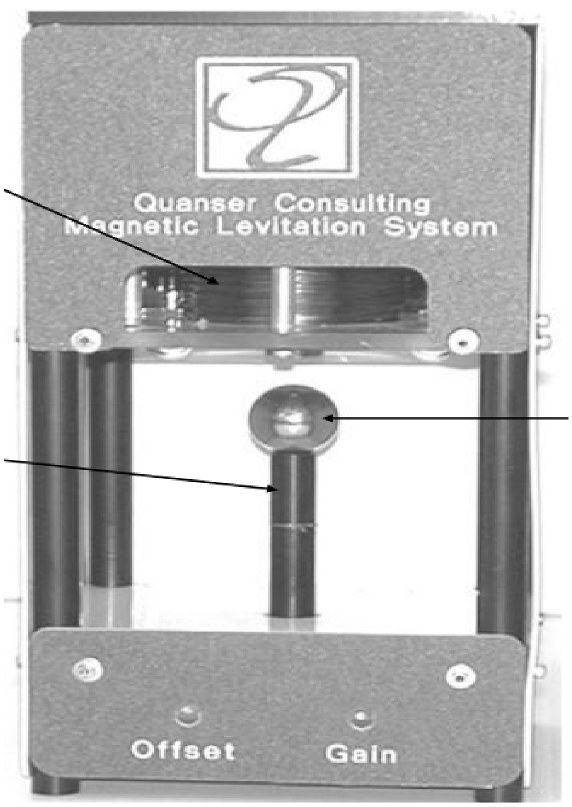
1. Objective

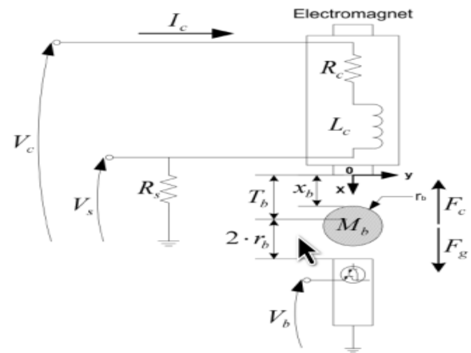
(a)Aim- The objective of this experiment is to model the magnetic levitation (MAGLEV) plant and to design a controller that levitates the ball from post and ball position tracks a desired trajectory.

(b)Software- MATLAB, Magnetic levitation kit, Q8-USB, UPM-2405 amplifier.

(c)Theory- The “MAGLEV” experiment consists of an electromagnet encased in a rectangular enclosure. One electromagnet pole faces a black post upon which a 2.54cm diameter steel ball rests. The ball elevation from the post is measured using a sensor embedded in the post. The post is designed such that with the ball at rest on its surface, it is 14 mm from the face of the electromagnet.



2. Mathematical Model



(a) Electrical System-The coil used in the electromagnet has an inductor and a resistance. The voltage applied to the coil results in a current governed by the differential equation.

$$v(t) = i(t)(R_c + R_s) + L\frac{di(t)}{dt} \qquad \textbf{(i)}$$

Applying laplace transform on equation-1

$$G_c(s) = \frac{I(s)}{V(s)} = \frac{K_{c_{dc}}}{\tau_c s + 1}$$

$$\text{Where,} \quad K_{c_{dc}} = \frac{1}{R_c + R_s}, \quad \tau_c = \frac{L_c}{R_c + R_s}$$

(b)Mechanical System-The force due to gravity applied to the ball is expressed by:

$$F_C + F_g = -\frac{1}{2}\frac{K_m I_c^2}{x_b^2} + M_b g \qquad \textbf{(ii)}$$

Applying Newton's second law of motion

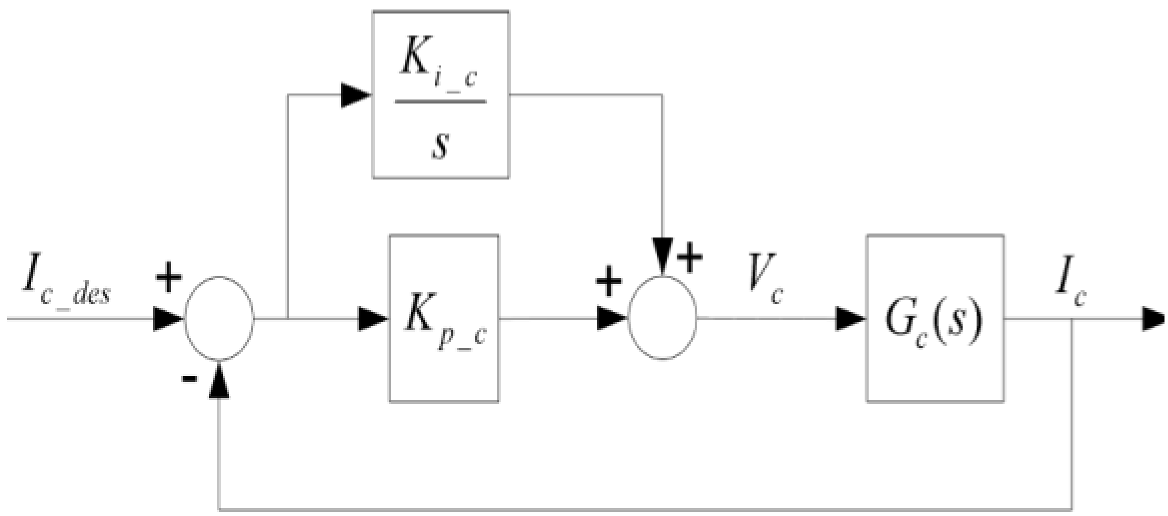
$$\frac{d^2 x_b}{dt^2} = g - \frac{1}{2}\frac{K_m I_c^2}{x_b^2 M_b}$$

At equilibrium: $F_c + F_g = 0$

$$I_{c0} = \sqrt{\frac{2M_b g}{K_m}}\, x_{b0} \qquad \textbf{(iii)}$$

3. Controller Design

(a)Coil current controller design: pole placement-Prior to control the steel ball position, the current flowing through the electromagnet needs to be controlled. The electromagnet current control loop consists of a Proportional-plus-Integral (PI) closed-loop scheme.



Transfer Function: $T_c(s) = \frac{I_c(s)}{I_{des}(s)}$

Poles: $P_{c1,2} = - 235 \pm 70i$

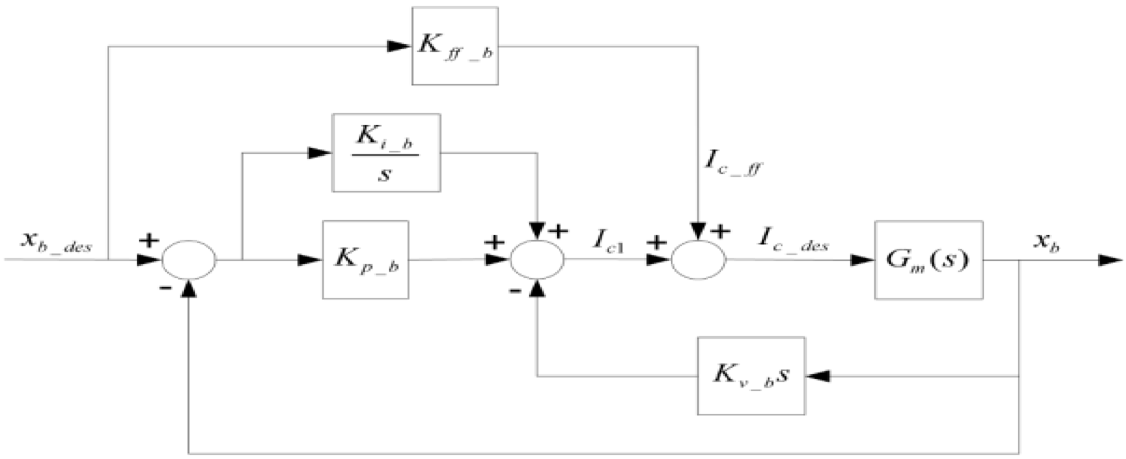
$$s^2 - (P_{c1} + P_{c2})s + P_{c1}P_{c2} = 0 \qquad \textbf{(iv)}$$

$$s^2 + 2\omega_n\zeta s + \omega_n^2 = 0 \qquad \textbf{(v)}$$

Comparing equation 4 and 5 we will get teh values of K_{pc} and K_{ic}.

The calculation can be verified in calculation sections below this section.

(b) Ball Position Controller Design: Pole Placement- The steel ball position is controlled by means of a Proportional-plus-Integral-plus-Velocity (PIV or PID) closed-loop scheme with the addition of a feed-forward action.



