**Probability** is a [measure](http://en.wikipedia.org/wiki/Measure_(mathematics)) or [estimation](http://en.wikipedia.org/wiki/Estimation) of the likeliness or [likelihood](http://en.wikipedia.org/wiki/Likelihood_function) that an [event](http://en.wikipedia.org/wiki/Event_(probability_theory)) will occur.[[1]](http://en.wikipedia.org/wiki/Probability#cite_note-1)

Probability is used to quantify an attitude of mind towards some proposition of whose truth we are not certain. [[2]](http://en.wikipedia.org/wiki/Probability#cite_note-Stuart_and_Ord_2009-2) The proposition of interest is usually of the form "Will a specific [event](http://en.wikipedia.org/wiki/Event_(probability_theory)) occur?" The attitude of mind is of the form "How certain are we that the event will occur?" The certainty we adopt can be described in terms of a numerical measure and this number, between 0 and 1 (where 0 indicates impossibility and 1 indicates certainty), we call probability. [[3]](http://en.wikipedia.org/wiki/Probability#cite_note-Feller-3) Thus the higher the probability of an event, the more certain we are that the event will occur. A simple example would be the toss of a fair coin. Since the the 2 outcomes are deemed equiprobable, the probability of "heads" equals the probability of "tails" and each probability is 1/2 or equivalently a 50% chance of either "heads" or "tails".

These concepts have been given an axiomatic [mathematical](http://en.wikipedia.org/wiki/Mathematics) derivation in [probability theory](http://en.wikipedia.org/wiki/Probability_theory) (see [probability axioms](http://en.wikipedia.org/wiki/Probability_axioms)), which is used widely in such [areas of study](http://en.wikipedia.org/wiki/Areas_of_study) as[mathematics](http://en.wikipedia.org/wiki/Mathematics), [statistics](http://en.wikipedia.org/wiki/Statistics), [finance](http://en.wikipedia.org/wiki/Finance), [gambling](http://en.wikipedia.org/wiki/Gambling), [science](http://en.wikipedia.org/wiki/Science) (in particular [physics](http://en.wikipedia.org/wiki/Physics)), [artificial intelligence](http://en.wikipedia.org/wiki/Artificial_intelligence)/[machine learning](http://en.wikipedia.org/wiki/Machine_learning) and [philosophy](http://en.wikipedia.org/wiki/Philosophy) to, for example, draw inferences about the expected frequency of events. Probability theory is also used to describe the underlying mechanics and regularities of [complex systems](http://en.wikipedia.org/wiki/Complex_systems).[[4]](http://en.wikipedia.org/wiki/Probability#cite_note-4)

Applications[[edit](http://en.wikipedia.org/w/index.php?title=Probability&action=edit&section=5" \o "Edit section: Applications)]

Probability theory is applied in everyday life in [risk](http://en.wikipedia.org/wiki/Risk) assessment and in trade on [financial markets](http://en.wikipedia.org/wiki/Financial_market). Governments apply probabilistic methods in [environmental regulation](http://en.wikipedia.org/wiki/Environmental_regulation), where it is called pathway analysis. A good example is the effect of the perceived probability of any widespread Middle East conflict on oil prices—which have ripple effects in the economy as a whole. An assessment by a commodity trader that a war is more likely vs. less likely sends prices up or down, and signals other traders of that opinion. Accordingly, the probabilities are neither assessed independently nor necessarily very rationally. The theory of [behavioral finance](http://en.wikipedia.org/wiki/Behavioral_finance) emerged to describe the effect of such [groupthink](http://en.wikipedia.org/wiki/Groupthink) on pricing, on policy, and on peace and conflict.[[16]](http://en.wikipedia.org/wiki/Probability#cite_note-16)

The discovery of rigorous methods to assess and combine probability assessments has changed society. It is important for most citizens to understand how probability assessments are made, and how they contribute to decisions.

