Formulae For Vector Algebra

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BASIC ALGEBRA OF VECTORS

Important Terms, Definitions & Formulae

01. Vector - Basic Introduction: A quantity having *magnitude* as well as the *direction* is called vector. It is denoted as $\overrightarrow{AB}$ or $\vec{a}$. Its magnitude (or *modulus*) is $\left|\overrightarrow{AB}\right|$ or $|\vec{a}|$ otherwise, simply AB or a.

❖ Vectors are denoted by symbols such as $\vec{a}$ or $\overline{a}$ or $\mathbf{a}$.

[Pictorial representation of vector]

02. Initial and Terminal points: The initial and terminal points mean that point from which the vector originates and terminates respectively.

03. Position Vector: The position vector of a point say $P(x, y, z)$ is $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and the magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$. The vector $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is said to be in its *component form*. Here x, y, z are called the *scalar components* or *rectangular components* of $\vec{r}$ and $x\hat{i}$, $y\hat{j}$, $z\hat{k}$ are the vector components of $\vec{r}$ along *x-, y-, z-* axes respectively.

❖ Also, $\overrightarrow{AB} = (\text{Position Vector of B}) - (\text{Position Vector of A})$. For example, let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$. Then, $\overrightarrow{AB} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$.

❖ Here $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along the axes OX, OY and OZ respectively. (The discussion about unit vectors is given later in the point 05(e).)

04. Direction ratios and direction cosines: If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{r}$ *i.e.,* x, y, z are called the *direction ratios* (abbreviated as d.r.'s) of vector $\vec{r}$. These are denoted by a, b, c *(i.e.* $a = x$, $b = y$, $c = z$ *; in a manner we can say that **scalar components of vector $\vec{r}$ and its d.r.'s both are the same**).*

Also, the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\hat{r}$ (which is the unit vector of $\vec{r}$) *i.e.,* $\frac{x}{\sqrt{x^2 + y^2 + z^2}}$, $\frac{y}{\sqrt{x^2 + y^2 + z^2}}$, $\frac{z}{\sqrt{x^2 + y^2 + z^2}}$ are called *direction cosines* (which is abbreviated as d.c.'s) of vector $\vec{r}$.

❖ These direction cosines are denoted by *l, m, n* such that $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ and $l^2 + m^2 + n^2 = 1 \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

❖ It can be easily concluded that $\frac{x}{r} = l = \cos\alpha$, $\frac{y}{r} = m = \cos\beta$, $\frac{z}{r} = n = \cos\gamma$.

Therefore, $\vec{r} = lr\hat{i} + mr\hat{j} + nr\hat{k} = r(\cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k})$. (Here $r = |\vec{r}|$.)

[See the ΔOAP in Fig.1]

❖ Angles α, β, γ are made by the vector $\vec{r}$ with the positive directions of *x, y, z*-axes respectively and these angles are known as the ***direction angles*** of vector $\vec{r}$).

➩ For a better understanding, you can visualize the Fig.1.

05. TYPES OF VECTORS

a) Zero or Null vector: Its that vector whose *initial* and *terminal points* are coincident. It is denoted by $\vec{0}$. Of course its magnitude is 0 (zero).

❖ Any non-zero vector is called a **proper vector**.

b) Co-initial vectors: Those vectors (two or more) having the *same initial point* are called the co-initial vectors.

c) Co-terminous vectors: Those vectors (two or more) having the *same terminal point* are called the co-terminous vectors.

d) Negative of a vector: The vector which has the *same magnitude* as the $\vec{r}$ but *opposite direction*. It is denoted by $-\vec{r}$. Hence if, $\overrightarrow{AB} = \vec{r} \Rightarrow \overrightarrow{BA} = -\vec{r}$. That is $\overrightarrow{AB} = -\overrightarrow{BA}, \overrightarrow{PQ} = -\overrightarrow{QP}$ etc.

e) Unit vector: It is a vector with the *unit magnitude*. The unit vector in the direction of vector $\vec{r}$ is given by $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$ such that $|\vec{r}| = 1$. So, if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then its unit vector is:

$$\hat{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}.$$

❖ Unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is: $\pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

f) Reciprocal of a vector: It is a vector which has the same direction as the vector $\vec{r}$ but magnitude equal to the reciprocal of the magnitude of $\vec{r}$. It is denoted as $\vec{r}^{\,-1}$. Hence $\left|\vec{r}^{\,-1}\right| = \dfrac{1}{|\vec{r}|}$.

