Information Sciences 181 (2011) 3323–3335

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Multi-objective robust PID controller tuning using two *lbests* multi-objective particle swarm optimization

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ARTICLE INFO

Article history:
Received 18 March 2010
Received in revised form 7 March 2011
Accepted 5 April 2011
Available online 8 April 2011

ABSTRACT

In this paper, two *lbests* multi-objective particle swarm optimization (2LB-MOPSO) is applied to design multi-objective robust Proportional-integral-derivative (PID) controllers for two MIMO systems, namely, distillation column plant and longitudinal control system of the super maneuverable F18/HARV fighter aircraft. Multi-objective robust PID controller design problem is formulated by minimizing integral squared error (ISE) and balanced robust performance criteria. During the search, 2LB-MOPSO can focus on small regions in the parameter space in the vicinity of the best existing fronts. As the *lbests* are chosen from the top fronts in a non-domination sorted external archive of reasonably large size, the offspring obtained can be more diverse with good fitness. The performance of various optimal PID controllers is compared in terms of the sum of ISE and balanced robust performance criteria. For the purpose of comparison, 2LB-MOPSO, NSGA-II as well as earlier reported Riccati, IGA and OSA methods are considered. The performance of PID controllers obtained using 2LB-MOPSO is better than that of others. In addition, Hypervolume-based comparisons are carried out to show the superior performance of 2LB-MOPSO over NSGA-II. The results reveal that 2LB-MOPSO yields better robustness and consistency in terms of the sum of ISE and balanced robust performance criteria than various optimal PID controllers.

1. Introduction

More than 90% of industrial controllers are still implemented based on PID control algorithms as no other controller matches the simplicity, clear functionality and ease of implementation. Several approaches have been reported in literature for tuning the parameters of PID controllers. Ziegler–Nichols and Cohen–Coon are the most commonly used conventional methods for tuning PID controllers [41,2]. A recent PID control survey [1], suggested the need for an efficient tuning method for multiple design objectives. Designing a PID controller to satisfy multiple objectives is an active research area [4,5,9,14,15,21,26,27,28,32–34,36,37,40,43]. Time domain performance [5,27,28,34], frequency domain performance [34] and robust performance criterion [4,14,15,36,37] based designs of PID controllers are reported in the literature.

Robustness is an important criterion in controller design because most of the real systems are vulnerable to external disturbance, measurement noise and model uncertainty [10,42]. There are two approaches to deal with robust optimal controller design problems. One is the structure specified controller [6,7,16,17,22,24] and the other is the output feedback controller [25]. Since the order of the robust output feedback controller is much higher than that of the system, it is not easy

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doi:10.1016/j.ins.2011.04.003

to implement the controller for higher order systems in practical engineering applications. To overcome this difficulty, the structure specified approach solves the robust optimal control problem from the suboptimal perspective [7].

Many researchers have employed mixed H_2/H_∞ optimal design strategy for optimal robust PID controller [6,7,16,17,22,24,44]. In [7], authors propose the design of simple genetic algorithm (GA) based mixed H_2/H_∞ controller for SISO system by minimizing ISE (H_2 norm) subjected to robust stability and disturbance attenuation constraints. GA is adopted to design structure specified H_∞ optimal controllers by minimizing the balanced robust performance criteria which combine the robust stability and disturbance attenuation performance measures, for practical applications [6]. But, their procedure requires prior domain knowledge, i.e. the Routh–Hurwitz criterion for reducing the size of the domain of every design parameter. In [24], authors suggest design of two-stage GA based optimal disturbance rejection PID controller for a servo motor system. In the first stage, GA is applied to determine H_∞ disturbance rejection value and in the second stage, GA is used to minimize ITSE performance index subjected to the disturbance rejection constraint. The orthogonal simulated annealing algorithm (OSA) is applied to design the optimal robust PID controller for MIMO system by minimizing combined ISE and balanced robust performance criteria [16]. An intelligent genetic algorithm (IGA) is proposed to the design of mixed H_2/H_∞ optimal robust PID controller [17]. In all of the above mentioned papers [6,7,16,17,22,24], the mixed H_2/H_∞ optimal robust PID controller design problem was solved using single objective optimization algorithms by combining different objectives or by treating some objectives as constraints.

The design of multi-objective robust PID controller using various multi-objective optimization algorithms is proposed in [4,14,15,36,37]. In [14], considering load disturbance rejection, robustness to model uncertainty and set point tracking as objectives and actuator limit as a constraint, Multi-objective robust control design (MRCD) genetic algorithm is used to solve the design problem. However, the weighting function for the design of robust controller and also robust stability and disturbance attenuation constraints are not considered. In [4,36,37], the multi-objective robust PID controller design problem is formulated as a solution of linear matrix inequalities. Recently, an intelligent multi-objective simulated annealing algorithm was proposed for designing robust PID controllers to satisfy disturbance attenuation, robust stability and set point tracking. The problem is treated as a three objective optimization problem. But, for performance analysis and ease of visualization of the Pareto front, disturbance attenuation and robust stability performance measures are combined into a balanced robust performance criterion. Pareto front is drawn between set point tracking and balanced robust performance criteria [18,3].

In this paper, two-*lbests* based multi-objective particle swarm optimizer (2LB-MOPSO) algorithm [45] is applied for the design of robust PID controller by minimizing ISE and balanced robust performance criterion for robust performance. For the design of optimal robust PID controller, two different systems, namely MIMO distillation column model [17,38] and MIMO super maneuverable F18/HARV fighter aircraft system [6,15–17] are considered. The two systems included in this paper have more than one objective conflicting with each other. Our aim is to find Pareto optimal trade-off solutions representing the best possible compromises among all objectives. The 2LB-MOPSO is developed taking into consideration two common goals, namely fast convergence to the Pareto front and good distribution of solutions along the front. The performance of 2LB-MOPSO is compared with that of NSGA-II by using the Hypervolume indicator [23].

