Objective

a) Aim

- To verify the effect of input waveform, loop gain, and system type upon steady-state errors.

b) Apparatus

- MATLAB, Simulink, and the Control System Toolbox.

Theory

a) Description

- Steady-state error is defined as the difference between the input (command) and the output of a system in the limit as time goes to infinity (i.e. when the response has reached steady state). The steady-state error will depend on the type of input (step, ramp, etc.) as well as the system type (0,I, or II).
- Steady-state error can be calculated from the open- or closed-loop transfer function for unity feedback systems.

E(s)=R(s)-Y(s)

H(s)=1 (Unity Feedback)

Steady state error can be calculated by:

$$e_{ss} = \lim_{s->0} sE(s) = \lim_{s->0} \frac{sR(s)}{1+G(s)}$$

Input Signals

a) Step Input = 1/s

$$e_{ss} = \frac{1}{1+ \lim_{s->0} G(s)} = \frac{1}{1+K_p}$$

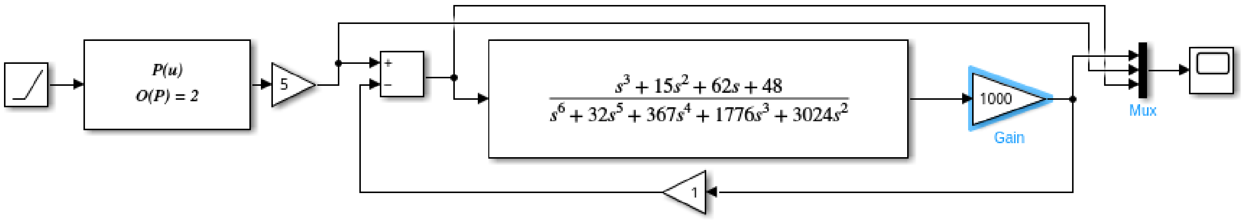
b) Ramp Input = 1/s^2

$$e_{ss} = \frac{1}{1+ \lim_{s->0} sG(s)} = \frac{1}{1+K_p}$$

c) Parabolic Input = 1/s^3

$$e_{ss} = \frac{1}{1+ \lim_{s->0} s^2G(s)} = \frac{1}{1+K_p}$$

System Design



Calculation

Calculation

(i) $\mathcal{E}(s) = R_s \cdot \frac{1}{1+G} = \frac{1}{s} \cdot \frac{1}{1+\frac{K}{s+a}}$

$$E(s) = \lim_{s \to 0} s \cdot \mathcal{E}(s) = \frac{s+a}{s+a}$$

$$E(s) = \frac{a}{a+k}$$

(ii) $E(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{1+\frac{k}{s+a}} \times \frac{1}{s^{\cancel{2}}}$

$$\lim_{s \to 0} \frac{s+a}{s^2+as+k} = \infty$$

(iii) $E(s) = \lim_{s \to 0} s \cdot \mathcal{E}(s)$

$$\frac{1}{s^3} \cdot \frac{1}{1+\frac{k}{s+a}}$$

$$E(s) = \lim_{s \to 0} s\, \mathcal{E}(s) = \infty$$

(iv) $\mathcal{E}(s) = \frac{1}{s} \cdot \frac{1}{1+\frac{k}{s(s+a)}} = \frac{1}{s} \cdot \frac{s(s+a)}{s(s+a)+k}$

$$E(s) = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{s(s+a)}{s(s+a)+k} = 0$$

(v) $$\mathcal{E}(s) = \frac{1}{s^2} \cdot \frac{1}{1+\frac{k}{s+a}} = \frac{1}{s^2} \cdot \frac{s(s+a)}{s(s+a)\cdot k}$$

$$\mathcal{E}(s) = \lim_{s\to 0} s\,E(s) = \frac{a}{k}$$

(vi) $$\mathcal{E}(s) = \frac{1}{s^3} \cdot \frac{1}{1+\frac{k}{s(s+a)}}$$

$$E(s) = \lim_{s\to 0} s \cdot \frac{1}{s^3} \cdot \frac{k(s+a)}{s(s+a)+k} = \infty$$

(7) $$\mathcal{E}(s) = \frac{1}{s} \cdot \frac{1}{1+\frac{k}{s^2(s+a)}} = \frac{1}{s} \cdot \frac{s^2(s+a)}{s^2(s+a)+k}$$

$$E(s) = \lim_{s\to 0} s\cdot E(s) = \frac{s^2(s+a)}{s^2(s+a)+k} = 0$$

(8) $$\mathcal{E}(s) = \frac{1}{s^2} \cdot \frac{s^2(s+a)}{s^2(s+a)+k}$$

$$E(s) = \lim_{s\to 0} \frac{s(s+a)}{s^2(s+a)+k} = 0$$

(9) $$\mathcal{E}(s) = \frac{1}{s^3} \cdot \frac{s^2(s+a)}{s^2(s+a)+k}$$

$$\mathcal{E}(s) = \lim_{s\to 0} s\,E(s) = a/k$$

(10) $$E(s) = \frac{1}{s} \cdot \frac{s^3(s+a)}{s^2(s+a)+K} \Rightarrow \lim_{s \to 0} sE(s) \to \infty$$

(11) $$E(s) = \frac{1}{s^3} \cdot \frac{s^3(s+a)}{s^3(s+a)+K}$$

$$E(s) = \lim_{s \to 0} s \cdot E(s) = 0$$

(12) $$E(s) = \frac{1}{s^2} \cdot \frac{s^2(s+a)}{s^3(s+a)+K}$$

$$\lim_{s \to 0} s \cdot E(s) = \frac{s^4(s+a)}{s^3(s+a)+Ks} = 0.$$

(6)

$\cancel{R(s) = \frac{5}{s}}$

$$R(s) = \frac{5}{s}$$

$$E(s) = \frac{5}{s} \cdot \frac{1}{1 + \frac{K(s+6)}{(s+4)(s+7)(s+9)(s+12)}}$$

$$E(s) = \lim_{s \to 0} s \cdot E(s)$$

$$= \lim_{s \to 0} s \cdot \frac{5}{s} \cdot \frac{(s+4)(s+7)\cdots(s+12)}{(s+4)\cdots(s+12)+K(s+6)}$$

$$E(s) = \frac{5 \cdot 4 \cdot 7 \cdot 9 \cdot 12}{4 \cdot 7 \cdot 9 \cdot 12 + 6K}$$

$$\boxed{E(s) = \frac{2520}{2520 + K}}$$

$\Rightarrow$ $R(s) = \frac{5}{s^2}$

$$E(s) = \lim_{s \to 0} s \cdot E(s) = \frac{1}{s} \cdot \left(\frac{E(s)}{s}\right)$$

$$= \frac{1}{s} \cdot \left(\frac{2520}{2520 + K}\right)$$

$$\lim_{s \to 0} \frac{1}{s} (\,)$$

$$\Rightarrow \infty.$$

$\Rightarrow$ $R(s) = \frac{5}{s^3}$

$$E(s) = \lim_{s \to 0} sE(s) = \frac{1}{s^2} \left(\frac{2520}{2520 + K}\right)$$

$$\lim_{s \to 0} \frac{1}{s^2} (\,) \Rightarrow \infty.$$

(7) $R(s) = \frac{5}{s}$

$$E(s) = \frac{5}{s} \cdot \frac{1}{1 + \frac{k(s+6)(s+8)}{s(s+4)(s+7)(s+9)(s+12)}}$$

$$= \frac{5}{s} \cdot \frac{s(s+4)(s+7)(s+9)(s+12)}{s(s+4)(s+7)(s+9)(s+12) + k(s+6)(s+8)}$$

