1 Presentation

The Quanser Aero Experiment experiment can be configured as a half-quadrotor system, as shown in Figure 1.1. In this set up, both the front and back rotors are horizontal to the ground and only motions about the yaw axis are enabled (i.e. the pitch axis is locked). By changing the direction and speed of the rotors, users can change the yaw axis angle.

Unmanned quadrotor vehicles are used for wide-variety of applications. Using a tethered half-quadrotor systems allows students and researchers to focus on the modeling, control, and parameter estimation in yaw-axis motion of quadrotors, which can then be applied to full quadrotor system.

Figure 1.1: Quanser Aero Experiment in half-quadrotor configuration

Topics covered:

- Find linear equations of motion describing the half-quadrotor yaw motions based on rotor voltage
- Derive the transfer function model
- Derive the linear state-space representation
- Design PD compensator to control the yaw angle.
- Simulate the PD control and run PD controller on the actual half-quadrotor system.
- Design state-feedback control using LQR optimization to control the yaw position.
- Simulate the state-feedback control and implement on half-quadrotor .
- Formulate a Kalman-based observer to estimate the state using IMU readings and use LQR to control the yaw

2 Modeling

The free-body diagram of the Quanser Aero Experiment in the half-quadrotor configuration is illustrated in Figure 2.1.

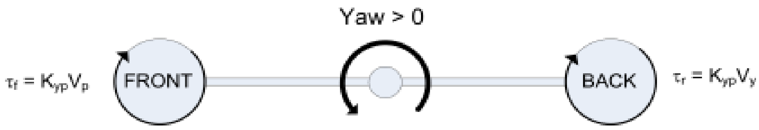


Figure 2.1: Simple free-body diagram of Quanser Aero Experiment

Similarly to when the Quanser Aero Experiment is in the 2 DOF Helicopter mode, the torque from the rotors causes the system to rotate about the yaw axis. We can use the following linear equations of motion to describe the yaw motion relative the DC motor voltages:

$$J_y\ddot{\psi} + D_y\dot{\psi} = \tau_y = -K_{yp}V_p - K_{yp}V_y, \tag{2.1}$$

where J_y is the moment of inertia about the yaw axis, D_y is the viscous damping coefficient about the yaw axis, K_{yp} is the cross-torque thrust gain identified in 2.2.3 of the Aero 2 DOF Laboratory Guide , V_p is the voltage applied to the front (pitch) motor, and V_y is the voltage applied to the back (yaw) motor. The cross-torque thrust gain K_{yp} is discussed and identified in Section 2.1 of the Aero 2 DOF Laboratory Guide . With both rotors facing down (i.e. like the front/pitch rotor in the 2 DOF helicopter configuration), only this cross-torque is to be considered and that is why it is used for both the front and back motor voltages. Model parameters are given in the Quanser Aero Experiment User Manual.

Because the pitch-axis is locked and only the yaw motions are considered, the same voltage is applied to both motors. Thus we can redefine the model in terms of a single control input: $u = V_p = V_y$. The equation of motion become

$$J_y\ddot{\psi} + D_y\dot{\psi} = -2K_{yp}u. \tag{2.2}$$

Recall that the cross-torque acting on the pitch from the yaw identified in Section 2.2 was negative, i.e. $K_{yp} < 0$. The negative sign in $-2K_{yp}u$ ensures the a positive voltage $u > 0$ results in a positive yaw response, $\dot{\psi} > 0$.

2.1 Transfer Function Model

The Laplace transform of the equation of motion (EOM) given in Equation 2.1 is

$$J_y\left(\Psi(s)s^2 - \psi(0^-)s - \dot{\psi}(0^-)\right) + D_y\left(\Psi(s)s - \psi(0^-)\right) = K_{yp}V_p(s) + K_{yy}V_y(s).$$

