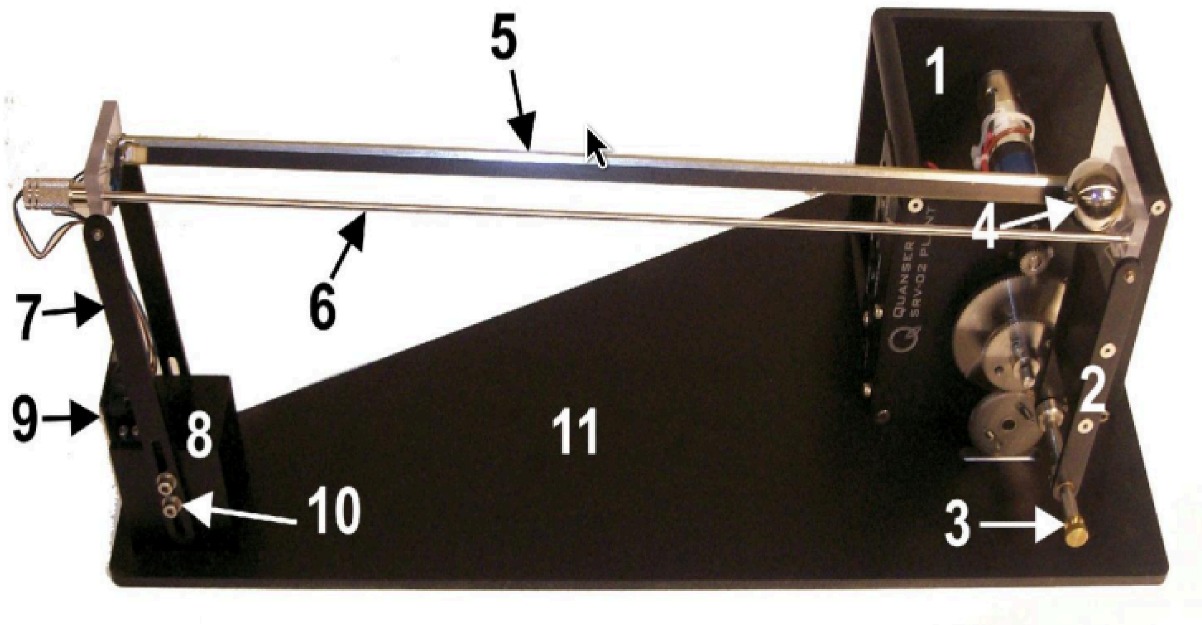
1. Objective

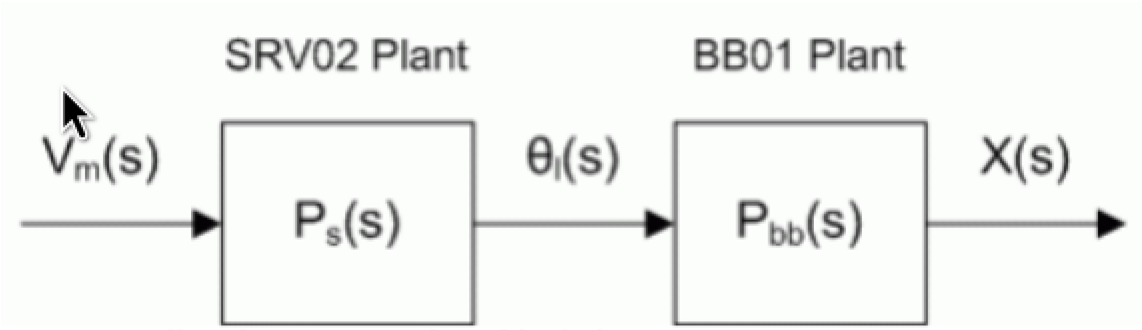
(a) Aim-The objective of the Ball and Beam experiment is to stabilise the ball to a desired position along the beam using the remote sensor unit. Using the proportional-derivative (PD) family, a cascade control system will be designed to meet a set of specifications.

(b) Software-MATLAB, Quanser SRV02 unit, Quanser Ball and Beam module, Q8-USB, UPM-2405 amplifier and remote sensor unit.

