Template-Guided Control for Legged Robots

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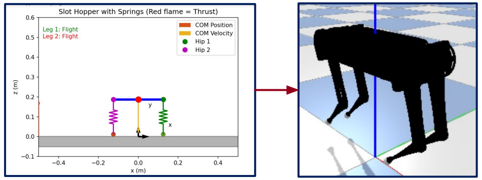
INTRODUCTION

- ***Motivation:*** Legged locomotion control is challenging due to high dimensionality, hybrid contact dynamics, and nonlinear body–leg interactions. One promising approach is template–anchor decomposition, where a low-dimensional template model captures essential locomotion mechanics and a higher-dimensional anchor robot implements the behavior.
- ***Objective:*** In this project, I implement the Slot Hopper as a reduced-order dynamic template and construct a kinematic anchoring map that transfers its COM motion, pitch dynamics, and leg actuation to a 2-link-per-leg quadruped model, enabling the generation of desired gait behaviors.



METHODS

- ***Slot Hopper Template:*** Implemented a vertically-constrained hybrid locomotion template with a sliding-pinned body, pitch rotation, and two independently actuated prismatic legs governed by alternating stance and flight dynamics.

$$Q = [z \; \dot{z} \; \phi \; \dot{\phi} \; r_1 \; \dot{r}_1 \; r_2 \; \dot{r}_2]^\top$$

Hip Heights: $[z_1 = z + d\,\phi, \qquad z_2 = z - d\,\phi]$

Flight Dynamics: $[\ddot{z} = -g, \qquad \ddot{\phi} = 0, \qquad \ddot{r}_i = 10(\rho - r_i)]$

Single Stance: $[\ddot{z}_1 = u_1, \qquad \ddot{z}_2 = -g - \dfrac{1-\kappa}{1+\kappa}(u_1 + g)]$

$[\ddot{z}_2 = u_2, \qquad \ddot{z}_1 = -g - \dfrac{1-\kappa}{1+\kappa}(u_2 + g)]$

METHODS (Continued)

Double Stance: $[\ddot{z} = \dfrac{u_1 + u_2}{2}, \qquad \ddot{\phi} = \dfrac{u_1 - u_2}{2d\kappa}]$

- ***Controller:*** Implemented an active damping controller that regulates vertical energy and stabilizes pitch using phase-based thrust modulation, enabling smooth and periodic hopping.

Phase-Energy Cord (Stance): $[p_i = \begin{bmatrix} -\dot{z}_i \\ (\rho - z_i)\,\omega \end{bmatrix}, \quad a_i = \|p_i\|, \quad \psi_i = \operatorname{atan2}(p_{i,2}, p_{i,1})]$

Phase-Energy Cord (Flight): $[a_i = \sqrt{2g(z_i - \rho) + \dot{z}_i^2}, \qquad \psi_i = \dfrac{a_i - \dot{z}_i}{2a_i}]$

Vertical Regulation: $[v_i = -\beta\,\dot{z}_i \; - \; k_a \cos(\psi_i)]$

Attitude Regulation: $[w_i = -(-1)^i \left(k_p \phi + k_d \dot{\phi}\right)]$

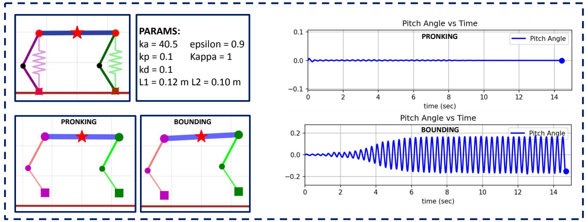
Total Thrust: $[u_i = \omega^2(\rho - z_i) \; + \; \epsilon\,(v_i + w_i)]$

- ***Anchoring:*** Deduced a kinematic anchoring scheme that transfers the template's COM and pitch to a planar two-link quadruped with fixed hip spacing. Template leg extensions are converted into foot targets, and closed-form 2-link inverse kinematics generates joint angles that reproduce the template's stance and flight geometry.

COM & Pitch: $[\begin{bmatrix} x^A_{\text{com}} \\ z^A_{\text{com}} \end{bmatrix} = \begin{bmatrix} 0 \\ z^T_{\text{com}} \end{bmatrix}, \qquad \phi^A = \phi^T]$

Anchor Hip Pos: $[h_i = \begin{bmatrix} x^A_{\text{com}} \\ z^A_{\text{com}} \end{bmatrix} + R(\phi^A) \begin{bmatrix} s_i\,d \\ 0 \end{bmatrix}, \qquad s_1 = +1, \;\; s_2 = -1] \qquad [x^A_{\text{foot},i} = x^A_{\text{hip},i}]$

Length Map: $[\ell^T_i(t) = z^T_{\text{hip},i}(t) - z^T_{\text{foot},i}(t)][z^A_{\text{foot},i}(t) = z^A_{\text{hip},i}(t) - \ell^T_i(t)] \qquad [f_i^{\text{target}} = \begin{bmatrix} x^A_{\text{hip},i} \\ z^A_{\text{foot},i} \end{bmatrix}]$



IK: $h_i = (x_h, z_h), \quad f_i^{\text{target}} = (x_f, z_f)][D = \sqrt{(x_f - x_h)^2 + (z_f - z_h)^2}]$

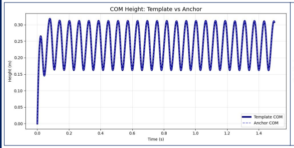
Theta(Knee): $[\theta_{k,i} = \pi - \cos^{-1}\left(\dfrac{L_1^2 + L_2^2 - D^2}{2L_1 L_2}\right)]$

Theta(Hip): $[\alpha = \operatorname{atan2}\left(x_f - x_h, \; -(z_f - z_h)\right)][\beta = \cos^{-1}\left(\dfrac{L_1^2 + D^2 - L_2^2}{2L_1 D}\right)][\theta_{h,i} = \alpha - \beta]$

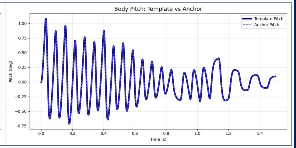
RESULTS

- ***Results:*** The anchored quadruped legs closely track the template foot placements during both stance and flight phases, and the joint angle profiles remain periodic, indicating consistent coordination between the front and rear limbs. Below are the trajectory plots for the bounding gait:

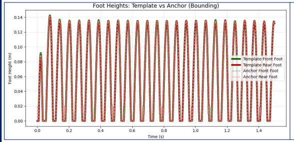
COM TRAJECTORY



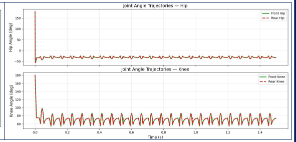
BODY PITCH



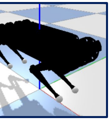
FOOT TRAJECTORY

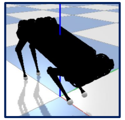


JOINT TRAJECTORY



3D SIMULATION RESULT ON THE MINI CHEETAH





FUTURE EXTENSION

- Extend to full dynamic anchoring for achieving closed-loop template-driven locomotion for diverse anchor designs. Test whole-body OSC in simulation and later can be extended to physical quadruped.

References: *De, Avik & Koditschek, Daniel. (2018). Vertical hopper compositions for preflexive and feedback-stabilized quadrupedal bounding, pacing, pronking, and trotting. The International Journal of Robotics Research. 37. 743-778. 10.1177/0278364918779874.*