Since the system starts from rest, all the initial conditions for the yaw are zero, therefore $\psi(0^-) = 0$ and $\dot{\psi}(0^-) = 0$. Because this is a MIMO system with one output and two inputs, we can represent the system as a set of two transfer functions: $\Psi(s)/V_p(s)$ and $\Psi(s)/V_y(s)$. Using this, we get the following transfer function model

$$\frac{\Psi(s)}{V_p(s)} = \frac{K_{py}}{J_y s^2 + D_y s} \text{ and } \frac{\Psi(s)}{V_y(s)} = \frac{K_{py}}{J_y s^2 + D_y s}. \tag{2.3}$$

When using the single-input EOM defined in Equation 2.2, we can simplify it to the following SISO transfer function model:

$$\frac{\Psi(s)}{U(s)} = \frac{-2K_{yp}}{J_p s^2 + D_p s}. \tag{2.4}$$

2.2 In-Lab Experiments

2.2.1 Estimating Viscous Damping Coefficients

Example lab results of finding the pitch and yaw damping coefficients are given in this section. Note that results may vary between different Quanser Aero Experiment systems.

Finding Yaw Damping:

1. Lock the pitch axis
2. Run the *q_aero_free_osc_response_yaw* QUARC controller to apply an impulse to the tail rotor and measure the corresponding yaw response.
3. See the sample response obtained in Figure

To find the time constant, examine the decaying response that starts at an initial maximum speed of $\omega_0 = 0.56$ rad/s at time $t_0 = 2.06$ s and eventually settles down to 0 rad/s. The response is similar to the decaying free-oscillation response discussed in Section 2.1.4. 37% of the final value between ω_0 and 0 rad/s is

$$\omega_1 = (1 - 0.63)\omega_0 = 0.207 \text{ rad/s.}$$

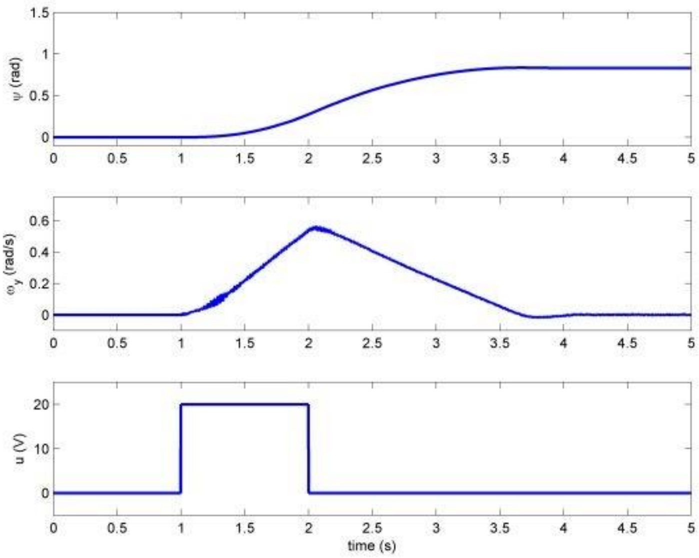


Figure : Free-oscillation response about pitch

which occurs at $t_1 = 3.06$ s. The time constant is therefore

$$\tau = t_1 - t_0 = 1.00\, s$$

and using Equation the damping is

$$D_y = \frac{J_t}{\tau}$$

2.2.3 Estimating the Cross-Thrust Gain Parameters

In this section the cross-torque gain parameters and estimated. Each axis is allowed to move freely and the response of each axis is examine when applying a torque to the other axis.

