EEL3040

Control System

Lab-9 Report

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Contents

1. Objective

(a) Aim:

To design and implement a state-feedback control system that will balance the pendulum in the upright, vertical position.

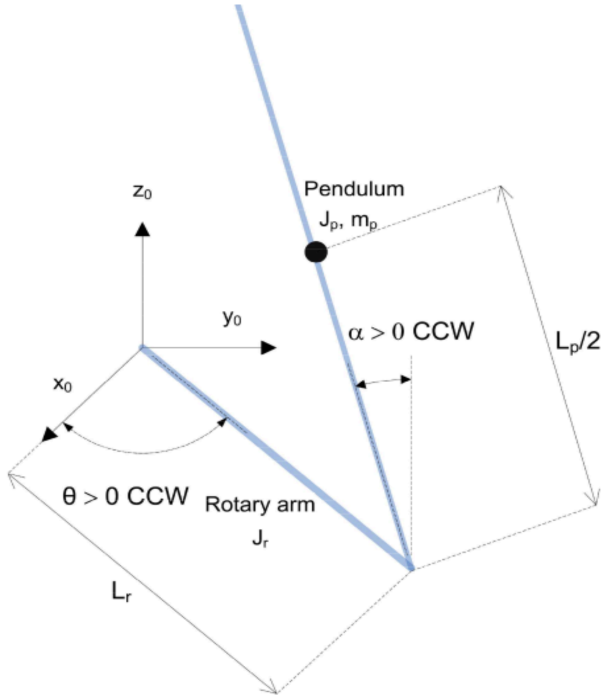
(b) Software:

Quanser SRV02-ET rotary servo, Quanser Rotary Pendulum Module (attached to SRV02), Quanser VoltPAQ power amplifier, Data-acquisition (DAQ) card, MATLAB.

2. Theory

(a) Description

The Coupled Tanks system is a re-configurable process control experiment. It can be used to demonstrate the level control process on a small scale. The coupled tank system is widely used in control theory. The control of liquid levels of significant importance in the industry and the research domain.



Useful equation that will be used.

$$x = Ax + Bu$$

$$y = Cx + Du$$

$$x^T = [\theta\ \alpha\ \theta\ \alpha]$$

$$y^T = [x_1\ x_2]$$

4. Balance Control

(a) Stability

The stability of a system can be determined from its poles

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$dets(sI - A) = 0$$

where det() is the determinant function, s is the Laplace operator, and I the identity matrix. These are the eigenvalues of the state-space matrix A.

(b) Controllability

If the control input u of a system can takes each state variable, xi where i = 1 . . . n, from an initial state to a final state then the system is controllable, otherwise it is uncontrollable.

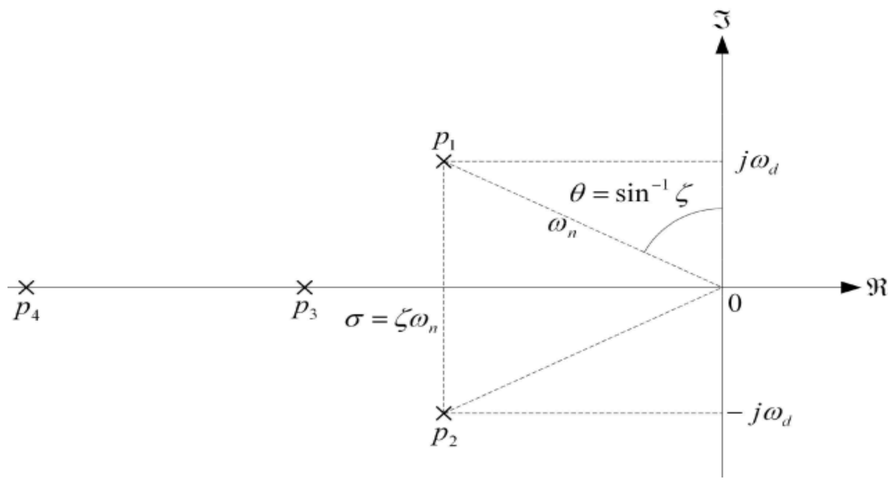
(c) Desired Poles

The inverted pendulum system has four poles. Poles p1 and p2 are the complex conjugate dominant poles and are chosen to satisfy the natural frequency, ω_n , and damping ratio, ζ. Let the conjugate poles be

$$p_1 = -\sigma + j\omega_d$$
$$p_2 = -\sigma - j\omega_d$$

Where $\sigma = \zeta\omega_n \qquad \omega_d = \omega_n\sqrt{1 - \zeta^2}$

The remaining closed-loop poles, p3 and p4, are placed along the real-axis to the left of the dominant poles.