g) Equal vectors: Two vectors are said to be equal if they have the same magnitude as well as direction, regardless of the positions of their initial points.

$$\text{Thus } \vec{a} = \vec{b} \Leftrightarrow \begin{cases} |\vec{a}| = |\vec{b}| \\ \vec{a} \text{ and } \vec{b} \text{ have same direction.} \end{cases}$$

Also, if $\vec{a} = \vec{b} \Rightarrow a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \Rightarrow a_1 = b_1, a_2 = b_2, a_3 = b_3$.

h) Collinear or Parallel vector: Two vectors $\vec{a}$ and $\vec{b}$ are *collinear* or *parallel* if there exists a non-zero scalar λ such that $\vec{a} = \lambda\vec{b}$.

❖ It is important to note that the respective coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{a}$ and $\vec{b}$ are proportional provide they are parallel or collinear to each other.
Consider $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then by using $\vec{a} = \lambda\vec{b}$, we can conclude that: $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3} = \lambda$.

❖ The d.r.'s of parallel vectors are same (or are in proportion).

❖ The vectors $\vec{a}$ and $\vec{b}$ will have same or opposite direction as λ is positive or negative.

❖ The vectors $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a}\times\vec{b} = \vec{0}$.

i) Free vectors: The vectors which can undergo parallel displacement without changing its magnitude and direction are called free vectors.

06. ADDITION OF VECTORS

a) Triangular law: If two adjacent sides (say sides AB and BC) of a triangle ABC are represented by $\vec{a}$ and $\vec{b}$ taken in same order, then the third side of the triangle taken in the reverse order gives the sum of vectors $\vec{a}$ and $\vec{b}$ *i.e.*, $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \Rightarrow \overrightarrow{AC} = \vec{a} + \vec{b}$. See Fig.2.

❖ Also since $\overrightarrow{AC} = -\overrightarrow{CA} \Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AA} = \vec{0}$.

❖ And $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0} \Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

b) Parallelogram law: If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and the direction by the two adjacent sides (say AB and AD) of a parallelogram ABCD, then their sum is given by that diagonal of parallelogram which is co-initial with $\vec{a}$ and $\vec{b}$ *i.e.*, $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}$. For the illustration, see Fig.3.

<u>Multiplication of a vector by a scalar</u>

Let $\vec{a}$ be any vector and k be any scalar. Then the product $k\vec{a}$ is defined as a vector whose magnitude is $|k|$ times that of $\vec{a}$ and the direction is

(i) same as that of $\vec{a}$ if k is positive, and (ii) opposite to that of $\vec{a}$ if k is negative.

07. PROPERTIES OF VECTOR ADDITION

- ***Commutative property:*** $\vec{a}+\vec{b}=\vec{b}+\vec{a}$.
  Consider $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ be any two given vectors.
  Then $\vec{a}+\vec{b}=(a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}=\vec{b}+\vec{a}$.
- ***Associative property:*** $\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)$
- ***Additive identity property:*** $\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$.
- ***Additive inverse property:*** $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$.

08. Section formula: The position vector of a point *say* P dividing a line segment joining the points A and B whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio *m:n*

(a) internally, is $\overrightarrow{OP}=\dfrac{m\vec{b}+n\vec{a}}{m+n}$

(b) externally, is $\overrightarrow{OP}=\dfrac{m\vec{b}-n\vec{a}}{m-n}$.

❖ Also if point P is the ***mid-point*** of line segment AB then, $\overrightarrow{OP}=\dfrac{\vec{a}+\vec{b}}{2}$.

PRODUCT OF VECTORS
[DOT PRODUCT & CROSS PRODUCT]

Important Terms, Definitions & Formulae

01. PRODUCT OF TWO VECTORS

a) Scalar product or Dot product: The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined by, $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0\le\theta\le\pi$. See Fig.4.

Consider $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$. Then $\vec{a}.\vec{b}=a_1b_1+a_2b_2+a_3b_3$.