The paper is organized as follows. The multi-objective robust PID controller is introduced in Section 2. Section 3 briefly describes 2LB-MOPSO algorithm. Section 4 introduces the two MIMO systems considered in this research. Sections 5 and 6 present the implementation of multi-objective optimal robust PID controller design for the two MIMO systems and the simulation results. Conclusions are given in Section 7.

2. Multi-objective robust PID controller

Consider a control system with n_i inputs and n_o outputs, as shown in Fig. 1, where $P(s)$ is the nominal plant, $\Delta P(s)$ is the plant perturbation, $G_c(s)$ is the controller, $r(t)$ is the reference input, $u(t)$ is the control input, $e(t)$ is the tracking error, $d(t)$ is the external disturbance, and $y(t)$ is the output of the system [17].

Without loss of generality, the plant perturbation $\Delta P(s)$ is assumed to be bounded by a known stable function matrix $W_1(s)$

$$\bar{\sigma}(\Delta P(j\omega)) \leqslant \bar{\sigma}(W_1(j\omega)), \quad \forall_\omega \in [0, \infty) \tag{1}$$

where $\bar{\sigma}(A)$ denotes the maximum singular value of a matrix $\boldsymbol{A}$.

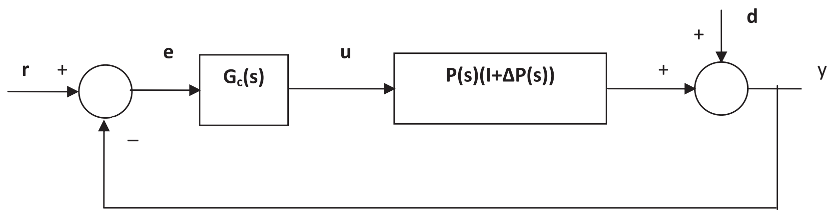

Fig. 1. Control system with plant perturbation and external disturbance.

2.1. Robust PID controller

If a controller $G_c(s)$ is designed in such a way that the nominal closed loop system ($\Delta P(s) = 0$ and $d(t) = 0$) is asymptotically stable, the robust stability performance satisfies the following inequality

$$J_a = \|W_1(s)T(s)\|_\infty < 1 \tag{2}$$

and the disturbance attenuation performance satisfies the following inequality

$$J_b = \|W_2(s)S(s)\|_\infty < 1 \tag{3}$$

Then the closed loop system is also asymptotically stable with $\Delta P(s)$ and $d(t)$, where $W_2(s)$ is a stable weighting function matrix specified by designers. $S(s)$ and $T(s) = I - S(s)$ are the sensitivity and complementary sensitivity functions of the system, respectively

$$S(s) = (I + P(s)G_c(s))^{-1} \tag{4}$$

$$T(s) = P(s)G_c(s)(I + P(s)G_c(s))^{-1} \tag{5}$$

H_∞-norm in (2) and (3) is defined as

$$\|A(s)\|_\infty \equiv ax_\omega \bar{\sigma}(A(j\omega)) \tag{6}$$

Robust controller design problem lies in how to find a controller to stabilize the closed loop system and attenuate the external disturbance. The principal criterion for this problem is simultaneously minimizing the robust stability performance J_a and disturbance attenuation performance J_b [6]. Minimization of balanced performance criterion $J_\infty = \left(J_a^2 + J_b^2\right)^{1/2}$ [17] is considered to minimize both J_a and J_b simultaneously. For improving the system performance, robust stability and disturbance attenuation together are often not enough in the control system design. Optimal tracking performance is also an appealing factor in many practical control engineering applications [7]. The most commonly used performance measure is the ISE because of its easy implementation and it being mathematically convenient both for analysis and computation. Minimization of ISE is a good compromise among reduction of rise time to limit the effect of large initial error, reduction of peak overshoot and reduction of settling time to limit the effect of small error lasting for a long time [31]. The minimization of tracking error ISE (J_2 i.e. H_2 norm) should be taken into account

$$J_2 = \int_0^\infty e^T(t)e(t)dt \tag{7}$$

where $e(t) = r(t) - y(t)$ is the error determined with $\Delta P(s) = 0$ and $d(t) = 0$.

In designing optimal robust controllers with fixed structures, both the tracking behavior and robust stability are considered [19]. In this paper, the PID controller is designed for obtaining better tracking performance by minimizing ISE i.e. J_2 and better robust performance against model uncertainty and disturbance by minimizing balanced robust performance criterion J_∞. Both the objectives J_2 and J_∞ are minimized simultaneously subjected to the robust stability and disturbance attenuation constraints by using multi-objective optimization algorithm 2LB-MOPSO. Thus the problem of multiobjective robust PID controller has been defined as follows:

$$\text{Minimize} \quad J = \{J_2, J_\infty\} \tag{8}$$

subjected to robust stability and disturbance attenuation constraints as defined in (2) and (3).

2.2. Robust PID controller structure

In general, transfer function of a structure-specified controller can be expressed as follows:

$$G_c(s) = \frac{N_c(s)}{D_c(s)} = \frac{B_m s^m + B_{m-1}s^{m-1} + \cdots + B_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{9}$$

where m and n are fixed orders of $N_c(s)$ and $D_c(s)$, respectively. Also B_k is defined in (10).

$$B_k = \begin{bmatrix} b_{k11} & \cdots & b_{k1n_i} \\ \vdots & \ddots & \vdots \\ b_{kn_0 1} & \cdots & b_{kn_0 n_i} \end{bmatrix} \quad \text{for} \quad k = 0, 1, \ldots, m \tag{10}$$

where n_i and n_o are the number of inputs and outputs, respectively.

