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STUDENT WORKBOOK

Inverted Pendulum Experiment for MATLAB®/Simulink® Users

Standardized for ABET* Evaluation Criteria

Developed by:
Jacob Apkarian, Ph.D., Quanser
Paul Karam, B.A.SC., Quanser
Michel Lévis, M.A.SC., Quanser

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1 Introduction

The objective of this laboratory is to design and implement a state-feedback control system that will balance the pendulum in the upright, vertical position.

Topics Covered

- Linearizing nonlinear equations of motion.
- Obtaining the linear state-space representation of the rotary pendulum plant.
- Designing a state-feedback control system that balances the pendulum in its upright vertical position using Pole Placement.
- Simulating the closed-loop system to ensure the specifications are met.
- Introduction to a nonlinear, energy-based swing up control.
- Implementing the controllers on the Quanser Rotary Pendulum plant and evaluating its performance.

Prerequisites

- Know the basics of MATLAB® and SIMULINK®.
- Understand state-space modeling fundamentals.
- Some knowledge of state-feedback.

2 Modeling

2.1 Background

2.1.1 Model Convention

The rotary inverted pendulum model is shown in Figure 2.1. The rotary arm pivot is attached to the Rotary Servo system and is actuated. The arm has a length of L_r, a moment of inertia of J_r, and its angle, θ, increases positively when it rotates counter-clockwise (CCW). The servo (and thus the arm) should turn in the CCW direction when the control voltage is positive, i.e. $V_m > 0$.

The pendulum link is connected to the end of the rotary arm. It has a total length of L_p and it center of mass is $\frac{L_p}{2}$. The moment of inertia about its center of mass is J_p. The inverted pendulum angle, α, is zero when it is perfectly upright in the vertical position and increases positively when rotated CCW.

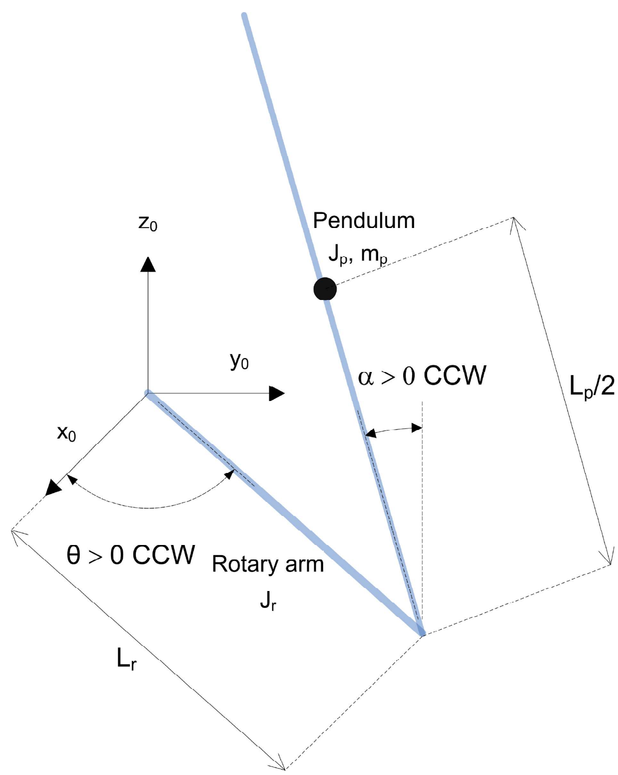

Figure 2.1: Rotary inverted pendulum conventions

2.1.2 Nonlinear Equations of Motion

Instead of using classical mechanics, the Lagrange method is used to find the equations of motion of the system. This systematic method is often used for more complicated systems such as robot manipulators with multiple joints.

More specifically, the equations that describe the motions of the rotary arm and the pendulum with respect to the servo motor voltage, i.e. the dynamics, will be obtained using the Euler-Lagrange equation:

$$\frac{\partial^2 L}{\partial t \partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

The variables q_i are called *generalized coordinates*. For this system let

$$q(t)^\top = \begin{bmatrix} \theta(t) & \alpha(t) \end{bmatrix} \tag{2.1}$$

where, as shown in Figure 2.1, $\theta(t)$ is the rotary arm angle and $\alpha(t)$ is the inverted pendulum angle. The corresponding velocities are

$$\dot{q}(t)^\top = \left[\frac{d\theta(t)}{dt}\,\frac{d\alpha(t)}{dt}\right]$$

Note: The dot convention for the time derivative will be used throughout this document, i.e. $\dot{\theta} = \frac{d\theta}{dt}$. The time variable t will also be dropped from θ and α, i.e. $\theta = \theta(t)$ and $\alpha = \alpha(t)$.

