CS 525 WINTER 2012: FINAL EXAM

For this exam you may use the textbook, your notes, and all course materials. Other resources are prohibited. You may not collaborate with anyone to obtain solutions. The solutions you submit must be your own.

You must submit your solutions to BbVista in typeset form as a single PDF-file (use LaTeX, LyX, Word, etc.).

All questions carry equal weight. Show your work, as partial credit will be given. You will be graded not only on the **correctness** of your answer, but also on the **clarity** with which you express it. Please, also provide some **redundancy**. For example, when specifying a TM for A_{CFG}, please add that you are referring to the machine described in Theorem 4.7 of the textbook. In general, provide some elaboration with your ideas.

Problem	Points	Score
1.a	5	
1.b	5	
1.c	5	
1.d	5	
2.	20	
3.a	6	
3.b	7	
3.c	7	
4.	20	
5.	20	
6.a	10	
6.b	10	
Total	120	

Good Luck!

(1) Answer each question for the following context-free grammar G.

$$\begin{array}{lcl} R & \to & XRX \mid S \\ S & \to & aTb \mid bTa \\ T & \to & XTX \mid X \mid \epsilon \\ X & \to & a \mid b \end{array}$$

   (a) What are the variables and terminals of G? Which is the start variable?
   (b) Give two examples of strings in $L(G)$, and one of a string *not* in $L(G)$.
   (c) True or False: $T \stackrel{*}{\Longrightarrow} XXX$.
   (d) Give a description in English of $L(G)$.

(2) Design a DFA that accepts all binary numbers that are multiples of 3. The input is read from left to right, that is, starting with the high-order bit. (Hint: The DFA has exactly 3 states).

(3) Show that the set of Turing-recognizable languages is closed under the following operations.
   (a) intersection
   (b) concatenation
   (c) star

(4) For any string w, let $w^{\mathcal{R}}$ denote the reverse string, that is, w spelled backwards. For any language L, let $L^{\mathcal{R}} = \{w^{\mathcal{R}} \mid w \in L\}$. Let

$$S_{\text{DFA}} = \{\langle M \rangle \mid M \text{ is a DFA such that } L(M) = L(M)^{\mathcal{R}}\}.$$

   Show that S_{DFA} is decidable.

(5) Show that there exists a language that cannot be recognized by any TM with an oracle for E_{TM}.

(6) Let SPATH $= \{\langle G, a, b, k \rangle \mid G$ is an undirected graph containing a simple path from a to b of length at most $k\}$, and let LPATH $= \{\langle G, a, b, k \rangle \mid G$ is an undirected graph containing a simple path from a to b of length at least $k\}$,
   (a) Show that SPATH $\in P$.
   (b) Show that LPATH is NP-complete. (Hint: Use the fact that the Hamiltonian path problem in undirected graphs is NP-complete).