Drug Dispenser Replenishment Optimization via Mixed Integer Programming in Central Fill Pharmacy Systems

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Abstract

This paper presents a study about drug dispenser replenishment control using mixed integer programming approach to reduce process operation cost in central fill pharmacy systems. Dispensing machine replenishment is one of the critical processes that influences the efficiency in pharmacy automation. The objective of this paper is to find an optimized replenishment strategy through mixed integer programming to minimize total cost, which includes inventory holding cost, shortage cost, device setup cost, and operational cost, considering of demand, resource, and cycle time constraints. Device reorder level is considered in this model, which can suggest an appropriate replenishment reorder time for decision making. The model is tested on two cases and four situations. The influences of different replenishment strategies to system total cost are explored and discussed. The results show that the proposed model can not only give valuable recommendations for replenishment planning, but also boosts the efficiency of the decision making process.

Keywords
Inventory management, Replenishment planning, Mixed integer programming

1. Introduction
With the rapid development and proliferation of pharmacy industry in the recent years, numerous robotic units have been implemented in pharmacy systems. Pharmacy automation has played a significant role in promoting higher quality patient care by reducing medication errors and increasing drug distribution efficiency [1]. Many large pharmacy facilities have applied the techniques, and can even handle more than 30,000 patient prescriptions per eight hours' shift. The system, typically called central fill pharmacy, consists of auto-dispensing, auto-packaging, and auto-transportation subsystems. Among these subsystems, the auto-dispensing system is a key procedure in terms of the overall system performance. In the dispensing system, medications are dispensed and transferred by the robot arm, and a number of parallel auto-dispensers are employed to deal with pill counting and vial dispensing. The dispensing machine is a storage and retrieval system, which requires an efficient inventory supply to maintain the system inputs. Parallel dispenser device replenishment is one of the most critical controllable attributes in the pharmacy automation system. Some research has been done to optimize the order scheduling process [3], however, there is a lack of research on controlling the inventory for multiple dispensing machines.

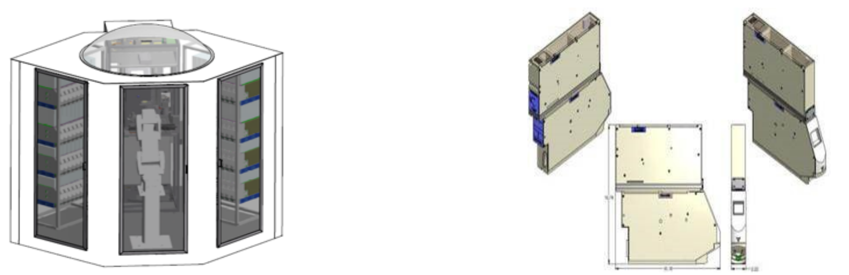
Device inventory control problem has been discussed in many fields over the last decades. The most well-known inventory control model is economic order quantity (EOQ), which was proposed by Ford W. Harris around 1913 [4]. However, the EOQ model can only solve one product problem with many assumptions, which make it simpler but unrealistic for many practical problems. Some researchers proposed various mathematical models to solve more complex inventory problems, such as multiple products, multiple planning period, stochastic demand, etc., with the consideration of different restrictions, such as shortage cost, space limitation, inventory management risk, etc. Considering of budget limitation and storage space constraint, a bi-objective model was built to identify optimal order quantity and reorder point by Parviz et al. The bi-objective model was proposed to minimize total cost, which included inventory holding cost, shortage cost, and ordering cost, and maximize service level, which basically reduced the waiting time of each order [5]. The model was tested using five heuristic algorithms, and the largest problem solved in their paper included 320 products. For multi-item multi-period inventory control problem, a mixed integer programming model was proposed by Pasandideh, et al. The objective was to minimize the total inventory cost, which consisted of ordering, holding, and purchasing costs, with the constraints of space limitation [6]. The final result was expected to get the optimal order quantity of different items in different periods. Parameter-tuned genetic algorithm and memetic algorithm were applied to solve the model in Pasandideh's paper.