(c) Theory



2. Assignment

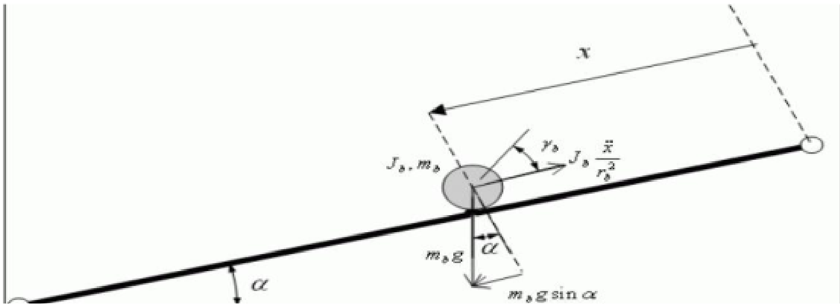


(a) Modeling of Ball and Beam system-The BB01 transfer function describes the displacement of the ball with respect to the load angle of the servo. The time-based motion equations are developed and, from these equations of motion, its transfer function is obtained. The SRV02 voltage-to-load angle plant transfer function was found to be

$$P(s) = \frac{K}{s(1+\tau s)}$$

(b) Nonlinear Equation of motion-The equation describing the motions of the ball, x, relative to the angle of the beam, α, is:

$$m_b\left(\frac{d^2x(t)}{dt^2}\right) = F_{x,t} - F_{x,r}$$



$$\frac{d^2x(t)}{dt^2} = f(\alpha(t))$$

(c) Adding SRV02 Dynamics-In this section, the equation of motion representing the position of the ball relative to the angle of the SRV02 load gear will be found. The obtained equation is nonlinear (includes a trigonometric term) and it will have to be linearized in order for the model tobe used for control design.

$$J = \frac{2mr^2}{5}$$

(d) Obtaining Transfer Function-The transfer function describing the servo voltage to ball position displacement will be calculated using the following steps as
Step-I: Finding the transfer function Pbb(s) of the BB01. Assuming all initial conditions are zero.
Step-II: Finding the complete SRV02+BB01 process transfer function P(s).
We will compare the equation with the standard equation of transfer function for second order systems.

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

3. Control Design for Desired Control Response

(a) Time-Domain Specifications-The time-domain specifications for controlling the position of the SRV02 load shaft are: The steady-state error (t_s) = 0, Peak time (t_p) = 0.15 sec and Percentage Overshoot (PO) = 5.0 %

$$t_s = -\frac{\ln(c_{ts}\sqrt{(1-\zeta^2)})}{\zeta\omega_n}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$PO\% = 100e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$$

b) Settling Time-The response of a second-order system, y(t), when subjected to a unit step reference, r(t), is shown in Figure 7. This response has a 5% settling time of 0.30 seconds. Thus the response settles within 5% of its steady-state value, which is between 0.95 and 1.05, in 0.30 seconds. Settling time is defined as $(t_s = t_1 - t_0)$

For BB1

$$\theta_e(s) = 2K_c X_d(s) - 2K_c X(s) - sK_c X(s)$$

$$\frac{2K_c X_d(s)}{X(s)} - (2K_c + sK_c) = \frac{\theta_e(s)}{X(s)} \rightarrow \text{given}$$

$$\frac{2K_c X_d(s)}{X(s)} - (2K_c + sK_c) = \frac{s^2}{K_{bb}}$$

$$\frac{X_d(s)}{X(s)} = \frac{1}{2K_c}\left(\frac{s^2}{K_{bb}} + 2K_c + sK_c\right)$$

$\zeta = 0.592$ $\quad \omega_n = 1.65$

$$TF = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\Rightarrow$ Comparing $\quad Z = \frac{\omega_n}{2\zeta} = \frac{1.65}{2 \times 0.59} = 1.39$

$$K_c = \frac{2\zeta\omega_n}{K_{bb}} = \frac{2 \times 0.597 \times 1.65}{0.418} = 4.67$$

Eqn ① $\quad V_m(t) = K_p\,(\theta_d(t) - \theta_e(t)] - K_v\,(\dot{\theta}_e(t))$

$\downarrow$ F(T)

$$V_m(s) = K_p\,(\theta_d(s) - \theta_e(s)) - K_v\,(\theta_e(s)s)$$

$$\frac{V_m(s)}{\theta_e(s)} = K_p\left[\frac{\theta_d(s)}{\theta_e(s)} - 1\right] - K_v(s)$$

$$\frac{(\tau s+1)s}{KK_p} + \frac{K_v(s)}{K_p} + 1 = \frac{\theta_d(s)}{\theta_l(s)}$$

