Formulae For Continuity & Differential Calculus

01. Formulae for Limits & Differential Calculus:

(LIMITS FOR SOME STANDARD FORMS)

a) $\lim\limits_{x\to 0}\cos x = 1$

b) $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$

c) $\lim\limits_{x\to 0}\dfrac{\tan x}{x} = 1$

d) $\lim\limits_{x\to 0}\dfrac{\sin^{-1} x}{x} = 1$

e) $\lim\limits_{x\to 0}\dfrac{\tan^{-1} x}{x} = 1$

f) $\lim\limits_{x\to 0}\dfrac{a^x - 1}{x} = \log_e a,\ a > 0$

g) $\lim\limits_{x\to 0}\dfrac{e^x - 1}{x} = 1$

h) $\lim\limits_{x\to 0}\dfrac{\log_e(1+x)}{x} = 1$

i) $\lim\limits_{x\to a}\dfrac{x^n - a^n}{x - a} = na^{n-1}$

j) $\lim\limits_{x\to 0}(1+kx)^{1/x} = e^k$, where k is any constant.

(DERIVATIVE OF SOME STANDARD FUNCTIONS)

a) $\dfrac{d}{dx}(x^n) = n\,x^{n-1}$

b) $\dfrac{d}{dx}(k) = 0,$ where k is any constant

c) $\dfrac{d}{dx}(a^x) = a^x \log_e a,\ a > 0$

d) $\dfrac{d}{dx}(e^x) = e^x$

e) $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\log_e a} = \dfrac{1}{x}\log_a e$

f) $\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

g) $\dfrac{d}{dx}(\sin x) = \cos x$

h) $\dfrac{d}{dx}(\cos x) = -\sin x$

i) $\dfrac{d}{dx}(\tan x) = \sec^2 x$

j) $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

k) $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

l) $\dfrac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\ \cot x$

m) $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}},\ x \in (-1,1)$

n) $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}},\ x \in (-1,1)$

o) $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2},\ x \in \mathrm{R}$

p) $\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1+x^2},\ x \in \mathrm{R}$

q) $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{x\sqrt{x^2-1}}$, where $x \in (-\infty,-1)\cup(1,\infty)$

r) $\dfrac{d}{dx}(\operatorname{cosec}^{-1} x) = -\dfrac{1}{x\sqrt{x^2-1}}$,where $x \in (-\infty,-1)\cup(1,\infty)$

*Do you know for trigonometric functions, angle '**x**' is in **Radians**?*

*Following derivatives should also be **memorized** by you for quick use:*

- $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$
- $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$
- $\dfrac{d}{dx}\left(x^x\right) = x^x(1+\log x)$

02. Important terms and facts about Limits and Continuity of a function:

➲ For a function $f(x)$, $\lim\limits_{x\to m} f(x)$ exists *iff* $\lim\limits_{x\to m^-} f(x) = \lim\limits_{x\to m^+} f(x)$.

- A function $f(x)$ is continuous at a point $x = m$ *iff* $\lim_{x \to m^-} f(x) = \lim_{x \to m^+} f(x) = f(m)$, where $\lim_{x \to m^-} f(x)$ is **Left Hand Limit** of $f(x)$ at $x = m$ and $\lim_{x \to m^+} f(x)$ is **Right Hand Limit** of $f(x)$ at $x = m$. Also $f(m)$ is the value of function $f(x)$ at $x = m$.
- A function $f(x)$ is *continuous* at $x = m$ *(say)* if, $f(m) = \lim_{X \to m} f(x)$ *i.e.,* a function is *continuous* at a point in its **domain** if the **limit value of the function** at that point ***equals*** the value of the function at the same point.
- For a continuous function $f(x)$ at $x = m$, $\lim_{x \to m} f(x)$ can be directly obtained by evaluating $f(m)$.
- **Indeterminate forms or meaningless forms:**

$$\frac{0}{0}, \frac{\infty}{\infty}, 0.\infty, \infty - \infty, 1^{\infty}, 0^0, \infty^0.$$

The first two forms i.e. $\frac{0}{0}, \frac{\infty}{\infty}$ *are the forms which are suitable for* ***L'Hospital Rule***.

03. Important terms and facts about Derivatives and Differentiability of a function:

- **Left Hand Derivative of $f(x)$ at $x = m$,**

$$Lf'(m) = \lim_{x \to m^-} \frac{f(x) - f(m)}{x - m} \text{ and,}$$

Right Hand Derivative of $f(x)$ at $x = m$,

$$Rf'(m) = \lim_{x \to m^+} \frac{f(x) - f(m)}{x - m}.$$

For a ***function to be differentiable*** *at a point, the* ***LHD and RHD at that point should be equal***.

- **Derivative of y w.r.t. x:** $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$.

Also for ***very- very small value h***, $f'(x) = \frac{f(x+h) - f(x)}{h}$, $(as\ h \to 0)$.

04. Relation between Continuity and Differentiability:

a) If a function is *differentiable* at a point, it is *continuous* at that point as well.
b) If a function is *not differentiable* at a point, it *may* or *may not be continuous* at that point.
c) If a function is *continuous* at a point, it *may* or *may not be differentiable* at that point.
d) If a function is *discontinuous* at a point, it is *not differentiable* at that point.

05. Rules of derivatives:

- Product or Leibnitz Rule of derivatives: $\frac{d}{dx}(u\,\mathrm{v}) = u\frac{d}{dx}(\mathrm{v}) + \mathrm{v}\frac{d}{dx}(u)$
- Quotient Rule of derivatives: $\frac{d}{dx}\left(\frac{u}{\mathrm{v}}\right) = \frac{\mathrm{v}\frac{d}{dx}(u) - u\frac{d}{dx}(\mathrm{v})}{\mathrm{v}^2}$.

Hii, All!
I hope this texture may have proved beneficial for you.
While going through this material, if you noticed any error(s) or, something which doesn't make sense to you, please bring it in my notice through SMS or Call at +91-9650 350 480 or Email at **theopgupta@gmail.com**.

With lots of Love & Blessings!

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