Computational Photography

Week 4, Spring 2009

Instructor: Prof. Ko Nishino

Binary Image Processing

Binary Images

- Images with only two values (0 or 1)
- Simple to process and analyze
- Very useful for industrial applications

Binary Images

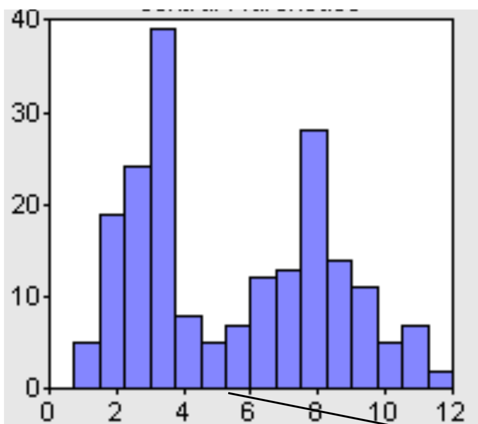
- Obtained from gray-scale (or color) image g(x, y) by thresholding

- Characteristic Function

$$b(x, y) = \begin{cases} 1 & \text{if } g(x, y) < T \\ 0 & \text{if } g(x, y) >= T \end{cases}$$

- Topics Discussed:
  - Geometric Properties
  - Continuous and Discrete Binary Images
  - Multiple Objects (Connectivity)
  - Sequential (iterative) processing

Selecting a Threshold

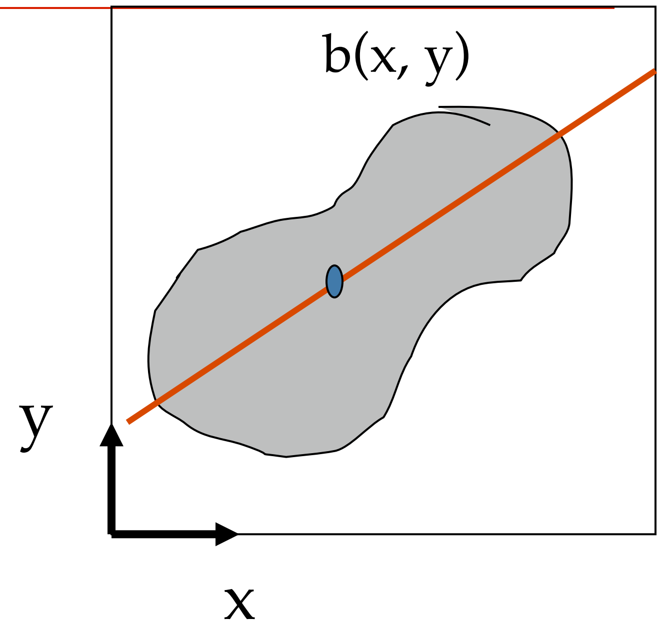


Bimodal Histogram

Threshold

Geometric Properties of Binary Images

- Assume:
  - b(x, y) is continuous
  - only one object



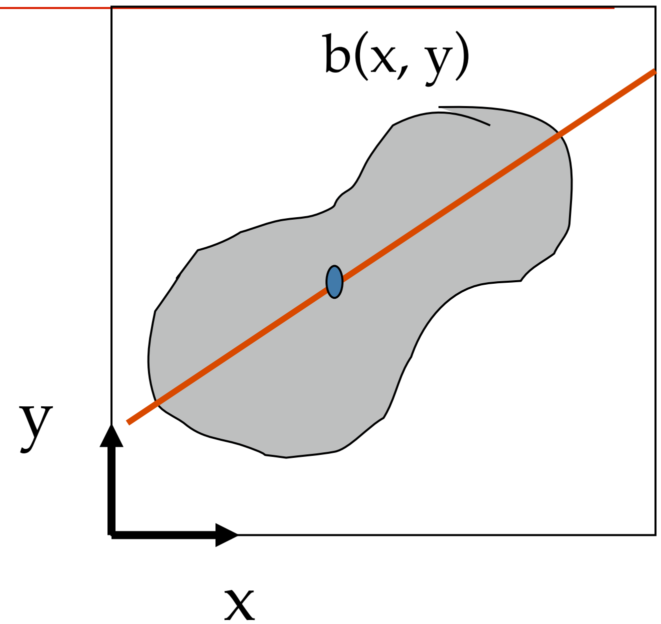
- Area: Zeroth Moment

$$A = \int \int b(x, y) dx dy$$

Geometric Properties of Binary Images

- Assume:

  b(x, y) is continuous
  only one object



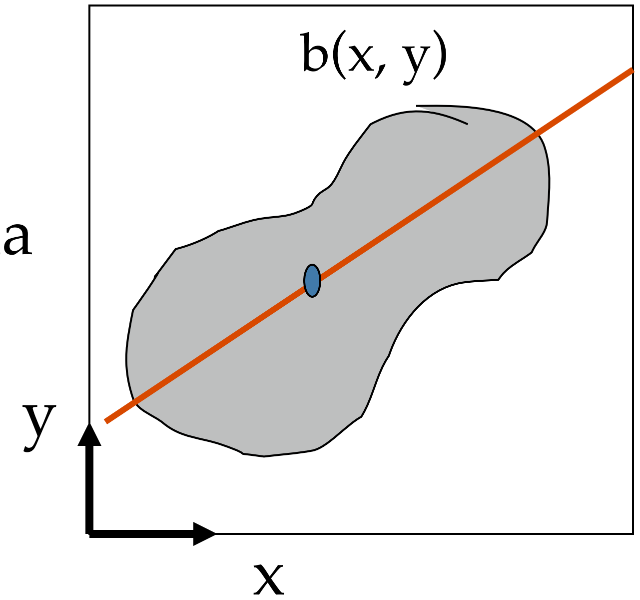
- Position: Center of Mass (First Moment)

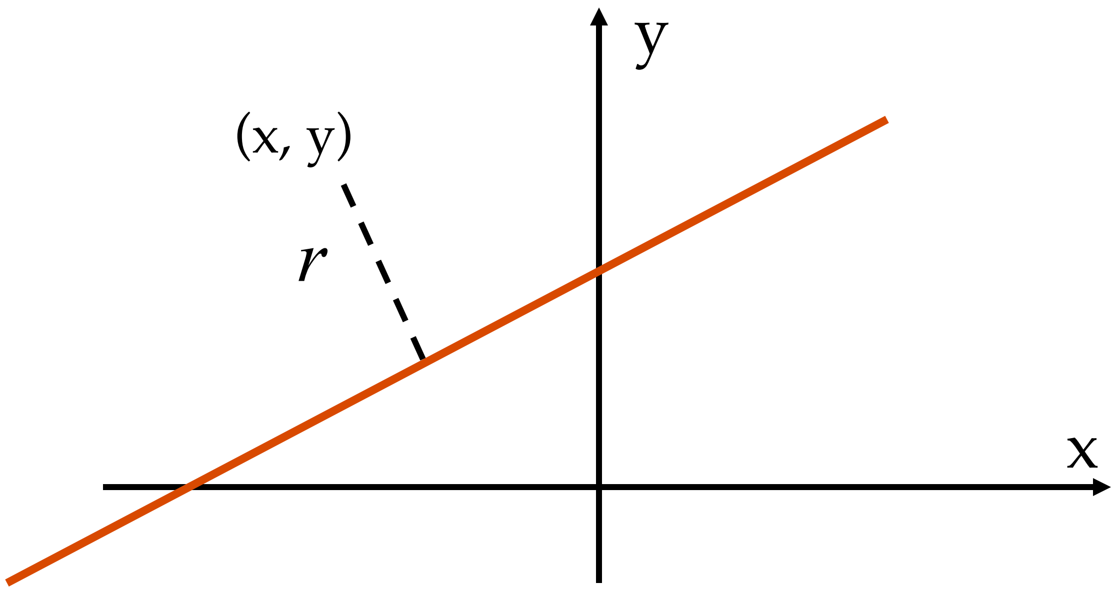
$$\bar{x} = \frac{1}{A} \int \int x b(x, y) dx dy$$

$$\bar{y} = \frac{1}{A} \int \int y b(x, y) dx dy$$

Geometric Properties of Binary Images

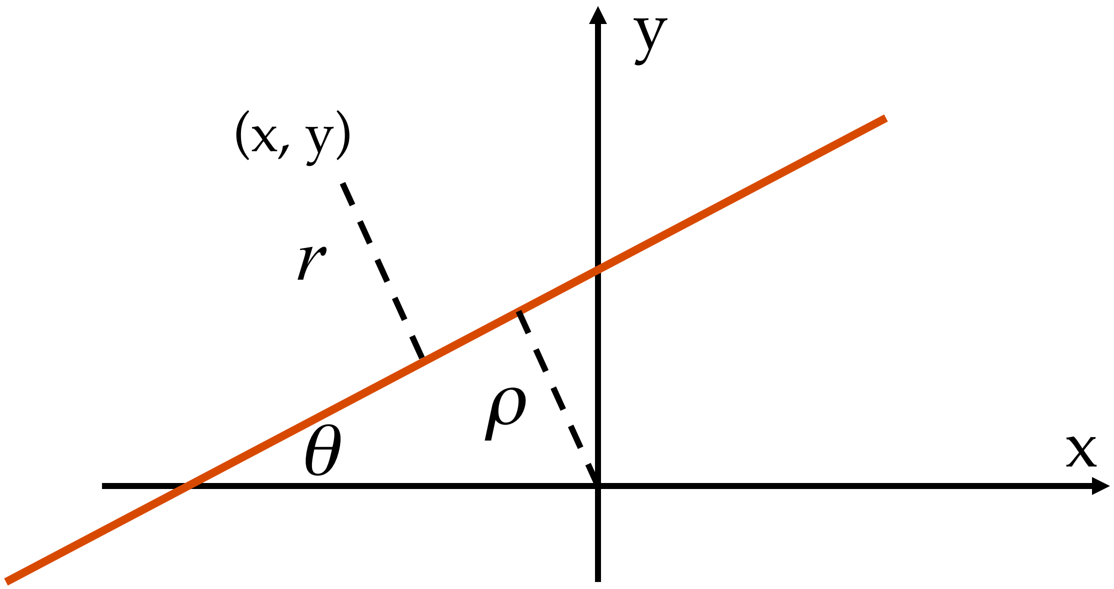
- Orientation: Difficult to define!
  - Axis of least second moment
  - For mass: Axis of minimum inertia