Different from these research, replenishment backup device amount and reorder time point are specially considered in this paper. In addition to minimize inventory holding cost and shortage cost, replenishment operation cost and device setup cost are included in the objective function. A mixed integer programming model is built to find the optimal reorder point and replenishment resource amount. The influences of different system configurations are also investigated and discussed. The structure of this paper is organized as: the background, mechanism about Robotic Drug Dispensing / Filling System (RDS) is introduced in Section 2; mathematical model description is presented in Section 3; Section 4 is the related experimental results and analysis, in which two testing cases are discussed; the final conclusion is conducted at Section 5.

2. Research Background

2.1 Robotic Drug Dispensing / Filling System

The RDS discussed in the paper consists of a robot arm and 8 shelves in the two side walls in one machine, which can place a total of 80 dispensing devices (10 devices on each shelf), as shown in Figure 1(a). These dispensing devices, also called dispensers, can store, count, and release drugs under computer control [7]. Each dispenser is fully programmable and employs software-controlled counting parameters to ensure accuracy and convenience in the dispenser / drug calibration process. Thus, as new drugs become available for auto-counting or a drug needs performing changeover, reassigning a dispenser on-site can be simply achieved by just scanning the bar code on both the dispenser and the drug stock bottle. For the automatic dispensing process, there are mainly two steps, dispensing and filling. Filling process is completed by robot arm, whose speed actually relates to the distance between dispenser and robot arm home position. If the dispenser is closer to robot arm home position, the machine cycle time for that related medication will become less.



(a) RDS System

(b) Dispenser and replenishment canister

Figure 1: RDS and dispenser overview

Each RDS contains automatically replenished, bottle and cap queues, an internal label print / apply capability, a camera to capture an image of the bottle contents for use at pharmacist verification, and an automatic capping device. In reality, the challenge of this system is that it should be dust-free, keep up with variety drug types, and solve the inventory replenishment trouble. Since the buffer size of each dispenser is limited, the dispensers must require frequent replenishment to keep working in high volume central fill pharmacy system. Reorder point and backup device amount become inevitable questions during the replenishment decision making process.

2.2 Dispenser Replenishment Mechanism

When RDS is designed, each dispenser is specific for one type of medication, which is normally classified by the National Drug Code (NDC). And in order to avoid drug contamination, a certain dispenser cannot be changed to serve any other drug types during the process. Therefore, workstation space becomes the most significant resource constraint that determines the number of drug types the system can serve [8]. Each dispenser has a basic storage capacity. However, the basic capacity might not satisfy the requirement when the demand becomes very large. Therefore, canisters are applied as an extra storage backup device to extend the basic dispenser storage capacity, as shown in Figure 1(b). When the storage lower than a predefined reorder level, the system will send out an alert to request replenishment. After canister replenished, an image will be took for verifications. All images of filled canisters are displayed to the pharmacist prior to allowing the canister to be used to replenish a dispenser. If the pharmacist rejects the replenishment, the canister will remain off-line until it is further investigated by a pharmacist. Therefore, the replenishment processing time is actually influenced by multiple processes in reality. In this research, all the verification processes about canister replenishment are considered as an operation time cost, which will be discussed in Section 4.

3. Methodology

In this paper, a mixed integer programming model is built for the parallel device system. The aim is to find an appropriate amount of backup canisters and an proper reorder point for each medication. Backup canister amount can offer an alternative to prepare inventory before work start, and reorder point provides a time criteria to start dispenser replenishing.

3.1 Assumptions and Notations

For the mathematical formulation, several assumptions are illustrated as following: Assumption 1: there are sufficient inventory ready for filling the canisters during the work time. Assumption 2: each dispenser can only serve one type of medication. And, medication, device, machine are interchangeable terminologies in this paper. For example, m types of medications means m devices or m dispensers. Assumption 3: all the dispensers have the same basic volume, and all the canisters have the same volume. Assumption 4: different drug has different size.
The meaning of each notation applied in the proposed model is explained as:

Index:	i	device (medication) number, $i = 1, \ldots, M$
	t	time point, $t = 1, \ldots, T$
Parameters:	p_i	pills per unit capacity contained for medication i
	q_i	device basic storage capacity for medication i
	d_{it}	demand level of medication i at the time point t
	Q	canister capacity
	T	work time length
	c	replenishment processing time
	M	total number of machines
	σ	resource constraint coefficient
	s	reorder point estimation tolerance
	α	replenishment operation cost per cycle
	β	inventory holding cost per unit
	γ	shortage cost per unit
	δ	canister setup cost
Variables:	I_{it}	device inventory level for medication i at the beginning of time t
	l_i	reorder level for medication i, a proportion of reorder inventory point to basic capacity
	n_i	the number of canisters required for medication i
	x_{it}	binary variable, $x_{it} = 1$ perform replenishment at time t for medication i, otherwise $x_{it} = 0$
	r_{it}	binary variable, $r_{it} = 1$ complete replenishment at time t for medication i, otherwise $r_{it} = 0$
	u_{it}	binary variable, $u_{it} = 1$ there is inventory shortage at time t for medication i, otherwise $u_{it} = 0$

3.2 Mixed Integer Programming Model

Based on the configurations of the real system, a mixed integer programming model is built. System cost is implemented in objective function formulation. For each cost, a coefficient is given, by which the optimal replenishment planning can be adjusted. In the constraint, reorder time point constraint, resource constraint, cycle time constraint, etc. are considered. The details of the model are given as following.

$$\min\ C = \alpha \sum_{i=1}^{M} \sum_{t=1}^{T} x_{it} + \beta \sum_{i=1}^{M} \sum_{t=1}^{T} (1 - u_{it}) I_{it} + \gamma \sum_{i=1}^{M} \sum_{t=1}^{T} (-u_{it} I_{it}) + \delta \sum_{i=1}^{M} n_i \tag{1}$$

s.t.

$$\sum_{t=1}^{T} d_{it} - p_i q_i - \sum_{t=1}^{T} r_{it} p_i Q \leq 0 \qquad \forall i \tag{2}$$

$$|x_{it} I_{it} - l_i p_i q_i| - s \leq 0 \qquad \forall i,t \tag{3}$$

$$\frac{c x_{it} x_{it'}}{n_i} + t x_{it} - t' x_{it'} \leq 0 \qquad \forall i,\ \ \forall t = 1,...,T-1,\ \ \forall t' = t+1,...,T \tag{4}$$

$$r_{i(t+c)} - x_{it} = 0 \qquad \forall i,\ \ \forall t = 1,...,T-c \tag{5}$$

$$I_{i(t-1)} - d_{it} + r_{it} p_i Q - I_{it} = 0 \qquad \forall i,\ \ \forall t = 2,...,T \tag{6}$$

$$u_{it} I_{it} \leq 0 \qquad \forall i,t \tag{7}$$

$$-(1-u_{it}) I_{it} \leq 0 \qquad \forall i,t \tag{8}$$

$$\sum_{i=1}^{M} n_i - \sigma M \leq 0 \tag{9}$$

$$I_{it} - p_i q_i = 0 \qquad \forall i,\ \ t = 1 \tag{10}$$

$$r_{it} = 0 \qquad \forall i,\ \ \forall t = 1,...,c \tag{11}$$

$$0 \leq l_i \leq 1 \qquad \forall i \tag{12}$$

$$r_{it}, x_{it} \in \{0,1\} \qquad \forall i,t \tag{13}$$

$$n_i \in Z^+ \qquad \forall i \tag{14}$$

Equation (1) minimizes the total cost including the replenishment operation cost $\alpha \sum_{i=1}^{M} \sum_{t=1}^{T} x_{it}$, inventory holding cost $\beta \sum_{i=1}^{M} \sum_{t=1}^{T} (1-u_{it}) I_{it}$, inventory shortage cost $\gamma \sum_{i=1}^{M} \sum_{t=1}^{T} (-u_{it} I_{it})$, and canister setup cost $\delta \sum_{i=1}^{M} n_i$. Equation (2) is the restriction on the total storage, which can make sure all demand can be served during work time. Equation (3) guarantees the dispensers are reordered when the inventory less then the defined reorder level at time t. Reorder level represents a proportion between the remain volume at reorder time point and dispenser basic capacity. Since $l_i p_i q_i$ may not be an integer, a tolerance parameter is used in Equation (3). Equation (4) restricts the replenishment processing time. Equation (5) records the replenishment finish time point. Equation (6) describes the inventory updates. Equation (7) and Equation (8) record system inventory shortage time point. Equation (9) assures the resource limitation. Equation (10) defines the initial inventory levels for all product at $t = 1$. Equation (11) initializes values for r_{it} between time 1 and c for all products. Equation (12), (13) and (14) denote the variables types.