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{KK_p}{s^2\tau + (1+KK_v)s + KK_p}$$

$$T/F = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$$

$$\omega_n^2 = \frac{K_p K}{\tau}, \quad K_p = \frac{28.91 \times 28.91 \times 0.0285}{1.76}$$

$$\boxed{K_p = 13.53}$$

$$2\zeta\omega_n = \frac{KK_v + 1}{\tau} \Rightarrow K_v = \frac{1}{K}(2\tau\zeta\omega_n - 1)$$

$$\boxed{K_v = 0.77}$$

(c) Steady-State Error-The steady-state error of the ball position is evaluated using a proportional compensator.

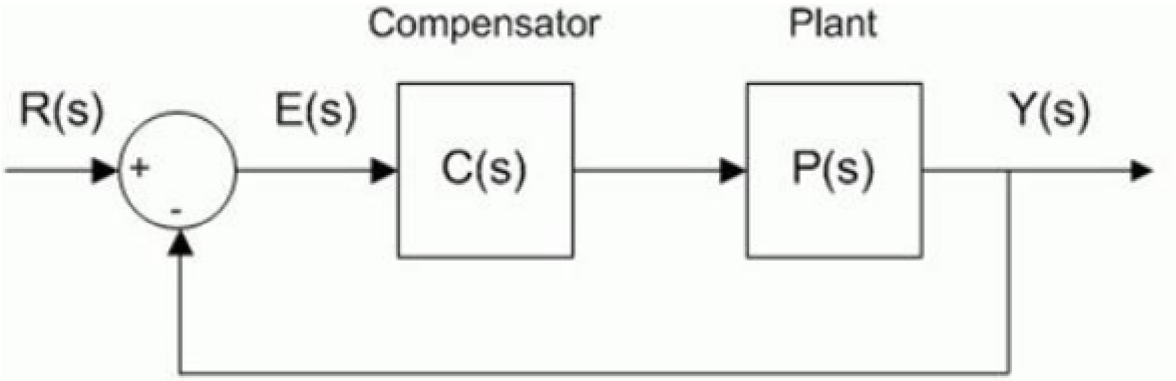
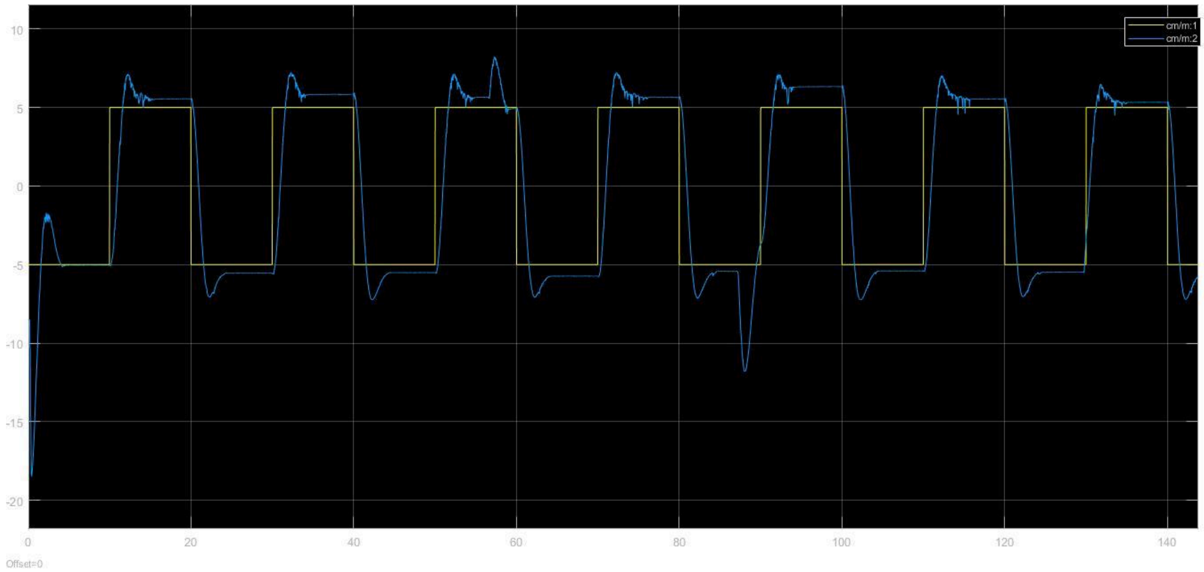
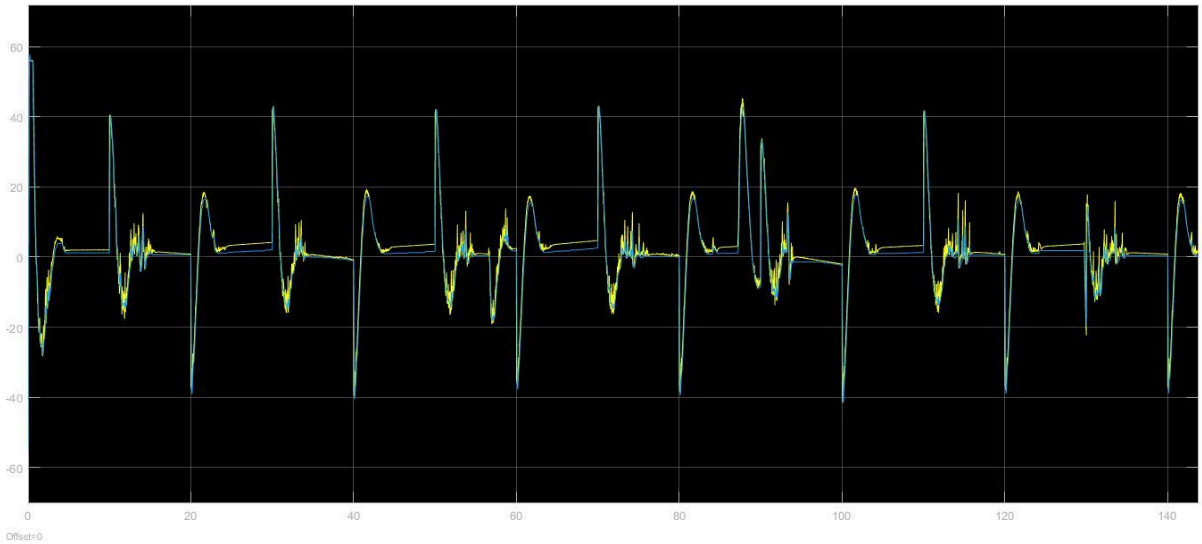