Another significant application of probability theory in everyday life is [reliability](http://en.wikipedia.org/wiki/Reliability_theory_of_aging_and_longevity). Many consumer products, such as [automobiles](http://en.wikipedia.org/wiki/Automobiles) and consumer electronics, use [reliability theory](http://en.wikipedia.org/wiki/Reliability_theory) in product design to reduce the probability of failure. Failure probability may influence a manufacturer's decisions on a product's [warranty](http://en.wikipedia.org/wiki/Warranty).[[17]](http://en.wikipedia.org/wiki/Probability#cite_note-17)

The [cache language model](http://en.wikipedia.org/wiki/Cache_language_model) and other [statistical language models](http://en.wikipedia.org/wiki/Statistical_Language_Model) that are used in [natural language processing](http://en.wikipedia.org/wiki/Natural_language_processing) are also examples of applications of probability theory.

*trigonometry* (from [Greek](http://en.wikipedia.org/wiki/Ancient_Greek) *[trigōnon](http://en.wiktionary.org/wiki/%CF%84%CF%81%CE%AF%CE%B3%CF%89%CE%BD%CE%BF%CE%BD" \o "wikt:τρίγωνον)*, "triangle" + *[metron](http://en.wiktionary.org/wiki/%CE%BC%CE%AD%CF%84%CF%81%CE%BF%CE%BD" \o "wikt:μέτρον)*, "measure"[[1]](http://en.wikipedia.org/wiki/Trigonometry#cite_note-1)) is a branch of [mathematics](http://en.wikipedia.org/wiki/Mathematics) that studies relationships involving lengths and [angles](http://en.wikipedia.org/wiki/Angles" \o "Angles)of [triangles](http://en.wikipedia.org/wiki/Triangle). The field emerged during the 3rd century BC from applications of [geometry](http://en.wikipedia.org/wiki/Geometry) to astronomical studies.[[2]](http://en.wikipedia.org/wiki/Trigonometry#cite_note-2)

The 3rd century astronomers first noted that the lengths of the sides of a right angle triangle and the [angles](http://en.wikipedia.org/wiki/Angle) between those sides have fixed relationships: that is, if at least the length of one side and the value of one angle is known all other angles and lengths can be determined algorithmically. These calculations soon came to be defined as the [trigonometric functions](http://en.wikipedia.org/wiki/Trigonometric_functions) and today are pervasive in both [pure](http://en.wikipedia.org/wiki/Pure_mathematics) and [applied](http://en.wikipedia.org/wiki/Applied_mathematics" \o "Applied mathematics)mathematics: fundamental methods of analysis such as the [Fourier Transform](http://en.wikipedia.org/wiki/Fourier_Transform), for example, or the [wave equation](http://en.wikipedia.org/wiki/Wave_equation), use trigonometric functions to understand [cyclical](http://en.wikipedia.org/wiki/Periodic_function) phenomena across a great many applications in fields as diverse as physics, mechanical and electrical engineering, music and acoustics, astronomy, ecology and biology. Trigonometry is also the foundation of the practical art of [surveying](http://en.wikipedia.org/wiki/Surveying).

Trigonometry is most simply associated with [planar](http://en.wikipedia.org/wiki/Planar) [right angle](http://en.wikipedia.org/wiki/Right_angle) triangles (a two dimensional triangle with one angle equal to 90 degrees). The applicability to non-right angle triangles exists but, since any non-right angle triangle (on a flat plane) can be bisected to create two right angle triangles, most problems can be reduced to calculations on right angle triangles. Thus the majority of applications relates to right angle triangles. One exception to this is [Spherical trigonometry](http://en.wikipedia.org/wiki/Spherical_trigonometry), the study of triangles on [spheres](http://en.wikipedia.org/wiki/Sphere), surfaces of constant positive [curvature](http://en.wikipedia.org/wiki/Curvature), in [elliptic geometry](http://en.wikipedia.org/wiki/Elliptic_geometry) (a fundamental part of[astronomy](http://en.wikipedia.org/wiki/Astronomy) and [navigation](http://en.wikipedia.org/wiki/Navigation)). Trigonometry on surfaces of negative curvature is part of [hyperbolic geometry](http://en.wikipedia.org/wiki/Hyperbolic_geometry).