(d) MATLAB Code

```
A = [0 0 1 0; 0 0 0 1; 0 80.3 -45.8 -0.930; 0 122 -44.1 -1.40];
B = [0; 0; 83.4; 80.3];
dim_A = size(A, 1);
dim_B = size(B, 1);

A_eig=eig(A);
Qc = ctrb (A, B);
k = rank (Qc);

poles_Desired = [ -2.8+2.86j -2.8-2.86j -30 -40];
char_poly_desired = poly(poles_Desired);
char_poly_desired_flp = fliplr (char_poly_desired);
char_poly_Sys = poly(A_eig);
char_poly_Sys2 = char_poly_Sys(:,2:5);

A_bar = [0 1 0 0; 0 0 1 0; 0 0 0 1; -1 * flip(char_poly_Sys2)];
B_bar = [0; 0; 0; 1];
Diff_matrix = A_bar - B_bar;

updated_char_poly = -1 * Diff_matrix(dim_A, :);
updated_char_poly(1) = 0;
K_bar = zeros(size (char_poly_desired));

for i = 1:length(char_poly_desired) - 1
    K_bar(i) = char_poly_desired_flp(i) + updated_char_poly(i);
end

K_bar = K_bar(1:end-1);

% Construct the augmented controllability matrix Qc_bar
Qc_bar = [B_bar A_bar*B_bar A_bar*A_bar*B_bar A_bar*A_bar*A_bar*B_bar];
Qc_bar_Inverse = inv(Qc_bar);
W = Qc* Qc_bar_Inverse;
K_bar = [19200 9843 1707 28.4];

% Compute the inverse of W
W_inverse = inv(W);

% Calculate the final control gain matrix K
K = K_bar* W_inverse;

% Display the values of Qc, poles_Desired, Qc_bar, W, K_bar, and K
disp('Controllability Matrix Qc:');
disp(Qc);
disp('Desired Poles:');
disp(poles_Desired);
disp('Controllability Matrix (for companion matrices A_bar & B_bar) Qc_bar:');
disp(Qc_bar);
disp('Transformation Matrix W:');
disp(W);
disp('K_bar:');
disp(K_bar);
disp('K:');
disp(K);
```

(d) MATLAB Code’s Output

```
Controllability Matrix Qc:
   1.0e+06 *

         0    0.0001   -0.0039    0.1883
         0    0.0001   -0.0038    0.1868
    0.0001   -0.0039    0.1883   -9.1039
    0.0001   -0.0038    0.1868   -9.0297

Desired Poles:
  -2.8000 + 2.8600i  -2.8000 - 2.8600i -30.0000 + 0.0000i -40.0000 + 0.0000i

Controllability Matrix (for companion matrices A_bar & B_bar) Qc_bar:
   1.0e+05 *

         0         0         0    0.0000
         0         0    0.0000   -0.0005
         0    0.0000   -0.0005    0.0233
    0.0000   -0.0005    0.0233   -1.1244

Transformation Matrix W:
   1.0e+03 *

   -3.7267    0.0421    0.0834   -0.0000
   -0.0000   -0.0002    0.0803   -0.0000
    0.0000   -3.7267    0.0421    0.0834
    0.0000   -0.0000   -0.0002    0.0803
```

```
K_bar:
   1.0e+04 *

    1.9200    0.9843    0.1707    0.0028

K:
   -5.1520   28.0319   -2.7009    3.1588
```

4. Calculation and Questions

(1) pot energy

$$E_p = \frac{1}{2} m_p g l_p \left(1 - \cos\alpha \right)$$

$$E_{Upr} = \frac{1}{2} m_p g l_p \left[1 - \cos 0 \right] \quad [\theta = 0]$$

$$E_p = 0.$$

$$\text{downwards} = \frac{1}{2} m g l_p \left[1 - \cos(180) \right] \quad [\alpha = 180]$$

$$= m g l_p$$

$$U = 0.127 \times 9.81 \times 0.337$$

$$\boxed{U = 0.42}$$

(2)

$$V_m = \frac{R_m m_r L_r u}{\eta_g K_g \eta_m K_t} + K_g K_m \dot{\theta}$$

$$V_m - K_g K_m \dot{\theta} = \frac{R_m m_r L_u u}{\eta_g K_g \eta_m K_t}$$

$$u = \frac{\eta_g K_g \eta_m K_t}{R_m m_r L_u}\left[V_m - K_g K_m \dot{\theta}\right]$$

$$u = \frac{(0.9)(70)(0.9)(0.00768)}{(2.6)(0.257)(0.216)}(5)$$

$$u = \frac{1.669}{0.144}$$

$$\boxed{u = 11.595}$$

3) Contr. acc^n when pendulum is hanging down is zero. So we basically need an external force, initially to swing the pendulum up.

