11	5	7	4		
Branch 12	6	8	1	1	1
Branch 13	7	8	2	1	2
				Sum	16

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