

**EEL3040**

Control System



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## Lab-4 Report

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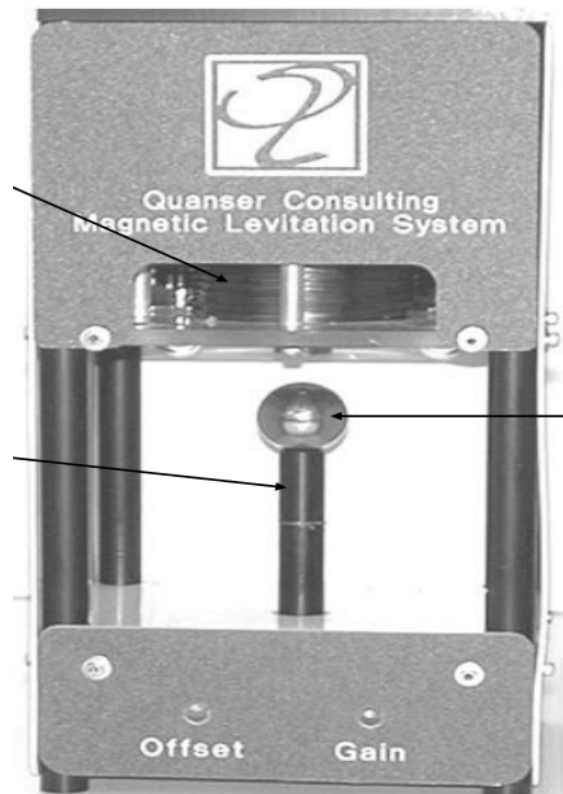
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## 1. Objective

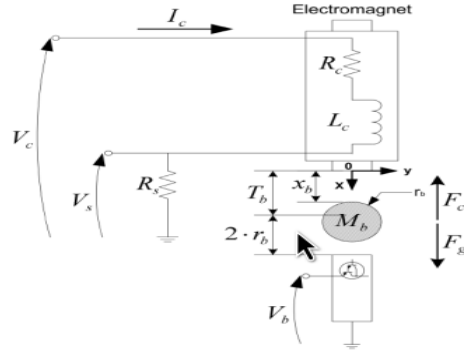
**(a) Aim-** The objective of this experiment is to model the magnetic levitation (MAGLEV) plant and to design a controller that levitates the ball from post and ball position tracks a desired trajectory.

**(b) Software-** MATLAB, Magnetic levitation kit, Q8-USB, UPM-2405 amplifier.

**(c) Theory-** The “MAGLEV” experiment consists of an electromagnet encased in a rectangular enclosure. One electromagnet pole faces a black post upon which a 2.54cm diameter steel ball rests. The ball elevation from the post is measured using a sensor embedded in the post. The post is designed such that with the ball at rest on its surface, it is 14 mm from the face of the electromagnet.



## 2. Mathematical Model



**(a) Electrical System**-The coil used in the electromagnet has an inductor and a resistance. The voltage applied to the coil results in a current governed by the differential equation.

$$v(t) = i(t)(R_c + R_s) + L \frac{di(t)}{dt} \quad (\text{i})$$

Applying laplace transform on equation-1

$$G_c(s) = \frac{I(s)}{V(s)} = \frac{K_{c_{dc}}}{\tau_c s + 1}$$

$$\text{Where, } K_{c_{dc}} = \frac{1}{R_c + R_s}, \quad \tau_c = \frac{L_c}{R_c + R_s}$$

