Drexel University
Department of Computer Science
CS 522: Data Structures and Algorithms

Name:____________________
Student ID:_______________
Winter 2013 (Online)

Midterm Exam (Online Section)

- This exam contains **5** problems, some with multiple parts. There are 8 pages to the exam (including 2 blank pages). You have **3:00** hours to earn 100 points (you can try all the problems).
- **Print out the exam** and write your name on the top of every page in this exam booklet.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem or use the last two blank pages. Do not put part of the answer to one problem on the back of the sheet for another problem.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat!
- You are given an additional **1:00** hour to scan or type-up your hand-written solutions and submit the resulting **PDF** file to BB Vista.
- Please note that the "Start Midterm Exam" assignment in BB Vista is used to timestamp your submission, so make sure you submit your exam in less than 4:00 hours.
- This exam is **closed book**. You may use one handwritten letter-size crib sheet.
- **Good Luck!**

Problem	Points	Grade
1.a.	10	
1.b.	10	
1.c.	10	
2.a.	10	
2.b.	10	
3.a.	10	
3.b.	10	
4.a.	15	
4.b.	15	
5.	20	
Sum	120	

1. Let $G = (V, E)$ be a directed graph with edge capacities $c : E \rightarrow R^+$, a source vertex s, and a sink vertex t. Suppose someone hands you an arbitrary function $f : E \rightarrow R$. Describe and analyze fast and simple algorithms to answer the following questions:

   (a) Is f a valid (s, t)-flow in G?

   (b) Is f a maximum (s, t)-flow in G?

   (c) Is f the **unique** maximum (s, t)-flow in G?

2. **True or False (explain your answer)**:

a. The following set of hash functions from universe $U = \{0, 1, 2, 3\}$ to $\{0, 1\}$ is a universal set of hash functions:

	0	1	2	3
h_1	1	0	0	1
h_2	0	1	0	1
h_3	1	0	1	0
h_4	0	1	1	0

b. Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, the expected number of colliding pairs of elements is $\Theta(n^2/m)$.

3. This problem has multiple parts:

   a. Given a graph $G = (V, E)$, with capacity function c, prove that for any pair of vertices u and v and any flow f for G, if c_f is the residual capacity with respect to flow f then:
   $$c_f(u, v) + c_f(v, u) = c(u, v) + c(v, u).$$

   b. Let f be a flow in network, and let α be a real number, the *scalar flow product* denoted by αf, is a function from $V \times V$ to R defined by $(\alpha f)(u, v) = \alpha \times f(u, v)$. Prove that the flows in a network forms a *convex set*. That is, show that if f_1 and f_2 are flows, then so is $\alpha f_1 + (1 - \alpha) f_2$ for all $0 \leq \alpha \leq 1$.

4. Assume we are in the comparison model with an initial set of m inputs $a_1, \ldots, a_m$ elements in a set S. There will be N steps. On each step the minimum is removed from S, and then a new input is presented and included in the set.

   a) Propose a data structure to store the set S, such that the amortized cost of a sequence of N operations, for $N \gg m$ is $O(\log m)$ per operation.

   b) Show that if $N = m$, it is not possible to have a constant amortized cost for a step.

5. A "*very sloppy heap*(**VSH**)" is a data structure for performing the operations **INSERT**(x) and **DELETE-SMALL()** on a set S.
The **DELETE-SMALL()** operation returns an element y which is among the $\lceil \frac{N}{2} \rceil$ smallest elements in the set S, where $N = |S|$. Which element gets returned is beyond users control, and the user is not informed as to the relative rank of y.
Explain how to implement a **VSH** in constant amortized time per operation and justify the running time. (If you can not do this, give the most efficient implementation possible and analyze its amortized cost.)