$$E(s) = \lim_{s \to 0} s\,E(s)$$

$$= \lim_{s \to 0} \frac{5 \cdot s(s+4) \cdots (s+12)}{s(s+4)(s+9) \cdot (s+12) + k(s+6)(s+8)}$$

$$E(s) = 0.$$

$\Rightarrow R(s) = \frac{5}{s^2}$

$$E(s) = \frac{5}{s^2} \cdot \frac{s(s+4)(s+7)(s+9)(s+12)}{s(s+4)(s+7)(s+9)(s+12) + K(s+6)(s+8)}$$

$$E(s) = \lim_{s \to 0} s \cdot E(s) = \frac{5 \cdot 4 \cdot 7 \cdot 9 \cdot 12 \cdot 12}{K \cdot 60 \cdot 5}$$

$$E(s) = \frac{315}{k}$$

$$\Rightarrow R(s) = \frac{5}{s^3}$$

$$E(s) = \lim_{s \to 0} = \frac{1}{s} \cdot \left[\frac{E(s)}{s^2} \right)$$

$$E(s) : \lim_{s \to 0} \frac{1}{s} \left(\frac{315}{k} \right)$$

$$E(s) \to \infty$$

(8)

$$R(s) = \frac{5}{s}$$

$$E(s) = \frac{5}{s} \cdot \frac{s^2(s+4)(s+7)(s+9)(s+12)}{s^2(s+4)(s+7)(s+9)(s+12) + k(s+1)(s+6)(s+8)}$$

$$E(s) = \lim_{s \to 0} s \cdot E(s) = 0$$

$$R(s) = \frac{5}{s^2}$$

$$E(s) = \frac{5}{\cancel{s^2}} \cdot \frac{\cancel{s^2}(s+4) \cdots (s+12)}{s^2(s+4) \cdots (s+12) + k(s+1)(s+6)(s+8)}$$

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$$E(s) = \lim_{s \to 0} s\,E(s) = \frac{5s \cdot (s+4) \cdot (s+12)}{s^3(s+4) \cdot (s+12) + K(s+1)(s+6)(s+8)}$$

$$E(s) = 0$$

$$R(s) = \frac{5}{s^3}$$

$$E(s) = \frac{5}{s^3} \cdot \frac{s^2(s+4) \cdot (s+12)}{s^2(s+4) \cdot (s+12) + K(s+1)(s+6)(s+8)}$$

$$\cancel{E(s) = \lim_{s \to 0} \frac{5}{s^2} \cdot 0 \cdot s}$$

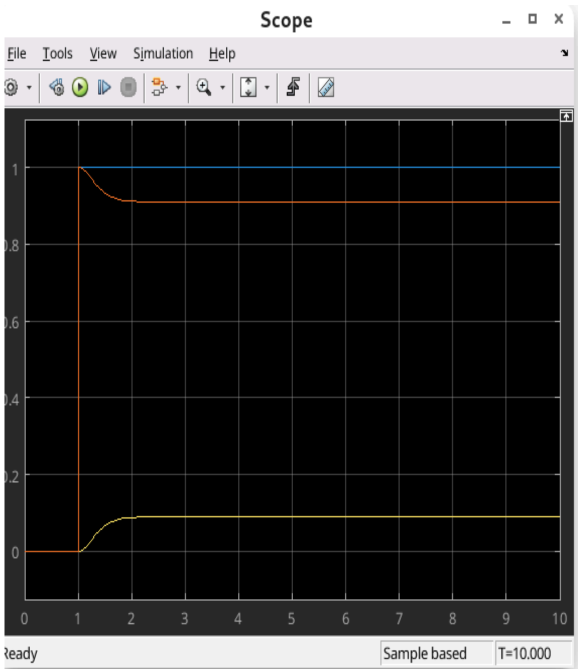
$$E(s) = \lim_{s \to 0} s \cdot E(s) = \frac{5 \cdot 4 \cdot 7 \cdot 9 \cdot 12 \cdot 12}{0 + K \cdot 1 \cdot 6 \cdot 8}$$

$$\boxed{E(s) = \frac{315}{K}}$$

Graphs

a) Question-6

Step Input



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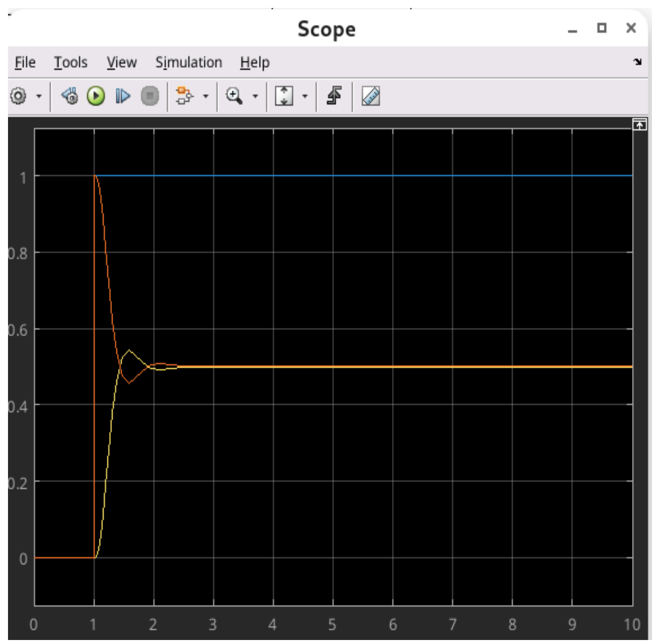

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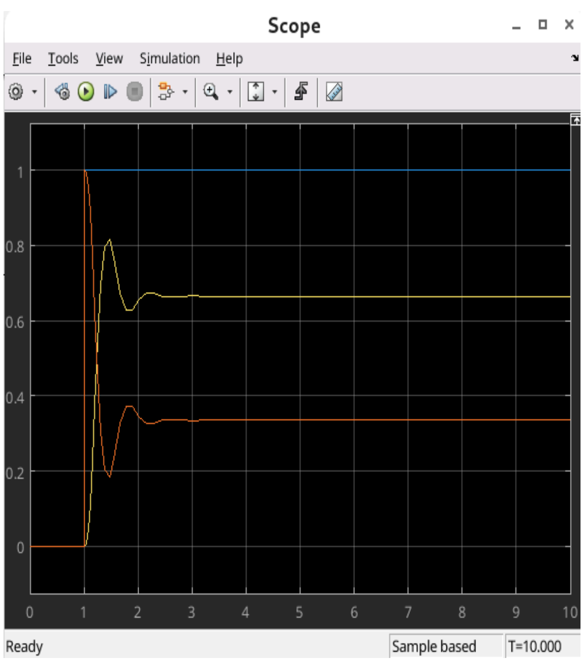