Figure 8: Unity feedback system.

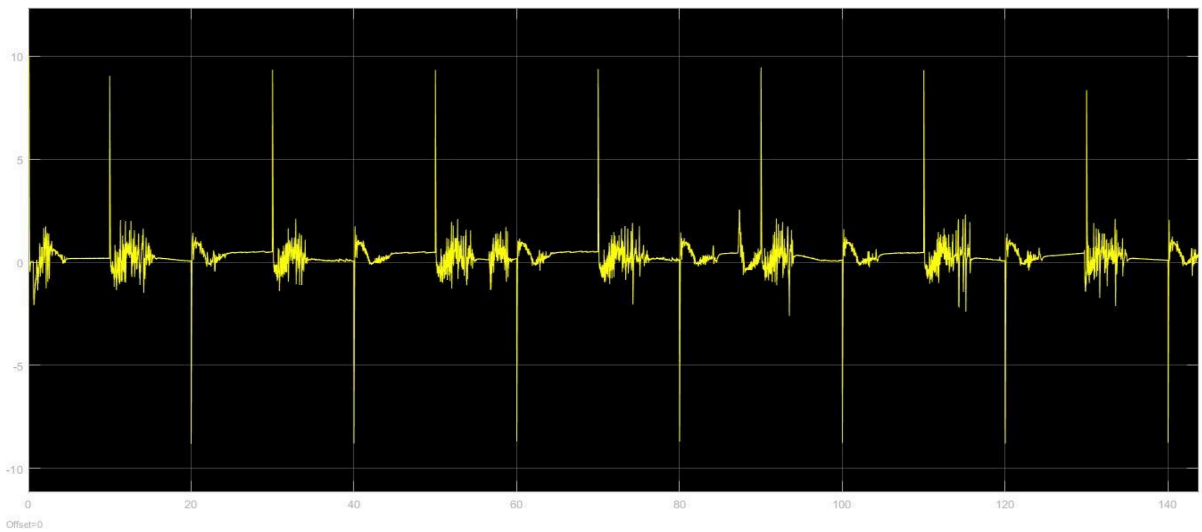
(d) Graphs-



Distance $X_l(s)$



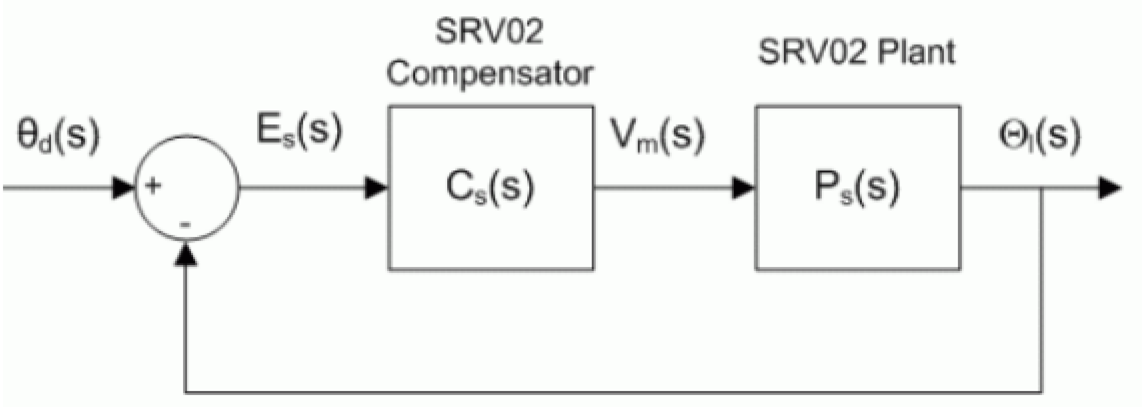
Theta $\theta(s)$



Voltage V(s)

4. Ball and Beam Cascade Control Design

(a) Inner Loop Design: SRV02 PV Position Controller-The proportional-velocity (PV) controller gains are computed for the SRV02 when it is in the high-gear configuration and based on the specifications.



Fundamental Equation for calculation

$$V_m(t) = K_p(\theta_d(t) - \theta_l(t)) - K_v\left(\frac{d^2\theta_l(t)}{dt^2}\right) \qquad \textbf{(i)}$$

$$\theta_l(s) = ZK_cX_d(s) - ZK_cX(s) - sK_cX(s) \qquad \textbf{(ii)}$$

The nominal model parameters, K and τ, when the SRV02 is in high-gear configurations are K = 1.76 rad/sec V and $\tau = 0.0285$ sec.

Solving the equation **(i)** and comparing it with the standard equation of transfer function for 2^{nd} degree differential equations as mentioned above.

$$V_m(t) = K_p[\theta_d(t) - \theta_L(t)] - K_v(\dot{\theta}_L(t))$$

$$\Big\downarrow \ \underline{\underline{LT}}$$