**(b) Mechanical System**-The force due to gravity applied to the ball is expressed by:

$$F_c + F_g = -\frac{1}{2} \frac{K_m I_c^2}{x_b^2} + M_b g \quad (\text{ii})$$

Applying Newton's second law of motion

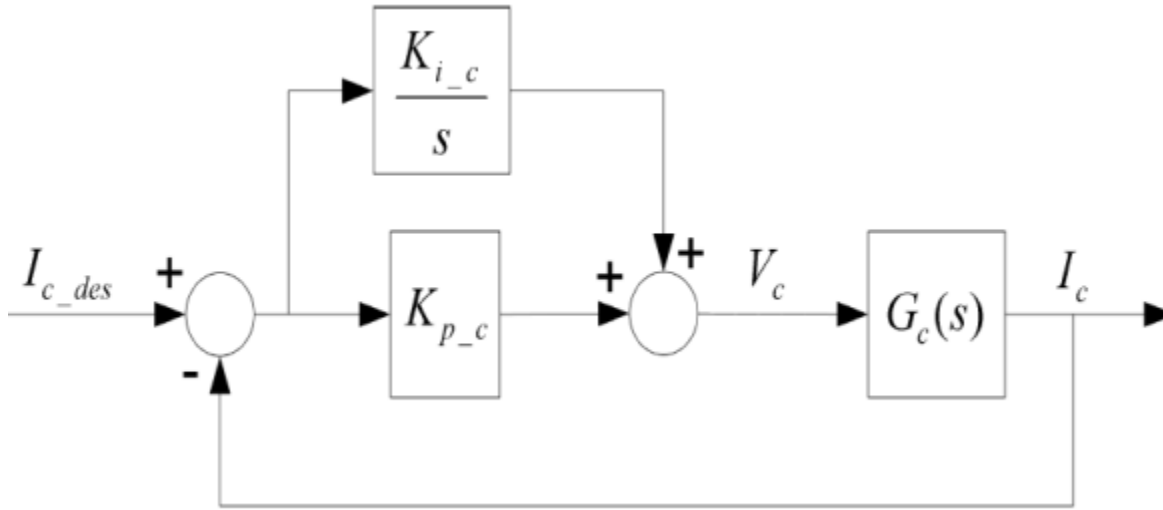
$$\frac{d^2 x_b}{dt^2} = g - \frac{1}{2} \frac{K_m I_c^2}{x_b^2 M_b}$$

At equilibrium:  $F_c + F_g = 0$

$$I_{c0} = \sqrt{\frac{2M_b g}{K_m}} x_{b0} \quad (\text{iii})$$

### 3. Controller Design

**(a)Coil current controller design: pole placement-**Prior to control the steel ball position, the current flowing through the electromagnet needs to be controlled. The electromagnet current control loop consists of a Proportional-plus-Integral (PI) closed-loop scheme.



$$\text{Transfer Function: } T_c(s) = \frac{I_c(s)}{I_{des}(s)}$$

$$\text{Poles: } P_{c1,2} = -235 \pm 70i$$

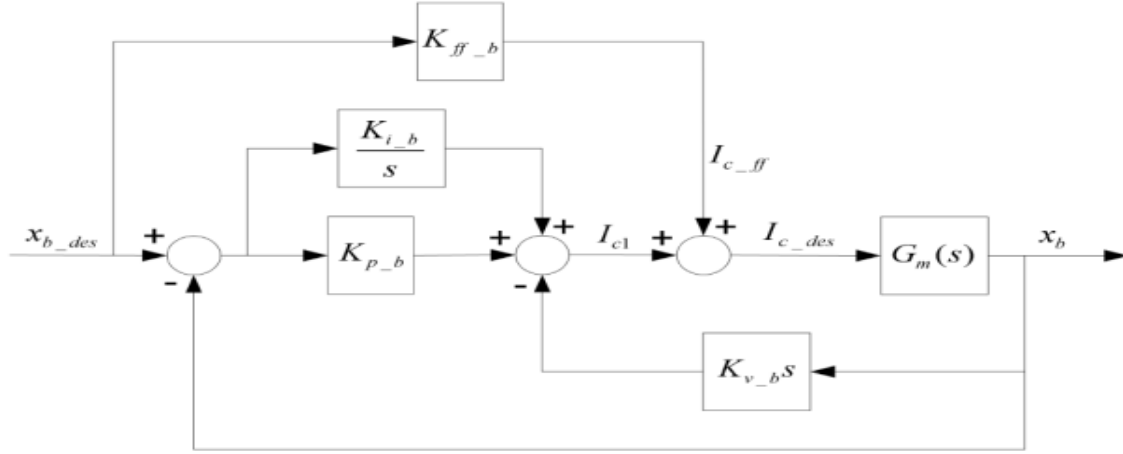
$$s^2 - (P_{c1} + P_{c2})s + P_{c1}P_{c2} = 0 \quad (\text{iv})$$

$$s^2 + 2\omega_n\zeta s + \omega_n^2 = 0 \quad (\text{v})$$

Comparing equation 4 and 5 we will get the values of  $K_{pc}$  and  $K_{ic}$ .

The calculation can be verified in calculation sections below this section.

**(b) Ball Position Controller Design: Pole Placement-** The steel ball position is controlled by means of a Proportional-plus-Integral-plus-Velocity (PIV or PID) closed-loop scheme with the addition of a feed-forward action.