Most of the conventional controllers used in industrial control systems have fundamental structures such as PID and lead/lag configurations. Such controllers are special cases of the structure-specified controllers. For the PID controller, we have $n = 1$, $m = 2$ and $a_0 = 0$, i.e.

$$G_c(s) = \frac{\boldsymbol{B}_2 s^2 + \boldsymbol{B}_1 s + \boldsymbol{B}_0}{s} \tag{11}$$

In case of the system with complicated plant and a single simple-structure controller, one seems to have less freedom of controller parameters to tune the system to achieve the H_∞ optimal design objectives. In this situation, we must increase the order m and n of the controller in (9).

3. Two-*lbests* based multi-objective particle swarm optimizer (2LB-MOPSO)

Particle swarm optimization (PSO) is a heuristic search technique proposed by Kennedy and Eberhart [12] who were inspired by the choreography of bird flocks, animal herds and fish schools. The relative simplicity of the PSO and the fact that it is a population-based technique has made it a good candidate for solving multi-objective optimization problems. When solving single-objective optimization problems, the leaders (i.e. *pbest*, *lbest*, *gbest*) [12,13,35,39] that each particle uses to update its position are completely determined once a neighborhood topology is chosen. However, when solving multi-objective optimization problems, due to the existence of numerous trade-off Pareto optimal solutions, each particle can have a set of leaders in the external archive from which *pbest*, *lbest*, *gbest* can be selected in order to update its position. Also, the content of the external archive is usually reported as the final Pareto approximation solution set at the end of evolution. Since the size of the external archive is usually limited to the finally required approximation solution set size, whenever the repository exceeds this size, crowding distance criterion is used. That is the particles located in less populated areas of objective space are given priority to remain in the archive over those in highly populated regions.

Recently, several MOPSO techniques are applied to PID controller design. In [30], a novel variant of a multi-objective particle swarm optimization algorithm is proposed to the design of PID controllers taking into account the classical design objectives of set-point tracking and output disturbance rejection. In order to overcome the drawbacks of current PID design methods for main steam temperature control of boiler unit in a thermal power plant, a new method is proposed for multi-objective PID controller to ensure reliability and robustness in its performance. Adaptively weighted PSO (AWPSO) technique is applied to the parameter optimization design [20]. In this paper, we employ a two *lbests* based MOPSO variant [45]. In order to select the first *lbest* for a particle, an objective is first randomly selected followed by a random selection of a bin of the chosen objective. Within this bin, the archived member with the lowest front number and the highest crowding distance is selected as the first *lbest*. The second *lbest* is selected from a neighboring non-empty bin with the lowest front number and the smallest Euclidean distance in the parameter space to the first *lbest*. As each particle's velocity is adjusted by the two *lbests* from two neighboring bins, the flight of each particle will be in the direction between the positions of two *lbests* and is orientated to improve upon the current solutions.

In 2LB-MOPSO, as the two *lbests* of every particle are selected somewhat randomly, and if a particle is assigned with a different pair of *lbests* in every generation, it will not be effective because the flight of each particle will become almost random in this scenario. Thus, after assigning a pair of *lbests* to a particle, the number of iterations the particle fails to contribute a solution to the archive $P(t)$ is recorded as *count*. If the *count* exceeds a given threshold, the particle is re-assigned to another pair of *lbests*. The 2LB-MOPSO is shown in Table 1.

4. Test systems

In order to validate the performance of the proposed 2LB-MOPSO based multi-objective robust PID optimal controller, two different MIMO systems, namely distillation column model (test system-I) and super maneuverable F18/HARV fighter aircraft system (test system-II) are considered and the details are given below.

4.1. Test system-I

We consider the methanol-ethanol distillation column by Chien and Ogunnaike [8]. In [17,38], robust PID controller was designed for this highly coupled distillation column model. In this system, tray temperatures are regulated, as inferential variables for composition, by manipulating liquid reflux and vapor boilup rates. The transfer function of the plant is given as follows:

$$P(s) = \begin{bmatrix} \frac{-33.98}{(98.02s+1)(0.42s+1)} & \frac{32.63}{(99.6s+1)(0.35s+1)} \\ \frac{-18.85}{(75.43s+1)(0.30s+1)} & \frac{34.84}{(110.5s+1)(0.03s+1)} \end{bmatrix} \tag{16}$$

where the outputs 1 and 2 are temperatures of tray 21 and tray 7 and inputs 1 and 2 are the liquid reflux and vapor boilup rates, respectively.

The plant uncertainties $\Delta P(s)$ are unknown but bounded by the following stable function:

$$W_1(s) = \frac{100s+1}{s+1000} I_{2\times 2} \tag{17}$$

and for attenuate disturbance, a weighting function $W_2(s)$ consisting of low-pass filter

Table 1
The procedure of the 2LB-MOPSO [45].