$$V_m(s) = K_p[\theta_d(s) - \theta_L(s)] - K_v s . \theta_L(s)$$

$$\frac{V_m(s)}{\theta_L(s)} = K_p\left[\frac{\theta_d(s)}{\theta_L(s)} - 1\right] - K_v s$$

$$\frac{(\tau s+1)s}{K} = K_p(x-1) - K_v s$$

$$\frac{(\tau s+1)s}{K} + K_v s = K_p(x-1)$$

$$\frac{(\tau s+1)s + KK_v s}{KK_p} = (x-1)$$

$$x = \frac{(\tau s+1)s + KK_v s + KK_p}{KK_p}$$

$$\frac{\theta_e(s)}{\theta_d(s)} = \frac{KK_p}{\tau s^2 + s + KK_v s + KK_p}$$

$$\boxed{\frac{\theta_e(s)}{\theta_d(s)} = \frac{KK_p}{\tau s^2 + (KK_v+1)s + KK_p}}$$

$$\frac{\theta_e(s)}{\theta_d(s)} = \frac{1.76K_p}{0.0285s^2 + (1.76K_v+1)s + (1.76K_p)}$$

motor

$$TF = \frac{(28.91)^2}{s^2 + 2\cdot(0.689)(28.91)s + (28.91)^2}$$

$$= \frac{\left(\frac{1.76}{0.0285}\right)K_p}{s^2 + \frac{(1.76K_v+1)}{0.0285}s + \left(\frac{1.76}{0.0285}\right)K_p}$$

$$K_p = \left(\frac{0.0285}{1.76}\right)(28.91)^2 = 13.534$$

$$K_v = \frac{(2)(0.689)(28.91)(0.0285) - 1}{1.76}$$

$$\left.\begin{array}{l}\boxed{K_v = 0.0769} \\ \boxed{K_p = 13.534}\end{array}\right\} \text{motor}$$