**Transfer Function:**  $T_b(s) = \frac{x_b(s)}{x_{bdes}(s)}$

**Poles:**  $P_{b1,2,3} = -2.5, -44, -51.6$

$$s^3 - \left(\sum_{i=1}^3 P_{bi}\right)s^2 + \left(\sum_{i,j} P_{bi}P_{bj}\right)s - P_{b1}P_{b2}P_{b3} = 0 \quad (\text{vi})$$

$$s^3 - \frac{2gK_{vb}s^2}{I_{c0}} - \left(\frac{2g}{x_{b0}} + \frac{2gK_{pb}}{I_{c0}}\right)s - \frac{2gK_{ib}}{I_{c0}} = 0 \quad (\text{vii})$$

On comparing the above two equation we will get the values of  $K_{vb}$ ,  $K_{pb}$ ,  $K_{ib}$

And we know that,  $K_{ff} = \frac{I_{c0}}{x_{b0}}$

Where  $x_{b0} = 6mm$ , and  $I_{c0}$  can be calculated from equation-3.

**(c) Calculations:**

**Electrical System**

Electrical Sys:-

$$G_c(s) = \frac{I(s)}{V(s)} = \frac{K_{cdc}}{\tau_c(s) + 1} \quad \left( K_{cdc} = \frac{1}{R_c + R_s} \right)$$

$$\tau_c = \frac{L_c}{R_c + R_s}$$

$$K_{cdc} = \frac{1}{R_c + R_s} = \frac{1}{10 + 1.2} = 0.909$$

$$\tau = \frac{L_c}{R_c + R_s} = \frac{412.5}{0.1 \times 1000} = 0.0375$$

$$G_c(s) = \frac{0.909}{(0.0375)s + 1} = \frac{0.909}{0.4125s + 11.0011}$$

## Mechanical and ElectroMechanical Systems

Mech. Sys.

$$F_c + F_g = -\frac{1}{2} \frac{k_m I_c^2}{x_b^2} + M_b g$$

$$\ddot{x}_b = g - \frac{1}{2} \frac{k_m I_c^2}{x_b^2 M_b}$$

$$x_b = x_{b1} + x_{b0}, \quad I_c = I_{c0} + I_{c1}$$

$$F_c = -F_g \quad (\text{Equilibrium}), \quad x_{b0} = 6 \times 10^{-3} \text{ m}$$

$$I_{c0} = \sqrt{\frac{2 M_b g}{k_m}} \cdot x_{b0} \quad (M_b = 0.068 \text{ kg})$$

$$(g = 9.81 \text{ m/s}^2)$$

$$\Rightarrow \boxed{I_{c0} = 0.8572 \text{ A}}$$

Electromech. model

$$\frac{\partial^2}{\partial t^2} x_{b1} = -\frac{1}{2} \frac{k_m I_{c0}^2}{M_b x_{b0}^2} + g + \frac{k_m I_{c0}^2 x_{b1}}{M_b x_{b0}^3} - \frac{k_m I_{c0} I_{c1}}{M_b x_{b0}^2}$$

$$x_{b1}(s) \cdot \frac{1}{s^2} = \frac{1}{s^2} \left( \frac{k_m I_{c0}^2}{2 M_b x_{b0}^2} + g + \frac{k_m I_{c0}^2 x_{b1}}{M_b x_{b0}^3} - \frac{k_m I_{c0} I_{c1}}{M_b x_{b0}^2} \right)$$

$$s^2 - \omega_b^2 = 0 \quad \omega_b^2 = \frac{g}{x_{b0}} = \frac{9.81}{6 \times 10^{-3}} = 1635 \text{ s}^{-2}$$

$$\omega_b = 40.43 \text{ rad/s}$$

$$\boxed{k_{bde} = 6.995 \times 10^3 \text{ N/m}}$$

$$\omega_b = \sqrt{\frac{2g}{x_{b0}}}$$

$$\boxed{\omega_b = 57.1834 \text{ rad/s}}$$

$$P_{C1,2} = -235 \pm 70i$$

$$s^2 + -(P_{C1} + P_{C2})s + P_{C1}P_{C2} = 0$$



## Block Diagram

Block diagram:-

$$G_c(s) = \frac{I_c}{I_{des} - I_c}$$

$$I_{des} - I_c = (K_p + K_i/s) (G_c(s)) = 2.5 \times 10^3$$

$$I_{des} = (I_{des} - I_c) (K_p + K_i/s) G_c(s)$$

$$0 = (I_{des} - I_c) \left( \frac{K_{ic} + K_{pc}}{s} \right) = V_{cs} \frac{K_{pc}}{s} - 2.5 \times 10^3$$