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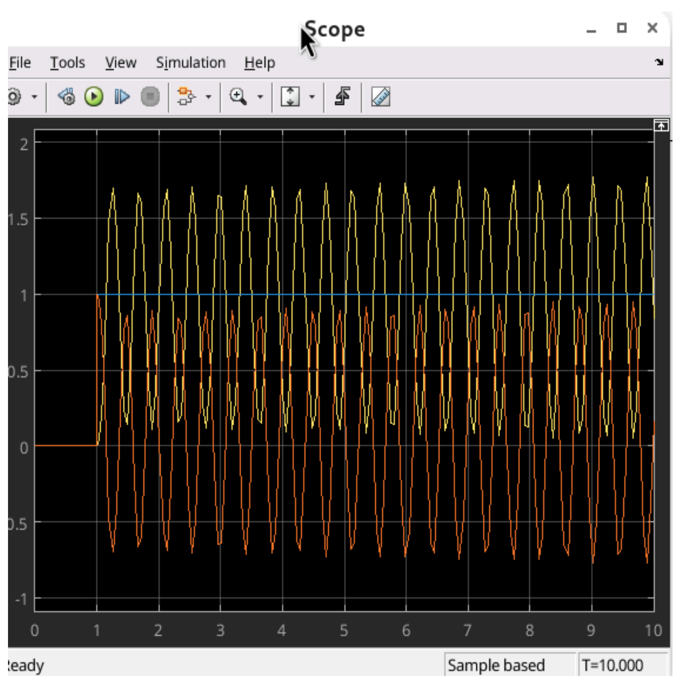
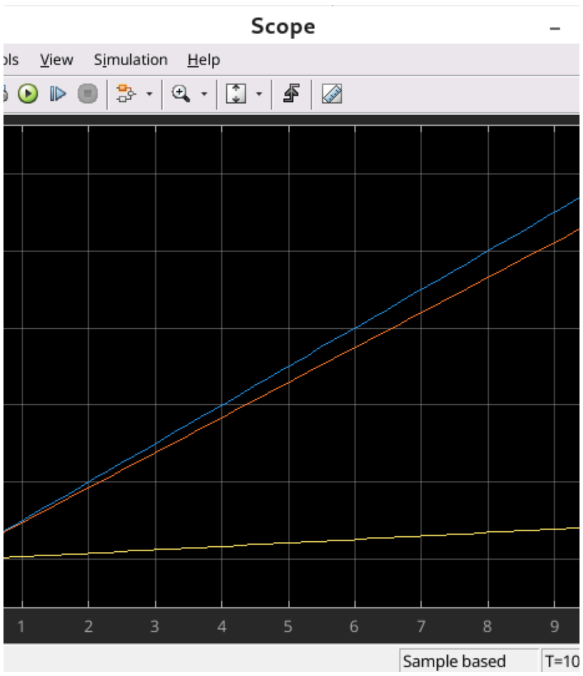


k=5000

a) Question-6 **Ramp Input**



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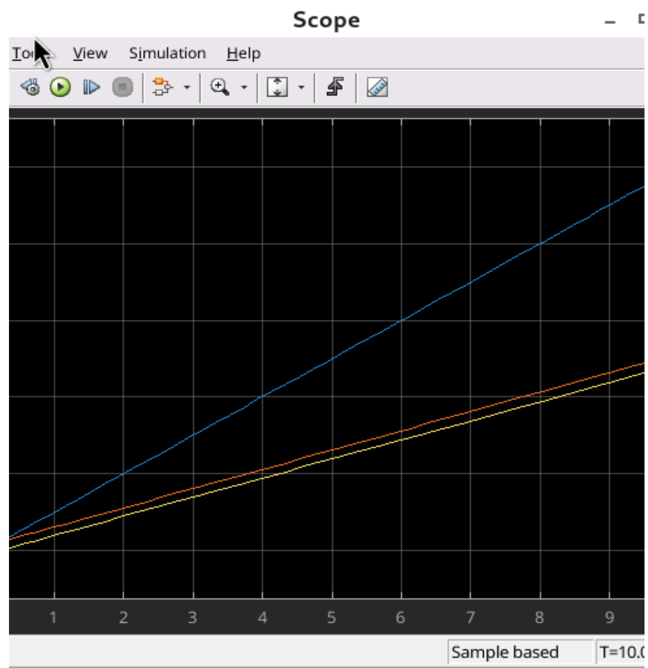

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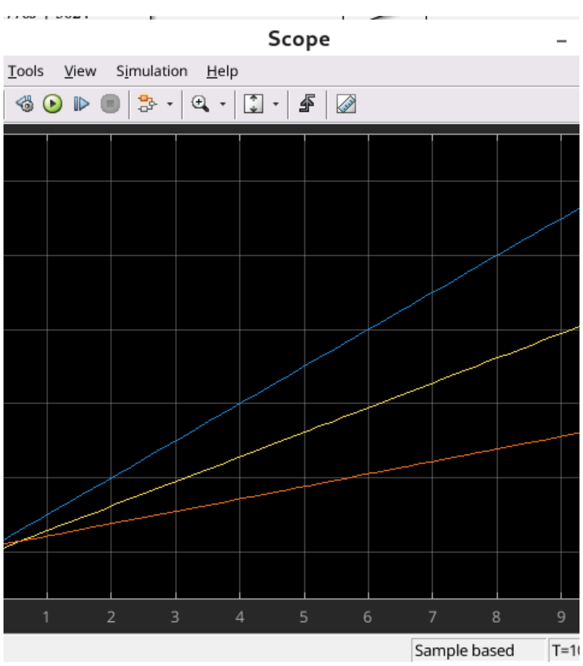

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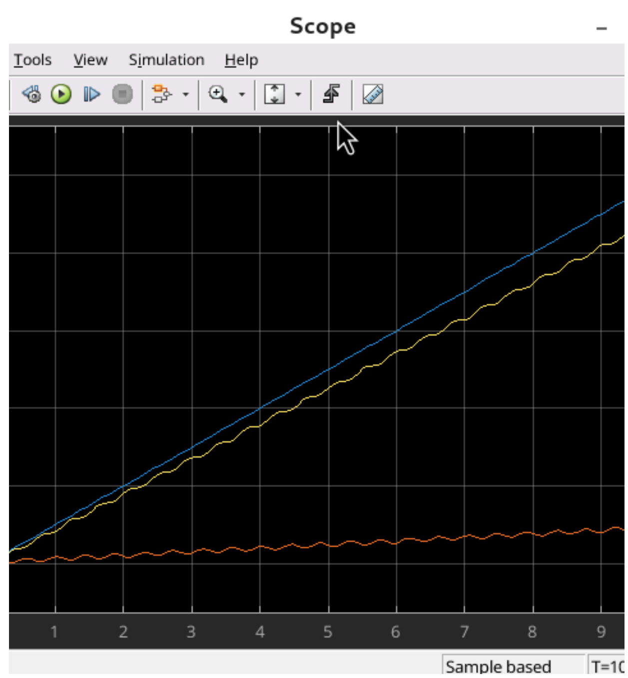
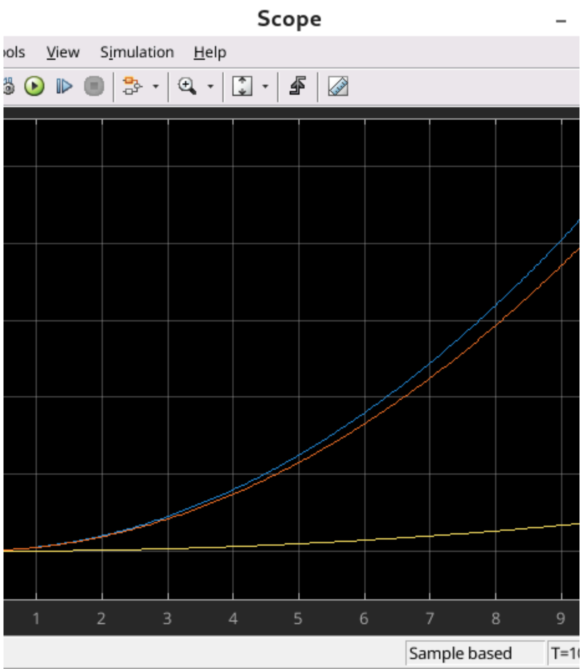


k=5000

a) Question-6 Polynomial Input



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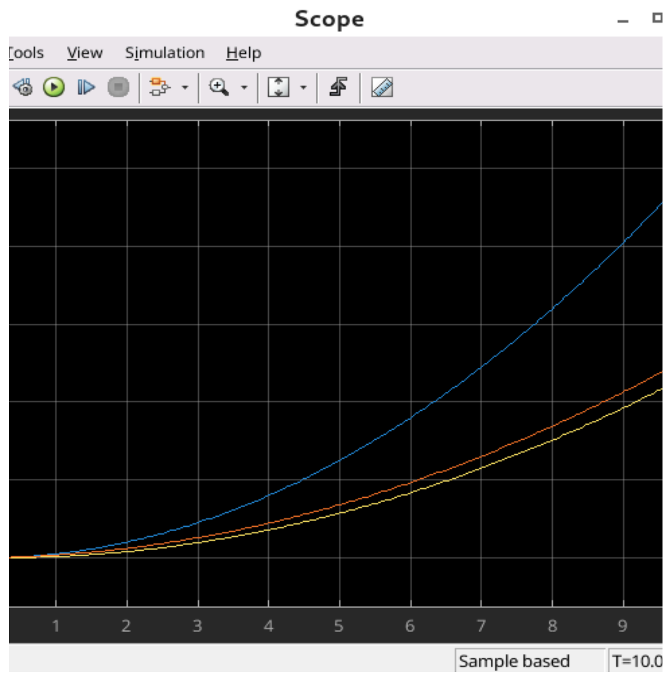

