CS 521: Data Structures and Algorithms 1
Fall 2011-2012 (Homework 2)

1. (10 Pts.) Show that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in any n-element binary heap .

2. (10 Pts.) What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about decreasing order?

3. (10 Pts.) Give an $O(n \lg k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists.

4. (15 Pts.) Banks often record transactions on an account in order of the times of the transactions, but many people like to receive their bank statements with checks listed in order by check number. People usually write checks in order by check number, and merchants usually cash them with reasonable dispatch. The problem of converting time-of-transaction ordering to check-number ordering is therefore the problem of sorting almost-sorted input. Argue that the procedure INSERTION-SORT would tend to beat the procedure QUICKSORT on this problem.

5. (15 Pts.) We can improve the running time of quicksort in practice by taking advantage of the fast running time of insertion sort when its input is"nearly" sorted. Upon calling quicksort on a subarray with fewer than k elements, let it simply return without sorting the subarray. After the top-level call to quicksort returns, run insertion sort on the entire array to finish the sorting process. Argue that this sorting algorithm runs in $O(nk + n \lg(n/k))$ expected time. How should we pick k, both in theory and in practice?

6. (10 Pts.) In the algorithm SELECT, the input elements are divided into groups of 5. Will the algorithm work in linear time if they are divided into groups of 7? Argue that SELECT does not run in linear time if groups of 3 are used.

7. (10 Pts.) Describe an $O(n)$-time algorithm that, given a set S of n distinct numbers and a positive integer $k \leq n$, determines the k numbers in S that are closest to the median of S.

8. (20 Pts.) The k-th quantiles of an n-element set are the $k-1$ order statistics that divide the sorted set into k equal-sized sets (to within 1). Give an $O(n \lg k)$-time algorithm to list the k-th quantiles of a set.

9. (**Extra Credit**) (25 Pts) Problem 7-5, page 188 of text book (CLRS, 3rd and 2nd editions).