Part 1_Ch 01_1 기초대수학

date	@2023/04/17
time	20:00
class	Monthly Task
category	Study
상태	☑
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Chapter 01. 기초대수학

1-1. Algebraic Properties

1) Commutative Property

연산의 object

a*b = b*a

ex) + , ×

ex). $10+3 = 3+10$ $\left[\begin{array}{l} 10-3 \neq 3-10 \\ 10\div3 \neq 3\div10 \end{array}\right]$ → Commutative Property 성립X

· $10\times3 = 3\times10$

2) Associative Property

(a*b)*c = a*(b*c)

ex) $(10+3)+2 = 10+(3+2)$

(×) $(10-3)-2 \neq 10-(3-2)$

3) Distributive Property

a*(b☆c) = (a*b)☆(a*c)

(b☆c)*a = (b*a)☆(c*a)

4) Identities and Inverses

- Identities(항등원): 어떤 값 과 연산(*)이 있을 때, (a) 이 값에 연산을 진행한 결과가 원래의 값과 동일하게 만드는 값

  a*e = a

  * : e는 *에 대한 identity

  ex) $a+e=a$

  $e=0$

  ex) $a*e=a$

  $e=\frac{a}{a}=1$

- Inverses(역원): 어떤 값(a)과 연산()*이 있을 때, 이 값에 연산을 진행한 결과 identity가 되게 만드는 값*

  *a**x = e

  * : x는 *에 대한 inverse

  ex) $a+x=e=0$ ← 덧셈에 대한 항등원 "0"

  $a+x=0$

  $x=-a$

  ex) $a*x=e=1$

  $a*x=1$

  $x=1/a=a^{-1}$

1-2. Sets : 집합

1) Definition

a collection of distinct and well-defined things(or elements)

↳ 사람에 따라 기준이 달라지지X

↳ ex). 숫자 {1/3, 1, 4, 3, 0.5 ...}

· 특정 txt , image ...

· 다양한 dataset이 들어갈 수 있음.

2) Notations

- Enumerating Elements(Roster Form) : 원소나열법

$$\text{Set} = \{\text{element}_1, \text{element}_2, \ldots, \text{element}_n\}$$

$$A = \{1, 2, 3, 4\}$$

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$C = \{red, green, blue\}$$

$$D = \{free, guarantee, help, prices, winner, chance\}$$

- Set Builder : 조건 제시

$$\text{Set} = \{\text{element} \mid \text{element's condition}\}$$

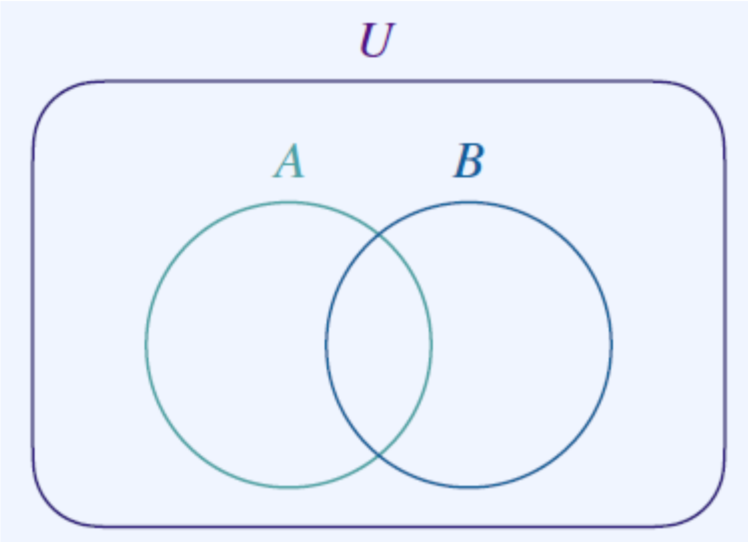
$$A = \{x \mid 1 \le x \in \mathbb{N} \le 4\}$$

$$B = \{x \mid x\text{는 } \mathbb{R}^{2\times 2}\text{의 standard basis vector}\}$$

$$C = \{x \mid x\text{는 빛의 삼원색}\}$$

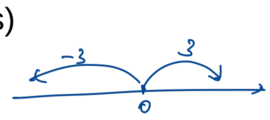
$$D = \{x \mid x\text{는 스팸메일에 자주 등장하는 단어}\}$$

- Venn Diagram



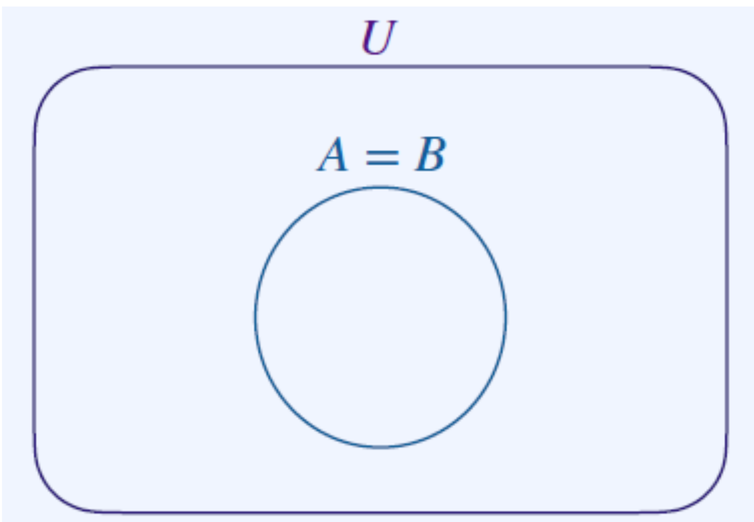
- Common Number Sets

Natural Numbers(자연수)	$\mathbb{N} = \{1, 2, 3, \ \dots\} = \{x \mid (x\text{는 자연수})\}$
Whole Number	$\mathbb{W} = \{0, 1, 2, \ \dots\} = \{x \mid (x\text{는 } 0 \lor x\text{는 자연수})\}$
Integers(정수)	$\mathbb{Z} = \{\dots \ , -2, \ -1, 0, 1, 2, \ \dots\} = \{x \mid (x\text{는 정수})\}$
Rational Numbers(유리수)	$\mathbb{Q} = \{\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{101}{100}, \ \dots\} = \{x \mid (x\text{는 유리수})\}$ $= \{x \mid x = \frac{p}{q}, \ (p, q \text{ are integers } \land q \neq 0)\}$
Irrational Numbers(무리수)	$\mathbb{I} = \{\pi, e, \sqrt{2}, \ \dots\} = \{x \mid (x\text{는 무리수})\} \ = \{x \mid \neg(x\text{는 유리수})\}$
Real Numbers(실수)	$\mathbb{R} = \{x \mid (x\text{는 실수})\} \ = \{x \mid (x\text{는 유리수} \lor x\text{는 무리수})\}$
Complex Numbers(복소수)	$\mathbb{C} = \{x \mid (x\text{는 복소수})\} \ = \{a + j \cdot b \mid (a, b\text{는 실수})\}$

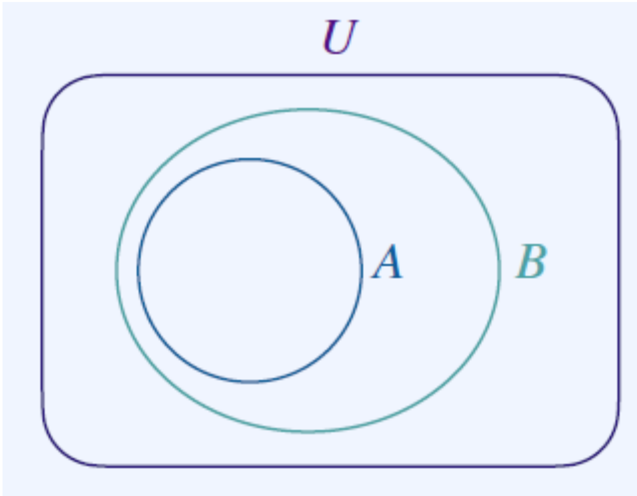
- Cardinality of Sets : 원소들의 개수 표현
  $|A|$ = (# elements)
  ex) $A = \{2, 3\} \Rightarrow |A| = 2$
  $|3| = 3$
  $|-3| = 3$
  

- Inclusion and Exclusion
  - (원소 a가 집합 A에 포함됨) = (a ∈ A)
  - (원소 a가 집합 A에 포함되지 않은) = (a ∉ A) ∉

