CS 525: Theory of Computation
Problem Set 6

Dustin Ingram, Aaron Rosenfeld, Eric Simon

March 7, 2012

6.4 **Solution:** For the purpose of contradiction, assume ${A_{TM}}'$ is decidable by a TM A'. Let N be a TM defined as:

$N =$ On input (m, c) where m is a TM and c is the code of m:

(a) Simulate A' on (m, c)

(b) If A' accepts *reject*; else *accept*.

Then, run N on input (N, m) where m is N's code. The output provides a contradiction: N accepts w if and only if N rejects w, proving that ${A_{TM}}'$ is undecidable relative to A_{TM}.

6.13 **Solution:** Unlike $\mathrm{Th}(\mathcal{N}, +, \times)$ wherein the possible results of $a \times b$ is unbounded, in $\mathrm{Th}(\mathcal{Z}_m, +, \times)$, all results of $a \times b$ must be bounded by $\mathcal{Z}_m$ and therefore the TM accepting $\mathrm{Th}(\mathcal{Z}_m, +, \times)$ can test every possibility, allowing a theorem to be decided.

6.14 **Solution:** Let $J = (A \times \{0\}) \cup (B \times \{1\})$. That is, a TM M deciding J decides words in the form $w' = (w, c)$. To decide if $w \in A$, pass M the input $(w, 0)$. This will accept if and only if the word is in A. Similarly, pass M the word $(w, 1)$ to decide if $w \in B$.