❖ **Properties / Observations of Dot product**

- $\hat{i}.\hat{i}=|\hat{i}||\hat{i}|\cos 0=1\Rightarrow\hat{i}.\hat{i}=1=\hat{j}.\hat{j}=\hat{k}.\hat{k}$.
- $\hat{i}.\hat{j}=|\hat{i}||\hat{j}|\cos\dfrac{\pi}{2}=0\Rightarrow\hat{i}.\hat{j}=0=\hat{j}.\hat{k}=\hat{k}.\hat{i}$.
- $\vec{a}.\vec{b}\in\mathrm{R}$, where R is real number *i.e.,* any scalar.
- $\vec{a}.\vec{b}=\vec{b}.\vec{a}$ (**Commutative property of dot product**).
- $\vec{a}.\vec{b}=0\Leftrightarrow\vec{a}\perp\vec{b}$.
- If $\theta=0$ then, $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|$. Also $\vec{a}.\vec{a}=|\vec{a}|^2=a^2$; as θ in this case is 0.

  Moreover if $\theta=\pi$ then, $\vec{a}.\vec{b}=-|\vec{a}||\vec{b}|$.
- $\vec{a}.\left(\vec{b}+\vec{c}\right)=\vec{a}.\vec{b}+\vec{a}.\vec{c}$ (**Distributive property of dot product**).
- $\vec{a}.\left(-\vec{b}\right)=-\left(\vec{a}.\vec{b}\right)=(-\vec{a}).\vec{b}$.
- **Angle between two vectors** $\vec{a}$ and $\vec{b}$ can be found by the expression given below:

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\text{or, } \theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right).$$

- **Projection of a vector** $\vec{a}$ on the other vector say $\vec{b}$ is given as $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)$ *i.e.*, $\vec{a}.\hat{b}$.

  This is also known as **Scalar projection** or **Component of** $\vec{a}$ **along** $\vec{b}$.

- **Projection vector** of $\vec{a}$ on the other vector say $\vec{b}$ is given as $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right).\hat{b}$.

  This is also known as the **Vector projection**.

- **Work done W** in moving an object from point A to the point B by applying a force $\vec{F}$ is given as $W = \vec{F}.\overrightarrow{AB}$.

b) Vector product or Cross product: The cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined by, $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{\eta}$, where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$ and $\hat{\eta}$ is a *unit vector perpendicular to both* $\vec{a}$ *and* $\vec{b}$. For better illustration, see Fig.5.
Consider $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.

$$\text{Then, } \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}.$$

❖ **Properties / Observations of Cross product**

- $\hat{i}\times\hat{i} = |\hat{i}||\hat{i}|\sin 0.\hat{j} = \vec{0} \Rightarrow \hat{i}\times\hat{i} = \vec{0} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}$.
- $\hat{i}\times\hat{j} = |\hat{i}||\hat{j}|\sin\frac{\pi}{2}.\hat{k} = \hat{k} \Rightarrow \hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$.
- Fig.6 at the end of chapter can be considered for memorizing the vector product of $\hat{i}, \hat{j}, \hat{k}$.
- $\vec{a}\times\vec{b}$ is a vector $\vec{c}$ *(say)* and this vector $\vec{c}$ *is perpendicular to both the vectors* $\vec{a}$ *and* $\vec{b}$.
- $|\vec{a}\times\vec{b}| = \left||\vec{a}||\vec{b}|\sin\theta\,\hat{\eta}\right| = |\vec{a}||\vec{b}|\sin\theta$ *i.e.,* $|\vec{a}\times\vec{b}| = ab\sin\theta$.
- $\vec{a}\times\vec{b} = \vec{0} \Leftrightarrow \vec{a} // \vec{b}$ or, $\vec{a} = \vec{0},\ \vec{b} = \vec{0}$.
- $\vec{a}\times\vec{a} = \vec{0}$.
- $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$ (**Commutative property does not hold for cross product**).
- $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c};\ (\vec{b}+\vec{c})\times\vec{a} = \vec{b}\times\vec{a} + \vec{c}\times\vec{a}$

  (**Distributive property of the vector product or cross product**).
- **Angle between two vectors** $\vec{a}$ and $\vec{b}$ in terms of Cross-product can be found by the expression given here: $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

$$\text{or, } \theta = \sin^{-1}\left(\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right).$$

- If $\vec{a}$ and $\vec{b}$ represent the *adjacent sides of a triangle*, then the **area of triangle** can be obtained by evaluating $\frac{1}{2}|\vec{a}\times\vec{b}|$.