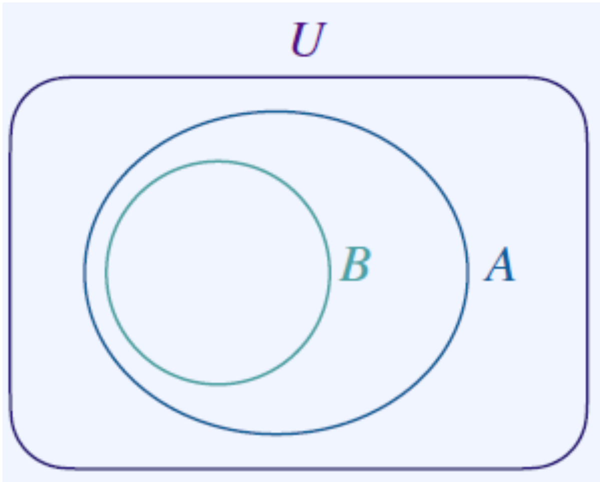
- Equal Sets
  A = B⟷[(∀a ∈ A) ∈ B] ∧ [(∀b ∈ B) ∈ A]



- Subsets and Supersets
  - Subset
    집합 의 모든 원소가 A 집합 B에 포함될 때, A는 B의 subset이라 한다.
    A ⊆ B⟷(∀a ∈ A) ∈ B
    A ⊆ B⟶|A| ≤ |B|



- Superset
  집합 B의 모든 원소가 집합 A에 포함될 때, A는 B의 superset이라 한다.
  $A \supseteq B \longleftrightarrow (\forall b \in B) \in A$
  $A \supseteq B \longrightarrow |A| \geq |B|$



- Proper Subsets
  집합 에 대해 가 의 subset이지만 완전히 A, B A B 같지는 않을 때, A는 B의 proper subset이라 한다. 적어도 B의 원소 중 하나는 A에 포함되지 않아야 한다.

  $A \subset B \longleftrightarrow [(\forall a \in A) \in B] \land [A \neq B]$
  $A \subset B \longrightarrow |A| < |B|$

- Proper Supersets
  집합 A, B에 대해 A가 B의 superset이지만 완전히 같지는 않을 때, A는 B의 proper superset이라 한다. 적어도 A의 원소 중 하나는 B에 포함되지 않아야 한다.

  $A \supset B \longleftrightarrow [(\forall b \in B) \in A] \land [A \neq B]$
  $A \sqsubseteq B \longrightarrow |A| \leq |B|$

- Set Operations : 집합의 연산
  - Operations on Sets
    일정한 규칙을 통해 새로운 집합을 만들어내는 과정

  - Unary Operations $f : A \longrightarrow B$
    - power set of sets
    - complement of sets

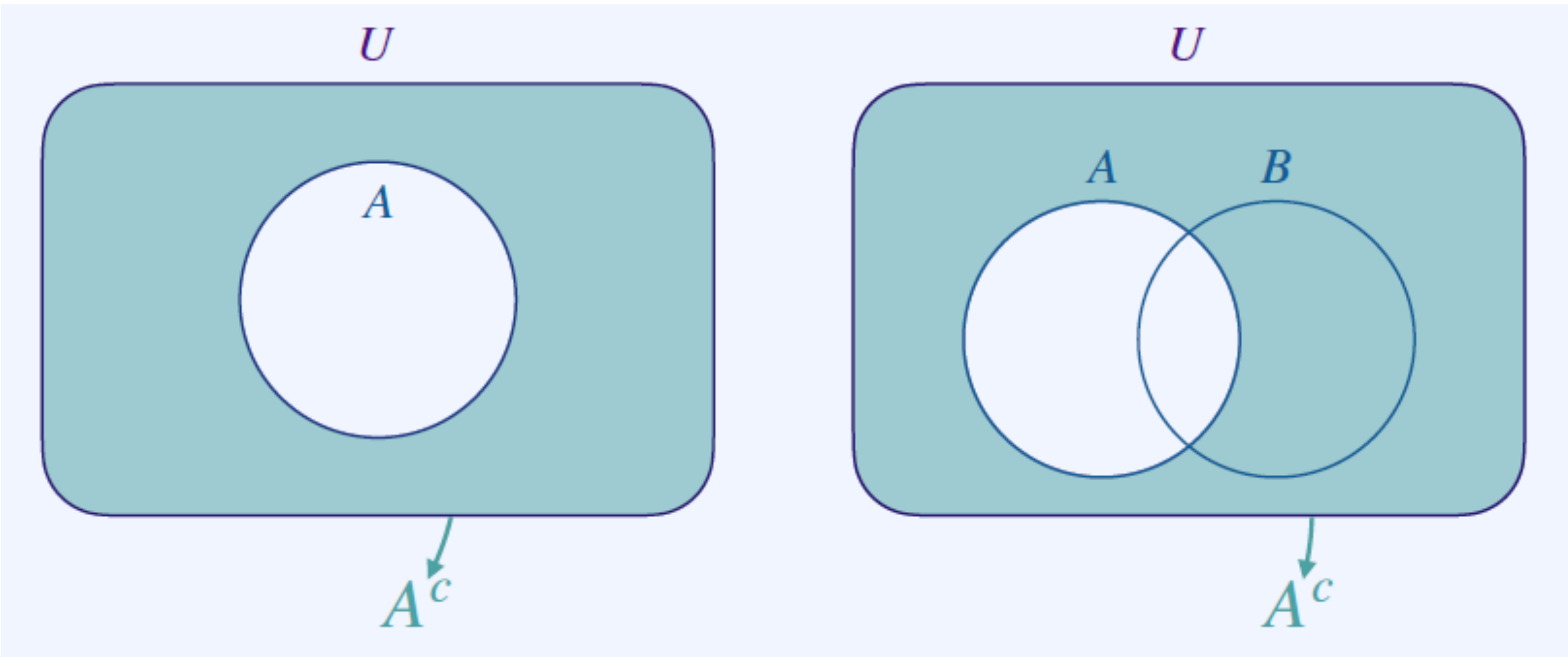
  - Binary Operations $f : A \times B \longrightarrow C$

- Intersection of sets
- union of sets
- set difference
- symmetric difference
- Cartesian product of sets

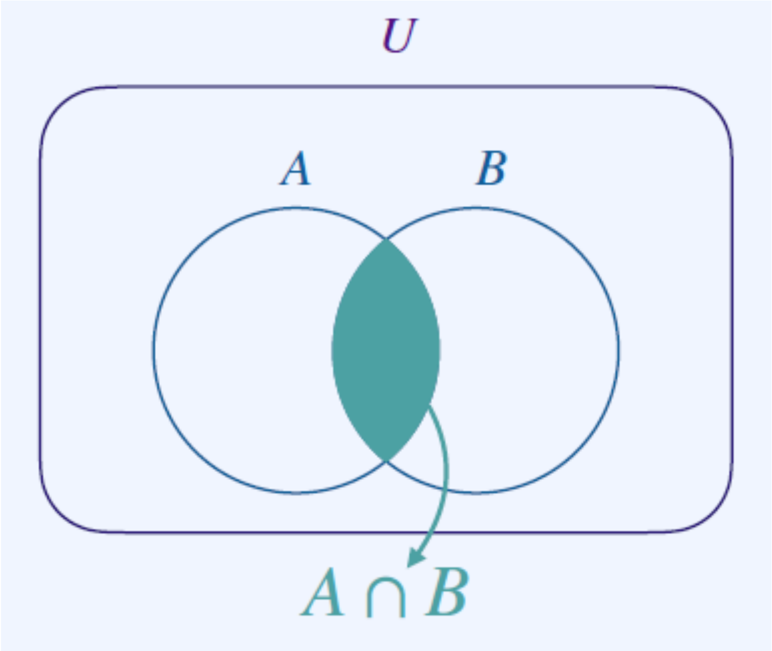
- Set Operations - Power Sets
  - Power Sets
    - 집합 A의 모든 subset들의 집합 𝒫(A)
      A= {1, 2}
    - 모든 원소들은 “집합”
      subset(A) = ∅, {1} {2} {1,2}
      → 4개
    - 𝒫(A) = {X | X ⊆ A}