Initialize the swarm: Initialize *NP* number of particles randomly and uniformly in the *D*-dimensional search space X_D. Evaluate the fitness values of all particles. Set all current positions to be *P*(0), the external archive
Select *lbests* from the *P*(0) external archive using the steps within "If *count*(*j*) > 5" below
Optimization Loop:
For *i* = 1to *Feval_max*
For *j* = 1 to *NP*
  If *count*(*j*) > 5
    For particle *j*, the first *lbest* is selected by (a) randomly selecting an objective, (b) randomly choosing a bin of the selected objective, (c) choosing the solution with the lowest front number and the largest crowding distance in the chosen bin. The second *lbest* is selected from the neighborhood of the first *lbest* in the parameter space
  Else
    For particle *j*, update the two *lbests* in the same bins used in the last iteration. The first *lbest* should have the lowest front number and the largest crowding distance in the bin. The second *lbest* is selected from the neighborhood of the first *lbest* in the parameter space
    Endif
  $V(j) = \omega^*(V(j) + c_1^* rand()^*(lbest(j) - particle(j)) + c_2^* rand()^*(lbest(j + NP) - particle(j)))$
  $V_i(d) = (min(V_{max}(d), V_i(d))$ and $max(-V_{max}(d), V_i(d)))$
  $X_{Di}(d) = X_{Di}(d) + V_{Di}(d)$
  If any dimension exceeds the search space
    Set the corresponding bound value to be the dimension value of *particle*(*j*);
  Endif
    Evaluate the fitness values of *particle*(*j*)
    $feval\_count = feval\_count + 1$
EndFor *j*
Do non-dominated sorting on combined *P*(*t*) and new particles *Q*(*t*) to obtain external archive *P*(*t* + 1)
Update *count* of each particle (*count* is the number of iterations the particle fails to contribute a solution to the external archive)
Stop if a stop criterion is satisfied (i.e. when *feval_count* reaches the *Feval_max*)
EndFor *i*

$$W_2(s) = \frac{s + 1000}{1000s + 1} I_{2\times 2} \tag{18}$$

is considered.

4.2. Test system-II

We consider the longitudinal control system of the super maneuverable F18/HARV fighter aircraft [6,15–17] in horizontal flight at an altitude of 15,000 (ft) with Mach number 0.24, airspeed $V_T = 238.7$ (ft/s), attack angle $\alpha = 25$ (deg), and pitch angle $\beta = 25$ (deg). The trim value of the path angle is $\beta - a = 0$ (deg) and the trim pitch rate is $q = 0$ (deg/s). The longitudinal dynamics of the system can be described as

$$\begin{aligned} \frac{dx}{dt} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{19}$$

where *A*, *B* and *C* are given as

$$A = \begin{bmatrix} -0.0750 & -24.0500 & 0 & -36.1600 \\ -0.0009 & -0.1959 & 0.9896 & 0 \\ -0.0002 & -0.1454 & -0.1677 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{20}$$

$$B = \begin{bmatrix} -0.0230 & 0 & -0.0729 & 0.0393 & -0.0411 & 0.1600 \\ -0.0002 & -0.0001 & -0.0004 & 0 & -0.0003 & -0.0003 \\ -0.0067 & -0.0007 & -0.0120 & 0.0007 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$x = [V_T \quad \alpha \quad q \quad \beta]^T \tag{23}$$

$$u = [\,u_{TV} \quad u_{AS} \quad u_{SS} \quad u_{LE} \quad u_{TE} \quad u_T\,]^T \tag{24}$$

where u_{TV}, u_{AS}, u_{SS}, u_{LE}, u_{TE}, u_T are the perturbations in symmetric thrust vectoring vane deflection, symmetric aileron deflection, symmetric stabilator deflection, symmetric leading edge flap deflection, symmetric trailing edge flap deflection, and throttle position, respectively. Note that the rank of matrix B is only three. By employing the pseudo-control technique, the six control inputs $(u_{TV}, u_{AS}, u_{SS}, u_{LE}, u_{TE}, u_T)$ are transformed into three linearly independent variables. Therefore, the system can be rewritten as,

$$\frac{dx}{dt} = Ax + B_v v \tag{25}$$

where B_v and v are given as,

$$B_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{26}$$

$$v = \begin{bmatrix} -0.0230 & 0 & -0.0729 & 0.0393 & -0.0411 & 0.1600 \\ -0.0002 & -0.0001 & -0.0004 & 0 & -0.0003 & -0.0003 \\ -0.0067 & -0.0007 & -0.0120 & -0.0006 & 0.0007 & 0.0005 \end{bmatrix} u \tag{27}$$

The bound $W_1(s)$ for the plant perturbation $\Delta P(s)$

$$W_1(s) = \frac{0.0125s^2 + 1.2025s + 1.25}{s^2 + 20s + 100} I_{3\times 3} \tag{28}$$

and for the attenuate disturbance, a weighting function $W_2(s)$ consisting of low-pass filter

$$W_2(s) = \frac{0.25s + 0.025}{s^2 + 0.4s + 10000000} I_{3\times 3} \tag{29}$$

are considered.

5. Implementation of multi-objective PID controller

5.1. Test system-I

For the highly coupled distillation column MIMO model, the PID controller structure is given in (11), where

$$\begin{aligned} \boldsymbol{B}_2 &= \begin{bmatrix} b_{2_{11}} & b_{2_{12}} \\ b_{2_{21}} & b_{2_{22}} \end{bmatrix} \\ \boldsymbol{B}_1 &= \begin{bmatrix} b_{1_{11}} & b_{1_{12}} \\ b_{1_{21}} & b_{1_{22}} \end{bmatrix} \\ \boldsymbol{B}_0 &= \begin{bmatrix} b_{0_{11}} & b_{0_{12}} \\ b_{0_{21}} & b_{0_{22}} \end{bmatrix} \end{aligned} \tag{30}$$

The design variables for this system are

$$X_D = [b_{2_{11}}, b_{2_{12}}, b_{2_{21}}, b_{2_{22}}, b_{1_{11}}, b_{1_{12}}, b_{1_{21}}, b_{1_{22}}, b_{0_{11}}, b_{0_{12}}, b_{0_{21}}, b_{0_{22}}] \tag{31}$$

with the lower and upper bounds of the design variables are taken as $-200 \leqslant X_{Di} \leqslant +200$ for $i = 1,2,3,4,5,\ldots,12$.