With the generalized coordinates defined, the Euler-Lagrange equations for the rotary pendulum system are

$$\begin{aligned}\frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial L}{\partial \theta} &= Q_1\\ \frac{\partial^2 L}{\partial t \partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} &= Q_2\end{aligned}$$

The Lagrangian of the system is described

$$L = T - V$$

where T is the total kinetic energy of the system and V is the total potential energy of the system. Thus the Lagrangian is the difference between a system's kinetic and potential energies.

The generalized forces Q_i are used to describe the non-conservative forces (e.g. friction) applied to a system with respect to the generalized coordinates. In this case, the generalized force acting on the rotary arm is

$$Q_1 = \tau - B_r\dot{\theta}$$

and acting on the pendulum is

$$Q_2 = -B_p\dot{\alpha}.$$

See Rotary Servo User Manual for a description of the corresponding Rotary Servo parameters (e.g. such as the back-emf constant, k_m). Our control variable is the input servo motor voltage, V_m. Opposing the applied torque is the viscous friction torque, or viscous damping, corresponding to the term B_r. Since the pendulum is not actuated, the only force acting on the link is the damping. The viscous damping coefficient of the pendulum is denoted by B_p.

The Euler-Lagrange equations is a systematic method of finding the equations of motion, i.e. EOMs, of a system. Once the kinetic and potential energy are obtained and the Lagrangian is found, then the task is to compute various derivatives to get the EOMs. After going through this process, the nonlinear equations of motion for the Rotary Pendulum are:

$$\begin{aligned}\left(m_pL_r^2 + \frac{1}{4}m_pL_p^2 - \frac{1}{4}m_pL_p^2\cos(\alpha)^2 + J_r\right)\ddot{\theta} - \left(\frac{1}{2}m_pL_pL_r\cos(\alpha)\right)\ddot{\alpha}&\\ + \left(\frac{1}{2}m_pL_p^2\sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha} + \left(\frac{1}{2}m_pL_pL_r\sin(\alpha)\right)\dot{\alpha}^2 &= \tau - B_r\dot{\theta} \qquad (2.2)\end{aligned}$$

$$\begin{aligned}-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta} + \left(J_p + \frac{1}{4}m_pL_p^2\right)\ddot{\alpha} - \frac{1}{4}m_pL_p^2\cos(\alpha)\sin(\alpha)\dot{\theta}^2&\\ -\frac{1}{2}m_pL_pg\sin(\alpha) &= -B_p\dot{\alpha}. \qquad (2.3)\end{aligned}$$

The torque applied at the base of the rotary arm (i.e. at the load gear) is generated by the servo motor as described by the equation

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m}. \qquad (2.4)$$

See Rotary Servo User Manual for a description of the corresponding Rotary Servo parameters (e.g. such as the back-emf constant, k_m).

Both the equations match the typical form of an EOM for a single body:

$$J\ddot{x} + b\dot{x} + g(x) = \tau_1$$

where x is an angular position, J is the moment of inertia, b is the damping, $g(x)$ is the gravitational function, and τ_1 is the applied torque (scalar value).

For a generalized coordinate vector q, this can be generalized into the matrix form

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \tag{2.5}$$

where D is the inertial matrix, C is the damping matrix, $g(q)$ is the gravitational vector, and τ is the applied torque vector.

The nonlinear equations of motion given in Equation 2.2 and Equation 2.3 can be placed into this matrix format.

2.1.3 Linearizing

Here is an example of how to linearize a two-variable nonlinear function called $f(z)$. Variable z is defined

$$z^\top = \begin{bmatrix} z_1 & z_2 \end{bmatrix}$$

and $f(z)$ is to be linearized about the operating point

$$z_0^\top = \begin{bmatrix} a & b \end{bmatrix}$$

The linearized function is

$$f_{lin} = f(z_0) + \left.\frac{\partial f(z)}{\partial z_1}\right|_{z=z_0} (z_1 - a) + \left.\frac{\partial f(z)}{\partial z_2}\right|_{z=z_0} (z_2 - b)$$

2.1.4 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \tag{2.6}$$

and

$$y = Cx + Du \tag{2.7}$$

where x is the state, u is the control input, A, B, C, and D are state-space matrices. For the rotary pendulum system, the state and output are defined

$$x^\top = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix} \tag{2.8}$$

and

$$y^\top = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \tag{2.9}$$

In the output equation, only the position of the servo and link angles are being measured. Based on this, the C and D matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.10}$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{2.11}$$

The velocities of the servo and pendulum angles can be computed in the digital controller, e.g. by taking the derivative and filtering the result though a high-pass filter.