  - Power Set and Cardinality
    - |𝒫(A)| = 2|A|

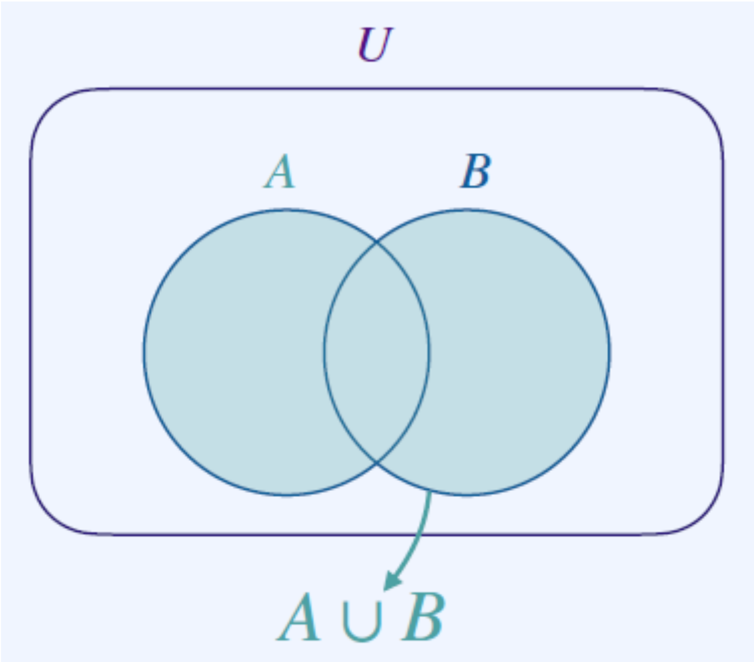
- Set Operations - Complements
  - A에 포함되지 않은 원소들을 모은 집합을 A의 complement이라 하고, Ac로 표현한다. → A^c
  - A = {x | x ∉ A}

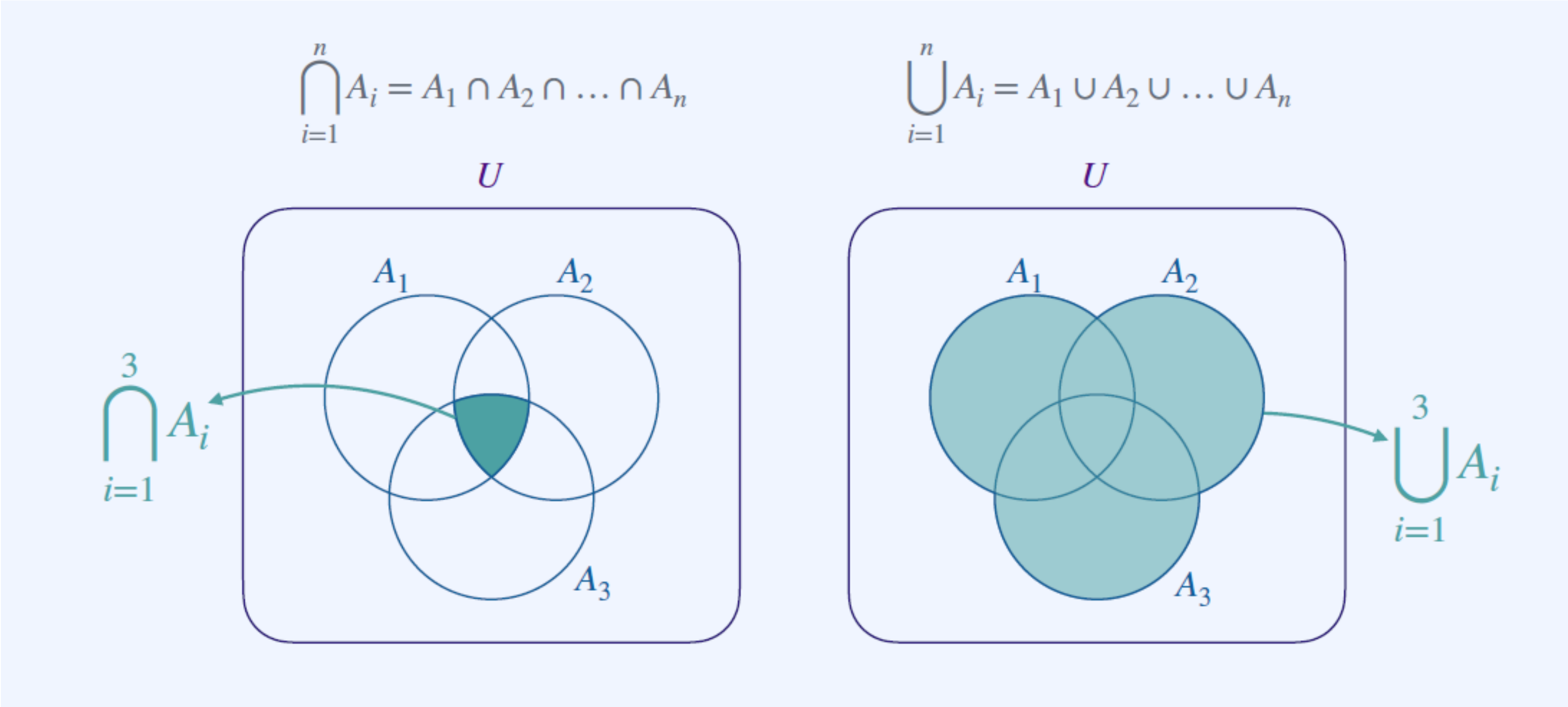


- Set Operations - Intersections and Unions
  - 집합 A, B에 모두 포함되는 원소들을 모든 집합을 A와 B의 intersection(교집합)이라고 부르고, A ∩ B로 나타낸다.
  - A ∩ B = {x | (x ∈ A) ∧ (x ∈ B)}
  - A ∩ B는 가끔 AB로 표현하기도 한다.



- 집합 A 또는(or) B에 포함되는 원소들을 모든 집합을 A와 B의 union(합집합)이라고 부르고, A ∪ B로 나타낸다.
- A ∪ B = {x | (x ∈ A) ∨ (x ∈ B)}





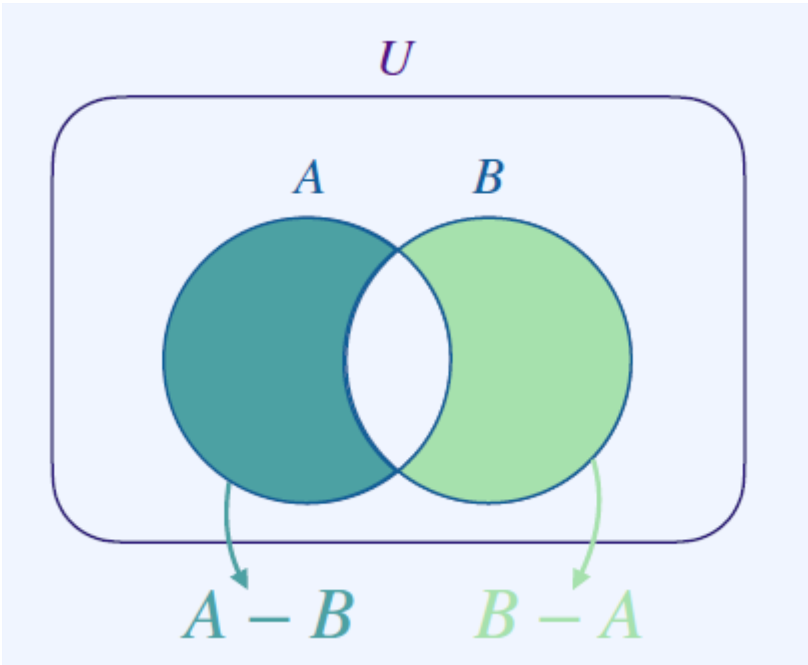
- Algebraic Properties of Intersections and Unions
  - The Algebraic Properties

- Commutative Law
  A ∪ B = B ∪ A   A ∩ B = B ∩ A

- Associative Law
  (A ∪ B) ∪ C = A ∪ (B ∪ C)   (A ∩ B) ∩ C = A ∩ (B ∩ C)

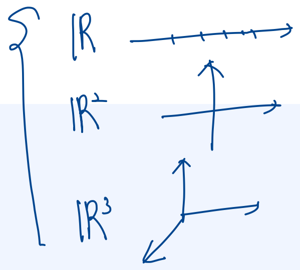
- Distributive Law
  A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)   A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
  A ∩ (B ∩ C) = (A ∩ B) ∩ (A ∩ C)   A ∪ (B ∪ C) = (A ∪ B) ∪ (A ∪ C)

- Set Operations - Set Differences
  - 집합 A, B에 대해 A는 포함되고, B에는 포함되지 않은 원소들을 모은 집합은 A - B로 나타내고, set difference(차집합)이라고 부른다.
  - A − B = {x | (x ∈ A) ∧ (x ∉ B)}
  - A − B는 A \ B로 표현하기도 한다.



