CS 540: High Performance Computing
Homework 1

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1. *Prove that the product of an m digit integer with an n digit integer can have either $m+n$ or $m+n-1$ digits.*

   As a base case, examine the range of all possible one-digit integers, [0...9]:

   $$[0, 1, 2, 3, ..., 63, 72, 81]$$

   Any integer longer than one digit can be represented as $10a + b$, where b is a one-digit integer. For example, 81 would be $10 * 8 + 1$.

   In this form, the product of two multi-digit integers would be:

   $$(10a + b) * (10c + d) = 100ac + 10(ad + bc) + bd$$

   Ideally I would then prove that multiplying by $10 * n$ would only add n digits, but I'm not sure how to prove that adding would'nt increase the number of digits.

2. *Prove the master theorem. To simplify the argument, assume that $n = b^k$.*

   We can determine an asymptotic bound on the master theorem using a recursion tree by dividing the total cost $f(n)$ a total of a times for a cost of $f(n/b)$ each. Therefore, for each level of j levels in the recursion tree (each time the cost is divided), we must add a cost of

   $$a^j f(n/b^j)$$

   Until we reach the base cost of $\Theta(1)$ totaling

   $$\Theta(n^{log_b(a)})$$

   Resulting in a total of

   $$T(n) = \sum_{j=0}^{log_b(n-1)} a^j f(n/b^j) + \Theta(n^{log_b(a)})$$

Using the formula for geometic series, this becomes

$$T(n) = aT(n/b) + \Theta(n)$$

Where $a \geq 1$ and $b > 1$.