2.4 Results

Fill out Table 2.1 with your answers from your modeling lab results - both simulation and implementation.

Description	Value
State-Space Matrix A	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 80.3 & -45.8 & -0.930 \\ 0 & 122 & -44.1 & -1.40 \end{bmatrix}$
State-Space Matrix B	$\begin{bmatrix} 0 \\ 0 \\ 83.4 \\ 80.3 \end{bmatrix}$
State-Space Matrix C	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
State-Space Matrix D	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
Open-loop poles	{-48.42, 7.06, -5.86, and 0 }

Table 2.1: Results

3 Balance Control

3.1 Specifications

The control design and time-response requirements are:

Specification 1: Damping ratio: $\zeta = 0.7$.

Specification 2: Natural frequency: $\omega_n = 4$ rad/s.

Specification 3: Maximum pendulum angle deflection: $|\alpha| < 15$ deg.

Specification 4: Maximum control effort / voltage: $|V_m| < 10$ V.

The necessary closed-loop poles are found from specifications 1 and 2. The pendulum deflection and control effort requirements (i.e. specifications 3 and 4) are to be satisfied when the rotary arm is tracking a ± 20 degree angle square wave.

3.2 Background

In Section Section 2, we found a linear state-state space model that represents the inverted rotary pendulum system. This model is used to investigate the inverted pendulum stability properties in Section 3.2.1. In Section 3.2.2, the notion of controllabitliy is introduced. The procedure to transform matrices to their companion form is described in Section 3.2.3. Once in their companion form, it is easier to design a gain according to the pole-placement principles, which is discussed in Section 3.2.4. Lastly, Section 3.2.6 describes the state-feedback control used to balance the pendulum.

3.2.1 Stability

The stability of a system can be determined from its poles ([2]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\det(sI - A) = 0$$

where *det()* is the determinant function, s is the Laplace operator, and I the identity matrix. These are the *eigenvalues* of the state-space matrix A.

3.2.2 Controllability

If the control input u of a system can take each state variable, x_i where $i = 1 \ldots n$, from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([2]).

Rank Test The system is controllable if the rank of its controllability matrix

$$T = \left[B \; AB \; A^2B \ldots A^nB\right] \tag{3.1}$$

equals the number of states in the system,

$$\text{rank}(T) = n.$$

3.2.3 Companion Matrix

If (A, B) are controllable and B is $n \times 1$, then A is similar to a companion matrix ([1]). Let the characteristic equation of A be

$$s^n + a_n s^{n-1} + \ldots + a_1.$$

Then the companion matrices of A and B are

$$\tilde{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \end{bmatrix} \tag{3.2}$$

and

$$\tilde{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{3.3}$$

Define

$$W = T\tilde{T}^{-1}$$

where T is the controllability matrix defined in Equation 3.1 and

$$\tilde{T} = [\tilde{B}\ \tilde{B}\tilde{A} \ldots \tilde{B}\tilde{A}^n].$$

Then

$$W^{-1}AW = \tilde{A}$$

and

$$W^{-1}B = \tilde{B}.$$

3.2.4 Pole Placement

If (A,B) are controllable, then pole placement can be used to design the controller. Given the control law $u = -Kx$, the state-space in Equation 2.6 becomes

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

To illustate how to design gain K, consider the following system

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -1 & -5 \end{bmatrix} \tag{3.4}$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.5}$$

Note that A and B are already in the companion form. We want the closed-loop poles to be at $[-1\ -2\ -3]$. The *desired* characteristic equation is therefore

$$(s+1)(s+2)(s+3) = s^3 + 6s^2 + 11s + 6 \tag{3.6}$$

For the gain $K = [k_1\ k_2\ k_3]$, apply control $u = -Kx$ and get

$$A - KB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 - k_1 & -1 - k_2 & -5 - k_3 \end{bmatrix}.$$

The characteristic equation of $A - KB$ is

$$s^3 + (k_3 + 5)s^2 + (k_2 + 1)s + (k_1 - 3) \tag{3.7}$$

Equating the coefficients between Equation 3.7 and the desired polynomial in Equation 3.6

$$\begin{aligned} k_1 - 3 &= 6 \\ k_2 + 1 &= 11 \\ k_3 + 5 &= 6 \end{aligned}$$

Solving for the gains, we find that a gain of $K = [9\ 10\ 1]$ is required to move the poles to their desired location.