→ Commutative law 가 성립되지 않는다.

- Set Operations - Cartesian Product
  - 집합 A, B에서 원소 a, b들을 각각 뽑아 (a, b)를 만들때, 모든 (a, b)들의 집합을 A X B라 한다.
  - A × B = {(a, b) | (a ∈ A) ∧ (b ∈ B)}



$A = \{a_1, a_2, \ldots, a_m\}$ → $|A| = m$

$B = \{b_1, b_2, \ldots, b_n\}$ → $|B| = n$

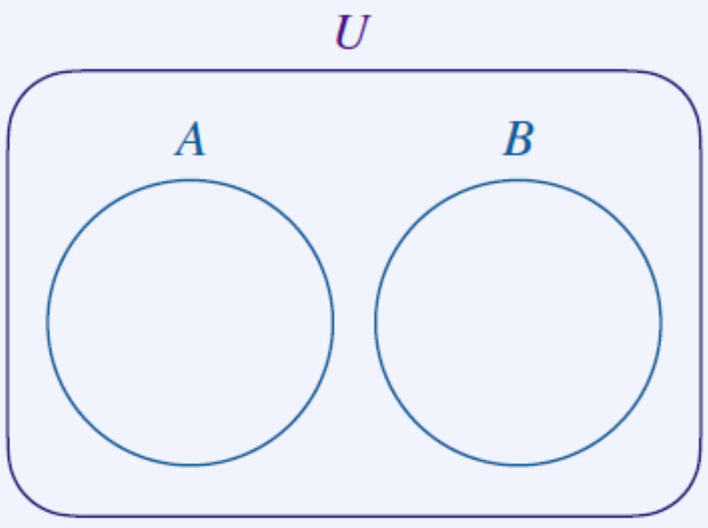
	b_1	b_2	$\cdots$	b_n
a_1	(a_1, b_1)	(a_1, b_2)	$\cdots$	(a_1, b_n)
a_2	(a_2, b_1)	(a_2, b_2)	$\cdots$	(a_2, b_n)
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
a_m	(a_m, b_1)	(a_m, b_2)	$\cdots$	(a_m, b_n)

Cartesian Product 의 결과

$$
\begin{aligned}
A \times B = \{&(a_1, b_1), (a_1, b_2), \ldots, (a_1, b_n) \\
&(a_2, b_1), (a_2, b_2), \ldots, (a_2, b_n) \\
&\vdots \\
&(a_m, b_1), (a_m, b_2), \ldots, (a_m, b_n)\}
\end{aligned}
$$

- Partitions
  - Disjoint Sets

- 집합 A, B에 대해 A ∩ B = ∅일 때 A, B는 disjoint set이다.
- disjoint는 mutually exclusive라고도 부른다.



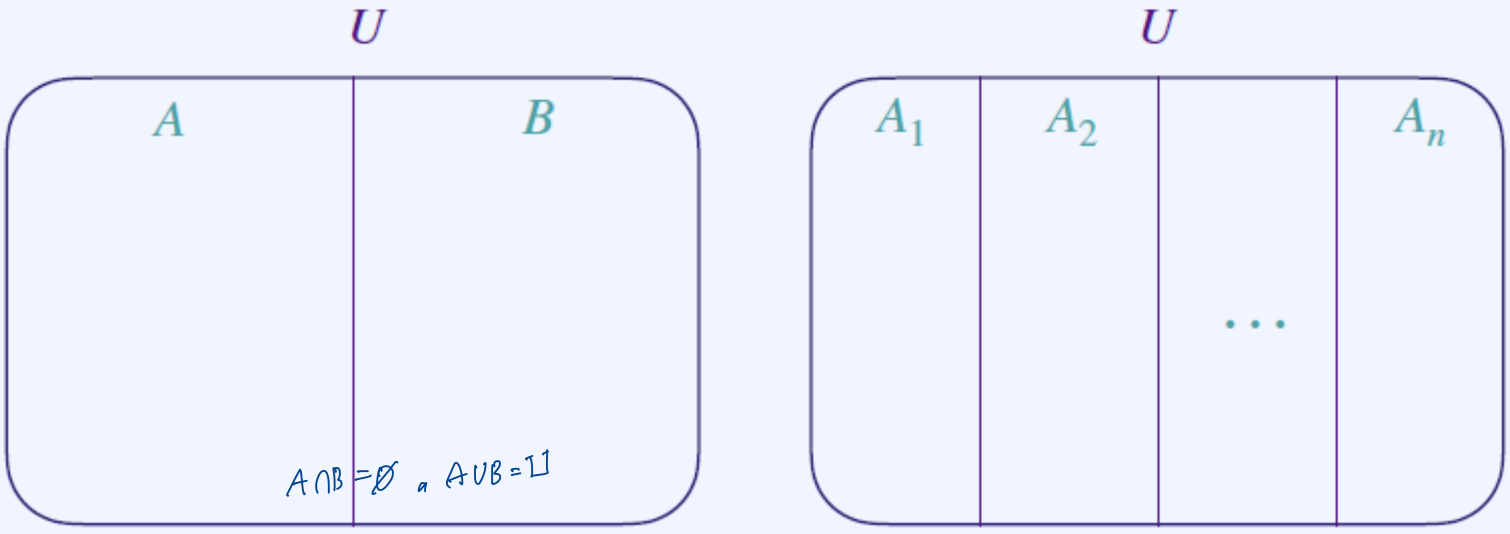
$A \cap B = \varnothing$

☆ A, B 가 상호배타적으로 떨어져서 ∩ 이 없음.

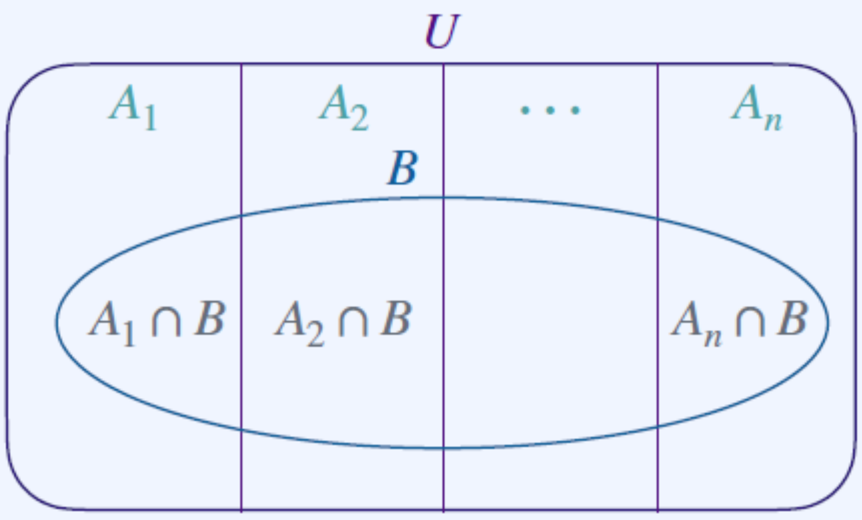
- Universal set(U)안에 n개의 집합 A1, A2, … , An 이 있고, U의 모든 원소들이 모두 단 하나의 Ai에만 포함될 때, {A1, A2, … , An}를 U의 partition이라 부른다.

