1. Objective

To use state-feedback using LQR (Linear-Quadratic Regulator) optimization to control the yaw position and implement it on the set-up of half-quadrotor.

(a) Aim

Quanser Aero Experiment in half-quadrotor configuration

(b) Software

Half-quadrotor, MATLAB

2. Theory

(a) Description

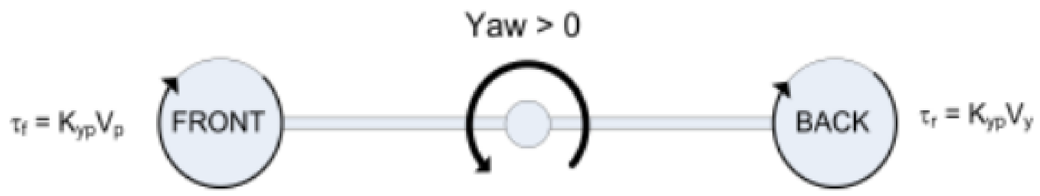
- In this setup, both the front and back rotors are horizontal to the ground and only motions about the yaw axis are enabled (i.e., the pitch axis is locked). By changing the direction and speed of the rotors, users can change the yaw axis angle.

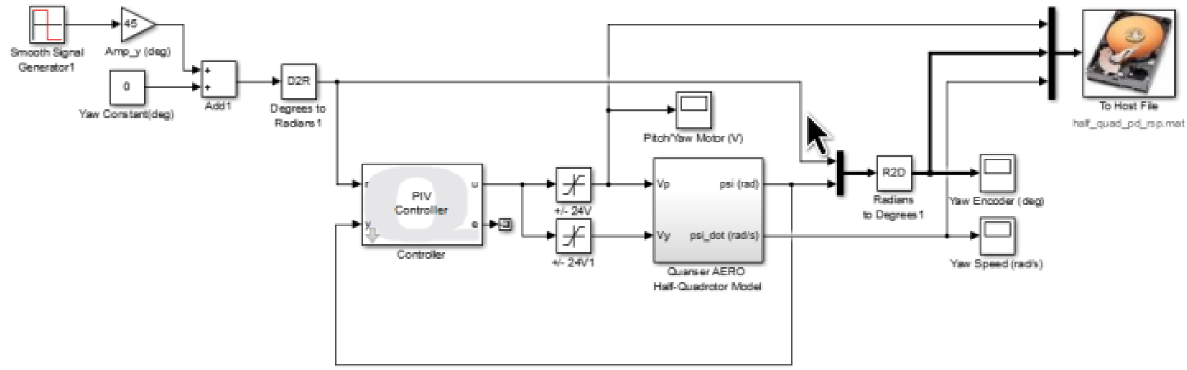
- A quadrotor helicopter (quadcopter) has four evenly spaced rotors at the corners of a square body. The helicopters help provide damping to stop the motion.
- There are 6 degrees of freedom, 3 translational and 3 rotational. The four rotor speeds are the inputs and are independent of each other as they are perpendicular.
- The torque from the rotors causes the system to rotate about the yaw axis. Equation of yaw motion :

$$\tau = J_y \frac{d^2(\psi)}{dt^2} + D_y \frac{d(\psi)}{dt} = -K_{yp} V_p - K_{py} V_y$$

where Jy is the moment of inertia about the yaw axis, Dy, is the viscous damping coefficient about the yaw axis, Kyp is the cross-torque thrust gain, Vp is the voltage applied to the front (pitch) motor, and Vy is the voltage applied to the back (yaw) motor.

