CS 525 WINTER 2012: MIDTERM EXAM 2

For this exam you may use the textbook, your notes, and all course materials. Other resources are prohibited. You may not collaborate with anyone to obtain solutions. The solutions you submit must be your own.

You must submit your solutions to BbVista in typeset form as a single PDF-file (use LaTeX, LyX, Word, etc.).

All questions carry equal weight. Show your work, as partial credit will be given. You will be graded not only on the **correctness** of your answer, but also on the **clarity** with which you express it. Please, also provide some **redundancy**. For example, when specifying a TM for A_{CFG}, please add that you are referring to the machine described in Theorem 4.7 of the textbook. In general, provide some elaboration with your ideas.

Good Luck!

(1) Let T be a binary tree that has a root, and assume that every node has exactly two children (note that that makes T infinite).
  (a) Show that the set of nodes in T is countable.
  (b) Using a diagonalization argument show that the set of all infinite paths from the root is uncountable.

(2) Show that the language

$$L_2 = \{\langle A\rangle \mid A \text{ is a DFA that has no useless state}\}$$

is decidable. (A useless state is a state that is not entered for any input string).

(3) Show that the language

$$L_3 = \{\langle M, N\rangle \mid M, N \text{ are TMs and } L(M) \subset L(N)\}$$

is undecidable.

(4) Show that the language

$$L_4 = \{\langle M\rangle \mid M \text{ is a TM and } |L(M)| = 1\}$$

is not Turing recognizable.

(5) Show that any infinite subset of MIN_{TM} is not Turing recognizable.

(6) Answer the following questions and give your reasons.
  (a) Is the statement $\forall x\ \exists y\ [x \cdot y = 1]$ a member of $\text{Th}(\mathbb{N}, \cdot)$?
  (b) Is the statement $\forall x\ \exists y\ [x \cdot y = 1]$ a member of $\text{Th}(\mathbb{Q}, \cdot)$?
  (c) Give a formula that defines the usual relation $\leq$, "less than or equal to", in $(\mathbb{R}, +, \cdot)$. (The only relations you may use in your definition are "$+$" and "$\cdot$").