We can generalize the procedure to design a gain K for a controllable (A,B) system as follows:

Step 1 Find the companion matrices $\tilde{A}$ and $\tilde{B}$. Compute $W = T\tilde{T}^{-1}$.

Step 2 Compute $\tilde{K}$ to assign the poles of $\tilde{A} - \tilde{B}\tilde{K}$ to the desired locations. Applying the control law $u = -Kx$ to the general system given in Equation 3.2,

$$\tilde{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_1 - k_1 & -a_2 - k_2 & \cdots & -a_{n-1} - k_{n-1} & -a_n - k_n \end{bmatrix} \tag{3.8}$$

Step 3 Find $K = \tilde{K}W^{-1}$ to get the feedback gain for the original system (A,B).

Remark: It is important to do the $\tilde{K} \rightarrow K$ conversion. Remember that (A,B) represents the actual system while the companion matrices $\tilde{A}$ and $\tilde{B}$ do not.

3.2.5 Desired Poles

The rotary inverted pendulum system has four poles. As depicted in Figure 3.1, poles p_1 and p_2 are the complex conjugate *dominant* poles and are chosen to satisfy the natural frequency, ω_n, and damping ratio, ζ, specifications given in Section 3.1. Let the conjugate poles be

$$p_1 = -\sigma + j\omega_d \tag{3.9}$$

and

$$p_2 = -\sigma - j\omega_d \tag{3.10}$$

where $\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the *damped* natural frequency. The remaining closed-loop poles, p_3 and p_4, are placed along the real-axis to the left of the dominant poles, as shown in Figure 3.1.

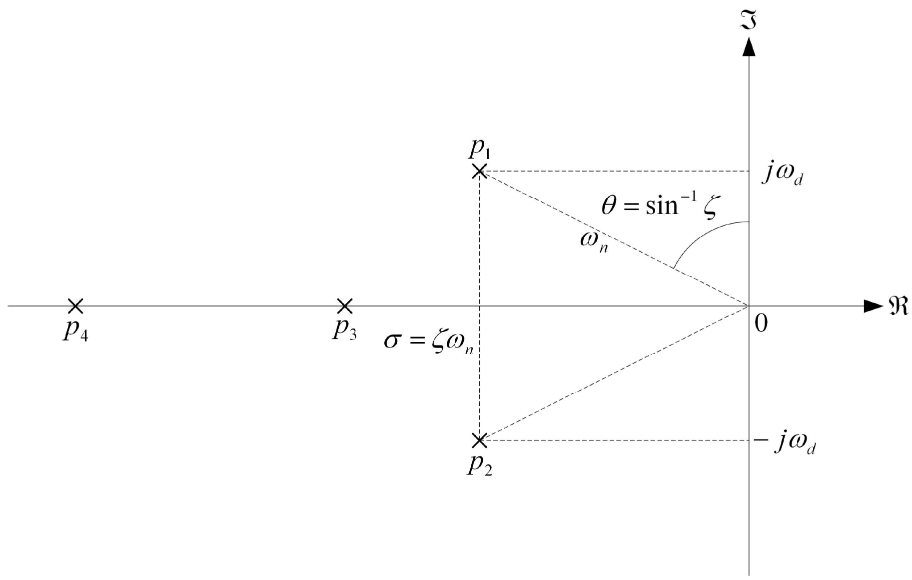

Figure 3.1: Desired closed-loop pole locations

3.2.6 Feedback Control

The feedback control loop that balances the rotary pendulum is illustrated in Figure 3.2. The reference state is defined

$$x_d = [\theta_d \; 0 \; 0 \; 0]$$

where θ_d is the desired rotary arm angle. The controller is

$$u = K(x_d - x). \tag{3.11}$$

Note that if $x_d = 0$ then $u = -Kx$, which is the control used in the pole-placement algorithm.