- Free Body Diagram of Quadcopter





Main and Final Equations used are:

$$\frac{Y(s)}{R(s)} = \frac{Kk_p/\tau}{s^2 + (1 + Kk_d)/\tau s + kk_p/\tau}$$

Where

$$K = \frac{2K_{yp}}{D_y} \qquad \tau = \frac{J_y}{D_y}$$

$$k_p = \tau\frac{\omega_n^2}{K} \qquad k_d = \frac{2\tau\omega_n\zeta - 1}{K}$$

$$\zeta = -\, log(\frac{PO}{100})\frac{1}{\sqrt{log(\frac{PO}{100})^2 + \pi^2}} \qquad \omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}}$$

3. Calculation:

$$J_y \ddot{\psi} + D_y \dot{\psi} = K_{yp} V_p + K_{yy} V_y$$

$$\boxed{J_y \ddot{\psi} + D_y \dot{\psi} = 2 K_{yp} u}$$

taking $V_p = V_y = u$

State space

$u_1 = \psi$ $\quad u_2 = \dot{\psi} = \dot{u}_1 \quad \dot{u}_1 = u$

$$\dot{u}_2 = \frac{2 K_{yp} u}{J_y} - \frac{D_y}{J_y} u_2$$

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -D_y/J_y \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2 K_{yp}}{J_y} \end{bmatrix} u$$

$$C = [1 \quad 0]$$

$$D = [0]$$

$$\tau_p = K_{pp} V_p + K_{py} V_y$$

$$\tau_y = K_{yp} V_p + K_{yy} V_y$$

$$J_p \ddot{\theta} + D_p \dot{\theta} + K_{sp} \theta = \tau_p$$

$$J_y \ddot{\psi} + D_y \dot{\psi} = \tau_y$$

$$= K_{yp} V_p + K_{yy} V_y$$

$$\Big\downarrow LT$$

$$J_y [\psi(s) s^2 - \psi(0) s - \dot{\psi}(0)] + D_y [\psi(s) s - \psi(0)] = K_{yp} V_p(s) + K_{yy} V_y(s)$$

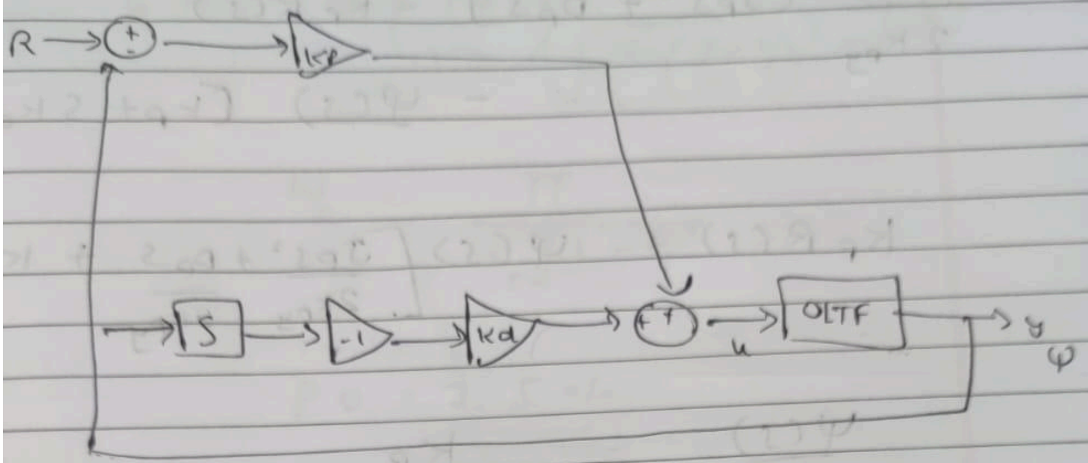
as $\psi(0^-) = 0$

$\dot{\psi}(0^-) = 0$

$$\frac{\psi(s)}{V_p(s)} = \frac{K_{py}}{J_y s^2 + D_y s},$$

$$\frac{\psi(s)}{V_y(s)} = \frac{K_{py}}{J_y s^2 + D_y s}$$

SISO: $\boxed{\dfrac{\psi(s)}{U(s)} = \dfrac{2 K_{py}}{J_p s^2 + D_p s}}$



$$OLTF = \frac{2K_{py}}{J_p s^2 + D_p s}$$

$\boxed{\frac{\psi}{R} = CLTF}$ Required.