$$A_i \cap A_j = \varnothing,\ i \neq j$$

$$\bigcup_{i=1}^{n} A_i = U$$



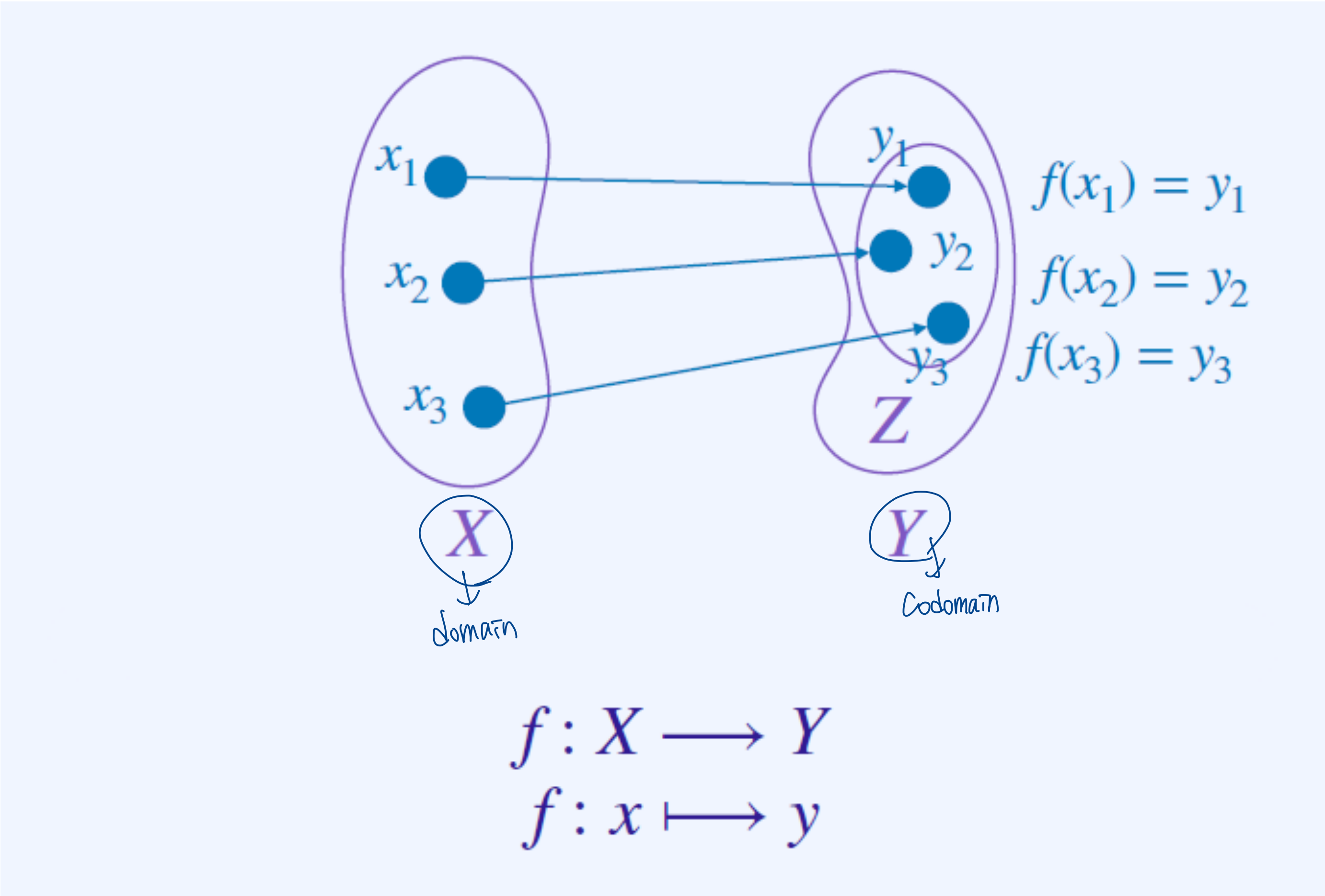
$$B = \bigcup_{i=1}^{n} (A_i \cap B)$$

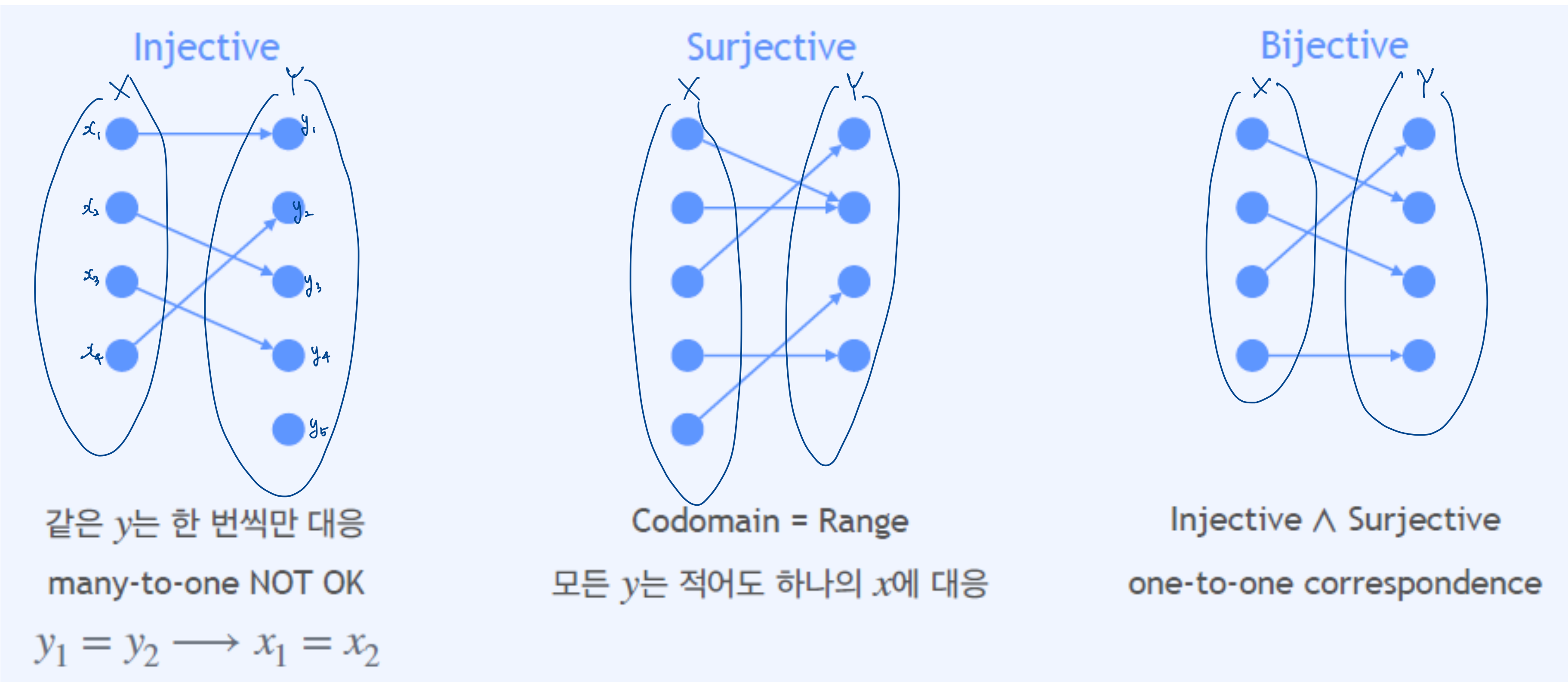


1-3. Functions

Functions

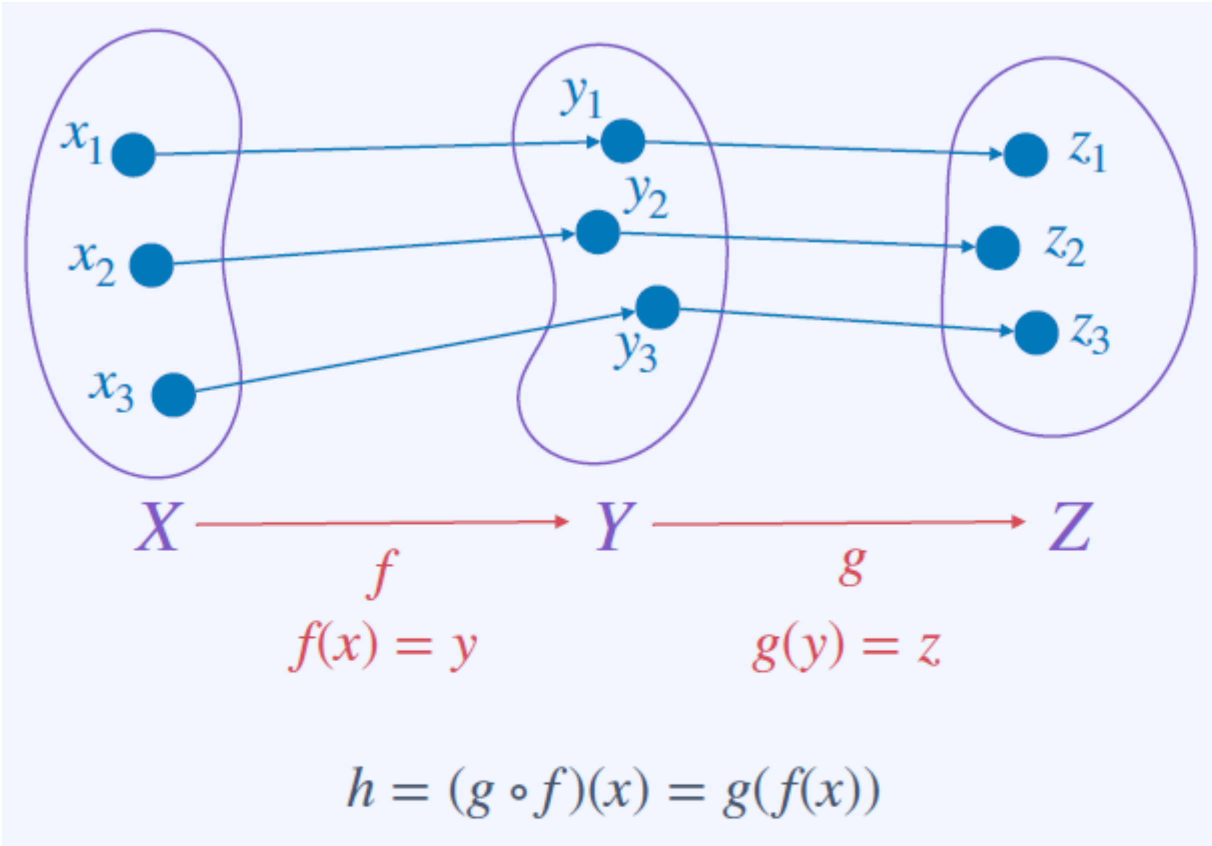
- 두 집합 X, Y에 대해 ∀x ∈ X가 y ∈ Y에 ~~오리지~~ 오로지 하나만 대응되는 관계