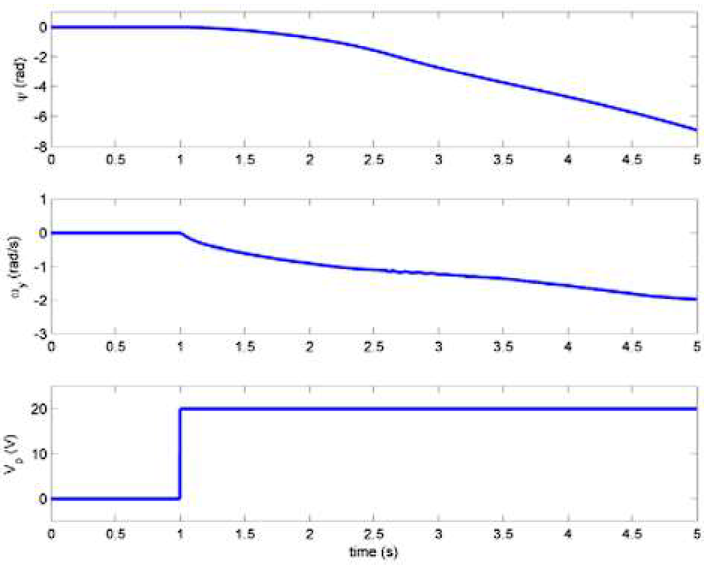


Figure 2.12: Yaw Step Response from Pitch Voltage

2. Run the *q_aero_step_response_yaw_from_pitch* QUARC controller to apply a step voltage 20V to the main rotor and examine the yaw response.

3. Sample response is given in Figure 2.12.

In this case, applying a positive torque about the pitch generates a *negative* torque about the yaw axis. Using Equation 2.24 and the response in Figure Figure 2.12, the cross-thrust gain of the yaw due to a voltage in the pitch rotor is

$$K_{yp} = \frac{J_y \frac{\Delta \Omega_y}{\Delta t} + D_y \Delta \omega_p}{V_p}$$

where $\Delta\omega_y = -1.95 - 0$ rad/s is the change in angular rate of the pitch axis over $\Delta t = 5 - 1 = 4$ s. The gain value is negative, again, because a positive pitch torque results in a negative torque about the yaw.

3 PD Control

3.1 Background

The same variation of the PD control that was used to control the 2 DOF Helicopter in Section 3.1 of the Aero 2 DOF Laboratory Guide is used to control the yaw-only half-quadrotor system. The PV control to stabilize the half-quadrotor yaw position to a setpoint is the same as presented (in Equation 3.1 of the Aero 2 DOF Laboratory Guide):

$$u = k_p\left(r(t) - y(t)\right) - k_d\dot{y}(t),$$

where $y = \psi(t)$ is the yaw angle, $r = \psi_d(t)$ is the yaw setpoint (reference command), and u is the voltage applied to the rotors. The closed-loop transfer function is denoted as $Y(s)/R(s) = \Psi_d(s)/\Psi(s)$. Assuming all initial conditions are zero, i.e. $\psi(0^-) = 0$ and $\dot{\psi}(0^-) = 0$, and taking the Laplace transform of the PV control given above yields

$$U(s) = k_p(R(s) - Y(s)) - k_d sY(s).$$

Substituting this into the $\Psi(s)/U(s)$ transfer function model in Equation 2.4

$$Y(s) = \frac{K}{s(\tau s+1)}(k_p(R(s) - Y(s)) - k_d sY(s)).$$

where

$$K = \frac{2K_{yp}}{D_y}$$

is the steady-state gain and

$$\tau = \frac{J_y}{D_y}$$

is the time constant of the half-quadrotor model. Solving for $Y(s)/R(s)$, we obtain the closed-loop expression

$$\frac{Y(s)}{R(s)} = \frac{Kk_p/\tau}{s^2 + (1+Kk_d)/\tau s + Kk_p/\tau}. \tag{3.1}$$

This is a second-order transfer function that can be mapped to the natural frequency and damping ratio parameters of the prototype second-order function (shown in Equation 3.3of the Aero 2 DOF Laboratory Guide). We can therefore design the PV gains according to specified natural frequency, ω_n, and damping ratio, ζ, using the gain equations:

$$k_p = \frac{\tau\omega_n^2}{K} \tag{3.2}$$

and

$$k_d = \frac{2\tau\zeta\omega_n - 1}{K}. \tag{3.3}$$

Given peak time and overshoot response specifications, use the overshoot and peak time equations given in Equation 3.6 and Equation 3.7 of the Aero 2 DOF Laboratory Guide to find the necessary damping ratio and natural frequency and then calculate the PV gains accordingly. See Section 3.1 for more information.

3.2 In-Lab: PD Control Design and Simulation

In this lab we will design PV gains according to a set of specifications and simulate the closed-loop response to ensure it matches those specifications. The PV control that is implemented in described in Section 3.1.