4. Experimental Results and Analysis

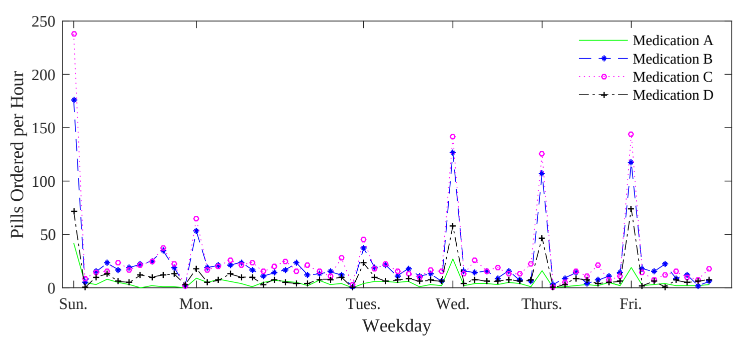


Figure 2: A weekly demand pattern for four medications

In real system, order arrival rate actually keeps changing during the day. Central fill pharmacy facilities only work during working time, while the orders are still received from drug stores. Therefore, numerous backlog orders will enter the system at the beginning of each workday. Figure 2 is an one week demand trend for four different mediations in a central fill pharmacy facility, which has two work shifts on Sunday and Monday, and one work shift on Tuesday, Thursday and Friday. During the model primal test, the computational time for 8 hours work time length problem is over 24 hours. In this experiment test, the system work time is scaled down to 30 minutes, and related parameters extracted based on real data are also rescaled. Two cases, single dispenser case and 20 dispensers case are discussed in this section. All the cases are solved using CPLEX solver with computational time less than 30 minutes.

4.1 Case 1: Single Dispenser Replenishment Problem

The first case aims to verify the mathematical model, and only a single product is considered. Firstly, considering $\alpha = \beta = \delta = 1$, the related parameters are given as $M = 1, Q = 5cc, T = 30\,minutes, d = 2pill/minute, p = 1pill/cc, q = 5cc$, $\sigma = 100$, $s = 1$. Two situations are tested for this case. The first situation considers $\gamma = \beta = 1$. In the second situation, the shortage cost importance is highlighted through setting $\gamma = 10$. Also, different replenishment processing times are tested on both situations, as show in Table 1 and 2.

Table 1: Optimal solutions based on $\alpha = \beta = \gamma = \delta = 1$

Replenishment processing time (c)	1	2	3	4	5
Reorder level (l)	0.4	0	0.4	0.8	Infeasible
Total Cost (C)	102	79	86	87	Infeasible
Number of canister required (n)	1	2	3	4	Infeasible
Replenishment cycle ($1/\bar{x}$) (minute)	2.73	2.73	2.73	2.73	Infeasible
Average inventory level ($\bar{v}$) (pill)	3.00	2.00	2.00	2.00	Infeasible
Percentage of shortage ($\bar{u}$)	0	0.2	0.2	0.2	Infeasible

From Table 1, we can see that the safety reorder level (l) and required canister amount can be successfully obtained for those cases that replenishment process time is less than 5. With the increase of replenishment processing time, more canisters are required. Also, inventory level tends to be lower because of the longer replenishment time. However, the inventory drain is actually eliminated by complementing more canisters. To finish all the demand, the upper bound of replenishment processing time is 5 minutes in this case.