$$u = K_{py}(r(t) - y(t)) - K_d \dot{y}(t)$$

$$\Big\downarrow LT$$

$$U(s) = K_{py}(R(s) - Y(s)) - K_d\, s Y(s)$$

$$U(s) = K_{py} R(s) - Y(s)[K_{py} + sK_d]$$

From OLTF

$$U(s) = \frac{\psi(s)\,[J_p s^2 + D_p s]}{\cancel{2K_{py}}\ 2K_{py}}$$

$$\frac{\psi(s)}{2K_{py}}\left[J_p s^2 + D_p s\right] = K_p R(s) - \psi(s)\left[K_p + s K_d\right]$$

$$K_p R(s) = \psi(s)\left[\frac{J_p s^2}{2K_{py}} + \frac{D_p s}{2K_{py}} + K_p + s K_d\right]$$

$$\frac{\psi(s)}{R(s)} = \frac{K_p}{\frac{J_p s^2}{2K_{py}} + \left[\frac{D_p}{2K_{py}} + K_d\right] s + K_p}$$

$$\frac{\psi(s)}{R(s)} = \frac{\frac{2K_{yp} \cdot K_p}{J_p}}{s^2 + \left[\frac{D_p}{J_p} + \frac{2K_{yp}K_d}{J_p}\right] s + \frac{2K_{yp}K_p}{J_p}}$$

$$\text{CLTF} = \frac{KK_p/\tau}{s^2 + \left[\frac{1 + KK_d}{\tau}\right] s + KK_p/\tau}$$

$$K = \frac{2K_{yp}}{D_y}$$

$$\tau = \frac{J_y}{D_y}$$

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$$\xi = -\ln\left(\frac{PO}{100}\right)\frac{1}{\sqrt{\left(\ln\left(\frac{PO}{100}\right)\right)^2+\pi^2}}$$

$$\omega_n = \frac{\pi}{t_p\sqrt{1-\xi^2}}$$

$PO = 7.5\%$

$$\xi = -\ln(7.5)\frac{1}{\sqrt{(\ln 7.5)^2+\pi^2}}$$

$$\boxed{\xi = 0.63}$$

$$\omega_n = \frac{\pi}{1.25\sqrt{1-(0.63)^2}}$$

$$\boxed{\omega_n = 3.256 \text{ rad/sec}}$$

$$2\omega_n\xi = \frac{1}{\tau_s} + \frac{2k_y p}{p_y}\cdot\frac{k_d}{\tau_s}$$

$$\boxed{K_d = \frac{2\tau\xi\omega_n - 1}{K}} \quad \ldots (i)$$

$$\frac{KK_p}{\tau} = \omega_n^2 \Rightarrow \boxed{K_p = \frac{\omega_n^2\tau}{K}} \quad \ldots (ii)$$

got values By comparing obtained eq with standard eq.

as

standard

$$CLTF = \frac{KK_P / \tau}{s^2 + \underbrace{\left[\frac{1 + KK_d}{\tau}\right]}_{s} + \frac{KK_P}{\tau}} \qquad CLTF = \frac{\omega_n^2}{s^2 + \underbrace{2\xi\omega_n}\, s + \omega_n^2}$$

compare.