$$0 = \frac{K_{pc}}{s} I_c - 2.5 \times 10^3 = \left( \frac{K_{pc}}{s} + K_{pc} \right) G_c(s)$$

$$G_c(s) = \frac{I_c}{V_c} = \frac{G_c(s)}{I_{des} - I_c}$$

$$\boxed{I_{des} = I_c + 1} \quad \dots (i)$$

Using eq (i)

$$\frac{K_{pc}}{s} (12.9) + \left( K_p + \frac{K_i}{s} \right) (G_c(s)) = 0$$

$$\text{Putting } (P_1, P_2) = j\omega \quad G_c(s)$$

$$\left[ 1 + \left( K_p + \frac{K_i}{s} \right) \right] \left( \frac{0.415}{-235 + 70i} \right) = 0$$

$$\left[ 1 + \left( K_p + \frac{K_i}{s} \right) \right] \left( \frac{0.415}{-235 + 70i} \right) = 0$$

Solving above eqn.

$$\boxed{K_{pc} = 182.876}$$

$$\boxed{K_{ic} = 24801.56}$$

## Controller Design

Controller design.

$$T_b(s) = G_b(s) / X_{bdes}(s)$$

$$P_{1,2,3} = (-2.5, -44, -51.6) \text{ rad/s}$$

$$\Rightarrow s^3 + \left( \sum_{i=1}^3 P_{bi} \right) s^2 + \left( \sum_{i \neq j} P_{bi} P_{bj} \right) s + \prod_{i=1}^3 P_{bi} = 0$$

$$\Rightarrow s^3 - \frac{29 K_{vb} s^2}{I_{co}} - \left( \frac{29}{(2) \times 60} + \frac{29 K_{pb}}{I_{co}} \right) s - \frac{29 K_{ib}}{I_{co}} = 0$$

Comparing above eqn

$$-98.1 = \frac{2(9.81) K_{vb}}{0.857}$$

$$\Rightarrow K_{vb} \approx 4$$

$$2.5 \times 44 + 51.6 \times 44 + 51.6 \times 2.5 = -\frac{2(9.81)}{0.857} - \frac{2(9.81) K_{pb}}{0.857}$$

$$(2) K_{pb} = (-252.81)$$

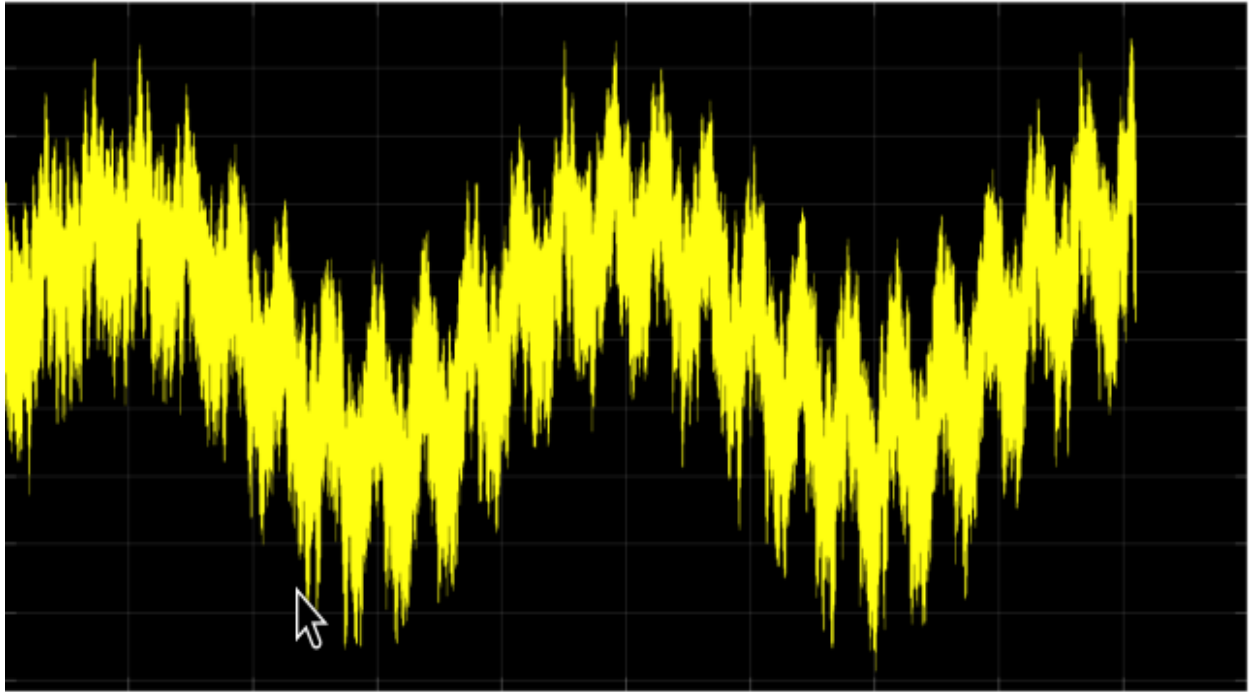
$$-2.5 \times 44 \times 51.6 = -\frac{2 \times 9.81 \times K_{ib}}{0.857}$$