Desired closed-loop response specifications:

1. Steady-state error: $e_{ss} \leq 2$ deg.
2. Peak time: $t_p \leq 2$ s.
3. Percent Overshoot: $PO \leq 7.5\%$.
4. No actuator saturation: $|V_y| \leq 24$V and $|V_p| \leq 24$V.

The closed-loop response is the simulated in Simulink® using the *s_aero_half_quad_pd_control* shown in Figure 3.1.

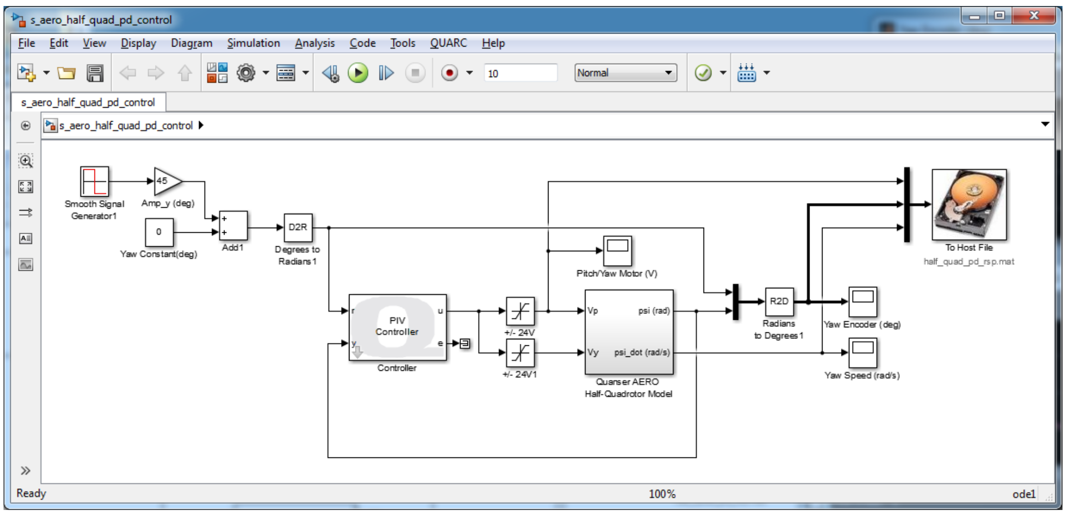


Figure 3.1: Simulink model used to simulate closed-loop PD response of half-quadrotor system

This model uses the QUARC PID block to implement the PV control for the pitch and yaw axes. The *Quanser AERO Half-Quadrotor Model* subsystem implements the transfer function describe in Section Section 2. We will design the PV gains for the peak time $t_p = 1.25$ rad/s and the overshoot of $PO = 7.5\%$. Similarly as done in 3.2 of the Aero 2 DOF Laboratory Guide , we use a lower peak time specification to generate the PV gain to ensure the peak time criteria above is satisfied. Given this, we would need a natural frequency and damping ratio of:

$$\zeta = -\log\left(\frac{PO}{100}\right)\frac{1}{\sqrt{\log\left(\frac{PO}{100}\right)^2 + \pi^2}}$$

and

$$\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}}$$

With the PV equations Equation 3.2 and Equation 3.3, this means we need the following PV gains to meet the peak time and overshoot design specifications

$$k_p = 43.2 \text{ V/rad and } k_d = 12.8 \text{ V-s/rad} \tag{3.4}$$

Running the PD control half-quadrotor simulation:

1. Open the Matlab script *half_quad_pd_design.m*. Similarly as in the PD design script used for the 2 DOF Helicopter in Section 3.2 of the Aero 2 DOF Laboratory Guide , this loads the model parameters set in the *quanser_aero_parameters.m* script using the default values found. You can update the thrust and damping parameters to those found for your Quanser Aero Experiment system.