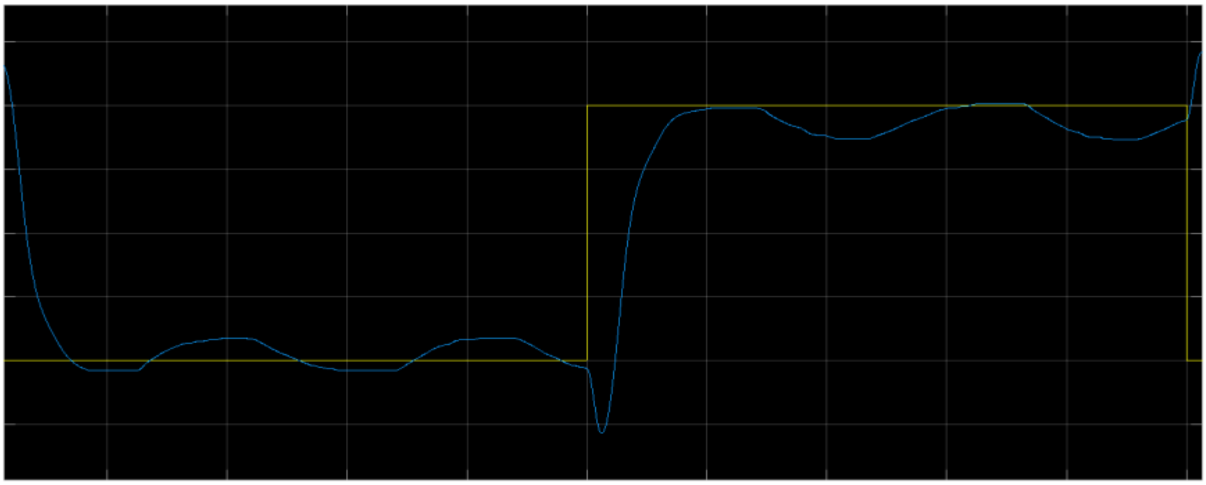
$$u = \text{satumax}\left(\mu(E - E_u)\, \text{sign}(\alpha \cos\alpha)\right)$$

$$= 20\,(0.42 - 0)\,\alpha\cos\alpha) = 8.4$$

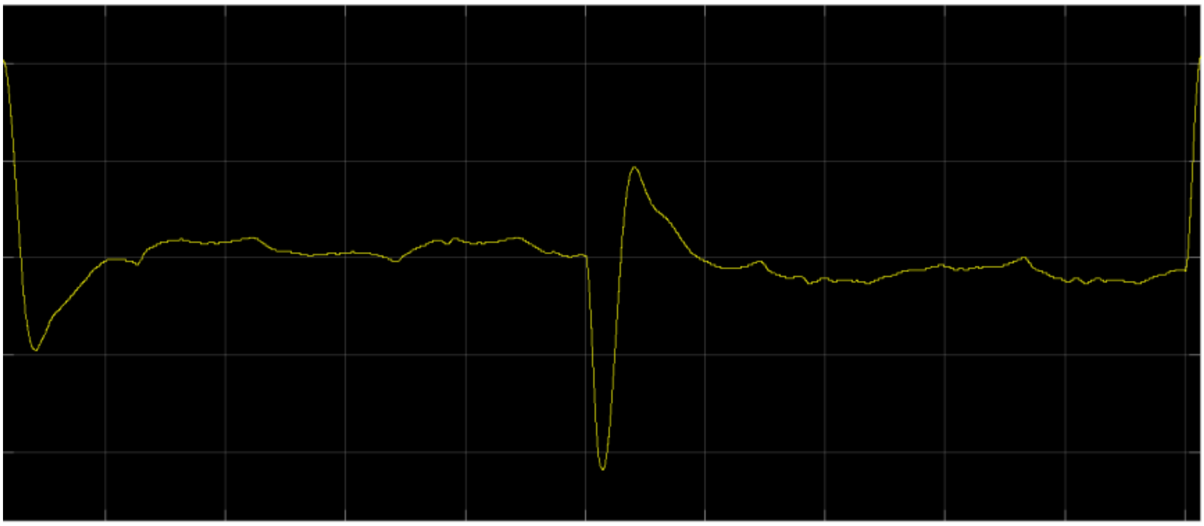
$$\boxed{u = 8.4}$$

max acc^n is 11.595 so, when $\mu = 20$ & $u = 8.4$ it will saturate the controller.