Table 2: Optimal solutions based on $\alpha = \beta = \delta = 1, \gamma = 10$

Replenishment processing time (c)	1	2	3	4	5
Reorder level (l)	0.2	0.6	0.8	0.8	Infeasible
Total Cost (C)	102	102	129	141	Infeasible
Number of canister required (n)	1	2	3	4	Infeasible
Replenishment cycle ($1/\bar{x}$) (minute)	2.73	2.73	2.73	2.73	Infeasible
Average inventory level ($\bar{v}$) (pills)	3.00	3.00	3.83	2.00	Infeasible
Proportion of inventory shortage ($\bar{u}$)	0	0	0	0.2	Infeasible

Through increasing the unit inventory shortage cost coefficient γ, the optimal replenishment strategy is a little different in Table 2. Comparing Table 1 and Table 2, it is obvious that the model tends to avoid inventory shortage based on the changes of inventory shortage proportion ($\bar{u}$). Also, the system's reorder level increases, which increase the average system inventory level. However, the total system cost also increases because holding cost goes up. Interestingly, replenishment cycle times are the same in both situations, which address that under the current demand pattern, essentially, average 2.73 minutes replenishment cycle is the requirement to finish all the orders.

4.2 Case 2: 20 Parallel Dispensers Replenishment Problem

In this case, a total of 20 parallel dispensers are tested. The same as before, two situations are considered for trade off inventory holding cost and shortage cost. The basic parameters are setting as following: $M = 20$, $Q = 5cc$, $T = 30$ minutes, $d = 4$ pill/minute for $t \in [1, 10]$, $d = 2$ pill/minute for $t \in [11, 20]$, $d = 1$ pill/minute for $t = t \in [21, 40]$, $c = 3$ minutes, p =random(1,10) pill/cc, $q = 5cc$, $\sigma = 100$, $s = 1$, $\alpha = \beta = \gamma = \delta = 1$. In this situation the total cost obtained is 10,491. The optimal replenishment strategy obtained for each dispenser is given as Table 3. Total canister required is 10, the average inventory level ($\bar{v}$) is 17.25 pills, and the average reorder level ($\bar{l}$) is 0.203. In addition, the unit cost

Table 3: Optimal replenishement palnning based on $\alpha = \beta = \gamma = \delta = 1$

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n	1	1	0	0	1	1	1	0	3	1	0	0	0	0	1	0	0	0	0	0
l	0.24	0.25	0	0.14	0.33	0.4	0.25	0	0.4	0.5	0	0.17	0.17	0	0.24	0.17	0.23	0.14	0.17	0.23

for inventory shortage is increased in the second situation, where $\alpha = \beta = \delta = 1, \gamma = 100$. The optimal replenishment planning is obtained in Table 4. In this result, average inventory level ($\bar{v}$) becomes larger, which is 18.12, to avoid inventory shortage. The reason is because that the total number of canister is increased to 16. However, the average inventory level($\bar{l}$) is reduced to 0.149 in this situation. Thus, the total cost is also increased as 15,782.

Table 4: Optimal replenishement palnning based on $\alpha = \beta = \delta = 1, \gamma = 100$

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n	3	1	0	0	1	3	1	0	3	1	0	0	0	0	3	0	0	0	0	0
l	0.08	0.25	0	0.1	0.47	0	0.25	0	0.4	0.5	0	0.13	0.13	0	0	0.13	0.17	0.1	0.13	0.17

4.3 Summary

The impacts of replenishment operation time and shortage cost coefficient are investigated from the tests. In practice, the trade-off for holding and shortage cost depends on real requirement. For instance, in the high integrated RDS system, it is better to have enough inventory to maintain the system work, because a single dispenser stop may cause the whole system stop. However, drug safety is also important, some drugs could be contaminated because of long time exposed to air. And some drugs are expensive and rarely to use. Therefore, holding cost becomes more significant in those cases. Another result is that replenishment operation time is a key factor that impacts replenishment planing.

5. Conclusion and Future Work

The paper studied a medication replenishment problem in RDS. An mixed integer programming model is built to find optimal order time and canister amount. The impacts of replenishment operation time and shortage cost coefficient are tested. Results show that given system configuration, the proposed model can offer a strategic replenishment planing to minimize system cost. However, the model is tested under scale data, heuristic methods can be implemented in the future to solve real large scale problem. This model can also be used in supply chain inventory management.

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