Trigonometry basics are often taught in [school](http://en.wikipedia.org/wiki/School) either as a separate course or as part of a [precalculus](http://en.wikipedia.org/wiki/Precalculus" \o "Precalculus) course.

Applications of trigonometry[[edit](http://en.wikipedia.org/w/index.php?title=Trigonometry&action=edit&section=6" \o "Edit section: Applications of trigonometry)]

[](http://en.wikipedia.org/wiki/File:Frieberger_drum_marine_sextant.jpg)

[http://bits.wikimedia.org/static-1.23wmf13/skins/common/images/magnify-clip.png](http://en.wikipedia.org/wiki/File:Frieberger_drum_marine_sextant.jpg)

[Sextants](http://en.wikipedia.org/wiki/Sextant) are used to measure the angle of the sun or stars with respect to the horizon. Using trigonometry and a[marine chronometer](http://en.wikipedia.org/wiki/Marine_chronometer), the position of the ship can be determined from such measurements.

*Main article:*[*Uses of trigonometry*](http://en.wikipedia.org/wiki/Uses_of_trigonometry)

There are an enormous number of uses of trigonometry and trigonometric functions. For instance, the technique of [triangulation](http://en.wikipedia.org/wiki/Triangulation) is used in [astronomy](http://en.wikipedia.org/wiki/Astronomy) to measure the distance to nearby stars, in [geography](http://en.wikipedia.org/wiki/Geography) to measure distances between landmarks, and in [satellite navigation systems](http://en.wikipedia.org/wiki/Satellite_navigation_system). The sine and cosine functions are fundamental to the theory of [periodic functions](http://en.wikipedia.org/wiki/Periodic_function) such as those that describe sound and [light](http://en.wikipedia.org/wiki/Light) waves.

Fields that use trigonometry or trigonometric functions include [astronomy](http://en.wikipedia.org/wiki/Astronomy) (especially for locating apparent positions of celestial objects, in which spherical trigonometry is essential) and hence [navigation](http://en.wikipedia.org/wiki/Navigation) (on the oceans, in aircraft, and in space), [music theory](http://en.wikipedia.org/wiki/Music_theory), [audio synthesis](http://en.wikipedia.org/wiki/Audio_synthesis), [acoustics](http://en.wikipedia.org/wiki/Acoustics), [optics](http://en.wikipedia.org/wiki/Optics), analysis of financial markets, [electronics](http://en.wikipedia.org/wiki/Electronics), [probability theory](http://en.wikipedia.org/wiki/Probability_theory), [statistics](http://en.wikipedia.org/wiki/Statistics), [biology](http://en.wikipedia.org/wiki/Biology), [medical imaging](http://en.wikipedia.org/wiki/Medical_imaging) ([CAT scans](http://en.wikipedia.org/wiki/CAT_scan) and [ultrasound](http://en.wikipedia.org/wiki/Ultrasound)), [pharmacy](http://en.wikipedia.org/wiki/Pharmacy), [chemistry](http://en.wikipedia.org/wiki/Chemistry), [number theory](http://en.wikipedia.org/wiki/Number_theory)(and hence [cryptology](http://en.wikipedia.org/wiki/Cryptology)), [seismology](http://en.wikipedia.org/wiki/Seismology), [meteorology](http://en.wikipedia.org/wiki/Meteorology), [oceanography](http://en.wikipedia.org/wiki/Oceanography), many [physical sciences](http://en.wikipedia.org/wiki/Physical_science), land [surveying](http://en.wikipedia.org/wiki/Surveying) and [geodesy](http://en.wikipedia.org/wiki/Geodesy), [architecture](http://en.wikipedia.org/wiki/Architecture), [phonetics](http://en.wikipedia.org/wiki/Phonetics),[economics](http://en.wikipedia.org/wiki/Economics), [electrical engineering](http://en.wikipedia.org/wiki/Electrical_engineering), [mechanical engineering](http://en.wikipedia.org/wiki/Mechanical_engineering), [civil engineering](http://en.wikipedia.org/wiki/Civil_engineering), [computer graphics](http://en.wikipedia.org/wiki/Computer_graphics), [cartography](http://en.wikipedia.org/wiki/Cartography), [crystallography](http://en.wikipedia.org/wiki/Crystallography) and [game development](http://en.wikipedia.org/wiki/Game_development).