- If $\vec{a}$ and $\vec{b}$ represent the *adjacent sides of a parallelogram*, then the **area of parallelogram** can be obtained by evaluating $\left|\vec{a}\times\vec{b}\right|$.
- If $\vec{p}$ and $\vec{q}$ represent the *two diagonals of a parallelogram*, then the **area of parallelogram** can be obtained by evaluating $\dfrac{1}{2}\left|\vec{p}\times\vec{q}\right|$.

If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then the diagonals $\vec{d}_1$ and $\vec{d}_2$ of the parallelogram are given as:

$$\vec{d}_1=\vec{a}+\vec{b}\,,\ \vec{d}_2=\vec{b}-\vec{a}\,.$$

02. Relationship between Vector product and Scalar product [Lagrange's Identity]

Consider two vectors $\vec{a}$ and $\vec{b}$. We also know that $\vec{a}\times\vec{b}=|\vec{a}|\left|\vec{b}\right|\sin\theta\,\hat{\eta}$.

Now, $\left|\vec{a}\times\vec{b}\right|=\left||\vec{a}|\left|\vec{b}\right|\sin\theta\,\hat{\eta}\right|$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|=|\vec{a}|\left|\vec{b}\right|\sin\theta$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2=|\vec{a}|^2\left|\vec{b}\right|^2\sin^2\theta$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2=|\vec{a}|^2\left|\vec{b}\right|^2\left(1-\cos^2\theta\right)$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2=|\vec{a}|^2\left|\vec{b}\right|^2-|\vec{a}|^2\left|\vec{b}\right|^2\cos^2\theta$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2=|\vec{a}|^2\left|\vec{b}\right|^2-\left(|\vec{a}|\left|\vec{b}\right|\cos\theta\right)^2$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2=|\vec{a}|^2\left|\vec{b}\right|^2-\left(\vec{a}\,.\vec{b}\right)^2$$

or, $\left|\vec{a}\times\vec{b}\right|^2+\left(\vec{a}\,.\vec{b}\right)^2=|\vec{a}|^2\left|\vec{b}\right|^2$.

❖ Note that $\left|\vec{a}\times\vec{b}\right|^2=\begin{vmatrix}\vec{a}\,.\vec{a} & \vec{a}\,.\vec{b}\\ \vec{a}\,.\vec{b} & \vec{b}\,.\vec{b}\end{vmatrix}$. Here the RHS represents a **determinant of order 2**.

03. Cauchy- Schwartz inequality:

For any two vectors $\vec{a}$ and $\vec{b}$, we always have $\left|\vec{a}.\vec{b}\right|\le|\vec{a}|\left|\vec{b}\right|$.

Proof: The given inequality holds trivially when either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ *i.e.*, in such a case $\left|\vec{a}.\vec{b}\right|=0=|\vec{a}|\left|\vec{b}\right|$.

So, let us check it for $|\vec{a}|\ne0\ne\left|\vec{b}\right|$.

As we know, $\vec{a}\,.\vec{b}=|\vec{a}|\left|\vec{b}\right|\cos\theta$

$$\Rightarrow \left(\vec{a}\,.\vec{b}\right)^2=|\vec{a}|^2\left|\vec{b}\right|^2\cos^2\theta$$

Also we know $\cos^2\theta\le1$ for all the values of θ.

$$\Rightarrow |\vec{a}|^2\left|\vec{b}\right|^2\cos^2\theta\le|\vec{a}|^2\left|\vec{b}\right|^2$$

$$\Rightarrow \left(\vec{a}\,.\vec{b}\right)^2\le|\vec{a}|^2\left|\vec{b}\right|^2$$

$$\Rightarrow \left|\vec{a}\,.\vec{b}\right|\le|\vec{a}|\left|\vec{b}\right|.$$ **[H.P.]**

04. Triangle inequality:

For any two vectors $\vec{a}$ and $\vec{b}$, we always have $\left|\vec{a}+\vec{b}\right|\le|\vec{a}|+\left|\vec{b}\right|$.

Proof: The given inequality holds trivially when either $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ *i.e.*, in such a case we have

$\left|\vec{a}+\vec{b}\right| = 0 = |\vec{a}|+|\vec{b}|$. So, let us check it for $|\vec{a}| \neq 0 \neq |\vec{b}|$.

