Control Systems

Lab 6

Observer Design

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1 Objective

To design a full order state observer, given system matrices A,B,C and input u

2 Theory

A device (or a computer program) that estimates the state variables is called a *state observer*. If the state observer estimates all the state variables of the system, regardless of whether some state variables are available for direct measurement, it is called a *full-order state observer*.

3 Procedure

State equation

$$\dot{x} = Ax + Bu$$

Output equation

$$y = cx$$

Here, x is the state variable, y is the measured output. A, B, C are state transition matrix, input matrix and output matrix respectively. The estimated state equation

$$\dot{\hat{x}} = A\hat{x} + bu$$

If the difference between the measured output and the estimated output is fed back to the system, we can speed up the estimation process and provide a useful state estimate. This method is known as **Luenberger state observer**. Equation for it is :

$$\dot{\hat{x}} = A\hat{x} + bu + m(y - \hat{y})$$

where m is an n x 1 real constant gain matrix. The state error vector

$$\tilde{x} = x - \hat{x}$$

Differentiating both sides,

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

Hence,

$$\dot{\tilde{x}} = Ax + bu - A\hat{x} - bu - mc(x - \hat{x}) = (A - mc)\tilde{x}$$

The characteristic equation of the error is given by

$$|sI - (A - mc)| = 0$$

The transposed auxiliary system is given by

$$\dot{\zeta}(t) = \mathbf{A}^T\zeta(t) + \mathbf{c}^T\eta(t)$$

$$\eta(t) = -\mathbf{m}^T\zeta(t)$$

- U matrix can be calculated as

$$\mathbf{U} = [\mathbf{c}^T \quad \mathbf{A}^T\mathbf{c}^T \quad (\mathbf{A}^T)^2\mathbf{c}^T \quad (\mathbf{A}^T)^3\mathbf{c}^T]$$

- Calculate P matrix where p1 is the last row of U^{-1}

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_1(\mathbf{A}^T) \\ \mathbf{p}_1(\mathbf{A}^T)^2 \\ \mathbf{p}_1(\mathbf{A}^T)^3 \end{bmatrix}$$

- m^T is obtained using this equation:

$$\mathbf{m}^T = [a_4 - \alpha_4 \quad a_3 - \alpha_3 \quad a_2 - \alpha_2 \quad a_1 - \alpha_1]\ \mathbf{P}$$

- With this **m**, the observer

$$\dot{\hat{x}} = (\mathbf{A}\text{-}\mathbf{mc})\hat{x} + \mathbf{b}u + \mathbf{m}y$$

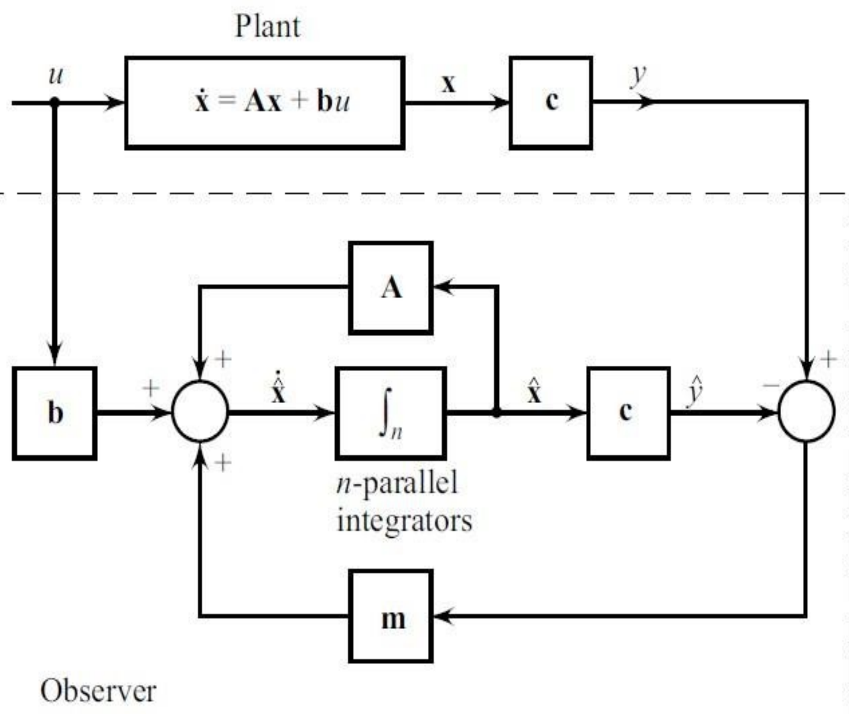

Figure 1: Full Order State Observer

4 MATLAB Code

Editor - C:\Users\DELL\Documents\MATLAB\Control Systems Lab\Lab 6\Matrix_Calc.m

Matrix_Calc.m B22EE024.m

```matlab
function [m] = Matrix_Calc(A, B, C, P0)
    % Calculate the observer matrix 'm'.

    n = size(A, 1);
    U = zeros(n, n);

    % Build the observability matrix U
    for i = 1:n
        U(:, i) = (transpose(A))^(i-1) * (transpose(C));
    end

    % Compute the inverse of U
    U_inv = inv(U);
    P1 = U_inv(n, :);
    P = zeros(n, n);

    % Build the observer polynomial matrix P
    for i = 1:n
        P(i, :) = P1 * (transpose(A)^(i-1));
    end

    eig_A = eig(transpose(A));
    A_coeff = poly(eig_A);
    P_coeff = poly(P0);
    Diff_Matrix = zeros(1, n);

    % Calculate coefficient differences
    for i = 1:n
        Diff_Matrix(:, 5-i) = P_coeff(i+1) - A_coeff(i+1);
    end

    % Compute the observer matrix 'm_trans'
    m_trans = Diff_Matrix * P;
    m = transpose(m_trans);

    % Display matrices and results
    disp("A: "); disp(A);
    disp("B: "); disp(B);
    disp("C: "); disp(C);
    disp("U: "); disp(U);
    disp("P: "); disp(P);
    disp("A_coeff: "); disp(A_coeff);
    disp("P_Coeff: "); disp(P_coeff);
    disp("m: "); disp(m);
end
```