# Arithmetic Progression

An **arithmetic progression** is a sequence of numbers such that the difference of any two successive members of the sequence is a constant.

***For example***, the sequence 1, 2, 3, 4, ... is is an arithmetic progression with common difference **1**.

***Second example:*** the sequence 3, 5, 7, 9, 11,... is an arithmetic progression  
with common difference **2**.  
***Third example:*** the sequence 20, 10, 0, -10, -20, -30, ... is an arithmetic progression  
with common difference **-10**.

#### Notation

We denote by **d** the common difference.

By **an** we denote the **n**-th term of an arithmetic progression.

By **Sn** we denote the sum of the first n elements of an arithmetic series.  
**Arithmetic series** means the sum of the elements of an arithmetic progression.

#### Properties

**a1 + an = a2 + an-1 = ... = ak + an-k+1**

and

**an = ½(an-1 + an+1)**

Sample: let 1, 11, 21, 31, 41, 51... be an arithmetic progression.

51 + 1 = 41 + 11 = 31 + 21  
and  
11 = (21 + 1)/2  
21 = (31 + 11)/2...

If the initial term of an arithmetic progression is ***a1*** and the common difference of successive members is ***d***, then the ***n-th*** term of the sequence is given by

**an = a1 + (n - 1)d, n = 1, 2, ...**

The sum ***S*** of the first ***n*** numbers of an arithmetic progression is given by the formula:

**S = ½(a1 + an)n**

where ***a1*** is the first term and ***an*** the last one.

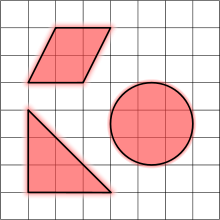
or

**S = ½(2a1 + d(n-1))n**

# Area

From Wikipedia, the free encyclopedia

*This article is about the geometric quantity. For other uses, see*[*Area (disambiguation)*](http://en.wikipedia.org/wiki/Area_(disambiguation))*.*

[](http://en.wikipedia.org/wiki/File:Area.svg)

[http://bits.wikimedia.org/static-1.23wmf13/skins/common/images/magnify-clip.png](http://en.wikipedia.org/wiki/File:Area.svg)

The combined area of these three[shapes](http://en.wikipedia.org/wiki/Shapes) is [approximately](http://en.wikipedia.org/wiki/Approximation) 15.57 [squares](http://en.wikipedia.org/wiki/Square).

**Area** is a [quantity](http://en.wikipedia.org/wiki/Quantity) that expresses the extent of a [two-dimensional](http://en.wikipedia.org/wiki/Two-dimensional) [surface](http://en.wikipedia.org/wiki/Surface) or [shape](http://en.wikipedia.org/wiki/Shape), or [planar lamina](http://en.wikipedia.org/wiki/Planar_lamina), in the [plane](http://en.wikipedia.org/wiki/Plane_(geometry)). Area can be understood as the amount of material with a given thickness that would be necessary to fashion a model of the shape, or the amount of [paint](http://en.wikipedia.org/wiki/Paint) necessary to cover the surface with a single coat.[[1]](http://en.wikipedia.org/wiki/Area#cite_note-MathWorld-1) It is the two-dimensional [analog](http://en.wikipedia.org/wiki/Analogy) of the [length](http://en.wikipedia.org/wiki/Length) of a [curve](http://en.wikipedia.org/wiki/Plane_curve) (a one-dimensional concept) or the [volume](http://en.wikipedia.org/wiki/Volume) of a [solid](http://en.wikipedia.org/wiki/Solid_geometry) (a three-dimensional concept).

