# CS 522: Data Structures and Algorithms II
# Homework 1

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1. **Solution:** If both INCREMENT and DECREMENT operations were included in the $k$-bit counter, an amortized analysis of the cost of $n$ operations would cost as much as $\Theta(nk)$ time because we would no longer be able to consider each operation as a consecutive INCREMENT, but rather as any combination of INCREMENTS and DECREMENTS. Thus, in a worse-case scenario, it would be possible to alternate $n$ times between two operations which cost $O(k)$ each, resulting in a total cost of $\Theta(nk)$.

2. **Solution:** To show that the amortized cost of TABLE-DELETE under this strategy is bounded above by a constant, we will consider two cases. We will use the potential function:

$$\Phi(T) = |2 \cdot T.num - T.size|$$

The first case is the one in which the table does not contract, and thus $num_i = num_{i-1} - 1$, $size_i = size_{i-1}$, and $c_i = 1$:

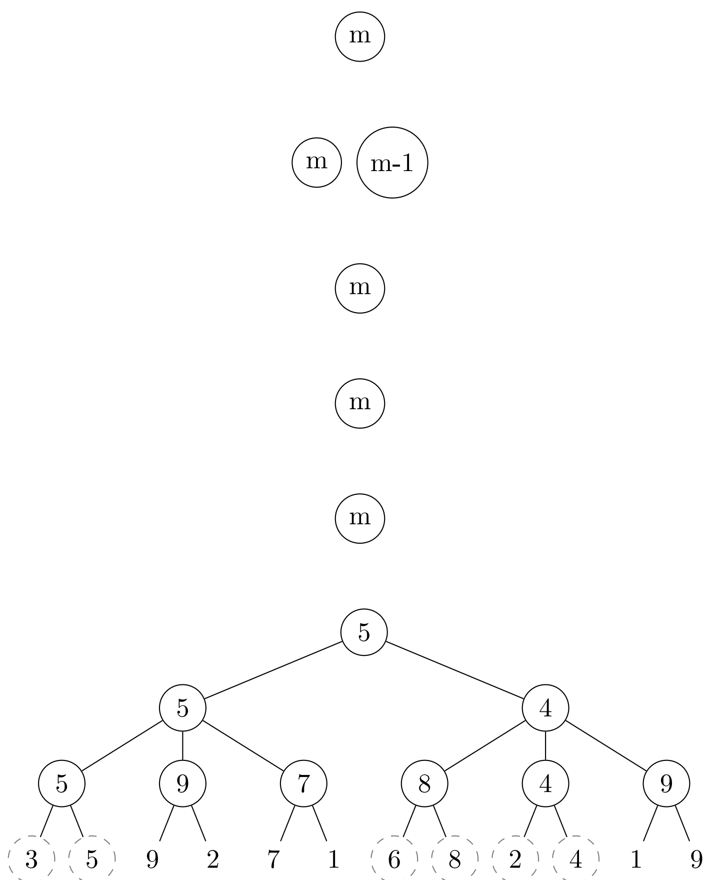
$$\begin{aligned}
\hat{c_i} &= c_i + \Phi_i - \Phi_{i-1} \\
\hat{c_i} &= 1 + |2 \cdot num_i - size_i| - |2 \cdot num_{i-1} - size_{i-1}| \\
\hat{c_i} &= 1 + |2 \cdot (num_{i-1} - 1) - size_{i-1}| - |2 \cdot num_{i-1} - size_{i-1}| \\
\hat{c_i} &= 1 + |-2| \\
\hat{c_i} &= 3
\end{aligned}$$

The second case is the one in which the table does contract, and thus $size_i = \frac{2}{3} size_{i-1}$, $num_{i-1} = \frac{1}{3} size_{i-1}$, and $c_i = num_i + 1$:

$$\hat{c}_i = c_i + \Phi_i - \Phi_{i-1}$$
$$\hat{c}_i = (num_i + 1) + |2 \cdot num_i - size_i| - |2 \cdot num_{i-1} - size_{i-1}|$$
$$\hat{c}_i = ((num_{i-1} - 1) + 1) + |2 \cdot (num_{i-1} - 1) - \frac{2}{3} size_{i-1}| - |2 \cdot num_{i-1} - size_{i-1}|$$
$$\hat{c}_i = (num_{i-1}) + | - 2 + \frac{1}{3} size_{i-1}|$$
$$\hat{c}_i = 2$$

Thus we see that the amortized cost of TABLE-DELETE is at most 3 and is thus bounded.

3. **Solution:** As follows:




4. **Solution:**

5. **Solution:**
6. **Solution:**
7. **Solution:**
8. **Solution:**