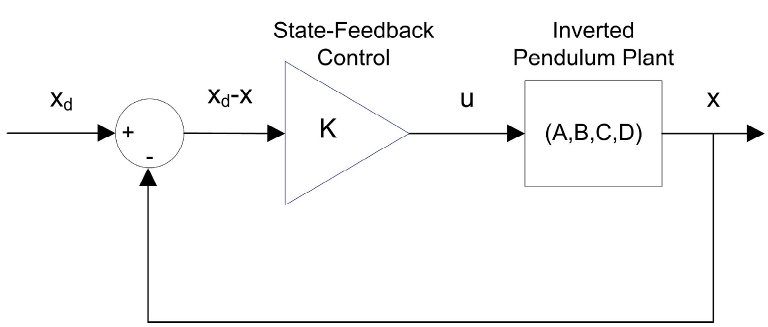

Figure 3.2: State-feedback control loop

When running this on the actual system, the pendulum begins in the hanging, downward position. We only want the balance control to be enabled when the pendulum is brought up around its upright vertical position. The controller is therefore

$$u = \begin{cases} K(x_d - x) & |x_2| < \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

where ϵ is the angle about which the controller should engage. For example if $\epsilon = 10$ degrees, then the control will begin when the pendulum is within ± 10 degrees of its upright position, i.e. when $|x_2| < 10$ degrees.

3.3 Pre-Lab Questions

1. Based on your analysis in Section 2.3, is the system stable, marginally stable, or unstable? Did you expect the stability of the inverted pendulum to be as what was determined?
2. Using the open-loop poles, find the characteristic equation of A.
3. Give the corresponding companion matrices $\tilde{A}$ and $\tilde{B}$. Do not compute the transformation matrix W (this will be done in the lab using QUARC®).
4. Find the location of the two dominant poles, p_1 and p_2, based on the specifications given in Section 3.1. Place the other poles at $p_3 = -30$ and $p_4 = -40$. Finally, give the desired characteristic equation.
5. When applying the control $u = -\tilde{K}x$ to the companion form, it changes $(\tilde{A}, \tilde{B})$ to $(\tilde{A} - \tilde{B}\tilde{K}, \tilde{B})$. Find the gain $\tilde{K}$ that assigns the poles to their new desired location.

3.5 Results

Fill out Table 3.1 with your answers from your control lab results - both simulation and implementation.

Description	Symbol	Value	Units
Pre Lab Questions			
Desired poles	DP		
Companion Gain	$\tilde{K}$		
Simulation: Control Design			
Transformation Matrix	W		
Control Gain	K		
Closed-loop poles	CLP		
Simulation: Closed-Loop System			
Maximum deflection	$\|\alpha\|_{max}$		deg
Maximum voltage	$\|V_m\|_{max}$		V
Implementation			
Control Gain	K		
Maximum deflection	$\|\alpha\|_{max}$		deg
Maximum voltage	$\|V_m\|_{max}$		V

Table 3.1: Results

4 Swing-Up Control

4.1 Background

In this section a nonlinear, energy-based control scheme is developed to swing the pendulum up from its hanging, downward position. The swing-up control described herein is based on the strategy outlined in [3]. Once upright, the control developed in Section Section 3 can be used to balance the pendulum in the upright vertical position.

4.1.1 Pendulum Dynamics

The dynamics of the pendulum can be redefined in terms of pivot acceleration as

$$J_p \ddot{\alpha} + \frac{1}{2} m_p g L_p \sin(\alpha) = \frac{1}{2} m_p L_p u \cos(\alpha). \tag{4.1}$$

The pivot accleration, u, is the linear acceleration of the pendulum link base. The acceleration is proportional to the torque of the rotary arm and is expressed as

$$\tau = m_r L_r u \tag{4.2}$$

where m_r is the mass of the rotary arm and L_r is its length, as shown in Section Section 2. The voltage-torque relationship is given in Equation 2.4.

4.1.2 Energy Control

If the arm angle is kept constant and the pendulum is given an initial position it would swing with constant amplitude. Because of friction there will be damping in the oscillation. The purpose of energy control is to control the pendulum in such a way that the friction is constant.