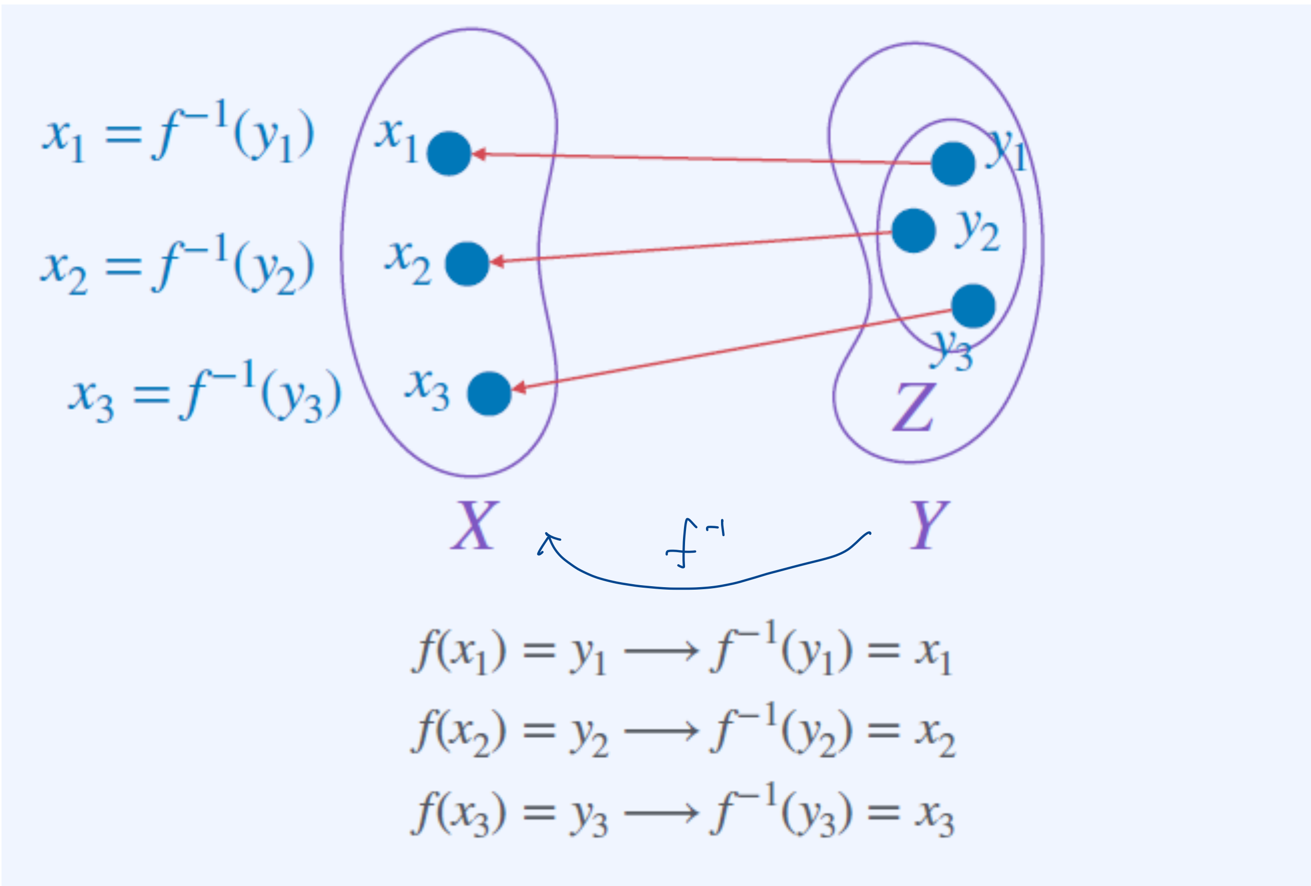


Composite Functions : 합성함수

☆ Deep learning은 합성함수와 비슷하다.



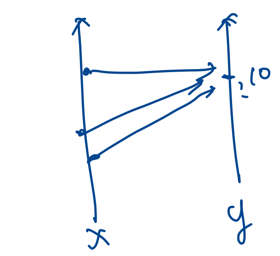
Inverse Functions : 역함수 → Bijective 조건을 만족해야지 역함수를 사용할 수 있다.



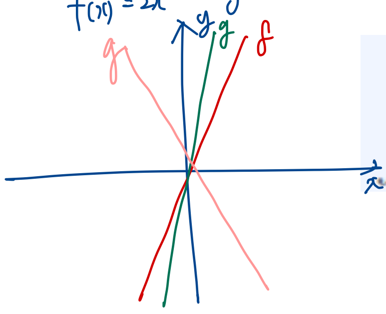
Algebraic Functions : 대수함수 : 함수끼리 사칙연산을 통해 새로운 함수를 만들 수 있음.

- Linear Functions

$f(x) = 10$



$f(x) = 2x$ $g(x) = 10x$ $h(x) = -2x$ ⇒ $g(x)$ 와 $f(x)$ 가 y축 기준으로 대칭

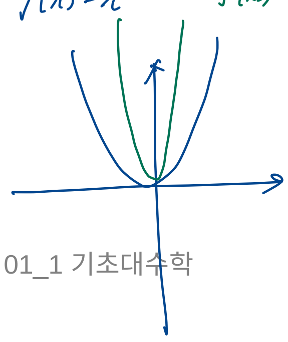


$$y = ax + b$$
$$f(x) = ax + b$$

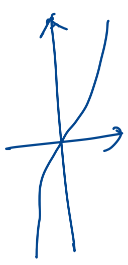
$|a| = 0 = (x축)$
$|a| = \infty = (y축)$

- Power Functions

$f(x) = x^2$ $f(x)' = x^4$



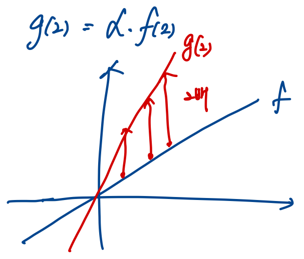
$g(x) = x^3$



$$y = x^n$$
$$g(x) = x^n$$

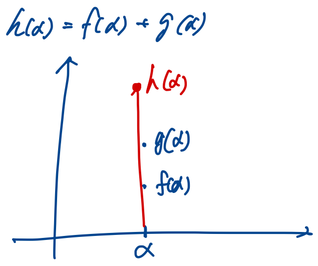
Arithmetic Operations of Functions

- Constant Multiple



$$g(x) = \alpha \cdot f(x)$$

- Addition



$$h(x) = f(x) + g(x)$$

Algebraic Functions

- Quadratic / Cubic Functions
  ㄴ 2차 함수 ㄴ 3차함수

$$y = ax^2 + bx + c \qquad y = ax^3 + bx^2 + cx + d$$

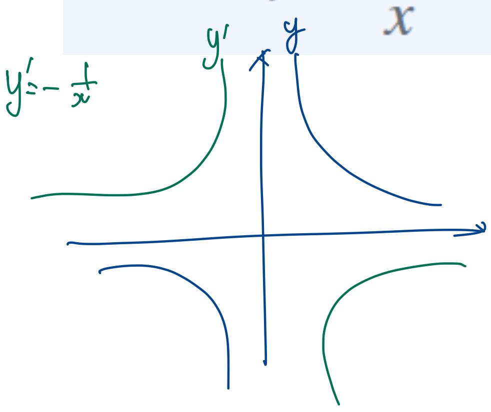
- Polynomial Functions : n차 다항함수

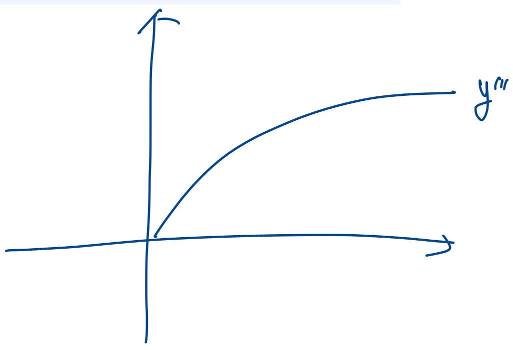
$$y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

유리함수 무리함수
- Rational / Irrational Functions

$$y = \frac{1}{x}$$: $x \cdot y$축은 점근선을 가진다.

$$y''' = \sqrt{x}$$ → 0을 포함한 양수를 가질수 있다.