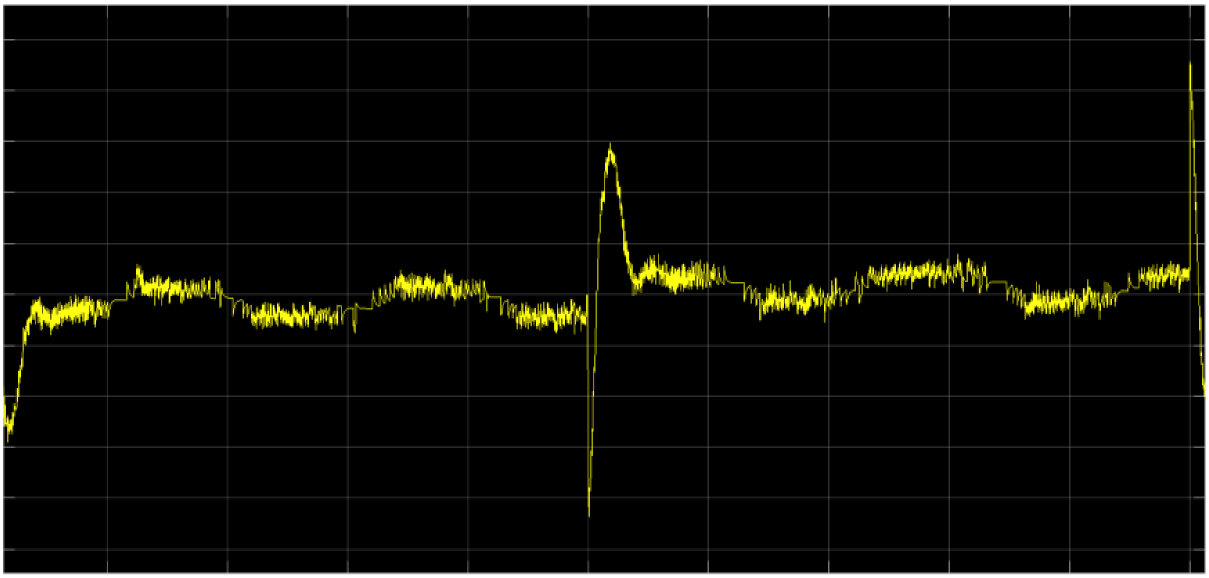
5. Plots



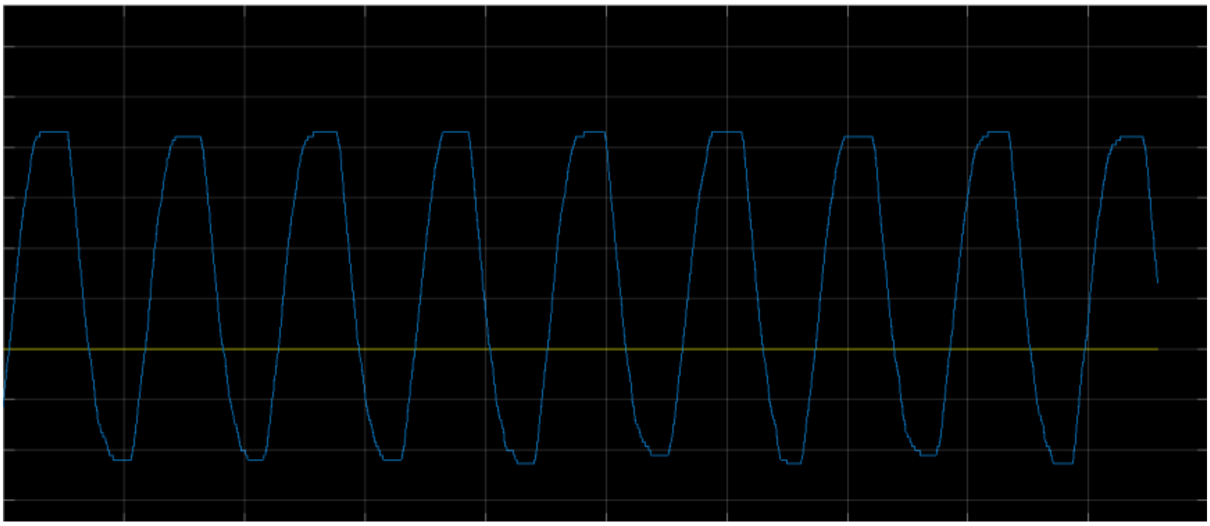
Case-1:θ