$$\theta_e(s) = Z K_c X_a(s) - Z K_c X(s) - s K_c X(s)$$

$$1 = Z K_c \cdot \frac{X_a(s)}{\theta_e(s)} - Z K_c \frac{K_{bb}}{s^2} - s K_c \cdot \frac{K_{bb}}{s^2}$$

$$1 = Z K_c \cdot \frac{X_a(s)}{s^2 X(s)} K_{bb} - Z K_c \frac{K_{bb}}{s^2} - \frac{K_c K_{bb} s}{s^2}$$

$$s^2 = \frac{Z K_c K_{bb}}{x} - Z K_c K_{bb} - K_c K_{bb} s$$

$$s^2 + K_c K_{bb} s + Z K_c K_{bb} = \frac{Z K_c K_{bb}}{x}$$

$$x = \frac{Z K_c K_{bb}}{s^2 + K_c K_{bb} s + Z K_c K_{sb}}$$

calcn

$$TF = \frac{(1.66)^2}{s^2 + 2(0.59)(1.66)s + (1.66)^2}$$

$$Z K_c K_{bb} = (1.66)^2$$

$$K_c K_{bb} = 2(0.59)(1.66)$$

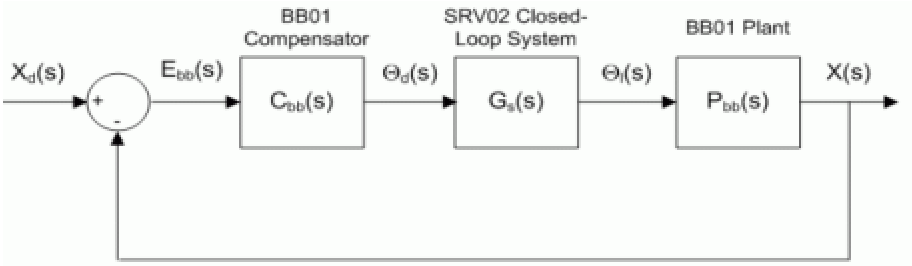
$$Z = \frac{(1.66)}{2(0.59)} = \frac{1.66}{1.18} = 1.406$$

$$K_c \cdot (0.418) = (1.18)(1.66)$$

$$\boxed{K_c = 4.686} \quad \boxed{Z = 1.406}$$

(b) Outer Loop Design-The inner loop that controls the position of the SRV02 load shaft is complete and the servo dynamics are now considered negligible. Thus, it is assumed that the desired load angle equals the actual load angle

