Neutrino Oscillations and the KamLAND Experiment

An example of quantum-mechanical dynamics leading to interference in a two-state system

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What are Neutrinos?

- Neutrinos are elementary particles with no charge and very small mass, much smaller than that of an electron. They interact only via the weak interaction and gravity, making them extremely difficult to detect as they pass through matter almost unimpeded.
- They occur in nature in three distinct flavours, electron neutrino (ν_e), muon neutrino (ν_μ) and tau neutrino (ν_τ). We will only be considering two flavours (ν_e and ν_μ) in this presentation to make it mathematically easier.
- In quantum mechanics, a particle's state is represented as a vector (or "ket") in a Hilbert space. Thus, neutrinos are quantum states. We can describe two bases for neutrinos:
  1. Flavor states: $|\nu_e\rangle, |\nu_\mu\rangle$
  2. Mass states: $|\nu_1\rangle, |\nu_2\rangle$

Flavour vs Mass States of Neutrinos

- Flavour states are interaction eigenstates. They are the states that participate in weak interactions ($|\nu_e\rangle, |\nu_\mu\rangle$). However, when neutrinos propagate through space, their evolution depends on their mass. Mass states are eigenstates of the free-particle Hamiltonian (they evolve with simple phase factor $e^{-iE_i t}$)
- The flavour and mass eigenstates are not aligned. Instead, they are related by a unitary mixing matrix, called the PMNS (Pontecorvo–Maki–Nakagawa–Sakata matrix) matrix U, as follows:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^{*} |\nu_i\rangle$$

where $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$.

Why Neutrino Oscillations happen?

- Each flavour state is a combination of mass states, as follows:

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle - \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = \sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

- In quantum mechanics, a stationary state evolves as:

$$|\psi(t)\rangle = e^{-iEt/\hbar}|\psi(0)\rangle$$

Each mass eigenstate $|\nu_i\rangle$ has its own energy E_i. So if you start with an electron neutrino (a superposition of ν_1 and ν_2):

$$|\nu_e(0)\rangle = \cos\theta|\nu_1\rangle - \sin\theta|\nu_2\rangle$$

then after some time t (or distance $L = ct$), it becomes:

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t/\hbar}|\nu_1\rangle - \sin\theta e^{-iE_2 t/\hbar}|\nu_2\rangle$$

Why Neutrino Oscillations happen?

- We can use the inverse of our mixing relations to write $|\nu_1\rangle$ and $|\nu_2\rangle$ in terms of $|\nu_e\rangle$ and $|\nu_\mu\rangle$:

$$
\begin{aligned}
|\nu_1\rangle &= \cos\theta|\nu_e\rangle + \sin\theta|\nu_\mu\rangle \\
|\nu_2\rangle &= -\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle
\end{aligned}
$$

Now plug these into $|\psi(t)\rangle$:

$$
\begin{aligned}
|\psi(t)\rangle = \cos\theta\, e^{-iE_1 t/\hbar}(\cos\theta\,|\nu_e\rangle + \sin\theta\,|\nu_\mu\rangle) \\
- \sin\theta\, e^{-iE_2 t/\hbar}(-\sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\mu\rangle)
\end{aligned}
$$

Simplify the coefficients of each flavor state:

$$
\begin{aligned}
|\psi(t)\rangle = \left[\, \cos^2\theta\, e^{-iE_1 t/\hbar} + \sin^2\theta\, e^{-iE_2 t/\hbar} \right] |\nu_e\rangle \\
+ \left[\, \cos\theta\, \sin\theta (e^{-iE_1 t/\hbar} - e^{-iE_2 t/\hbar}) \right] |\nu_\mu\rangle
\end{aligned}
$$

Deriving the Probability Functions

- The **amplitude** to find it as an electron neutrino is the coefficient of $|\nu_e\rangle$:

$$A(\nu_e \to \nu_e) = \cos^2\theta e^{-iE_1 t/\hbar} + \sin^2\theta e^{-iE_2 t/\hbar}$$

The **probability** is the modulus squared:

$$P(\nu_e \to \nu_e) = |A(\nu_e \to \nu_e)|^2$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{(E_2 - E_1)t}{2\hbar}\right)$$

- For a relativistic neutrino (speed c), we can approximate its energy as:

$$E_i \approx pc + \frac{m_i^2 c^3}{2p}$$

Thus:

$$E_2 - E_1 \approx \frac{\Delta m^2 c^3}{2p}$$

where $\Delta m^2 = m_2^2 - m_1^2$.