Transfer Function: $T_b(s) = \frac{x_b(s)}{x_{bdes}(s)}$

Poles: $P_{b1,2,3} = -2.5, -44, -51.6$

$$s^3 - (\sum_{i=1}^{3} P_{bi})s^2 + (\sum_{i,j}^{3} P_{bi}P_{bj})s - P_{b1}P_{b2}\,P_{b3} = 0 \qquad \textbf{(vi)}$$

$$s^3 - \frac{2gK_{vb}s^2}{I_{c0}} - (\frac{2g}{x_{b0}} + \frac{2gK_{pb}}{I_{c0}})s - \frac{2gK_{ib}}{I_{c0}} = 0 \qquad \textbf{(vii)}$$

On comparing the above two quation we will get the values of K_{vb}, K_{pb}, K_{ib}

And we know that, $K_{ff} = \frac{I_{c0}}{x_{b0}}$

Where $x_{b0} = 6mm$, and I_{c0} can be calculated from equation-3.

(c) Calculations:

Electrical System

Electrical Sys:-

$$G_c(s) = \frac{I(s)}{V(s)} = \frac{K_{dc}}{\tau_c(s)+1} \quad \left(K_{dc} = 1/R_c+R_s \qquad \tau = LC/R_c+R_s \right)$$

$$K_{d(s)} = \frac{1}{R_c+R_s} = \frac{1}{10+1} = 0.909$$

$$\tau = \frac{LC}{R_c+R_s} = \frac{(412.5)}{1000} \cdot (0.909) = 0.0375$$

$$G_c(s) = \frac{0.0909}{(0.0375)s+1} = \frac{1}{0.4125s + 11.0011}$$

Mechanical and ElectroMechanical Systems

Mech. Sys.

$$F_c + F_g = -\frac{1}{2}\frac{K_m I_c^2}{x_b^2} + M_b g$$

$$\ddot{x}_b = g - \frac{1}{2}\frac{K_m I_c^2}{x_b^2 M_b}$$

$x_b = x_{b_1} + x_{b_0}$, $I_c = I_{c_0} + I_{c_1}$

$F_c = -F_g$ (Equillibrium) , $x_{b0} = 6\times10^{-3}$m

$$I_{c0} = \sqrt{\frac{2M_b g}{K_m}} \cdot x_{b0}$$

($M_b = 0.068$ Kg)

($g = 9.81$ m/s²)

$$\Rightarrow \boxed{I_{c0} = 0.8572 \text{ A}}$$

Electromech. model

$$\frac{\partial^2}{\partial t^2} x_{b_1} = -\frac{1}{2}\frac{K_m I_{c0}^2}{M_b x_{b0}^2} + g + \frac{K_m I_{c0}^2 x_{b_1}}{M_b x_{b0}^3} - \frac{K_m I_{c0} I_{c}}{M_b x_{b0}^2}$$

$$\frac{x_{b_1}(s)}{I_{c_1}(s)} \text{ or } G_{b_1}(s) \text{ for } K_b, \; \frac{\omega_b^2}{s^2 - \omega_b^2}$$

$$K_{bdc} = \frac{x_{b0}}{I_{c0}} = \frac{6\times10^{-3}}{0.85762} = 6.996\times10^{-3}$$

$$\boxed{K_{bdc} = 6.996\times10^{-3}}$$

$$\omega_b = \sqrt{\frac{2g}{x_{b0}}}$$

$$\boxed{\omega_b = 57.1834}$$

$P_{c_{1,2}} = -235 \pm 70i$

$$s^2 - (P_{c_1} + P_{c_2})s + P_{c_1}P_{c_2} = 0.$$

Block Diagram

Block diagram:-

$G_f(s) = I_c / I_{cdes}$

$I_{cdes} - I_c = (K_p + K_i/s)\,(G(s))$

$I_{cdes} = (I_{des} - I_c)\,(K_p + \frac{K_i}{s})\,G(s)$

$(I_{cdes} - I_c)\,(\frac{K_{ic}}{s} + K_{pc}) = V_c$

$I_c = (\frac{K_i}{s} + K_p)\,G(s)$

$G_c(s) = \frac{I_c}{V_c} = \frac{G(s)}{I_{cdes} - I_c}$

$\boxed{I_{cdes} = I_1 + 1}$ . . . (i)

Using eq (i)

