CS 525: Theory of Computation
Problem Set 3

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5.13 **Solution:** Let $E = \{\langle M\rangle | M$ is TM with a useless state$\}$. Assume that TM R decides E. Construct S that uses R to decide E_{TM}, thus arriving at a contradiction, as follows:

$S =$ on input $\langle M\rangle$:

(a) Run R on M. If there are no useless states, then the accepting state is reachable by some input, and therefore R rejects.

(b) If there are useless states in M, then we must determine if the accepting state is one of them. For the set of states Q, create sets of states where each set represents a case where at least one non-accepting state is missing.

(c) For each of the sets of states created in step (a), create a machine M_i that is identical to M except that it only contains the subset of states.

(d) Run R on each machine M_i:

   i. If at least one instance of M_i is found with no useless states, then the accepting state in this instance of M_i is reachable. If we can reach the accepting state in M_i then we can reach the accepting state in M, since M has all the transitions and states that M_i contains. S will reject.

   ii. If every instance of M_i has a useless state, then there is no way to reach the accepting state in M. $L(M)$ is and therefore S will accept.

S decides if $L(M)$ is empty, which is a contradiction, therefore R does not exist.

5.14 **Solution:** Let $D = \{\langle M\rangle | M$ is TM which attempts to move its head left when its head is on the left-most tape cell when run on some input $\}$. Assume that TM R decides D. Construct S that uses R to decide A_{TM}, thus arriving at a contradiction.

$S =$ on input $\langle M, w\rangle$:

(a) Create a new machine, M_2, which is identical to M but with some differences:

   i. For every state q_i, make a state q_{Fi} which moves the tape head to the left and transitions to state q_i;

   ii. Place a dot over the left-most tape cell;

   iii. For every transition that moves the tape head to the left, make a duplicate of the transition that operates on the "dotted" input, and which writes a dotted version of the same output, moves the tape head to the right, and then moves to state q_{Fi} if the original transition moved to state q_i.

(b) M_2 will accept an input w if and only if M accepts w. The only difference is that M_2 will never move the tape head to the left from the left-most position.

(c) Create TM T using M_2 such that:

   $T =$ on any input $\langle x \rangle$:

   i. Simulate M_2 on input w;

   ii. If M_2 accepts w, move the tape head all the way to the left until it reaces the left-most position, and then attempt to move to the left again.

(d) Use R to decide if T ever attempts to move the tape head to the left from the left-most position. If R accepts, then M_2 accepts w and so M accepts w. If R rejects, then M does not accept w.

S decides if M accepts w, which is a contradiction, therefore R does not exist.

5.15 **Solution:** Let Q be the states of M and Γ be the tape alphabet. Let $D = \{\langle M \rangle | M$ is TM which attempts to move its head left when run on some input $\}$.

Simulate M on input w:

(a) Simulate M for $|\Gamma| \cdot |Q|^{|w|}$ steps.

(b) At each step mark the current tape position.

(c) If the tape head ever reads a marked tape position, *accept*.

(d) If M rejects or accepts the input, or the number of steps is exceeded without encountering a marked tape position, *reject*.