

# CS 525: Theory of Computation

## Problem Set 5, Re-do

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6.15 **Solution:** For any language  $A$ , we can create a language  $B$  that consists of the set of all strings that map to Turing Machines that decide  $A$ . If we have an oracle for  $B$ , then the following Turing Machine,  $T^B$ , decides  $A$ :

$T^B =$  On input  $w$ , where  $w$  is a string:

- (a) Enumerate strings and test for membership in  $B$ . Start with strings of length 1, and then length 2, and so on.
- (b) If  $A$  is decidable, then eventually a string  $\langle M \rangle$  in  $B$  will be found since some Turing Machine must decide  $A$ .
- (c) Convert the string  $\langle M \rangle$  into TM  $M$ , and simulate  $\langle M, w \rangle$ . Since  $M$  decides  $A$ , it will halt on  $w$ . If  $M$  accepts, accept. If  $M$  rejects, reject.

If such an oracle for  $B$  exists, then  $A$  is decidable. Therefore,  $A \leq_T B$ . On the other hand, if an oracle for  $A$  exists, we can't use it to decide  $B$ .