5.2. Test system-II

For the longitudinal control system of the super maneuverable F18/HARV fighter aircraft the PID controller structure is given in (11), where

$$\boldsymbol{B}_2 = \begin{bmatrix} b_{2_{11}} & b_{2_{12}} & b_{2_{13}} \\ b_{2_{21}} & b_{2_{22}} & b_{2_{23}} \\ b_{2_{31}} & b_{2_{32}} & b_{2_{33}} \end{bmatrix}$$
$$\boldsymbol{B_1} = \begin{bmatrix} b_{1_{11}} & b_{1_{12}} & b_{1_{13}} \\ b_{1_{21}} & b_{1_{22}} & b_{1_{23}} \\ b_{1_{31}} & b_{1_{32}} & b_{1_{33}} \end{bmatrix} \tag{32}$$
$$\boldsymbol{B_0} = \begin{bmatrix} b_{0_{11}} & b_{0_{12}} & b_{0_{13}} \\ b_{0_{21}} & b_{0_{22}} & b_{0_{23}} \\ b_{0_{31}} & b_{0_{32}} & b_{0_{33}} \end{bmatrix}$$

The design variables for this system are

$$\begin{aligned} X_D = [&b_{2_{11}}, b_{2_{12}}, b_{2_{13}}, b_{2_{21}}, b_{2_{22}}, b_{2_{23}}, b_{2_{31}}, b_{2_{32}}, b_{2_{33}}, b_{1_{11}}, b_{1_{12}}, b_{1_{13}}, b_{1_{21}}, b_{1_{22}}, b_{1_{23}}, b_{1_{31}}, b_{1_{32}}, b_{1_{33}}, \\ &b_{0_{11}}, b_{0_{12}}, b_{0_{13}}, b_{0_{21}}, b_{0_{22}}, b_{0_{23}}, b_{0_{31}}, b_{0_{32}}, b_{0_{33}}] \end{aligned} \tag{33}$$

The lower and upper bounds of the design variables are taken as

$$-20,000 \leqslant X_{D_i} \leqslant +20,000 \quad \text{for} \quad i = 1, 2, 3, 4, 5, \ldots, 27 \tag{34}$$

6. Simulation results

All the simulations are carried on Pentium Core2duo PC operating @2.2 GHz with 3 GB RAM. In order to validate the performance of the 2LB-MOPSO based multi-objective robust PID controller, highly coupled distillation column and super maneuverable F18/HARV fighter aircraft MIMO test systems are considered. In order to show the superior performance of 2LB-MOPSO over other existing established MOEA, NSGA-II [11] is applied to test the same control systems. Owing to the randomness of the evolutionary algorithms, 25 independent trials are conducted. The best, mean, and worst value of best solutions with ISE combined with balanced robust performance criteria from 25 trials obtained by 2LB-MOPSO and NSGA-II are presented in Table 4. For the purpose of comparison, earlier reported results of OSA [16], IGA methods [17] are taken. The best solutions of the obtained Pareto fronts by those algorithms are used to compare the performance of the multi-objective algorithms in Tables 2 and 3. The Hypervolume indicator [23] is used to assess the approximated Pareto fronts obtained by 2LB-MOPSO and NSGA-II. The Hypervolume indicator measures the Hypervolume of that portion of the objective space that is weakly dominated by an approximation set A, and it is to be maximized. Here, we consider the Hypervolume difference to a reference set R, and we will refer to this indicator as H, which is defined as $H = H(R) - H(A)$, where smaller values correspond to higher quality – in contrast to the original Hypervolume. In order to compare the results of the two algorithms, we combined two sets of results obtained by two algorithms over all 25 runs, applying the non-dominance sorting on them to obtain the overall non-dominated solutions as the reference set R. The Hypervolume indicator H takes into account both the convergence performance to the Pareto front and distribution of solutions in approximating the front. It indicates better performance with smaller indicator value. Maximum number of functional evaluations (Max_Feval) is used as the stopping criterion. The best obtained Pareto fronts using NSGA-II and 2LB-MOPSO are shown in Fig. 2.

Table 2
Best performance of various PID controllers in terms of $J = J_2 + J_\infty$ for test system-I.

Method	Test system-I		
	J_2	J_∞	J
Riccati based [17,38]	42.6753	1.1588	43.8341
IGA [17]	16.7777	0.8520	17.6297
NSGA-II [11]	10.5009	1.0244	11.5253
2LB-MOPSO [45]	9.7505	1.0179	10.7685

Table 3
Best performance of various PID controllers in terms of $J = J_2 + J_\infty$ for test system-II.

Method	Test system-II		
	J_2	J_∞	J
OSA [16]	0.0019	0.4374	0.4393
IGA [17]	0.0092	0.1396	0.1488
NSGA-II [11]	3.5037e−03	0.1154	0.1189
2LB-MOPSO [45]	6.7506e−05	0.0828	0.0829

Max_Feval is set at 10,000 for test systems-I and II. As both problems have constraints, we apply the epsilon-constraint handling method [29] to solve them. For 2LB-MOPSO, the parameters are set as in the [45]: *count* and *n_bins* are defined as 5 and 10; population size $NP = 50$, inertia weight $\omega = 0.729$; $c_1 = c_2 = 2.05$; $V_{\max} = 0.25(X_{D\max} - X_{D\min})$. Both NSGA-II and 2LB-MOPSO have the same size 100 for the external archive for the two test systems.