$1 + (K_p + \frac{K_i}{s})\,(G(s)) = 0$

putting $(P_1 \& P_2)$ in $G(s)$

$\left[1 + \left(K_p + \frac{K_i}{-235+70i}\right)\left(\frac{1}{0.415(-235+70i) + 11.011}\right)\right] = 0$

$\left[1 + \left(K_p + \frac{K_i}{-235-70i}\right)\left(\frac{1}{0.415(-235-70i) + 11.011}\right)\right] = 0$

Solving above eqn.

$$\boxed{\begin{aligned} K_{pc} &= 182.876 \\ K_{ic} &= 24801.56 \end{aligned}}$$

Controller Design

Controller design.

$T_b(s) = x_b(s) / x_{b\,des}(s)$

$P_{1,2,3} = (-2.5, -44, -51.6)$ rad/s

$\Rightarrow s^3 (\sum_{i=1}^{3} P_{bi}) s^2 + (\sum_{i \neq j} P_{bi} P_{bj}) s + \prod_{i=1}^{3} P_{bi} = 0$

$\Rightarrow s^3 - \frac{2g K_{vb} s^2}{I_{co}} - \left(\frac{2g}{x_{b0}} + \frac{2g k_{pb}}{I_{co}} \right) s - \frac{2g K_{ib}}{I_{co}} = 0$

Comparing above eqn

$-98.1 = \frac{2(9.81) K_{vb}}{0.857} \Rightarrow \boxed{K_{vb} \simeq 4}$

$2.5 \times 44 + 51.6 \times 44 + 51.6 \times 2.5 = -\frac{2(9.81)}{6mm} - \frac{2(9.81) K_{pb}}{0.857}$

$\boxed{K_{pb} = -252.81}$

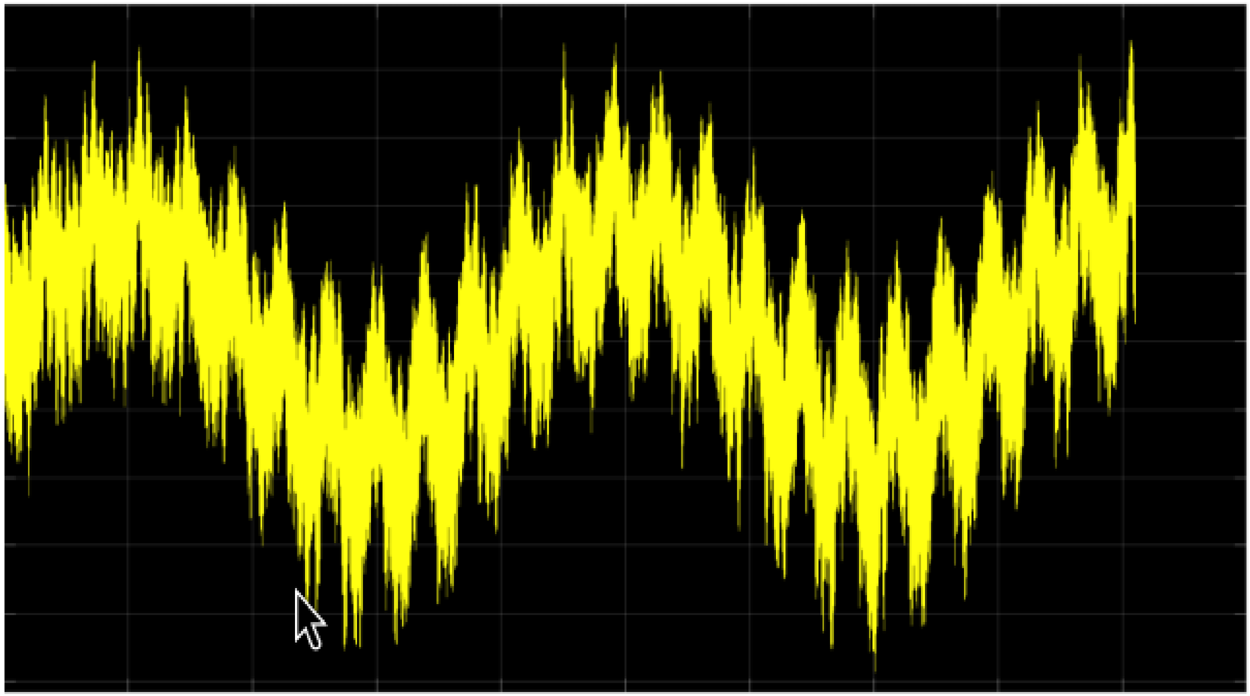
$-2.5 \times 44 \times 51.6 = -\frac{2 \times 9.81 \times K_{ib}}{0.857}$