Minimize: $$E = \int\int r^2 b(x,y)\,dx\,dy$$

Which equation of line to use?



$$y = mx + b\,?\qquad 0 \le m \le \infty$$

We use:

$$x\sin\theta - y\cos\theta + \rho = 0$$

θ ρ

are finite

Minimizing Second Moment

Find θ and ρ that minimize E for a given b(x,y)

We can show that: $r = x\sin\theta - y\cos\theta + \rho$

So, $E = \int\int (x\sin\theta - y\cos\theta + \rho)^2 b(x,y)\,dx\,dy$

Using $\frac{dE}{d\rho} = 0$ we get: $A(\bar{x}\sin\theta - \bar{y}\cos\theta + \rho) = 0$

Note: Axis passes through the center $(\bar{x}, \bar{y})$

So, change co-ordinates: $x' = x - \bar{x}, y' = y - \bar{y}$

Minimizing Second Moment

We get: $$E = a\sin^2\theta - b\sin\theta\cos\theta + c\cos^2\theta$$

where,

$$\begin{aligned} a &= \int\int (x')^2 b(x,y)\,dx'\,dy' \\ b &= 2\int\int (x',y')b(x,y)\,dx'\,dy' \\ c &= \int\int (y')^2 b(x,y)\,dx'\,dy' \end{aligned}$$

- second moments w.r.t $(\bar{x}, \bar{y})$

We are not done yet!!

Minimizing Second Moment

$$E = a\sin^2\theta - b\sin\theta\cos\theta + c\cos^2\theta$$

Using $\frac{dE}{d\theta} = 0$ we get: $tan2\theta = \frac{b}{a-c}$

$$\sin 2\theta = \pm\frac{b}{\sqrt{b^2+(a-c)^2}} \quad \cos 2\theta = \pm\frac{a-c}{\sqrt{b^2+(a-c)^2}}$$

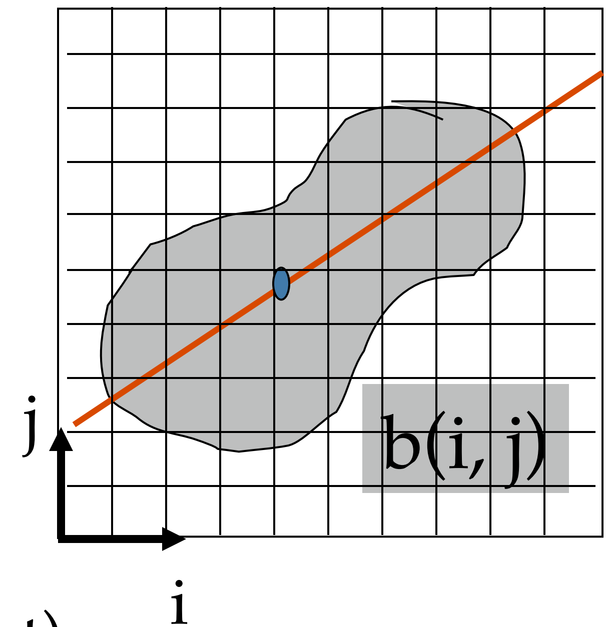
Solutions with +ve sign must be used to minimize E.
(-ve signe gives maximum E)

$$\frac{E_{min}}{E_{max}} \longrightarrow roundedness$$

Discrete Binary Images

- Assume:

  b(x, y) is discrete
  only one object



- Area: Zeroth Moment

$$A = \sum \sum b_{ij}$$

- Position: Center of Mass (First Moment)

$$\bar{x} = \frac{1}{A} \sum \sum i b_{ij} \quad \bar{y} = \frac{1}{A} \sum \sum j b_{ij}$$

- Second Moments:

$$a' = \sum \sum i^2 b_{ij} \quad b' = 2 \sum \sum i j b_{ij} \quad c' = \sum \sum j^2 b_{ij}$$

Note: a′,b′,c′ are defined w.r.t origin