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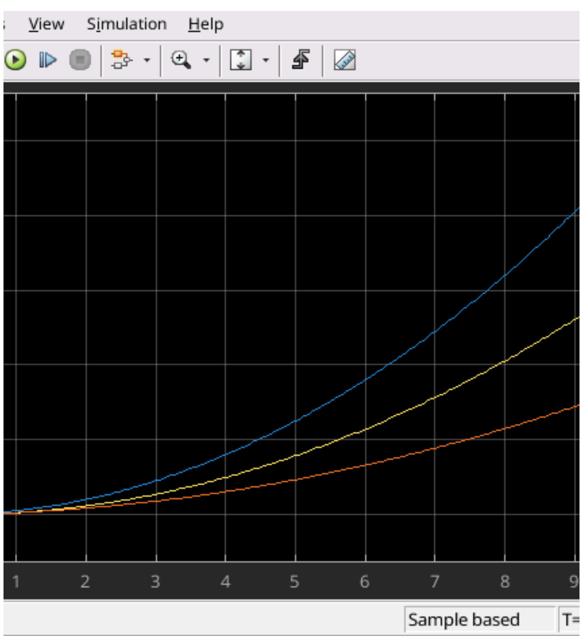

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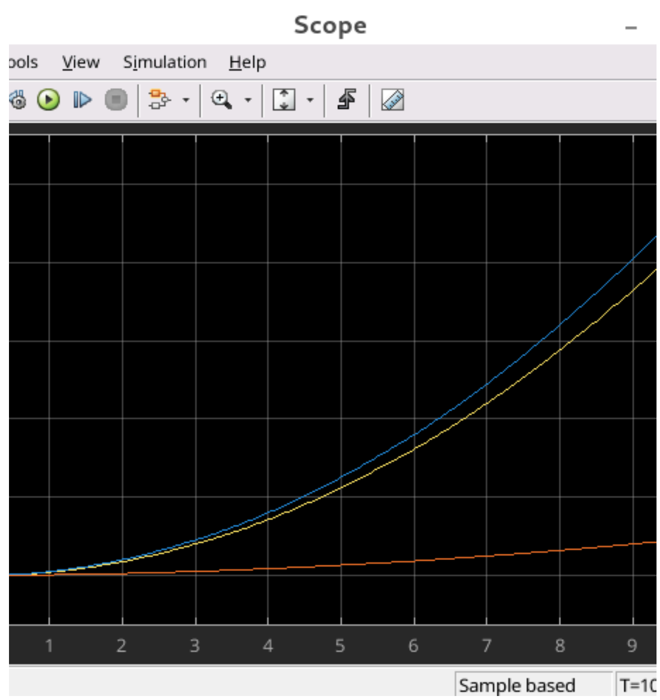
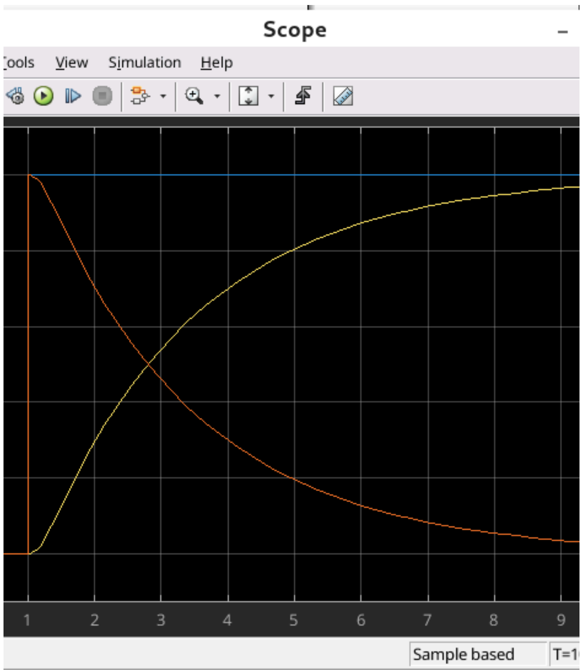


k=5000

b) Question-7 **Step Input**



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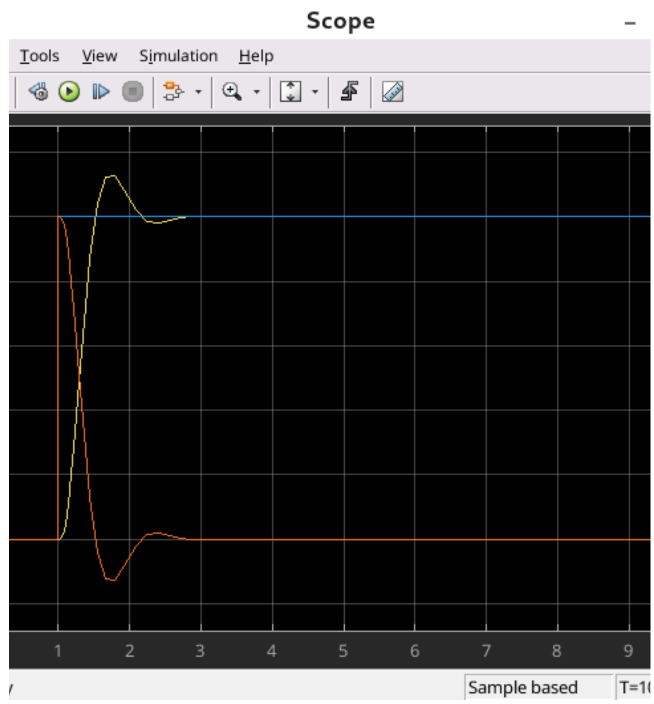

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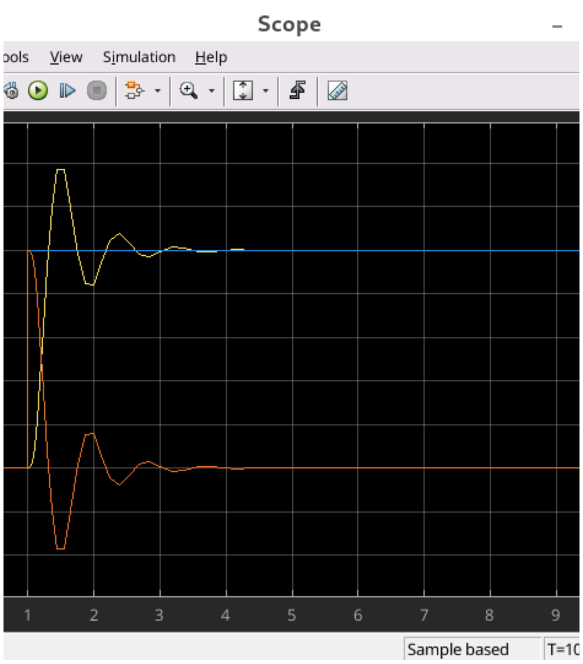