Deriving the Probability Functions

- Now, since neutrinos travel close to the speed of light, $t \approx L/c$:

$$\frac{(E_2 - E_1)t}{2\hbar} \approx \frac{\Delta m^2 c^3 L}{4E\hbar}$$

(using $E \approx pc$).

- Plug that in:

$$\boxed{P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 c^4 L}{4E\hbar c} \right)}$$

This is the neutrino survival probability.

- Similarly, the probability of detecting a **muon neutrino** instead of an electron neutrino is:

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 c^4 L}{4E\hbar c} \right)$$

and note that:

$$P(\nu_e \to \nu_e) + P(\nu_e \to \nu_\mu) = 1$$

Neutrino Oscillations

- Thus, the probability of finding it as a particular flavor depends on how these two mass components interfere - just like two waves combining with a phase difference.
  This is a pure quantum interference pattern - oscillating between 0 and 1 as the phase difference builds up with distance.
- Neutrino oscillation is like a double-slit experiment, but in "mass space" instead of position space:
  - The two mass states ($|\nu_1\rangle$ and $|\nu_2\rangle$) play the role of the two slits.
  - The phase difference acts like the path length difference.
  - The oscillating probability pattern is the interference fringe.

The KamLAND Experiment

- KamLAND stands for Kamioka Liquid-scintillator Anti-Neutrino Detector. It is a neutrino detector located deep underground in the Kamioka mine in Japan. KamLAND uses a large spherical tank (13 meters in radius) filled with 1 kiloton of liquid scintillator (a material that emits light when charged particles pass through it).
- When a $\bar{\nu}_e$ interacts with a proton in the scintillator, it produces a positron and a neutron:

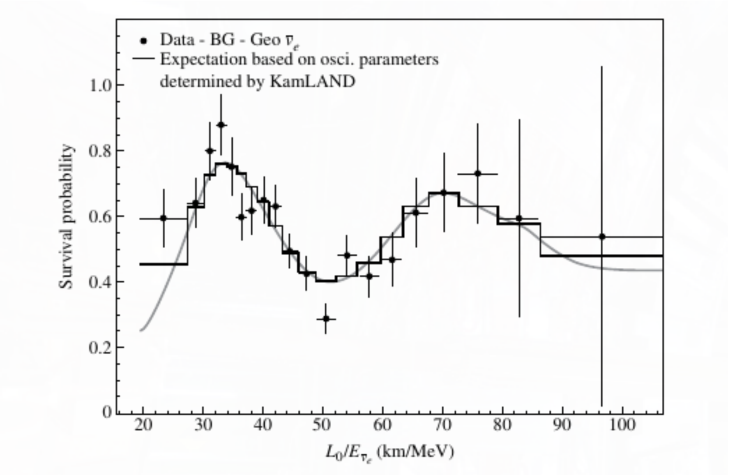
$$\bar{\nu}_e + p \to e^+ + n$$

- The positron creates an immediate light flash (prompt signal). The neutron is captured by another nucleus a few hundred microseconds later, releasing another light flash (delayed signal). This "delayed coincidence" signature makes $\bar{\nu}_e$ events easy to identify and distinguish from background noise.

The Oscillation Plot

- x-axis: L/E - distance divided by neutrino energy (in km/MeV). This ratio determines how much phase difference builds up between the mass states.
- y-axis: Survival probability - the probability that a $\bar{\nu}_e$ produced at a reactor is still a $\bar{\nu}_e$ when it reaches KamLAND.
- The data points (black dots) are the measured probabilities from KamLAND's observations, corrected for background signals (like from the Earth's natural radioactivity). The solid curve shows the theoretical prediction for neutrino oscillations using the best-fit oscillation parameters (Δm^2 and θ). The curve oscillates as L/E increases, just like a wave.

The Oscillation Plot



Why is the KamLAND Experiment Important?

- This experiment confirmed two crucial things: Neutrinos do oscillate (so flavor changes happen). Therefore, neutrinos must have mass, because oscillation requires non-zero mass differences ($\Delta m^2 \neq 0$).
- Neutrino oscillation is of great theoretical and experimental interest, as the precise properties of the process can shed light on several properties of the neutrino. In particular, it implies that the neutrino has a non-zero mass, which requires a modification to the Standard Model of particle physics.
- The experimental discovery of neutrino oscillation, and thus neutrino mass, by the Super-Kamiokande Observatory and the Sudbury Neutrino Observatories was recognized with the 2015 Nobel Prize for Physics.

Thank You!