The area of a shape can be measured by comparing the shape to [squares](http://en.wikipedia.org/wiki/Square) of a fixed size.[[2]](http://en.wikipedia.org/wiki/Area#cite_note-AF-2) In the [International System of Units](http://en.wikipedia.org/wiki/International_System_of_Units) (SI), the standard unit of area is the [square metre](http://en.wikipedia.org/wiki/Square_metre) (written as m2), which is the area of a square whose sides are one [metre](http://en.wikipedia.org/wiki/Metre" \o "Metre) long.[[3]](http://en.wikipedia.org/wiki/Area#cite_note-B-3) A shape with an area of three square metres would have the same area as three such squares. In [mathematics](http://en.wikipedia.org/wiki/Mathematics), the [unit square](http://en.wikipedia.org/wiki/Unit_square) is defined to have area one, and the area of any other shape or surface is a [dimensionless](http://en.wikipedia.org/wiki/Dimensionless_quantity) [real number](http://en.wikipedia.org/wiki/Real_number).

There are several well-known [formulas](http://en.wikipedia.org/wiki/Formula) for the areas of simple shapes such as [triangles](http://en.wikipedia.org/wiki/Triangle), [rectangles](http://en.wikipedia.org/wiki/Rectangle), and [circles](http://en.wikipedia.org/wiki/Circle). Using these formulas, the area of any[polygon](http://en.wikipedia.org/wiki/Polygon) can be found by [dividing the polygon into triangles](http://en.wikipedia.org/wiki/Polygon_triangulation).[[4]](http://en.wikipedia.org/wiki/Area#cite_note-bkos-4) For shapes with curved boundary, [calculus](http://en.wikipedia.org/wiki/Calculus) is usually required to compute the area. Indeed, the problem of determining the area of plane figures was a major motivation for the [historical development of calculus](http://en.wikipedia.org/wiki/History_of_calculus).[[5]](http://en.wikipedia.org/wiki/Area#cite_note-5)

For a solid shape such as a [sphere](http://en.wikipedia.org/wiki/Sphere), [cone](http://en.wikipedia.org/wiki/Cone_(geometry)), or [cylinder](http://en.wikipedia.org/wiki/Cylinder_(geometry)), the area of its boundary surface is called the [surface area](http://en.wikipedia.org/wiki/Surface_area).[[1]](http://en.wikipedia.org/wiki/Area#cite_note-MathWorld-1)[[6]](http://en.wikipedia.org/wiki/Area#cite_note-MathWorldSurfaceArea-6) Formulas for the surface areas of simple shapes were computed by the [ancient Greeks](http://en.wikipedia.org/wiki/Greek_mathematics), but computing the surface area of a more complicated shape usually requires [multivariable calculus](http://en.wikipedia.org/wiki/Multivariable_calculus).

Area plays an important role in modern mathematics. In addition to its obvious importance in [geometry](http://en.wikipedia.org/wiki/Geometry) and calculus, area is related to the definition of [determinants](http://en.wikipedia.org/wiki/Determinant) in [linear algebra](http://en.wikipedia.org/wiki/Linear_algebra), and is a basic property of surfaces in [differential geometry](http://en.wikipedia.org/wiki/Differential_geometry).[[7]](http://en.wikipedia.org/wiki/Area#cite_note-doCarmo-7) In [analysis](http://en.wikipedia.org/wiki/Analysis), the area of a subset of the plane is defined using [Lebesgue measure](http://en.wikipedia.org/wiki/Lebesgue_measure" \o "Lebesgue measure),[[8]](http://en.wikipedia.org/wiki/Area#cite_note-Rudin-8) though not every subset is measurable.[[9]](http://en.wikipedia.org/wiki/Area#cite_note-9) In general, area in higher mathematics is seen as a special case of [volume](http://en.wikipedia.org/wiki/Volume) for two-dimensional regions.[[1]](http://en.wikipedia.org/wiki/Area#cite_note-MathWorld-1)

Area can be defined through the use of axioms, defining it as a function of a collection of certain plane figures to the set of real numbers. It can be proved that such a function exists.