$$\Rightarrow K_d = \frac{2\tau\xi\omega_n - 1}{K}$$

$$K_p = \frac{\tau\omega_n^2}{K}$$

$J_p = 0.022$

$\omega_0 = 8.6 \times 10^{-1} = 0.86$

$\omega_1 = (1 - 0.63)\,\omega_0 = 0.3182$

$t_0 = 1.995 \quad t_1 = 3.369$

$\tau = t_1 + t_0 = 1.375$

$V_p = 20V$ $\quad D_y = \frac{J_y}{\tau} = \frac{0.022}{1.375}$

$D_y = 0.016$

$\Delta t = 5 - 1 = 4$ $\quad \cancel{\Delta\omega_y = 2.44}$

$\Delta\omega_y = -2.44$

$$K_{yp} = \frac{0.022 \times \frac{-2.44}{4} + 0.016 - 2.44}{20}$$

$$= \frac{-(0.0132 + 0.03904)}{20}$$

$$\boxed{K_{yp} = -0.0026}$$

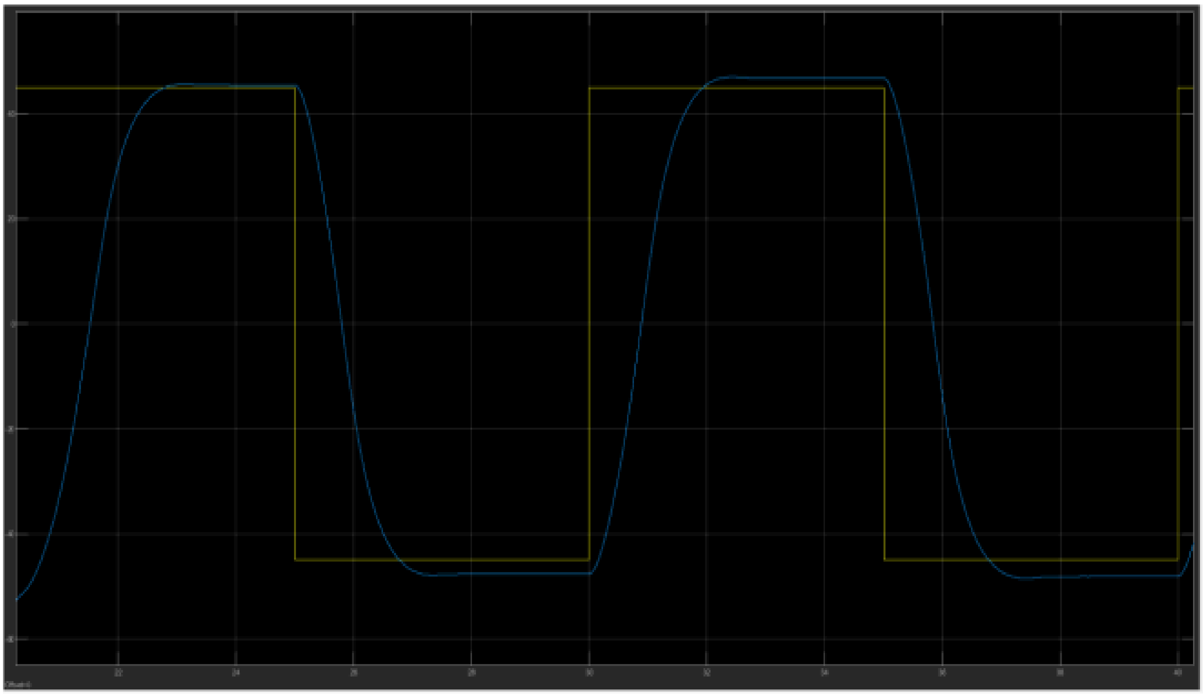
$$K_p = \frac{\omega_n^2 J_y}{2K_{py}} = \frac{(3.256)^2 \cdot (0.022)}{2 \times (0.0026)}$$