6.1. Results for test system-I

2LB-MOPSO is used to design a multi-objective robust PID controller for distillation column plant by minimizing ISE for unit step reference input and balanced robust performance criteria. The best obtained Pareto front out of 25 trials is shown in Fig. 2. For the purpose of comparison, the obtained Pareto front using NSGA-II method is also given. The parameters J_2, J_∞ and $J = J_2 + J_\infty$ were considered to enable a convenient comparison of the 2LB-MOPSO with existing methods such as Riccati based and IGA [17][38] and also NSGA-II method [11]. Best PID controller in terms of J using 2LB-MOPSO is given in (35).

$$G_c(s) = \frac{\begin{bmatrix} -9.1462 & -3.0599 \\ -0.4071 & 0.4046 \end{bmatrix} s^2 + \begin{bmatrix} -91.2012 & 18.5016 \\ -20.0168 & 20.4916 \end{bmatrix} s + \begin{bmatrix} -72.2656 & 104.9353 \\ -63.7446 & 92.2075 \end{bmatrix}}{s} \tag{35}$$

The step responses of the system with plant perturbation $\Delta P(s)$ in (17) using IGA-based, NSGA-II based and 2LB-MOPSO based controllers are shown in Figs. 3–5, respectively. From the step response characteristics, it is clear that the step responses (y_1 and y_2) of the best optimal controller designed using 2LB-MOPSO are more robust than those designed using

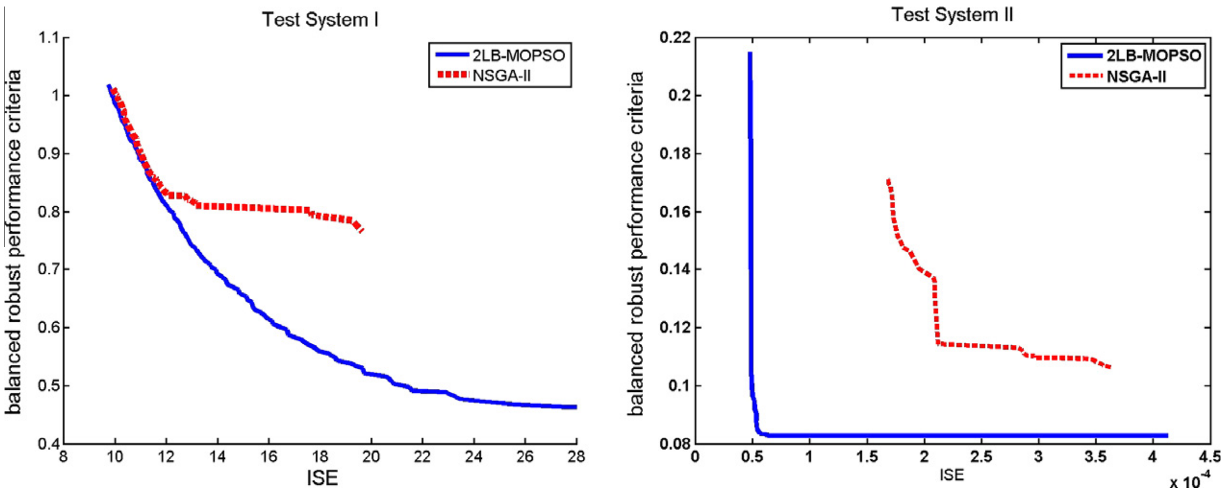

Fig. 2. Best obtained Pareto fronts for test systems-I and II using 2LB-MOPSO and NSGA-II.

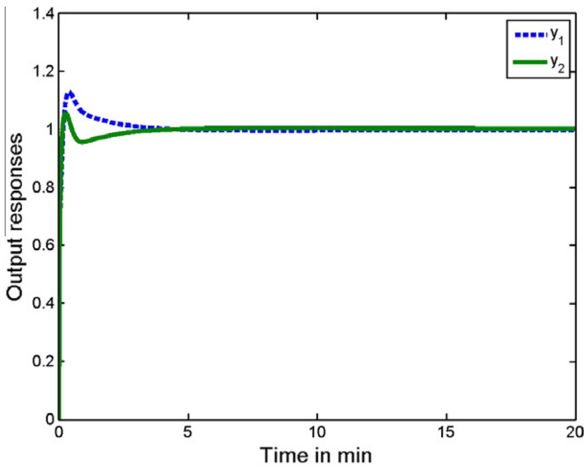

Fig. 3. Step response of test system-I with plant perturbation using IGA method.

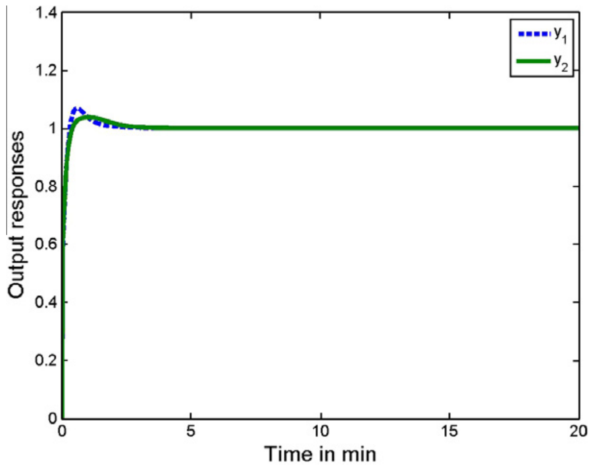

Fig. 4. Step response of test system-I with plant perturbation using NSGA-II method.