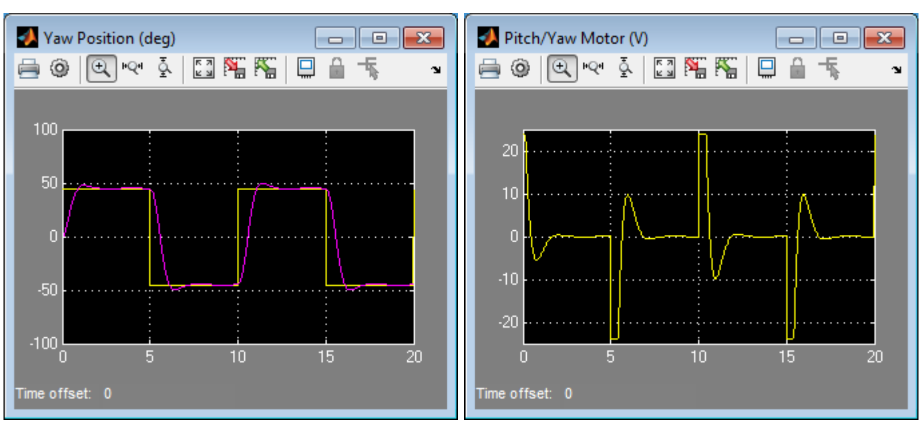
```
%% Load parameters
quanser_aero_parameters;
%
%% Control Specifications
% Peak time and overshoot specifications
PO = 7.5;
tp = 1.25;
% Damping ratio from overshoot specification.
zeta = -log(PO/100) * sqrt( 1 / ( ( log(PO/100) )^2 + pi^2 ) );
% Natural frequency from specifications (rad/s)
wn = pi / ( tp * sqrt(1-zeta^2) );
%
%% PV Design for Half-Quadrotor yaw control
kp = -Jy*wn^2/(2*Kyp)
kd = -(2*Jy*zeta*wn - Dy) / (2*Kyp)
```

2. Run the script to generate PV control gains for each axis. The gains depend on the model parameters and the specifications set in PO and t_p. Using the standard model parameters and the specifications above, the following control gains are generated:

```
kp =

 43.2128


kd =

 12.8045
```

3. Enter those gains into the *PIV Controller* block.

4. Run the *s_aero_half_quad_pd_control* Simulink diagram to simulated the closed-loop response of the Quanser Aero Experiment using the transfer function model in Section Section 2. See the sample response given in Figure 3.2.



(a) Yaw Angle (b) Motor Voltage

Figure 3.2: Sample PV simulated yaw closed-loop response for half-quadrotor

5. Examine the obtained closed-loop response and see if it matches the desired specifications given in Section 3.1.

Response Analysis
The obtained simulated response is shown in Figure 3.3.

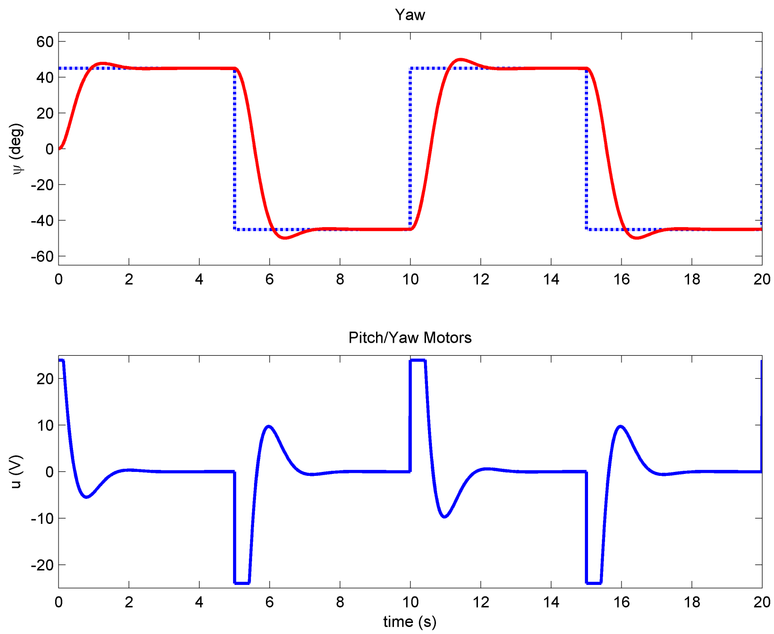


Figure 3.3: Sample PV simulated closed-loop response

The peak time, overshoot, and steady-state error of the simulated response are:

1. Steady-state error: $e_{ss} = 0 < 2$ deg.
2. Peak time: $t_p = 11.4 - 10 = 1.4$ s < 2s.
3. Percent Overshoot: $PO = (49.9 - 45)/90 = 5.4\% < 7.5\%$.
4. Actuator saturation: $|V_y| \leq 24$V and $|V_p| \leq 24$V.

The response matches the criteria above. The motors do get saturated, but only for a short instant so this is still acceptable. The saturation blocks prevent higher voltages as well.

3.3 In-Lab: Running PD on System

In this section the PD control is implemented on the half-quadrotor using the *q_aero_half_quad_pd_control* Simulink diagram shown in Figure 3.4 with QUARC® .

Running the PV control on the actual half-quadrotor :

1. Lock the pitch axis. Ensure the yaw axis is free to rotate.
2. Similarly as done in Section 3.2, generate the PV control gains using the Matlab script *half_quad_pd_design.m*.
3. Enter those gains into the *PIV Controller* block.

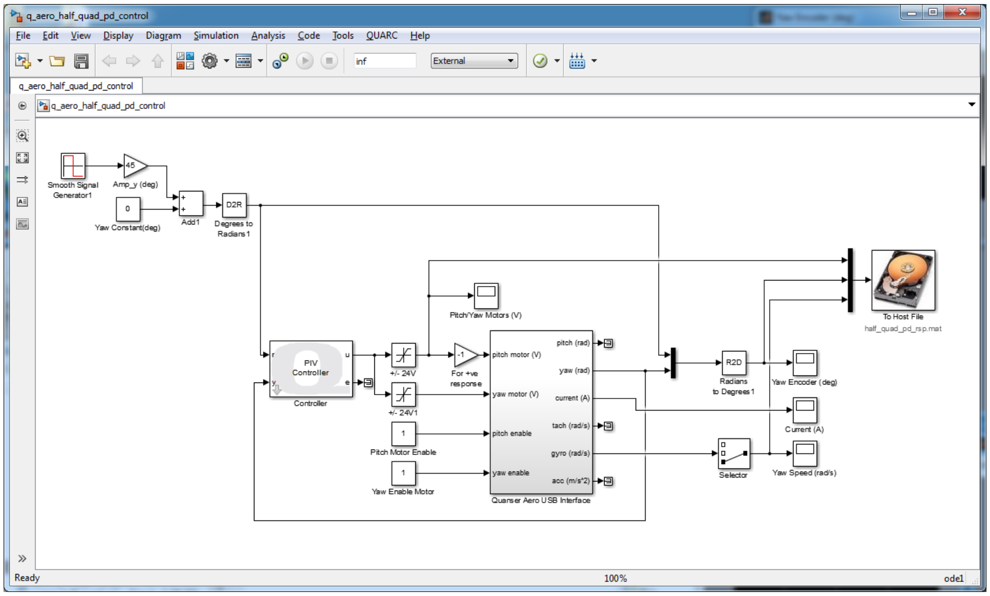
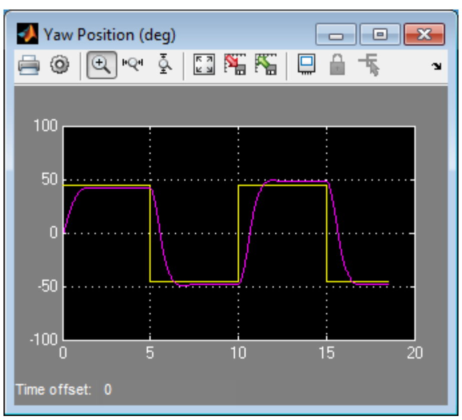


Figure 3.4: Simulink model used to run PV controller on half-quadrotor system using QUARC