$$K_d = \frac{2\xi\omega_n J_y - D_y}{2K_{py}}$$

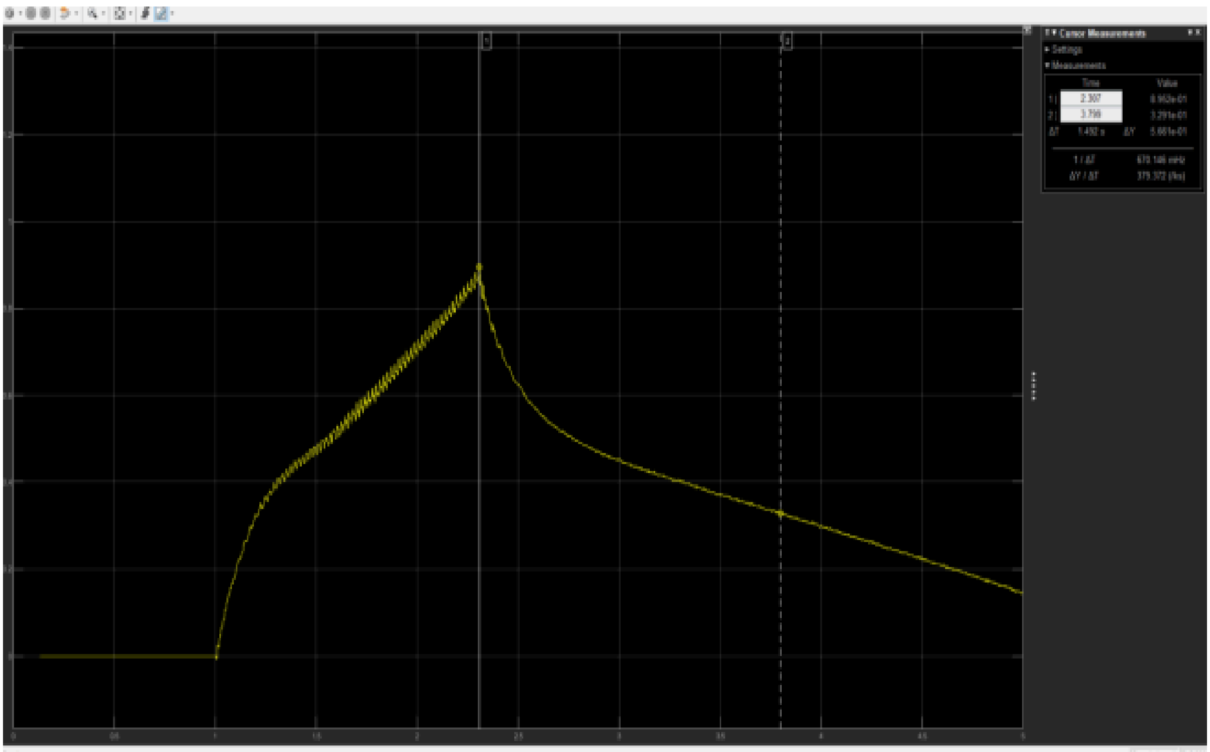
$$\boxed{K_d = 14.34 \text{ rad/s}}$$

$$\boxed{K_p = 44.85 \text{ rad/s}}$$

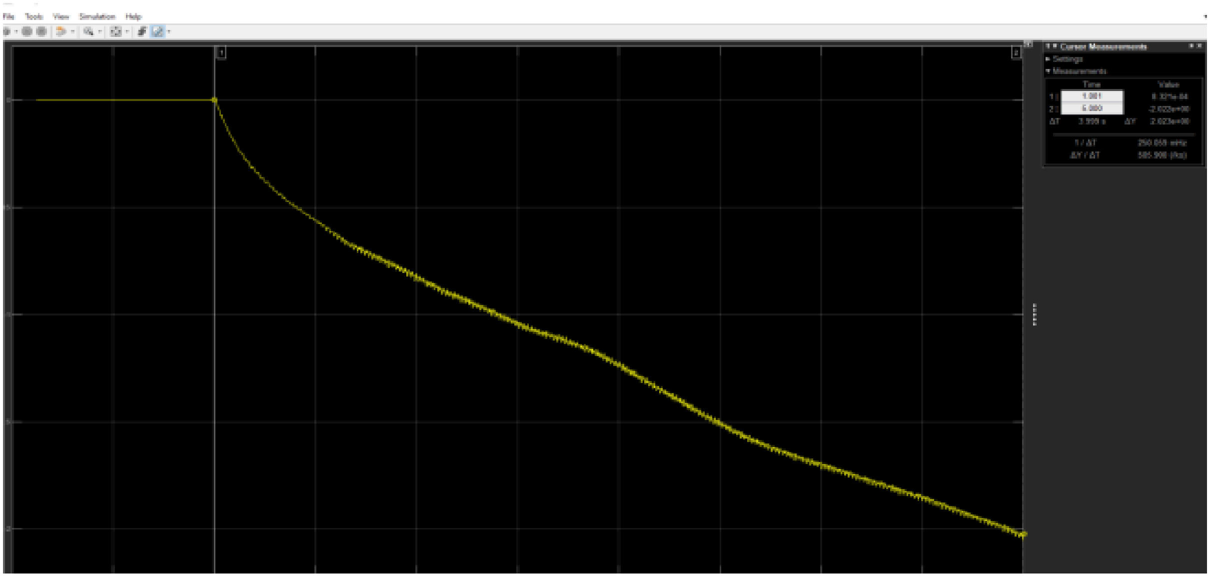
4. Plot:



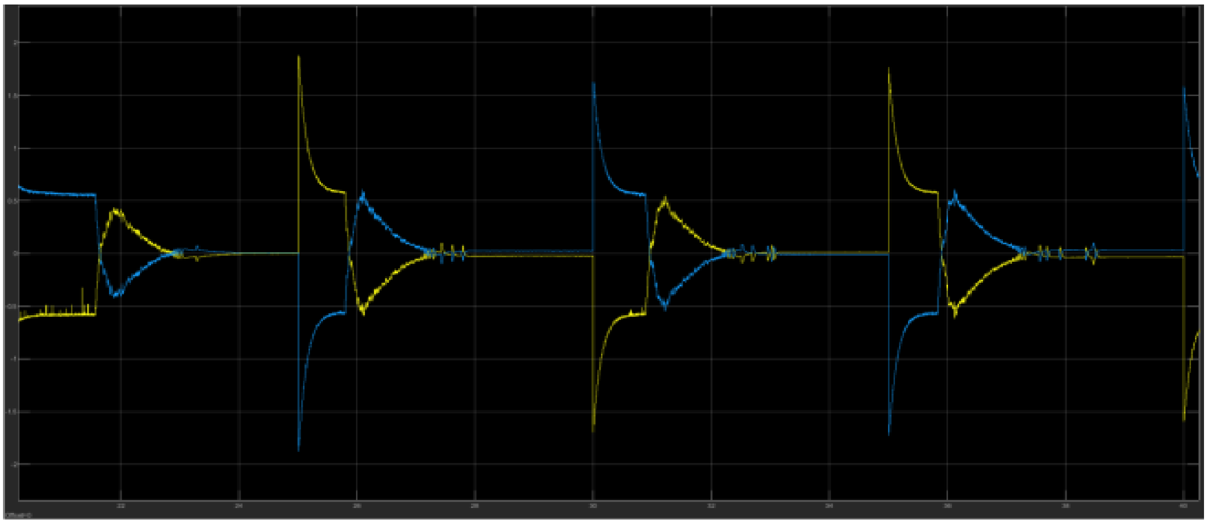
Yaw Position



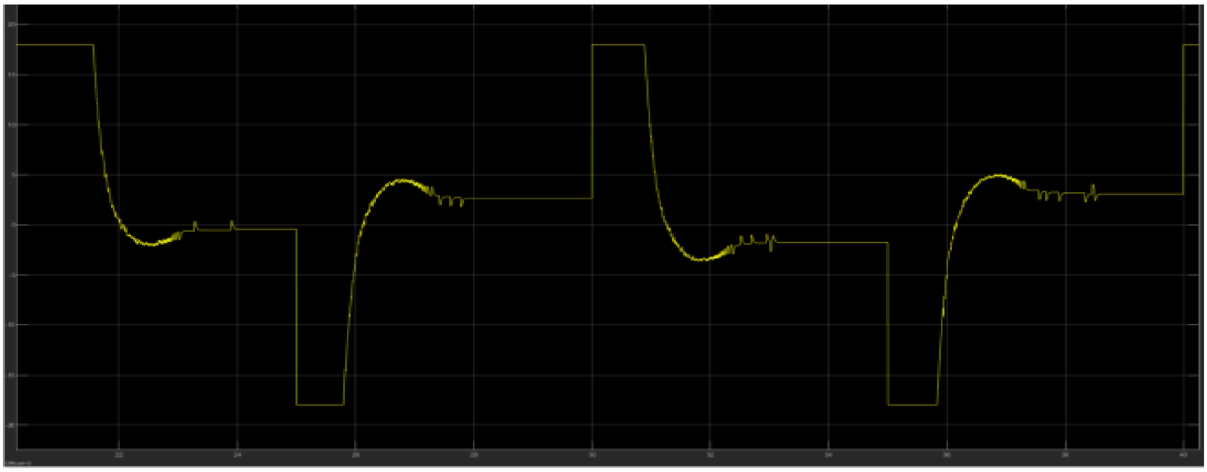
Yaw Speed



Kyp



Current



Pitch Voltage