$$K_{ib} = -248.09$$

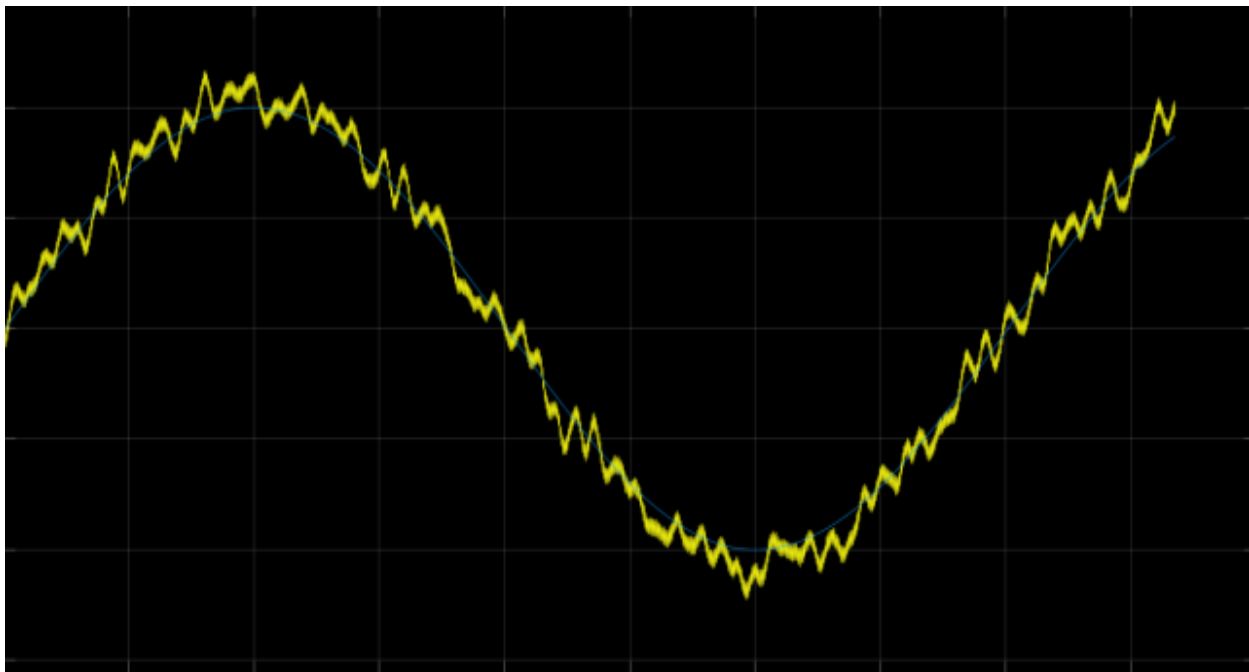
$$K_{ff} = \frac{I_{co}}{X_{bo}} = \frac{0.857}{6 \times 10^{-2}}$$

$$K_{ra} = 142.936$$

**(d)Graphs**



**Current( $I_c$ )**



**Displacement( $x_b$ )**

#### 4. Assignment

##### (a) Important Formula's

$$PO = \exp\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi\right)100 \quad , \quad T_s = \frac{4}{\zeta\omega_n}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad , \quad P_{c1,2} = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}$$

$$T_r = \frac{\pi - \beta}{\omega_d} \quad , \quad \beta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

##### (b) Controller gains : Value's Table

Controller Gain	Values
$K_{pc}$	182.876
$K_{ic}$	24801.56
$K_{ffb}$	142.9336
$K_{pb}$	-252.61
$K_{ib}$	-248.09
$K_{vrb}$	-4.24828

##### (c) Conclusion:

The laboratory experiment results demonstrate that by determining and utilizing the necessary constants for the control system, we can achieve precise positioning of the ball ( $x_b$ ) within the system, even when varying the desired position. This remarkable capability is a testament to the effectiveness of control systems in minimizing errors and ensuring accurate positioning.