Digital Signal Processing
Instituto Superior Técnico
Lab assignment 2 - Spectral Analysis with the Discrete Fourier Transform
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Introduction

The Discrete Fourier Transform (DFT) is a Fourier representation used for finite duration sequences. The set of DFT coefficients is itself a finite duration sequence, which corresponds to samples, equally spaced in frequency, of the Fourier Transform of the signal. The DFT has a number of useful properties and there are efficient algorithms for its computation, which are known as Fast Fourier Transform (FFT). For these reasons, the DFT is very useful for analyzing and designing systems in the Fourier domain.

The DFT analysis and synthesis equations are the following:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}, \quad 0 \leq k \leq N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}, \quad 0 \leq n \leq N-1$$

Notes

In this assignment, you'll often find between brackets, as in {`command`}, suggestions of Matlab commands that may be useful to perform the requested tasks. You should use Matlab's help, when necessary, to obtain a description of how to use these commands.

Experimental work

1. **Spectral analysis of a synthetic signal.**

   R1.a) Create a synthetic signal $x(n)$ with duration M=512, corresponding to the following sum of sinusoids:

   $$x(n) = 5\cos(w_0 n + 1) + 2\cos(2w_0 n + 2) + 3\cos(5w_0 n + 3) \quad 0 \leq n \leq M-1$$

   Use, in this expression, $w_0 = 5.2 \times \frac{2\pi}{M}$ rad.

   R1.b) Plot the signal. Comment on what you observe. {`plot`}

   R1.c) Compute the signal's DFT of length N=512. Plot the signal's magnitude and phase spectra. Comment on what you observe. {`fft`}, {`fftshift`},{`abs`}, {`angle`}, {`xlabel`}, {`ylabel`}

   R1.d) Manually identify the 3 largest peaks in the one sided magnitude spectrum and indicate their index and corresponding magnitude, frequency and phase. From this information, and without using the inverse FFT, reconstruct the input sequence, $x_r(n)$.

   R1.e) Visualize both $x(n)$ and $x_r(n)$ on the same plot. Comment on what you observe. {`plot`} {`hold`}

   R1.f) Repeat items c), d) and e) using a DFT of length N=1024. Comment on the effect of DFT length.

2. **Spectral analysis of a real voice signal.**

   R2.a) Load the sound file `Howmanyroads.wav` and listen to the sound (don't forget to use the appropriate sampling frequency in the soundsc command). {`audioread`} {`soundsc`}

   R2.b) Identify the speaker.

R2.c) Obtain a segment of the voice signal of length M=2048, starting from sample 48500.

R2.d) Compute the signal's DFT of length N=2048 and perform the necessary steps in order to reconstruct the voice signal from the three most important sinusoids.

R2.e) Compare the original signal with its reconstruction and comment.