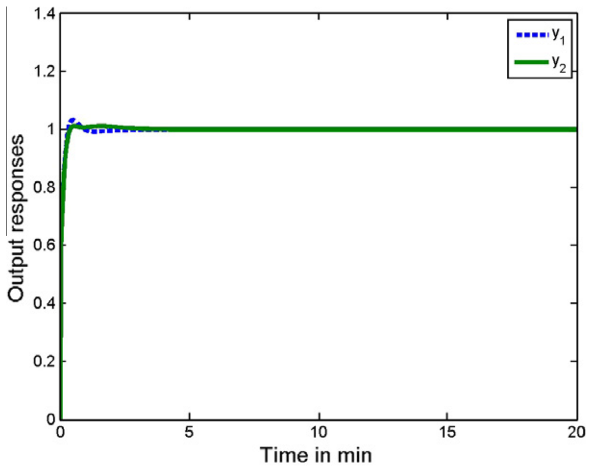

Fig. 5. Step response of test system-I with plant perturbation using 2LB-MOPSO method.

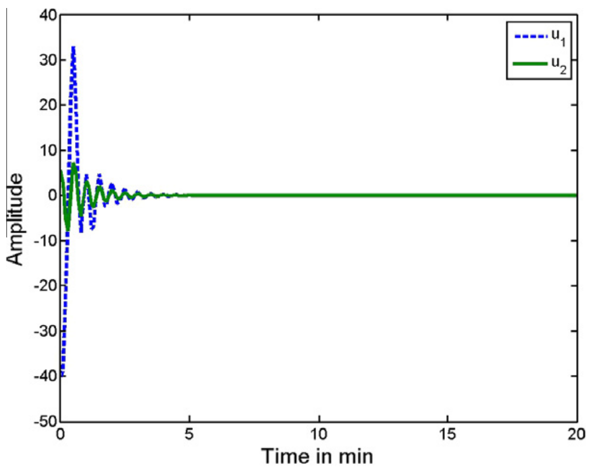

Fig. 6. The controller outputs (u_1 and u_2) using reported IGA method.

other methods against plant perturbation. The controller outputs (u_1 and u_2) using reported IGA based method and 2LB-MOPSO based controller are given in Figs. 6 and 7, respectively. It is clear that 2LB-MOPSO based controller output is better than that generated by the IGA-based method.

6.2. Results for test system-II

2LB-MOPSO algorithm is used to design a multi-objective robust PID controller for F18/HARV fighter aircraft system by minimizing ISE for the reference input $r(t) = [0, \quad 1 - e^{-3t}, \quad 1 - e^{-6t}]$ and balanced robust performance criteria. The best obtained Pareto front out of 25 trials for the test system-II is shown in Fig. 2. For the purpose of comparison, NSGA-II method and the already reported OSA and IGA based controllers are considered. The best PID controller obtained using 2LB-MOPSO is given in (36).

$$G_c(s) = \frac{\begin{bmatrix} 6.7047e+003 & -1.1386e+004 & -1.7767e+004 \\ -6.5330e+003 & -6.4182e+003 & 8.5945e+001 \\ 2.0000e+004 & -2.0000e+004 & 2.0000e+004 \end{bmatrix} s^2}{s} + \frac{\begin{bmatrix} 2.0000e+004 & -2.0000e+004 & -2.0000e+004 \\ -2.0000e+004 & -2.0000e+004 & -8.0985e+003 \\ 6.1823e+003 & -2.0000e+004 & 2.0000e+004 \end{bmatrix} s}{s} + \frac{\begin{bmatrix} 2.0000e+004 & -1.5947e+004 & -2.0000e+004 \\ -1.7599e+004 & -6.7764e+003 & -5.4814e+003 \\ -3.7193e+003 & -2.0000e+004 & 5.2109e+003 \end{bmatrix}}{s} \tag{36}$$

Tables 2 and 3 show the best performance of various PID controllers in terms of J.2LB-MOPSO results are significantly better than earlier reported results obtained by the Riccati based, the OSA based, IGA based and NSGA-II based methods. Moreover, all the results obtained by 2LB-MOPSO satisfy the constraints in Eqs. (2) and (3). Table 4 shows the best, mean, and

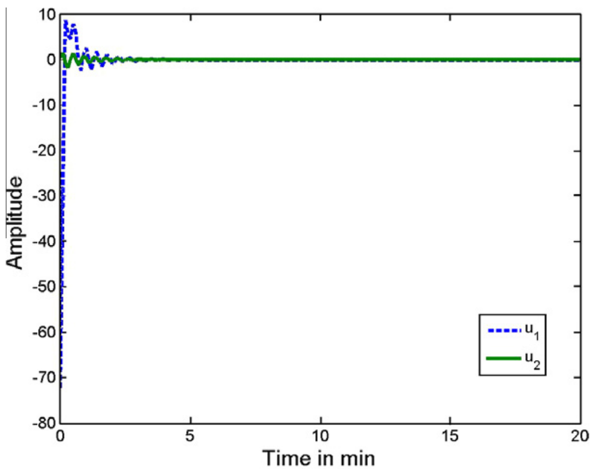

Fig. 7. The controller outputs (u_1 and u_2) using 2LB-MOPSO method.

Table 4
Statistical performance of 2LB-MOPSO and NSGA-II in terms of J for test systems-I and II.