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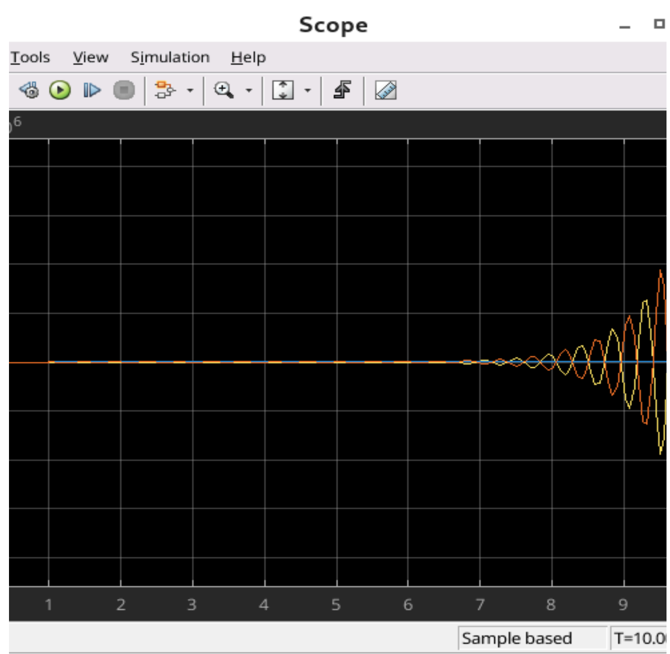
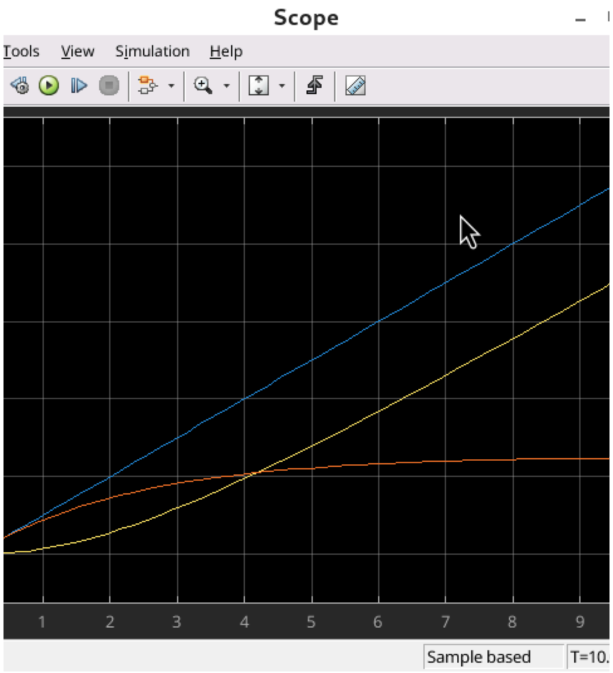


k=5000

b) Question-7 **Ramp Input**



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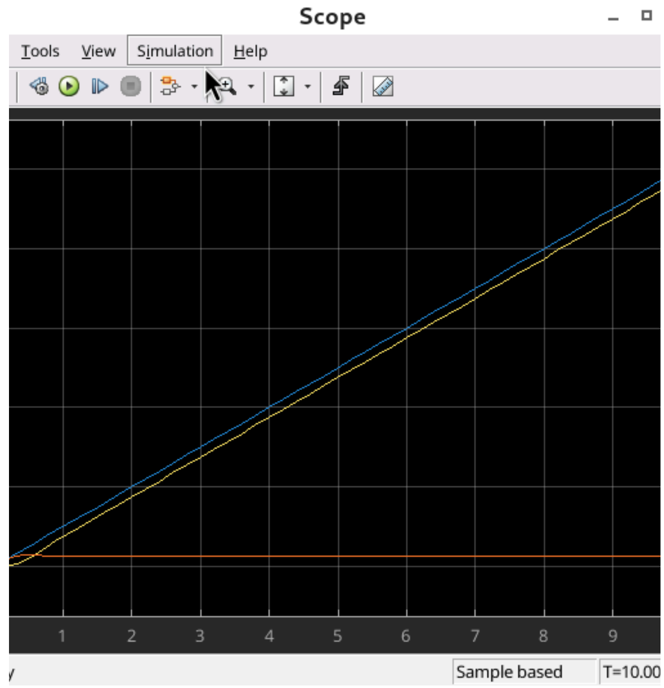

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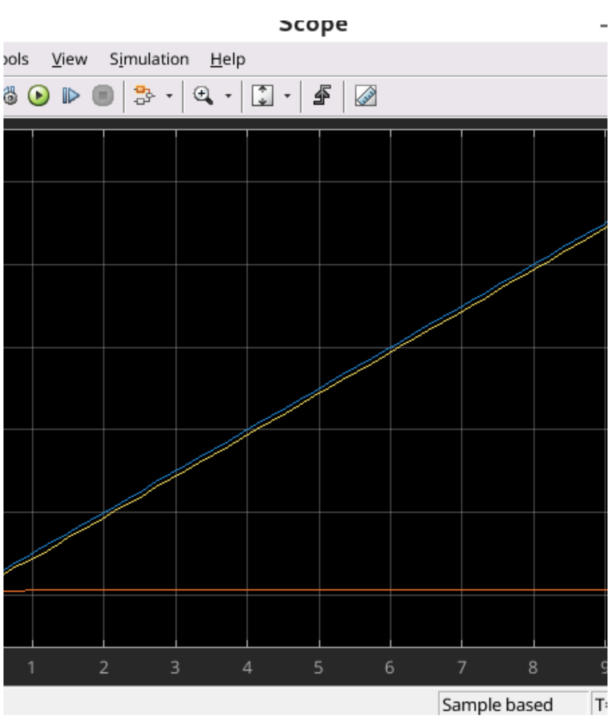

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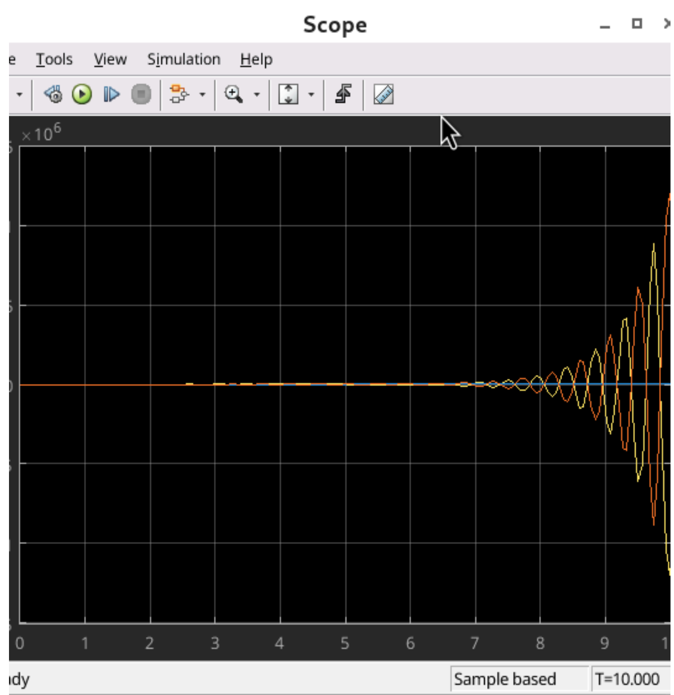
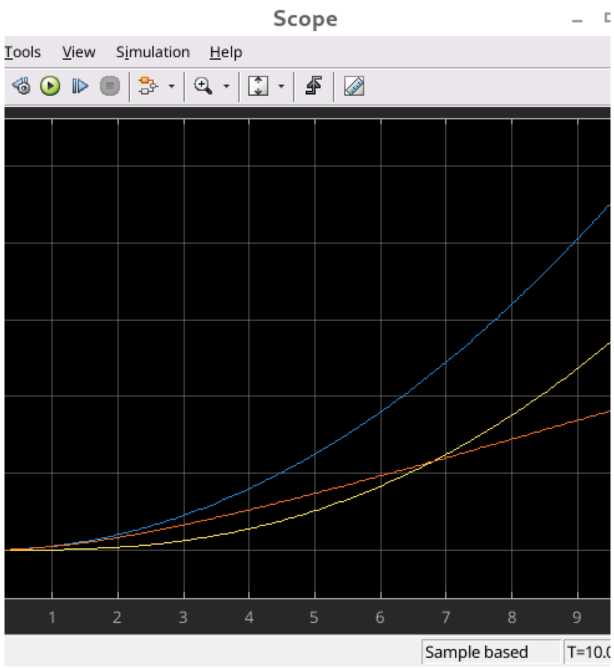