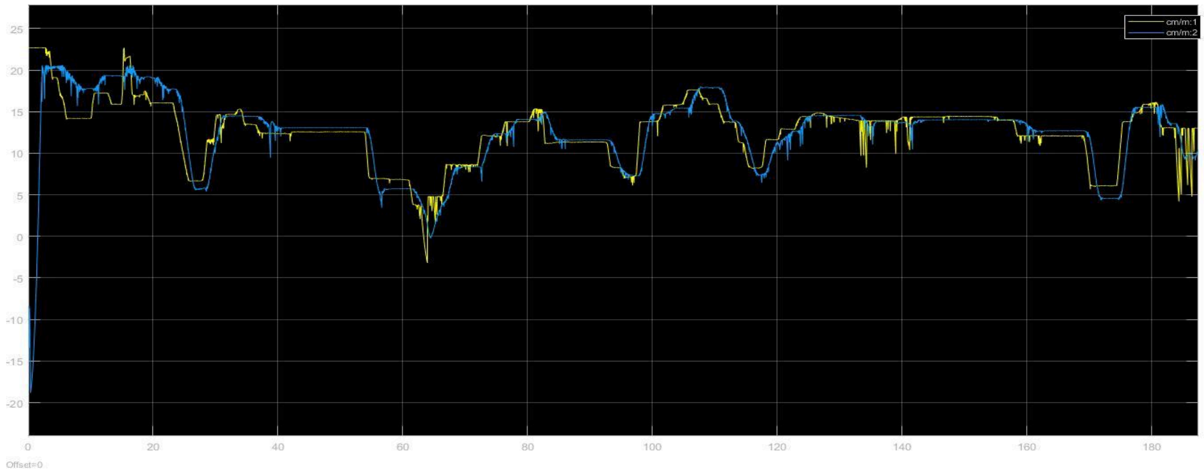
$$\theta_l(t) = \theta_d(t)$$



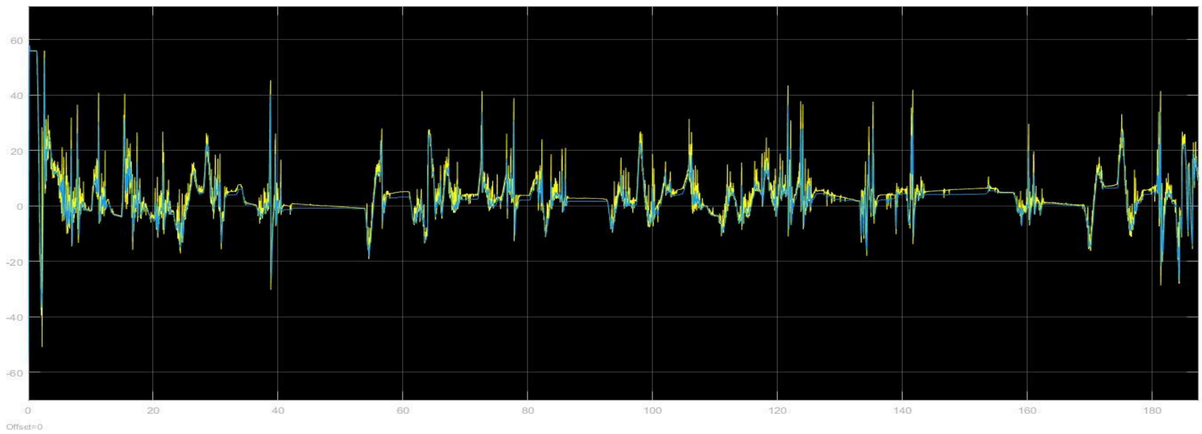
The above solved value for K_c, K_p *and* Z are obtained by solving equation **(ii)** as mentioned above. Also the related equation which is needed to arrive at these result are:

$$K_c = \frac{2\zeta\omega_n}{K_{bb}} \quad , \quad z = \frac{\omega_n^2}{K_{bb}K_c}$$

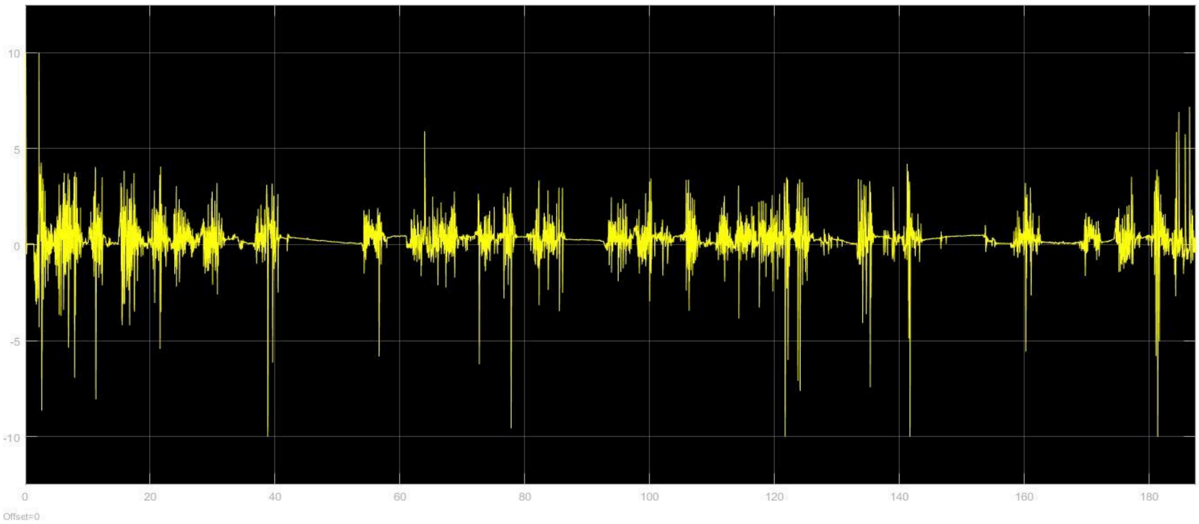
(c) Graphs-



Distance $X_l(s)$



Theta $\theta(s)$



Voltage V(s)