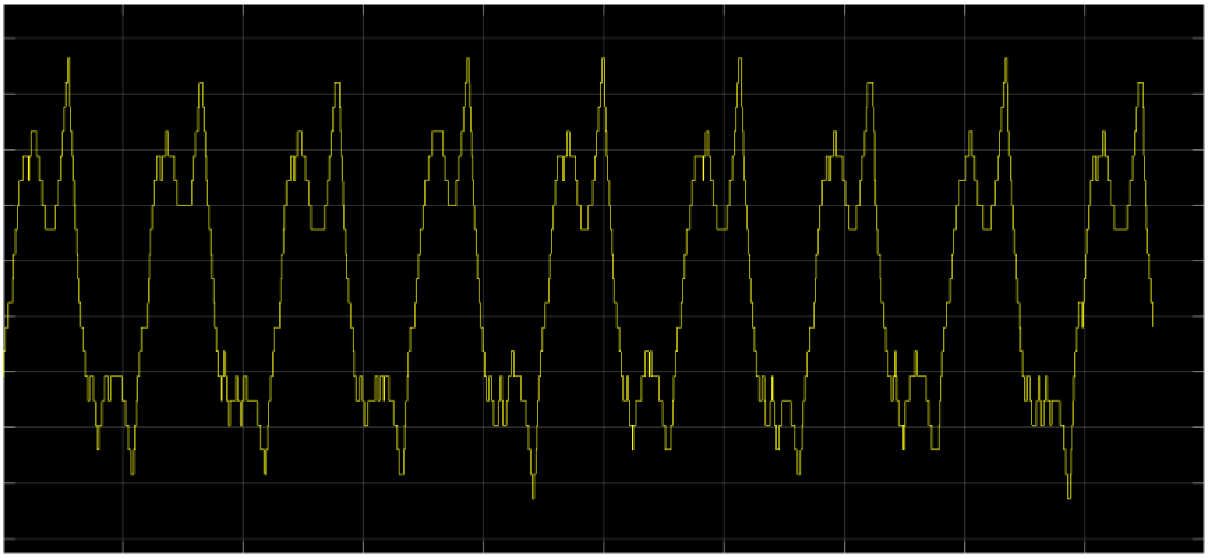
Case-1:α



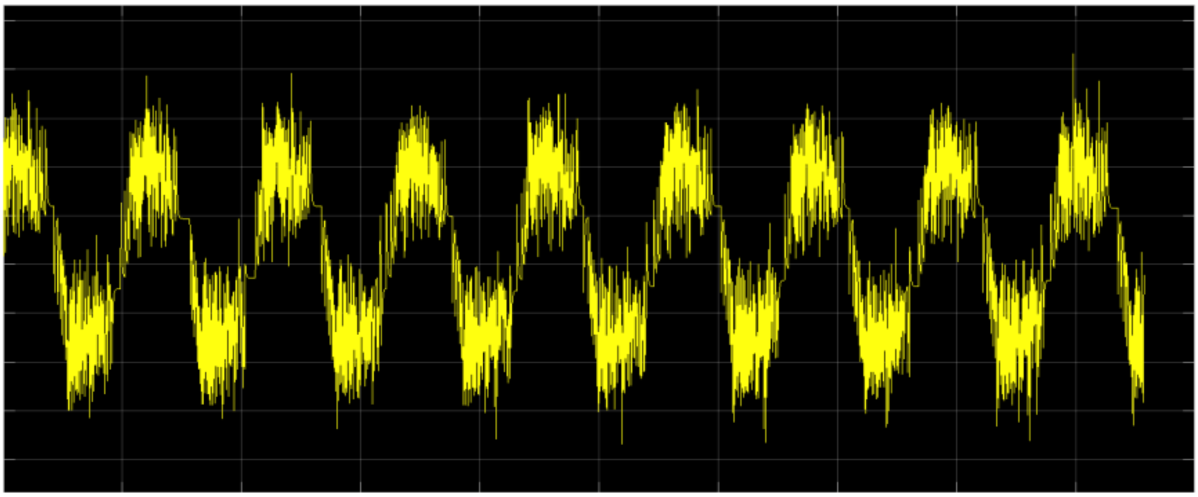
Case-1:V



Case-2:θ



Case-2:α



Case-2:V