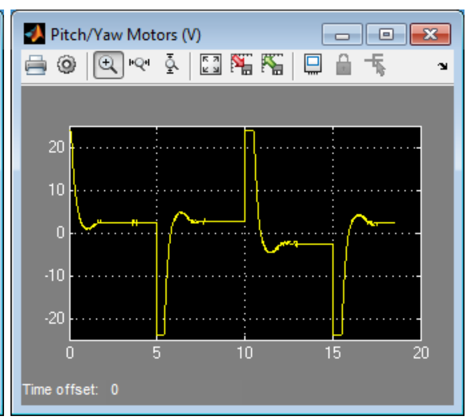
4. Build and run the *q_aero_half_quad_pd_control* QUARC controller to implement the PV controller.

5. Examine the obtained closed-loop response and see if it matches the desired specifications given in Section 3.1.

6. Sample scope response is given in Figure 3.5.



(a) Yaw Angle



(b) Pitch/Yaw Motor Voltage

Figure 3.5: Sample PV closed-loop response on Quanser Aero Experiment

7. If it does not match the desired specifications, then you can tune your PV control with the script by adjusting the desired peak time and overshoot specifications. To do this, adjust the t_p and PO parameters in the Matlab script, run the *quanser_aero_pv_design.m* script to generate a new gains, enter them in the **PIV Controller** blocks (or enter the Matlab parameter names) and run the QUARC controller again.

Response Analysis
The obtained response is shown in Figure 3.6.

The peak time, overshoot, and steady-state error of the response on the actual system are:

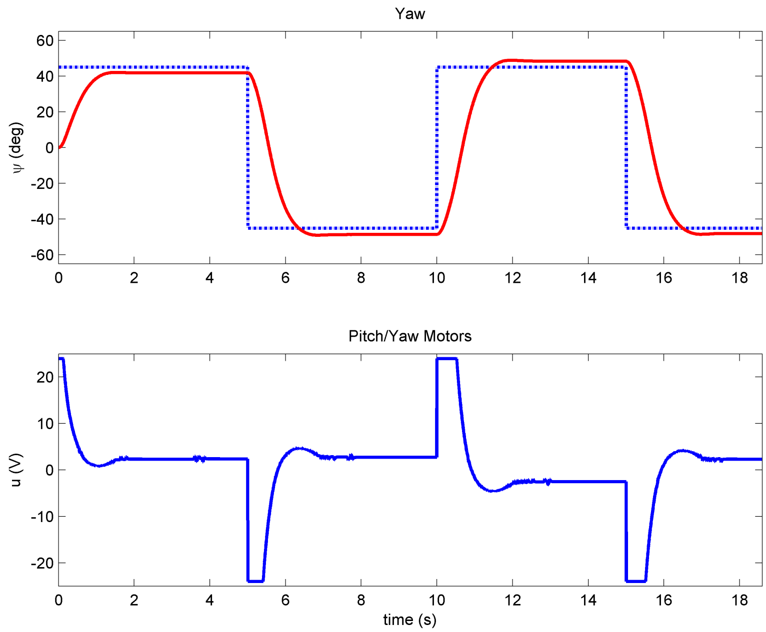


Figure 3.6: Sample PV closed-loop response on half-quadrotor

1. Steady-state error: $e_{ss} = 48.3 - 45 = 3.3 > 2$ deg.
2. Peak time: $t_p = 11.9 - 10 = 1.9$ s < 2s.
3. Percent Overshoot: $PO = 0$% < 7.5% .
4. Actuator saturation: $|V_y| \leq 24$V and $|V_p| \leq 24$V.

The response matches all the control requirements given above except for the steady-state error. Compared to the simulated response in Figure 3.3, the response on the actual system has steady-state error and less overshoot. Both of these observations are probably due to the unmodeled friction (e.g. Coulomb friction) about the yaw axis. Increasing the proportional gain, introducing an integrator, or using a more advanced friction-compensation scheme could minimize this. Lastly, the motors do get saturated but only for a short instant so this is still acceptable. The saturation blocks prevent higher voltages as well.