CS 610: Midterm
Winter 2013

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0. (a) We consider three probabilities:

    - The probability of being in the initial start state, S_0, where:

    $$S_0 = (\texttt{LanguageA}/\text{Anti-Polka})$$
    $$P_0 = 0.7 \cdot 0.6 = 0.42$$

    - The probability of a transition from S_0 to S_1, where:

    $$S_1 = (\texttt{LanguageB}/\text{Anti-Polka})$$
    $$P_1 = 0.15 \cdot 0.7 = 0.105$$

    - The transition from S_1 to S_2, where:

    $$S_1 = (\texttt{LanguageC}/\text{Dummy})$$
    $$P_1 = 0.7 \cdot 0.3 = 0.21$$

    Thus, the probability that *at the beginning of the day* the machine will produce chips in the order $S_0 \to S_1 \to S_2$ is:

    $$\begin{aligned} P_0 \cdot P_1 \cdot P_2 &= 0.42 \cdot 0.105 \cdot 0.21 \\ &= 0.009261 \\ &\approx 1\% \end{aligned}$$

   (b) We consider the first, heavy TV. For simplicity, we use the following notation:

    $$\begin{aligned} P(C|W_h) &= P(\text{chip}_1, \text{chip}_2|\text{weight} = \text{heavy}) \\ P(W_h|C) &= P(\text{weight} = \text{heavy}|\text{chip}_1, \text{chip}_2) \\ P(W_h) &= P(\text{weight} = \text{heavy}) \\ P(C_s) &= P_{start}(\text{chip}_1, \text{chip}_2) \end{aligned}$$

    The probability for the chips at the start of the run, $P(C_s)$ is given in part (a) and is as follows:

	`LanguageA`	`LanguageB`	`LanguageC`
Anti-Polka	0.42	0.14	0.14
Dummy	0.18	0.06	0.06

We are given $P(W_h|C)$ by the table in the question. $P(W_h)$ is given as follows:

$$P(W_h) = \sum (P(W_h|C) \cdot P(C_s)) = \mathbf{0.486}$$

Using Bayes' theorem:

$$P(C|W_h) = \frac{P(W_h|C) \cdot P(C_s)}{P(W_h)}$$

Thus, the resulting values for $P(C|W_h)$ for the first, heavy TV are as follows:

$P(\text{Anti-Polka}, \texttt{LanguageA}|\text{weight} = \text{heavy})$	**0.60**
$P(\text{Anti-Polka}, \texttt{LanguageB}|\text{weight} = \text{heavy})$	**0.14**
$P(\text{Anti-Polka}, \texttt{LanguageC}|\text{weight} = \text{heavy})$	**0.12**
$P(\text{Dummy}, \texttt{LanguageA}|\text{weight} = \text{heavy})$	**0.11**
$P(\text{Dummy}, \texttt{LanguageB}|\text{weight} = \text{heavy})$	**0.01**
$P(\text{Dummy}, \texttt{LanguageC}|\text{weight} = \text{heavy})$	**0.01**

For the second, light TV, we use the following

$$\begin{aligned} P(C|W_l) &= P(\text{chip}_1, \text{chip}_2|\text{weight} = \text{light}) \\ P(W_l|C) &= P(\text{weight} = \text{light}|\text{chip}_1, \text{chip}_2) \\ P(W_l) &= P(\text{weight} = \text{light}) \\ P(C_t) &= P_{transition}(\text{chip}_1, \text{chip}_2) \end{aligned}$$

We no longer need to consider the probabilities of the starting state, but rather the transition between any two states:

	AP, `LA`	AP, `LB`	AP, `LC`	D, `LA`	D, `LB`	D, `LC`
AP, `LA`	0.490	0.105	0.105	0.210	0.045	0.045
AP, `LB`	0.105	0.490	0.105	0.045	0.210	0.045
AP, `LC`	0.105	0.105	0.490	0.045	0.045	0.210
D, `LA`	0.210	0.045	0.045	0.490	0.105	0.105
D, `LB`	0.045	0.210	0.045	0.105	0.490	0.105
D, `LC`	0.045	0.045	0.210	0.105	0.105	0.490

However, this probability of transition is assuming that we are equally likely to be in any given preceding state. As we have shown in the first half of the question where the first TV is heavy, in this case there is a known probability for being in a certain state before the transition. We must multiply this across the transition table to get the probability of transition if the first TV was heavy:

	AP, `LA`	AP, `LB`	AP, `LC`	D, `LA`	D, `LB`	D, `LC`
AP, `LA`	0.296	0.064	0.064	0.127	0.027	0.027
AP, `LB`	0.015	0.071	0.015	0.006	0.030	0.006
AP, `LC`	0.012	0.012	0.056	0.005	0.005	0.024
D, `LA`	0.023	0.005	0.005	0.054	0.012	0.012
D, `LB`	0.001	0.003	0.001	0.001	0.006	0.001
D, `LC`	0.001	0.001	0.003	0.001	0.001	0.006

And then sum the probabilities that result in the same state to get $P(C_t)$ for a TV following a heavy TV:

	`LanguageA`	`LanguageB`	`LanguageC`
Anti-Polka	0.348	0.154	0.143
Dummy	0.196	0.082	0.077

Again, using Bayes' theorem:

$$P(C|W_l) = \frac{P(W_l|C) \cdot P(C_t)}{P(W_l)}$$

and:

$$P(W_l) = \sum (P(W_l|C) \cdot P(C_t)) = \mathbf{0.211}$$

Gives us the following probabilities for $P(C|W_h)$ for the second, light TV:

$P(\text{Anti-Polka}, \texttt{LanguageA}|\text{weight} = \text{light})$	**0.165**
$P(\text{Anti-Polka}, \texttt{LanguageB}|\text{weight} = \text{light})$	**0.146**
$P(\text{Anti-Polka}, \texttt{LanguageC}|\text{weight} = \text{light})$	**0.204**
$P(\text{Dummy}, \texttt{LanguageA}|\text{weight} = \text{light})$	**0.186**
$P(\text{Dummy}, \texttt{LanguageB}|\text{weight} = \text{light})$	**0.116**
$P(\text{Dummy}, \texttt{LanguageC}|\text{weight} = \text{light})$	**0.182**

(c)