Test system	Statistical performance on J	2LB-MOPSO [45]	NSGA-II [11]
I	Best	10.7685	11.5252
	Mean	10.7710	12.4924
	Worst	10.7786	20.4050
II	Best	0.0829	0.1189
	Mean	0.0829	0.1255
	Worst	0.0831	0.1456

worst value of best solutions with ISE combined with balanced robust performance criteria out of each approximate Pareto front from 25 trials for 2LB-MOPSO and NSGA-II. It can be observed that the results of 2LB-MOPSO is relatively more stable with smaller standard deviations indicating superior robust performance of 2LB-MOPSO based optimal PID controller over that of NSGA-II based. To demonstrate the robust performance capability of the 2LB-MOPSO and IGA based optimal controllers, the system was simulated with the reference inputs $r(t) = \lfloor 0, \quad 1 - e^{-3t}, \quad 1 - e^{-}6t \rfloor$ and external disturbance of $d(t) = 0.01e^{-0.2t}\cos(3162.3t)[1, \quad 1, \quad 1]^T$ and plant perturbation $\Delta P(s)$ as given in (28). The output responses (y_1, y_2 and y_3) of the system with the IGA-based and the 2LB-MOPSO based methods are shown in Figs. 8 and 9, respectively. Figs. 8 and 9 clearly indicate that the robust performance of 2LB-MOPSO based optimal PID controller is better than the performance of IGA based controller. The best, mean, worst and standard deviation values of Hypervolume indicator H for the approximate Pareto fronts out of 25 trials obtained by 2LB-MOPSO and NSGA-II are shown in Table 5. As the Hypervolume indicator captures both the convergence performance to the Pareto front and distribution of solutions in the approximated front, the Hypervolume indicator values of obtained Pareto front can be directly compared. Smaller Hypervolume indicator H values correspond to higher quality. Hence, it can easily be observed that the Hypervolume indicators obtained by 2LB-MOPSO are much better than the results obtained by NSGA-II on test systems with respect to all comparison categories with small standard deviations. The Hypervolume indicators presented in Table 5 indicate better robust performance of 2LB-MOPSO based optimal PID controller with faster convergence to the Pareto front and better distribution of solutions along the front than those generated by NSGA-II. The superior performance can also be observed from Fig. 2, in which the best Pareto fronts obtained by 2LB-MOPSO dominate the ones obtained by NSGA-II.

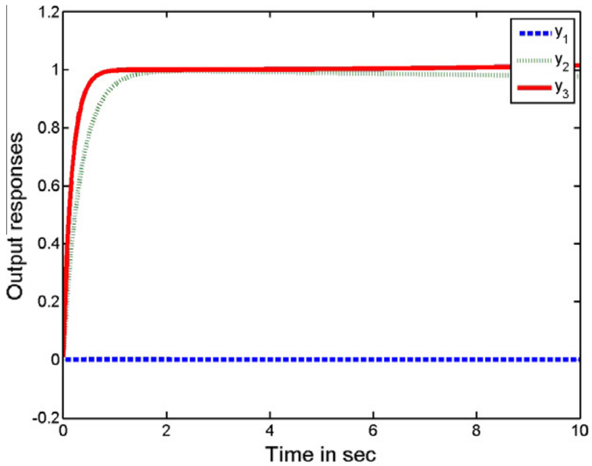

Fig. 8. Response of test system-II with plant perturbation and external disturbance using IGA method.

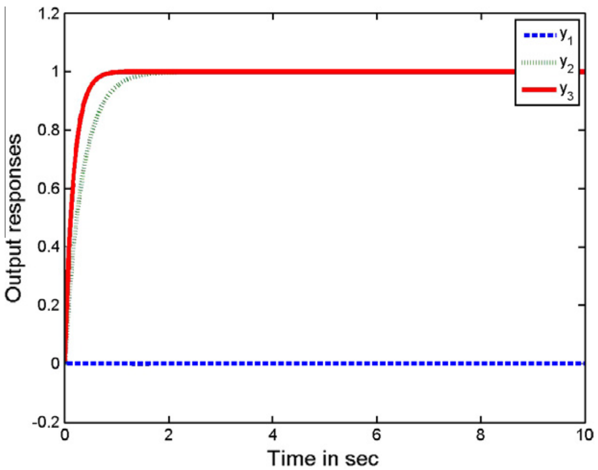

Fig. 9. Response of test system-II with plant perturbation and external disturbance using 2LB-MOPSO method.

Table 5
Hypervolume indicators H obtained by 2LB-MOPSO and NSGA-II for test systems-I and II.

Test system	Algorithms	Best	Mean	Worst	STD
I	2LB-MOPSO [45]	1.8185e−005	**3.4397e−002**	7.7381e−002	2.9096e−002
	NSGA-II [11]	4.2141e−001	5.3263e−001	6.0805e−001	5.8706e−002
II	2LB-MOPSO [45]	8.6349e−007	**3.1670e−006**	6.1601e−006	2.0662e−006
	NSGA-II [11]	7.6992e−003	1.5240e−002	3.2741e−002	8.5482e−003

The better mean value in each comparison category is highlighted in bold in this table.

7. Conclusion

2LB-MOPSO is applied to design multi-objective robust PID controllers. Multi-objective robust controller design problem is formulated by minimizing ISE and balanced robust performance criteria. Two MIMO test systems, namely distillation column plant and longitudinal control system of the super maneuverable F18/HARV fighter aircraft are taken to validate the performance of the 2LB-MOPSO. For the purpose of comparison, 2LB-MOPSO and NSGA-II as well as earlier reported Riccati, IGA and OSA methods are considered. Statistical performances of PID controllers designed using 2LB-MOPSO in 25 independent trials are reported. In addition, Hypervolume testing is also conducted on the results obtained by 2LB-MOPSO and NSGA-II. The performance of various PID controllers is compared with respect to sum of the ISE and balanced robust performance criteria. Comparative studies reveal that 2LB-MOPSO results are better than earlier reported results and the results obtained by NSGA-II. The proposed 2LB-MOPSO gives consistent performance in terms of lesser mean value of sum of ISE and balanced robust performance criteria.

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