Then consider $\left|\vec{a}+\vec{b}\right|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$

$$\Rightarrow \left|\vec{a}+\vec{b}\right|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

For $\cos\theta \le 1$, we have: $2|\vec{a}||\vec{b}|\cos\theta \le 2|\vec{a}||\vec{b}|$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta \le |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|$$

$$\Rightarrow \left|\vec{a}+\vec{b}\right|^2 \le \left(|\vec{a}|+|\vec{b}|\right)^2$$

$$\Rightarrow \left|\vec{a}+\vec{b}\right| \le |\vec{a}|+|\vec{b}|.$$ **[H.P.]**

SCALAR TRIPLE PRODUCT OF VECTORS

Important Terms, Definitions & Formulae

01. SCALAR TRIPLE PRODUCT:

If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors, then the scalar product of $\vec{a}\times\vec{b}$ with $\vec{c}$ is called scalar triple product of $\vec{a}, \vec{b}$ and $\vec{c}$.

Thus, $(\vec{a}\times\vec{b}).\vec{c}$ is called the scalar triple product of $\vec{a}, \vec{b}$ and $\vec{c}$.

❖ **Notation for scalar triple product:** The scalar triple product of $\vec{a}, \vec{b}$ and $\vec{c}$ is denoted by $[\vec{a}\ \vec{b}\ \vec{c}]$. That is, $(\vec{a}\times\vec{b}).\vec{c} = [\vec{a}\ \vec{b}\ \vec{c}]$.

Scalar triple product is also known as ***mixed product*** *because in scalar triple product, both the signs of dot and cross are used.*

Consider $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.

Then, $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

❖ **Properties / Observations of Scalar Triple Product**

- $(\vec{a}\times\vec{b}).\vec{c} = \vec{a}.(\vec{b}\times\vec{c})$. That is, the position of dot and cross can be interchanged without change in the value of the scalar triple product (provided their cyclic order remains the same).
- $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$. That is, the value of scalar triple product doesn't change when cyclic order of the vectors is maintained.

  Also $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{b}\ \vec{a}\ \vec{c}]$; $[\vec{b}\ \vec{c}\ \vec{a}] = -[\vec{b}\ \vec{a}\ \vec{c}]$. That is, the value of scalar triple product remains the same in **magnitude but changes the sign** when cyclic order of the vectors is altered.
- For any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and scalar λ, we have $[\lambda\vec{a}\ \vec{b}\ \vec{c}] = \lambda[\vec{a}\ \vec{b}\ \vec{c}]$.
- The value of scalar triple product is zero if any two of the three vectors are identical. That is, $[\vec{a}\ \vec{a}\ \vec{c}] = 0 = [\vec{a}\ \vec{b}\ \vec{b}] = [\vec{a}\ \vec{b}\ \vec{a}]$ etc.
- Value of scalar triple product is zero if any two of the three vectors are parallel or collinear.
- Scalar triple product of $\hat{i}, \hat{j}$ and $\hat{k}$ is 1 (*unity*) *i.e.,* $[\hat{i}\ \hat{j}\ \hat{k}] = 1$.

- If $[\vec{a}\ \vec{b}\ \vec{c}]=0$ then, the non-parallel and non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are **coplanar**.

Volume Of Parallelopiped

❖ If $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three co-terminus edges of a parallelopiped, then its volume can be obtained by: $[\vec{a}\ \vec{b}\ \vec{c}]=(\vec{a}\times\vec{b}).\vec{c}$. That is,

$(\vec{a}\times\vec{b}).\vec{c}=$ Base area of Parallelopiped $\times$ Height of Parallelopiped on this base.

❖ If for any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, we have $[\vec{a}\ \vec{b}\ \vec{c}]=0$, then volume of parallelepiped with the co-terminus edges as $\vec{a}$, $\vec{b}$ and $\vec{c}$, is zero. This is possible only if the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are co-planar.

Hii, All!

I hope this texture may have proved beneficial for you.

While going through this material, if you noticed any error(s) or, something which doesn't make sense to you, **please** bring it in my notice through SMS or Call at **+91-9650 350 480** or Email at **theopgupta@gmail.com**.

With lots of Love & Blessings!