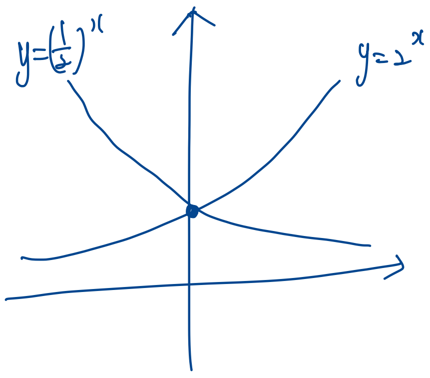




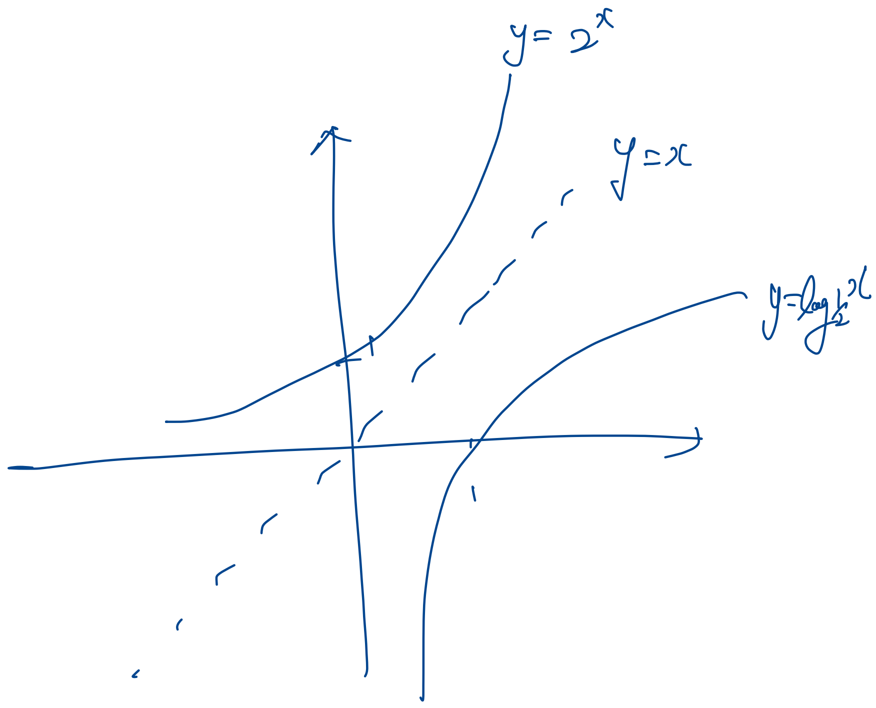
Transcendental Functions

- Exponential Functions : 지수함수

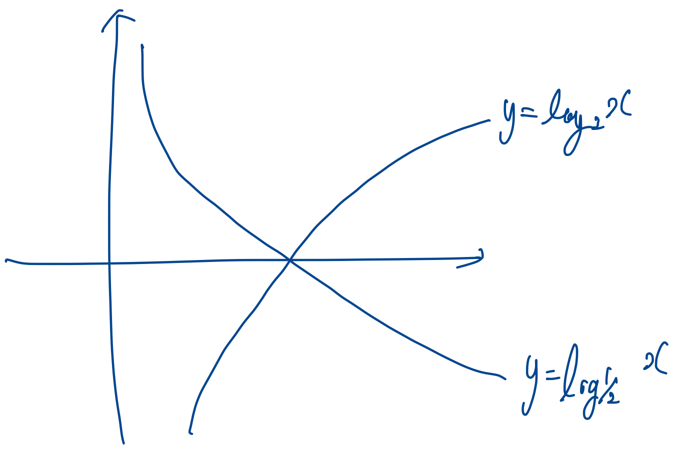
$y = 2^x$, 3^x 4^x ...



$$y = a^x$$



- Logarithmic Functions : 로그함수



$$y = log_a(x)$$

- Trigonometric Functions : 삼각함수

$\frac{1}{\sin(x)} = \operatorname{cosec}(x)$

$\frac{1}{\cos(x)} = \sec(x)$

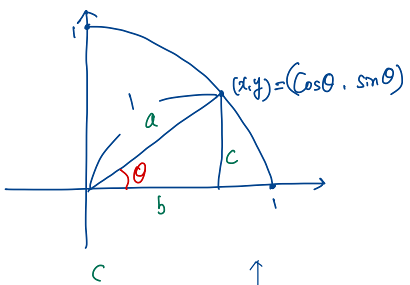
$\frac{1}{\tan(x)} = \cot(x)$

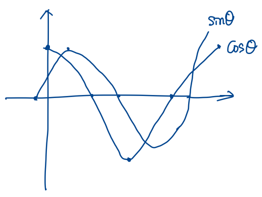
$$y = sin(x) \qquad y = cos(x) \qquad y = tan(x)$$

$\begin{cases} -1 \leq \sin(x) \leq 1 \\ -1 \leq \cos(x) \leq 1 \end{cases}$

$= \frac{\sin(x)}{\cos(x)}$

$\Rightarrow$ 기울기





$\begin{cases} \sin\theta = \frac{c}{a} \\ \cos\theta = \frac{b}{a} \\ \tan\theta = \frac{c}{b} = \frac{\sin\theta}{\cos\theta} \end{cases}$

- Hyperbolic Functions

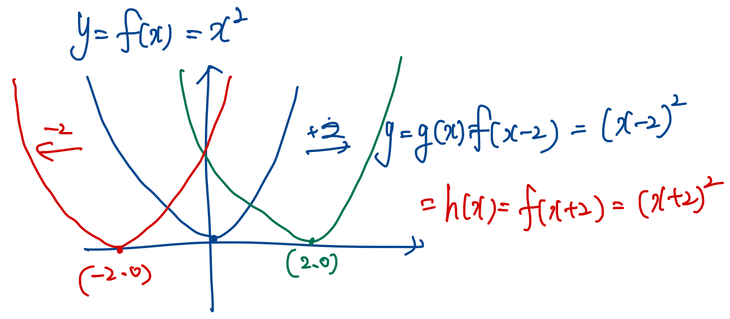
자연상수

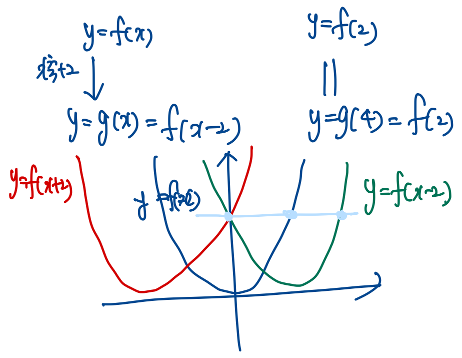
$$sinh(x) = \frac{e^x - e^{-x}}{2} \qquad cosh(x) = \frac{e^x + e^{-x}}{2} \qquad tanh(x) = \frac{sinh(x)}{cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