k=5000

b) Question-7 Ploynomial Input



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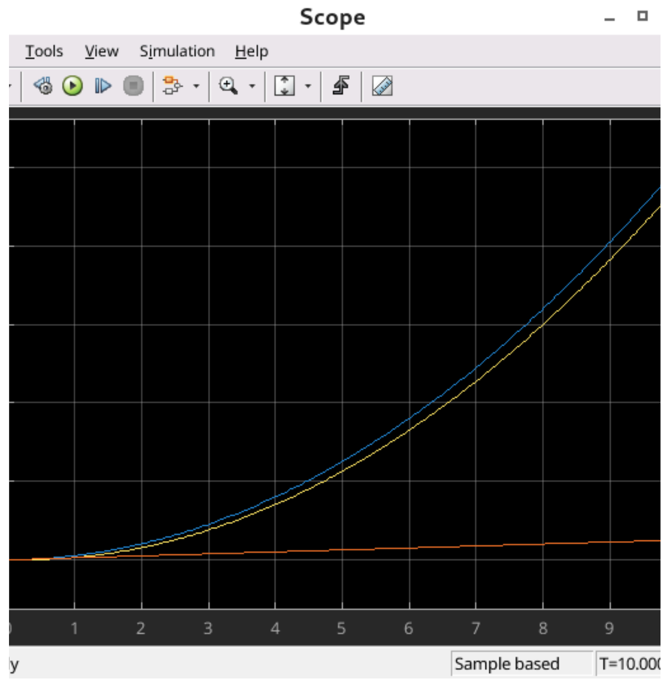

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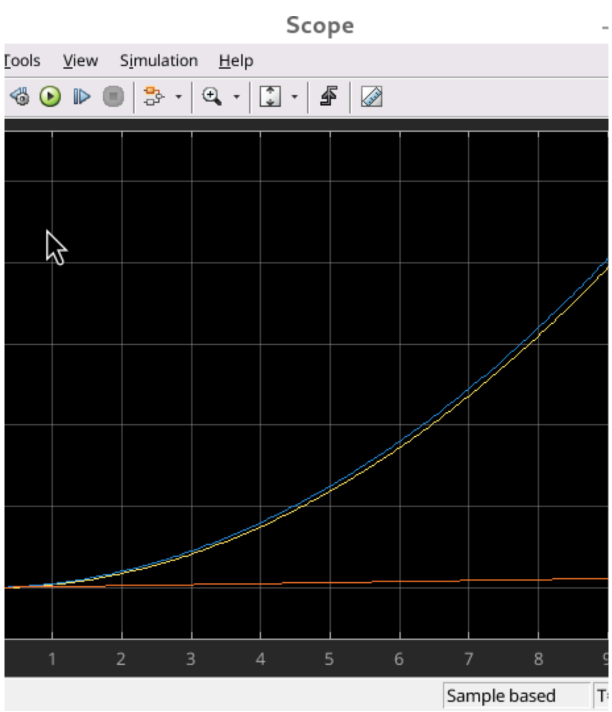

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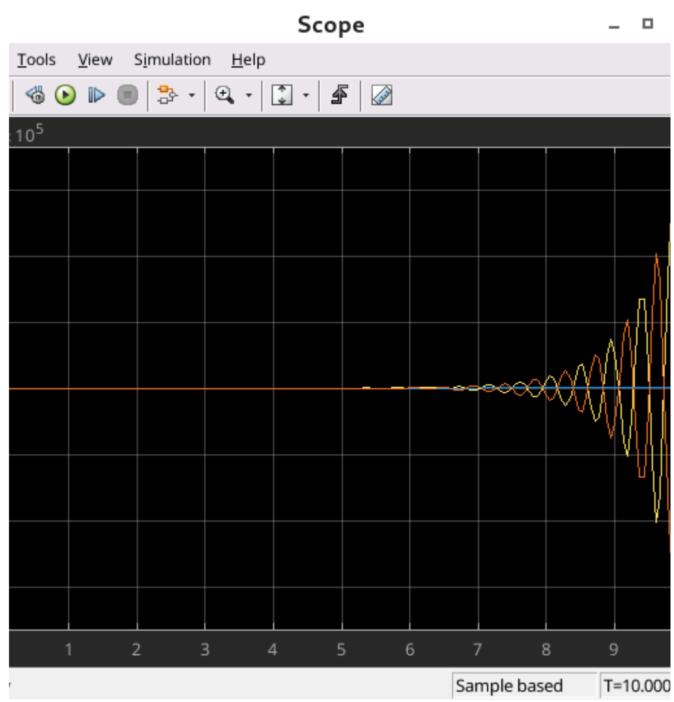
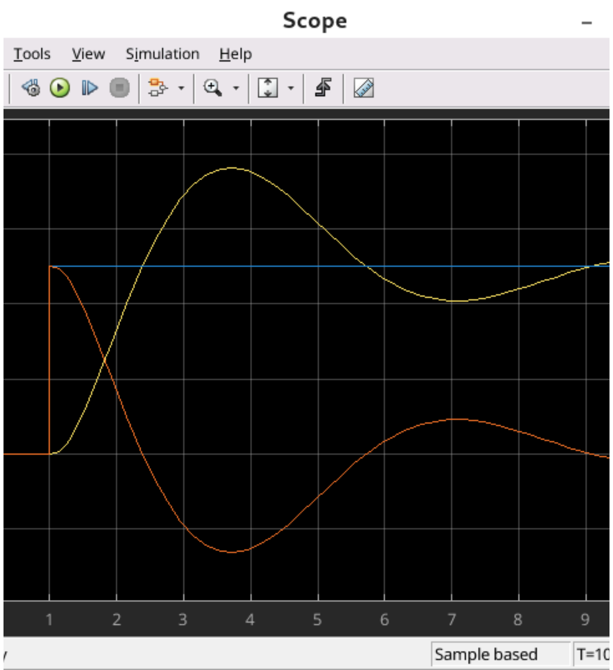


K=5000

c) Question-8 **Step Input**



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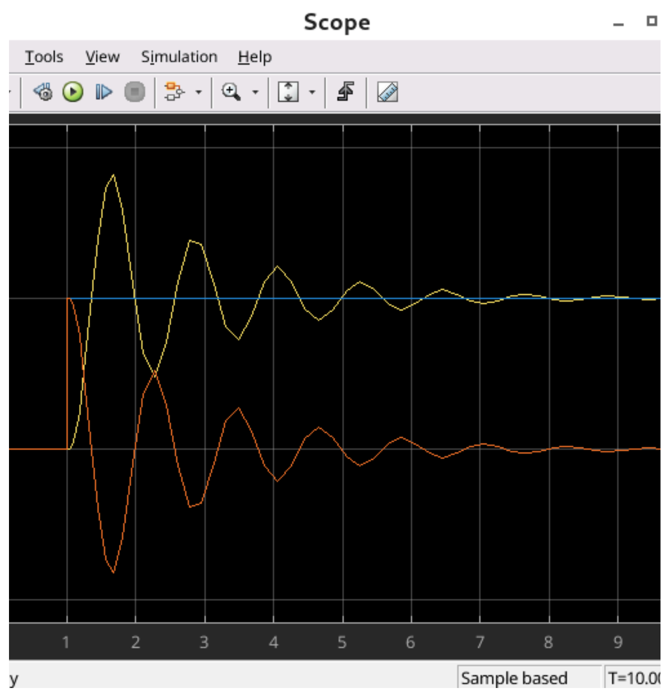