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VARIOUS FIGURES RELATED TO THE VECTOR ALGEBRA

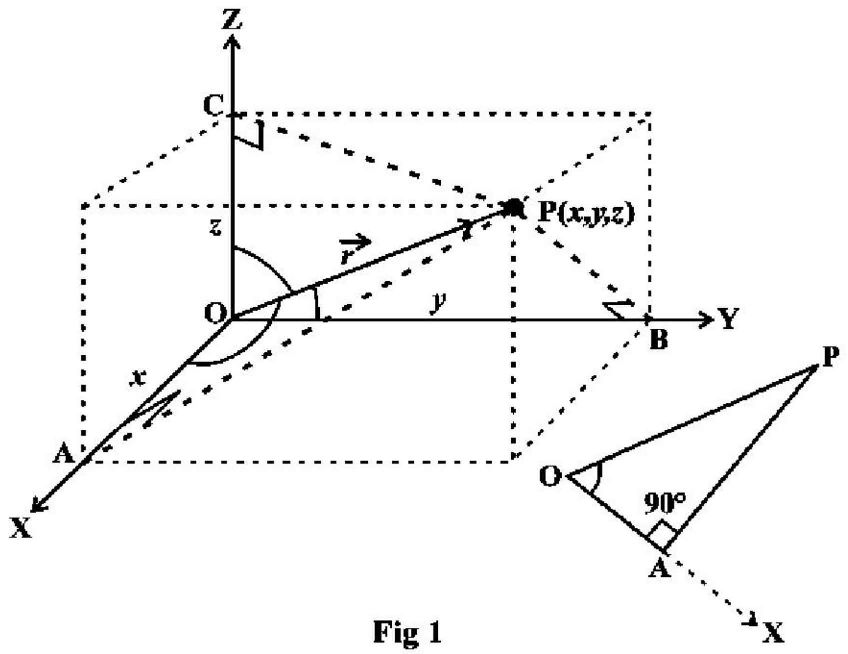

Fig 1

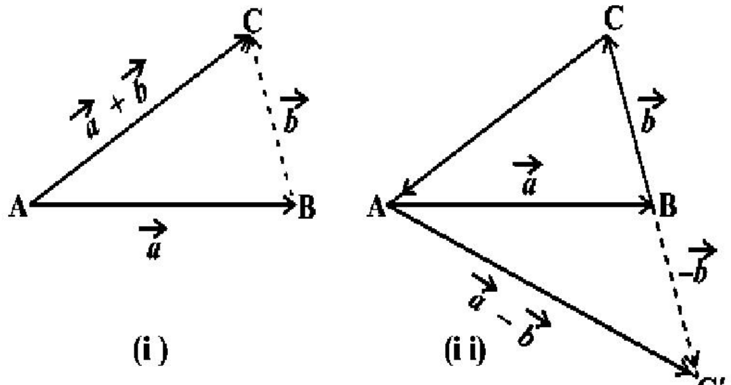

Fig 2

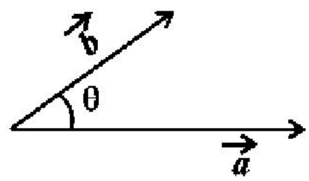

Fig 4

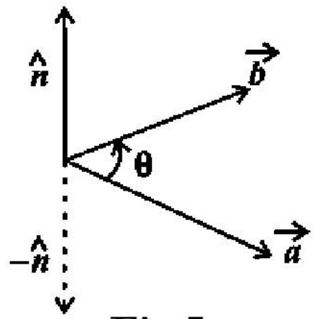

Fig 5

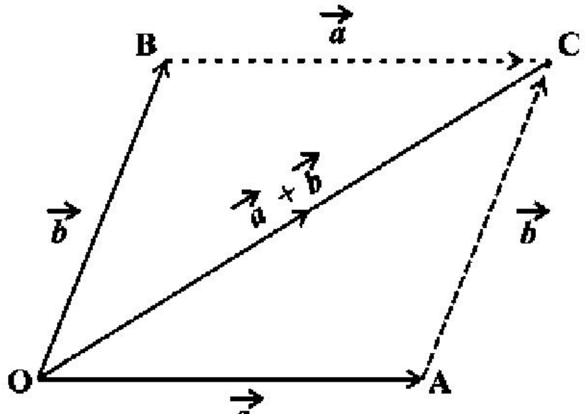

Fig 3

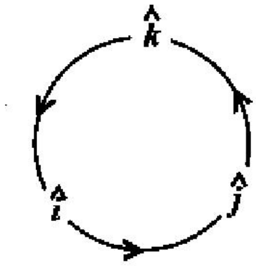

Fig 6