대칭이동 확장·축소

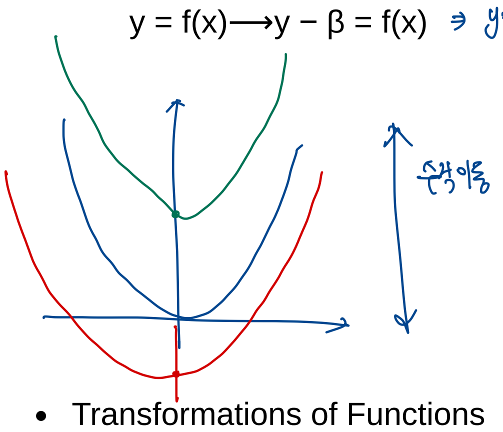
Translations and Transformations

- Translations of Functions
  - Horizontal Translations : 평행이동
    y = f(x)⟶y = f(x − α)

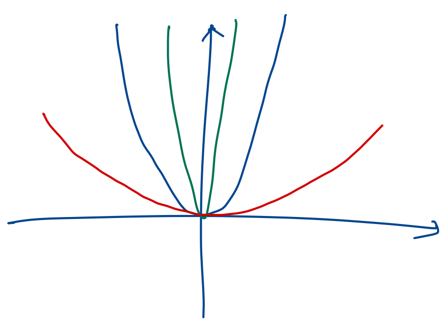




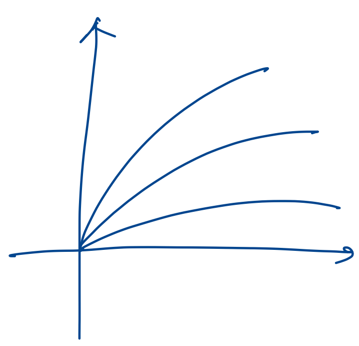
  - Vertical Translations : 수직이동
    y = f(x)⟶y − β = f(x) ⇒ $y=f(x)+\beta$

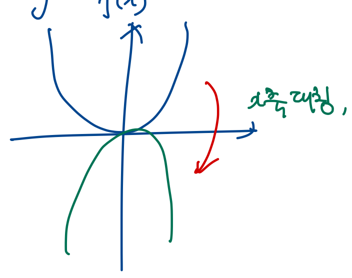


- Transformations of Functions
  - Horizontal Transformations
    y = f(x)⟶y = f(α · x) ⇒ y축 대칭



  - Vertical Transformations (축소)
    y = f(x)⟶β · y = f(x) ⇒ x축 대칭 ⇒ $y=-f(x)$





- Reflections of Functions :원점을 기준으로 대칭

Horizontal Reflections
$y = f(x) \longrightarrow y = f(-x)$

Vertical Reflections
$y = f(x) \longrightarrow -y = f(x)$

Rotation across the Origin
$y = f(x) \longrightarrow y = -f(-x)$

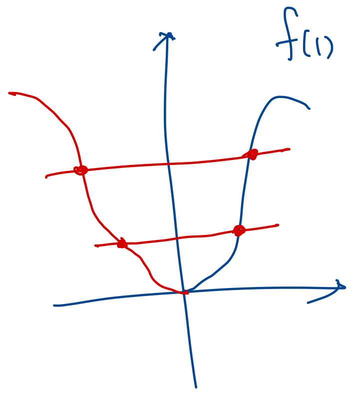
- Even / Odd Functions

우함수
Even Functions
$f(-x) = f(x)$

ex) $\cos(x) = \cos(-x)$

$f(2) = f(-2)$

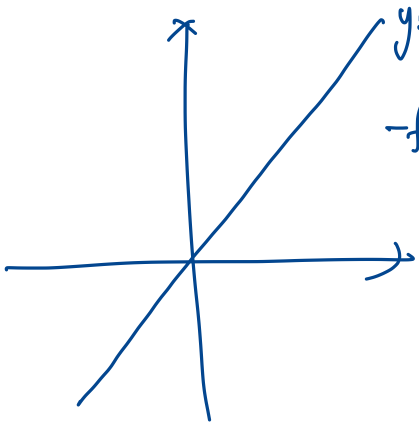
$f(1) = f(-1)$



y축을 기준으로 대칭

기함수
Odd Functions :원점을 기준으로 대칭
$f(-x) = -f(x)$

ex) $\cdot \sin(-x) = -\sin(x)$
$\cdot \tan(-x) = -\tan(x)$



$y = f(x) = 2x$

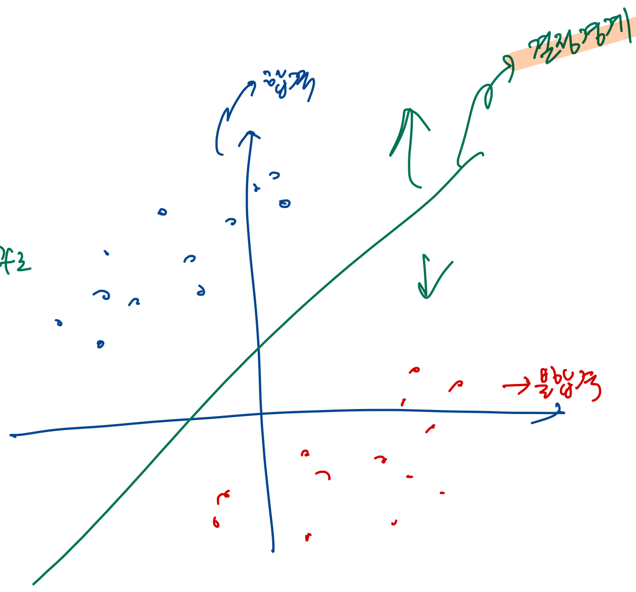
$-f(-x) = 2(-x) \Rightarrow 2x.$

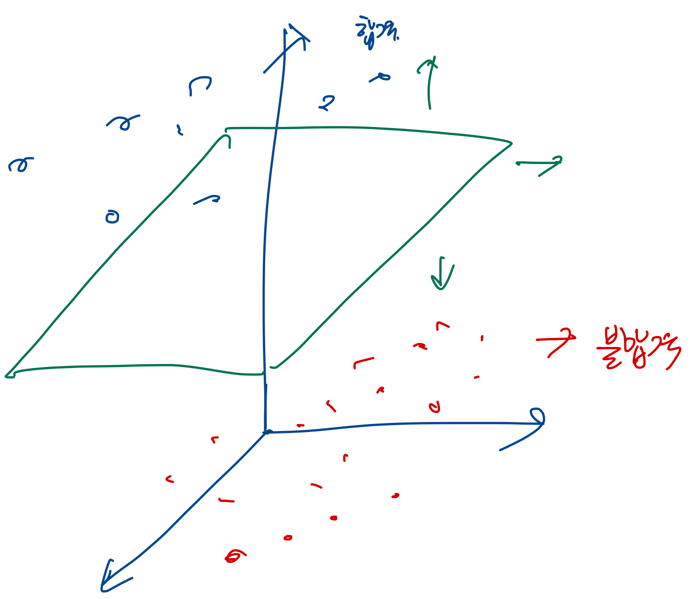
- Linear Functions

① 다른 의미
- linear D.B.
- linearity (선형)

② 선형.

* 시그모이드 함수: 연속적인 Data를 on/off로 바꿔 줌





- Linear Functions

$2X$ 짜장면 $\rightarrow \boxed{f} \rightarrow 2X\ y = f($짜장$)$

짜 $\rightarrow$, 탕 $\rightarrow \boxed{f} \rightarrow f($짜$) + f($탕$)$

Homogeneity

$$f(\alpha x) = \alpha f(x)$$

Additivity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

Linearity → Homogeneity 와 Additivity 의 모든 조건 적용

$$f(\alpha x_1 + \beta x_2) = \alpha f(x_1) + \beta f(x_2)$$

$\alpha x_1 \rightarrow \square \rightarrow \alpha f(x_1)$
$+ \qquad\qquad +$
$\beta x_2 \rightarrow \square \rightarrow \beta f(x_2)$

$$f(x) = ax$$

$$f(\alpha x_1 + \beta x_2) = \alpha f(x_1) + \beta f(x_2)$$

$$g(x) = ax + b$$

$$g(\alpha x_1 + \beta x_2) \neq \alpha g(x_1) + \beta g(x_2)$$

Parametric Models

- Multivariate Functions

$$f(x, y) = z \qquad f(x_1, x_2, \dots, x_n) = y$$
$$f(\mathbf{x}) = y$$

$$f(\mathbf{x}; \theta) = y$$

- Weighted Sum

$$f(x_1, x_2, \dots x_n \ ; \ w_1, w_2, \dots, w_n) = w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$

- Affine Functions

$$f(x_1, x_2, \ldots x_n \ ; \ w_1, w_2, \ldots, w_n, b) = w_1x_1 + w_2x_2 + \ldots + w_nx_n + b$$

- Artificial Neurons