The potential and kinetic energy of the pendulum is

$$E_p = \frac{1}{2} m_p g L_p (1 - \cos(\alpha)) \tag{4.3}$$

and

$$E_k = \frac{1}{2} J_p \dot{\alpha}^2.$$

The pendulum parameters are described in Section Section 2 and their values are given in Rotary Pendulum User Manual. In the potential energy calculation, we assume the center of mass to be in the center of the link, i.e. $\frac{L_p}{2}$. Adding the kinetic and potential energy together give us the total pendulum energy

$$E = \frac{1}{2} J_p \dot{\alpha}^2 + \frac{1}{2} m_p g L_p (1 - \cos\alpha). \tag{4.4}$$

Taking its time derivative we get

$$\dot{E} = \dot{\alpha} \left(J_p \ddot{\alpha} + \frac{1}{2} m_p g L_p \sin\alpha \right). \tag{4.5}$$

To introduce the pivot acceleration u and eventually, our control variable, solve for $\sin\alpha$ in Equation 4.1 to obtain

$$\sin(\alpha) = \frac{1}{m_p g L_p} (-2 J_p \ddot{\alpha} + m_p L_p u \cos(\alpha)).$$

Substitute this into $\dot{E}$, found in Equation 4.5, to get

$$\dot{E} = \frac{1}{2} m_p L_p u \dot{\alpha} \cos\alpha$$

One strategy that will swing the pendulum to a desired reference energy E_r is the proportional control

$$u = (E - E_r)\dot{\alpha}\cos\alpha.$$

By setting the reference energy to the pendulum potential energy, i.e. $E_r = E_p$, the control will swing the link to its upright position. Notice that the control law is nonlinear because the proportional gain depends on the pendulum angle, α, and also notice that the control changes sign when $\dot{\alpha}$ changes sign and when the angle is ± 90 degrees.

For energy to change quickly the magnitude of the control signal must be large. As a result, the following swing-up controller is implemented

$$u = sat_{u_{max}}(\mu(E - E_r)\text{sign}(\dot{\alpha}\cos\alpha)) \tag{4.6}$$

where μ is a tunable control gain and $sat_{u_{max}}$ function saturates the control signal at the maximum acceleration of the pendulum pivot, u_{max}. Taking the sign of $\dot{\alpha}\cos\alpha$ allows for faster switching.

In order to translate the pivot acceleration into servo voltage, first solve for the voltage in Equation 2.4 to get

$$V_m = \frac{\tau R_m}{\eta_g K_g \eta_m k_t} + K_g k_m \dot{\theta}.$$

Then substitute the torque-acceleration relationship given in Equation 4.2 to obtain the following

$$V_m = \frac{R_m m_r L_r u}{\eta_g K_g \eta_m k_t} + K_g k_m \dot{\theta}. \tag{4.7}$$

4.1.3 Self-Erecting Control

The energy swing-up control can be combined with the balancing control in Equation 3.11 to obtain a control law which performs the dual tasks of swinging up the pendulum and balancing it. This can be accomplished by switching between the two control systems.

Basically the same switching used for the balance control in Equation 3.12 is used. Only instead of feeding 0 V when the balance control is not enabled, the swing-up control is engaged. The controller therefore becomes

$$u = \begin{cases} K(x_d - x) & |x_2| < \epsilon \\ sat_{u_{max}}(\mu(E - E_r)\text{sign}(\dot{\alpha}\cos\alpha)) & \text{otherwise} \end{cases} \tag{4.8}$$

4.2 Pre-lab Questions

1. Evaluate the potential energy of the pendulum when it is in the downward and upright positions.

2. Compute the maximum acceleration deliverable by the Rotary Servo. Assume the maximum equivalent voltage applied to the DC motor is 5 V such that

$$V_m - K_g k_m \dot{\theta} = 5. \tag{4.9}$$

   The Rotary Servo motor parameters are given in Rotary Servo User Manual.

3. Find the controller acceleration when the pendulum is initially hanging down and motionless. From a practical viewpoint, what does this imply when the swing-up control is activated?

4. Assume the pendulum is starting to swing from the downward position in the positive direction. Calculate the acceleration the swing-up controller will generate when $\mu = 20$. Does this saturate the controller?

4.4 Results

Fill out Table 4.1 with your answers from your swing-up control lab results.

Description	Symbol	Value	Unit
Pre Lab Questions			
Potential Energy	E_p		J
Maximum Acceleration of Servo	u_{max}		m/s^2
Implementation			
Control Gain	K		
Reference Energy	E_r		J
Control Maximum Acceleration	u_{max}		m/s^2
Proportional Gain	μ		

Table 4.1: Results