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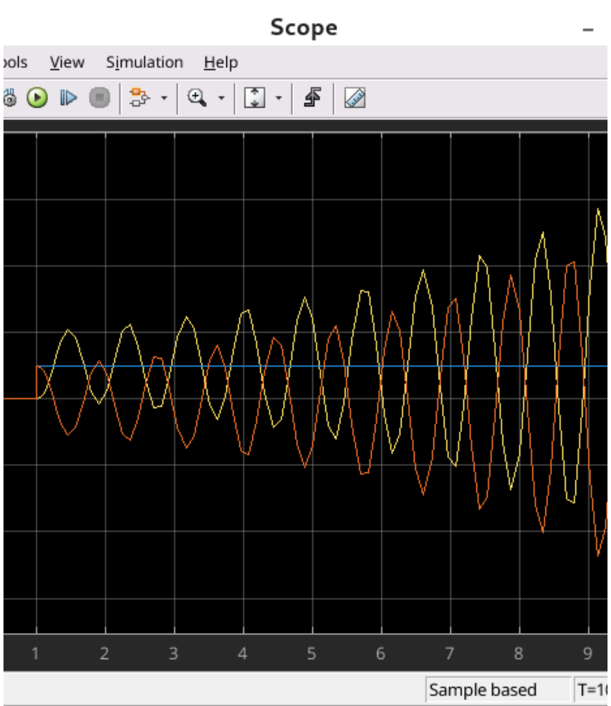

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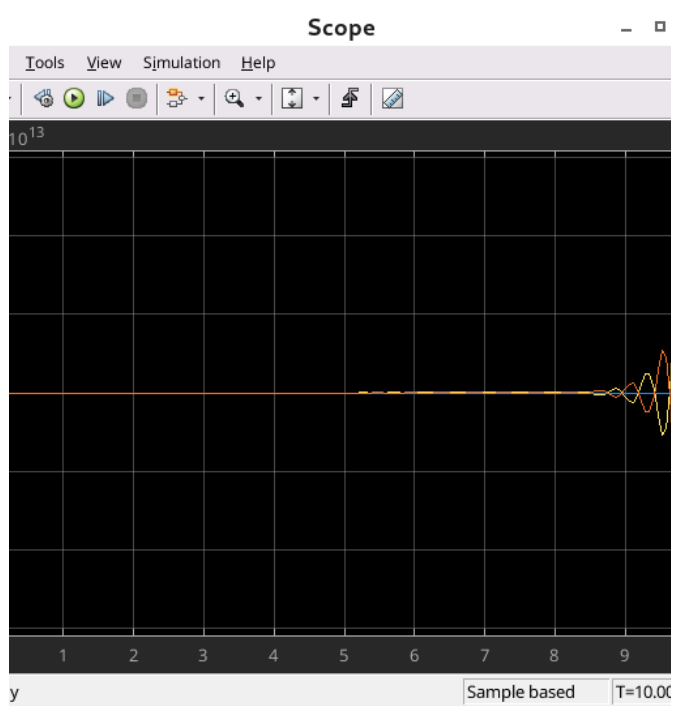
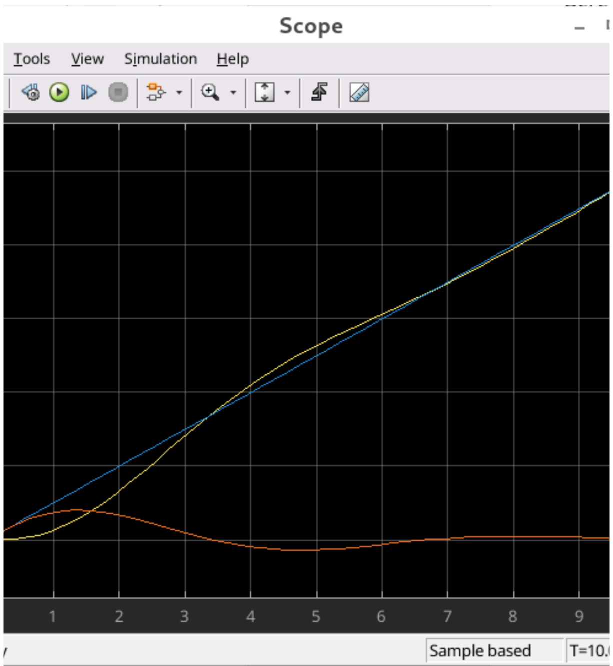


K=5000

c) Question-8 Ramp Input



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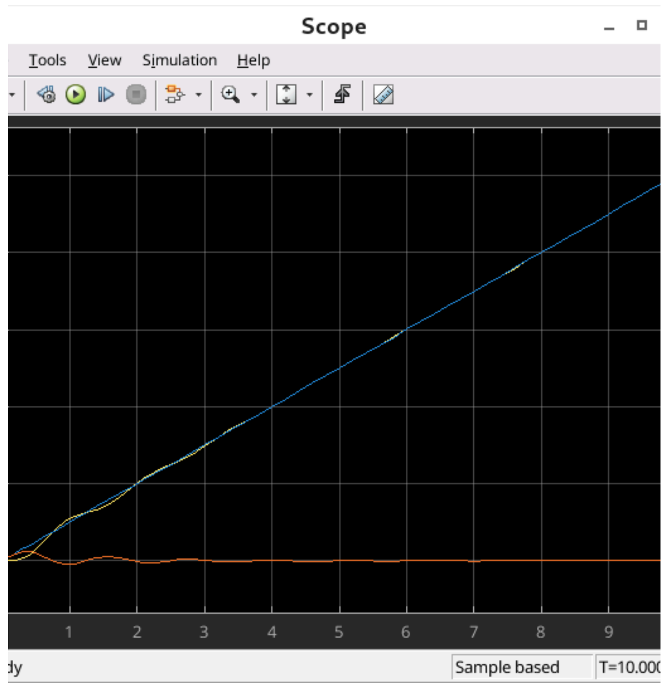

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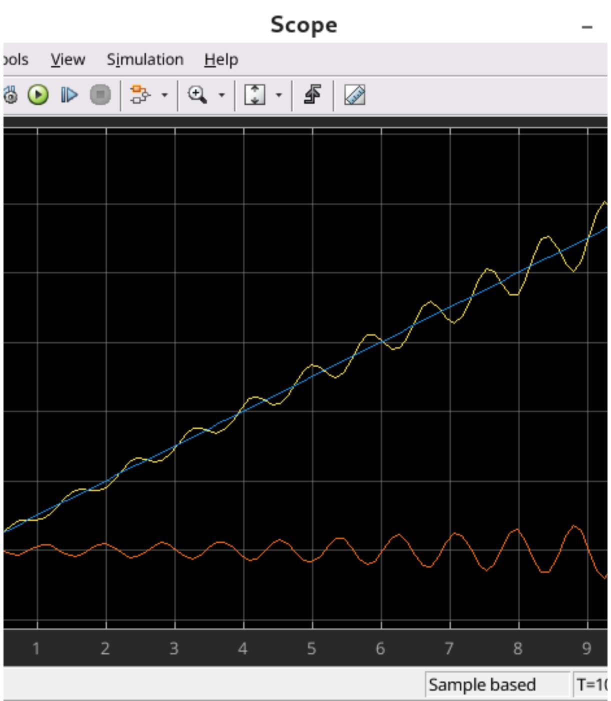