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Matrix_Calc.m | B22EE024.m

```matlab
A = [0 1 0 0; 16.3106 0 0 0; 0 0 0 1; -1.0637 0 0 0];
B = [0; -1.4458; 0; 0.9639];
C = [0 0 1 0];
P0 = [-2 -2+1j -2-1j -3];

[m] = Matrix_Calc(A,B,C,P0);
```

Calling the Function to calculate the m matrix

5 MATLAB Code's Output

```
>> B22EE024
A:
         0    1.0000         0         0
   16.3106         0         0         0
         0         0         0    1.0000
   -1.0637         0         0         0

B:
         0
   -1.4458
         0
    0.9639

C:
     0     0     1     0

U:
         0         0   -1.0637         0
         0         0         0   -1.0637
    1.0000         0         0         0
         0    1.0000         0         0

P:
         0   -0.9401         0         0
   -0.9401         0         0         0
         0  -15.3338         0    1.0000
  -15.3338         0    1.0000         0

A_coeff:
    1.0000   -0.0000  -16.3106         0         0

P_Coeff:
     1     9    31    49    30

m:
 -184.0701
 -753.6564
    9.0000
   47.3106
```

6 Calculation

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where, $A = \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0 & 0.2 & 0.2 & 0 \\ 0 & 0.5 & 0.2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0.5 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$$C = \begin{bmatrix} 0.5 & 0.2 & 1 & 0 \end{bmatrix}$$

Observer pole locations : $-0.3 \pm 0.5j$, -1, -1.5

$\therefore$ Corresponding characteristic equation:

$$s^4 + 3.1s^3 + 3.34s^2 + 1.75s + 0.51 = 0$$

$\therefore a_1 = 3.1$, $a_2 = 3.34$, $a_3 = 1.75$, $a_4 = 0.51$

Controllability matrix

$$U = \begin{bmatrix} C^T & A^TC^T & (A^T)^2C^T & (A^T)^3C^T \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.2 & 0.79 & 0.528 & 0.4586 \\ 1 & 0.24 & 0.2060 & 0.1468 \\ 0 & 1 & 1.24 & 1.446 \end{bmatrix}$$

$$\Rightarrow U^{-1} = \begin{bmatrix} -5.0731 & 0.6574 & 3.4051 & 1.2 \\ -31.3268 & 8.2267 & 14.0181 & 6.8 \\ 117.4728 & -22.4254 & -54.2513 & -28 \\ -79.0729 & 13.5414 & 36.8282 & 20 \end{bmatrix}$$

Now, P_1 = last row of matrix U^{-1}

$$\Rightarrow P_1 = \begin{bmatrix} -79.0729 & 13.5414 & 36.8282 & 20 \end{bmatrix}$$

$$P = \begin{bmatrix} P_1 \\ P_1(A^T) \\ P_1(A^T)^2 \\ P_1(A^T)^3 \end{bmatrix} = \begin{bmatrix} -79.0729 & 13.5414 & 36.8282 & 20 \\ -72.3022 & 10.0739 & 34.1363 & 20 \\ -67.2653 & 8.842 & 31.8642 & 20 \\ -62.8442 & 8.1413 & 30.7939 & 20 \end{bmatrix}$$

$$|SI - A^T| = S^4 + \alpha_1 S^3 + \alpha_2 S^2 + \alpha_3 S + \alpha_4$$

$$= S^4 - 2.4S^3 + 1.74S^2 - 0.28S - 0.06$$

$$\Rightarrow \alpha_1 = -2.4 \quad \alpha_2 = 1.74, \ \alpha_3 = -0.28, \ \alpha_4 = -0.06$$

$$\therefore m^T = [a_4 - \alpha_4 \quad a_3 - \alpha_3 \quad a_2 - \alpha_2 \quad a_1 - \alpha_1] P$$

$$= [-645.1128 \quad 87.0928 \quad 310.6378 \quad 194]$$

$$\dot{\hat{x}} = (A - mc)\hat{x} + bu + my$$

m = observer gain.

7 Conclusion

- **Effectiveness of State Observers:** The experiment demonstrated the practical utility of full-order state observers. These observers proved to be valuable tools for estimating unmeasured states within dynamic systems. Engineers and students alike learned that state observers play a critical role in enhancing control and monitoring strategies, improving overall system performance.

- **Validation of Theoretical Foundations:** The consistency between manually calculated results and those generated by the code emphasized the importance of solid theoretical knowledge. This experiment underscored the need for a strong understanding of underlying mathematical principles in engineering applications. It served as a real-world validation of theoretical concepts in control and estimation.

- **Automation's Role in Efficiency:** The code used in the experiment showcased the power of automation in engineering. By automating the complex task of observer matrix calculations, the code not only reduced the potential for errors but also expedited the design process. This experience highlighted the efficiency gains achievable through computational tools and programming, a valuable lesson for engineers and students.

Overall, the lab experiment provided a hands-on opportunity to apply theoretical knowledge to practical engineering challenges. It reinforced the importance of state observers in control systems, the significance of theory in real-world applications, and the advantages of automation for efficiency and accuracy in engineering tasks.