$\boxed{K_{ib} = -248.09}$

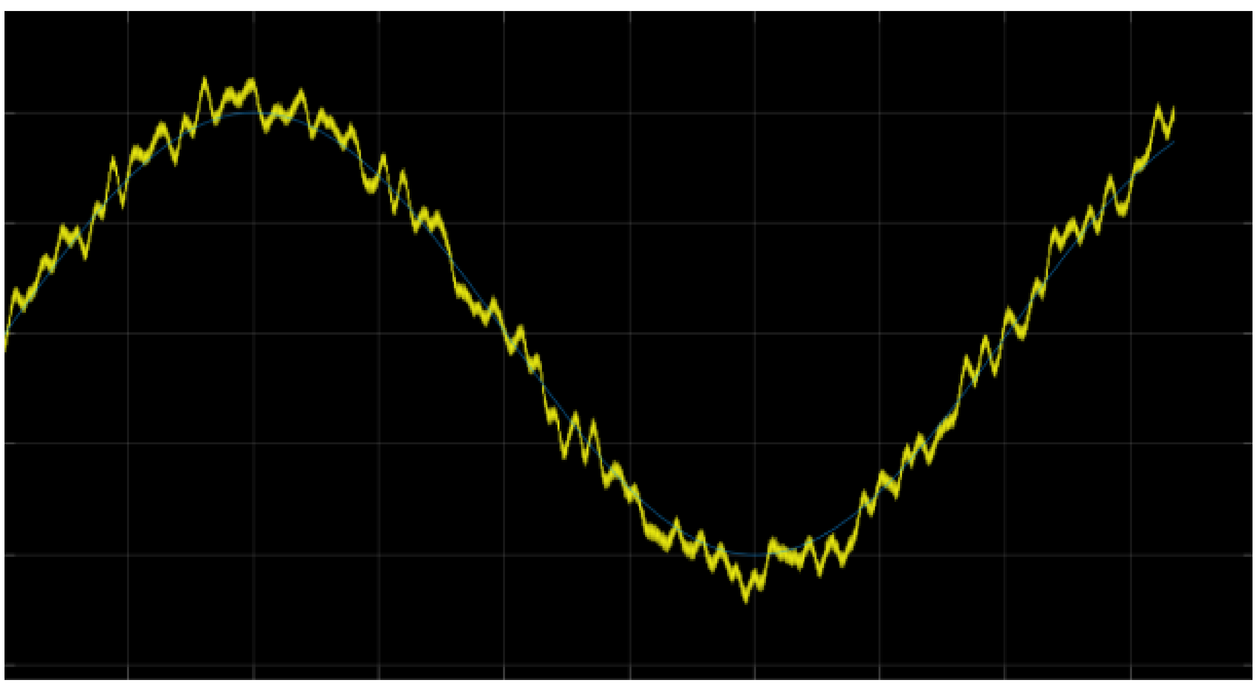
$K_{ff} = \frac{I_{co}}{x_{b0}} = \frac{0.857}{6 \times 10^{-3}}$

$\boxed{K_{ff} = 142.936}$

(d)Graphs



Current(I_c)



Displacement(x_b)

4. Assignment

(a) Important Formula's

$$PO = exp(-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi)100 \quad , \quad T_s = \frac{4}{\zeta\omega_n}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad , \quad P_{c1,2} =- \zeta\omega_n \pm \sqrt{\zeta^2 - 1}$$

$$T_r = \frac{\pi-\beta}{\omega_d} \quad , \quad \beta = tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})$$

(b) Controller gains : Value's Table

Controller Gain	Values
K_{pc}	182.876
K_{ic}	24801.56
K_{ffb}	142.9336
K_{pb}	-252.61
K_{ib}	-248.09
K_{vvb}	-4.24828

(c) Conclusion:

The laboratory experiment results demonstrate that by determining and utilizing the necessary constants for the control system, we can achieve precise positioning of the ball (x_b) within the system, even when varying the desired position. This remarkablecapability is a testament to the effectiveness of control systems in minimizing errors and ensuring accurate positioning.