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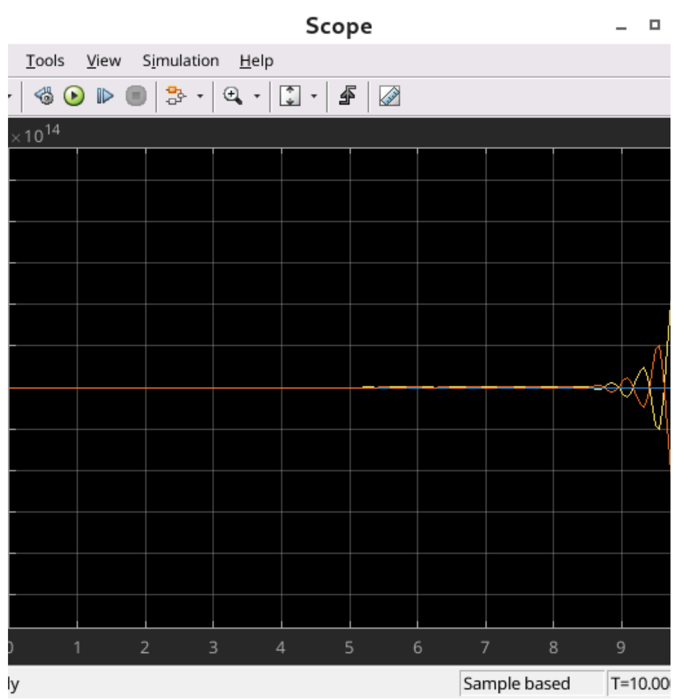
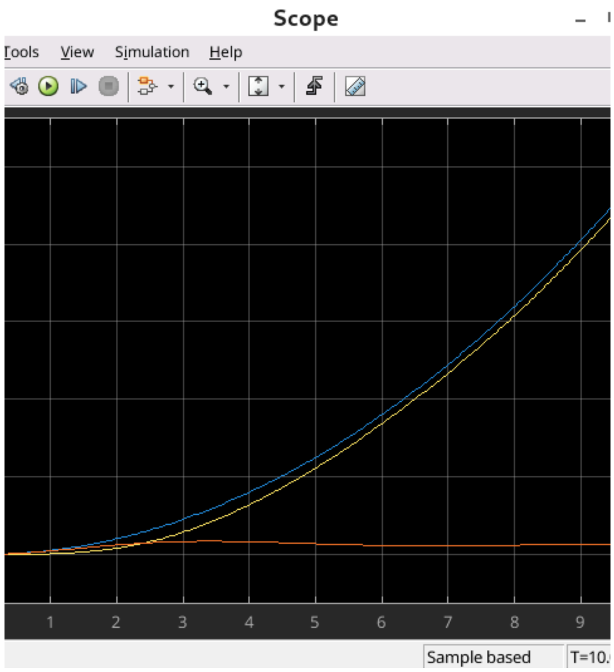


K=5000

c) Question-8 Polynomial Input



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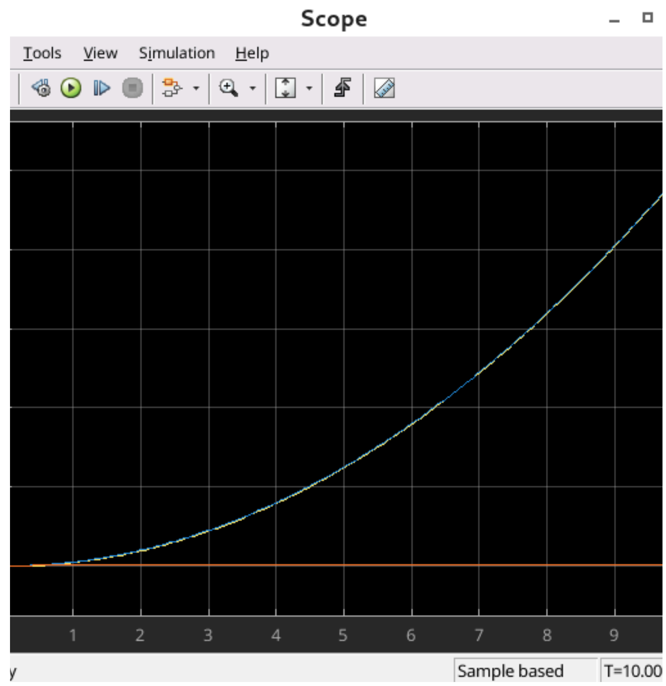

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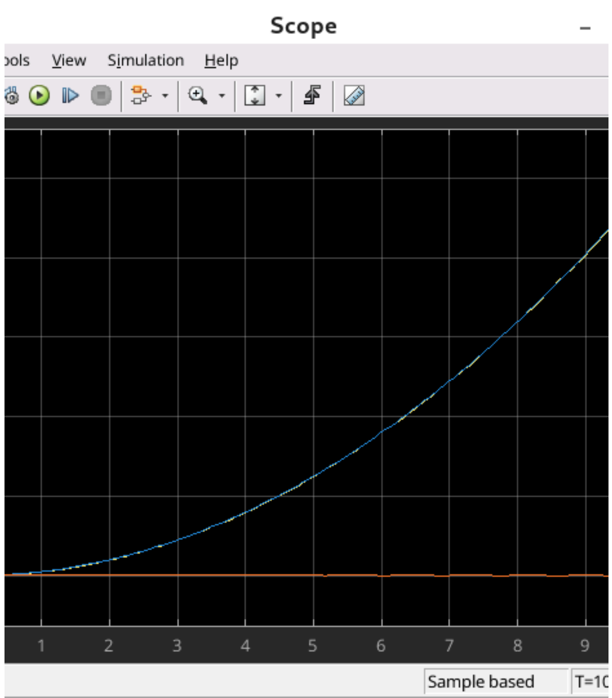

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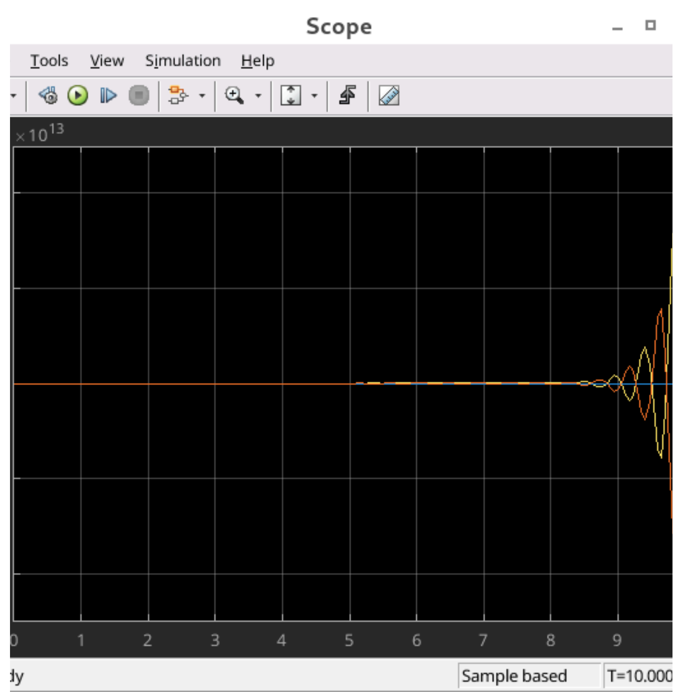

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