# Operation Planning of Elective Patients with a Minimal Confidence Level of Not Overcoming the Total Time

***Abstract*—This paper considers the operation scheduling and planning of elective patients in a Surgery Department of a Hospital. We assume an ordered list of patients that should be planned for surgery in two available rooms, each room being possible to be used for a specific duration of time per day. Based on the average durations of surgeries that have been computed by considering historical information from hospital, we propose a Mixed Integer Linear Programming (MILP) problem to obtain a given utilization rate per room. In order to improve the solution we extend the operation planning by using a Mixed Integer Quadratic Constraint Programing (MIQCP) model that considers the durations of the surgeries and the time between operations as random variables with given average durations and standard deviations. MIQCP model allows planning of the OR optimizing maximum occupation rate by assuming a given confidence level of not overcoming the proposed daily time. Based on the results obtained by using MIQCP, the computational time to solve the scheduling problem is reduced by using a new model (MIQCP-P). This model schedules with a given occupation rate of OR rooms and fixing the minimum confidence of no overcoming the total time. The results are tested on some real data from the hospital and some simulation results are provided.**

## I. Introduction

Linear programming models were developed during World War II to make plans or proposals of training time, logistics or deployment of combat units. After the war, many industries began to use it in their daily planning. Subsequently, it was observed that, through proper system modeling, linear programming can be applied to different fields of application.

In this paper, different mathematical programing models for scheduling of non-urgent surgeries in a hospital department are used and compared. Scheduling of surgeries can be seen as the planning of a production system: (a) there is a waiting list of patients representing the system demand and (b) there exists a limited number of surgeons and a limited number of operating rooms (OR) representing the capacity of the production system. For our application, the bottleneck of the resources are the OR. In particular, there exists two OR for non-emergency surgeries in the Surgery Department, each one being available only a certain number of hours per day. Furthermore, the operating room is the most expensive and limited material resource in a surgical service, being therefore extremely important to obtain its maximum performance. The operating rooms have permanently human resources and if the maximum performance is not obtained, the mentioned staff do not have fully committed their labor day, and the OR have nevertheless economic resources consumption. The purpose of the mathematical models is to optimize the use of the OR. Different surgeries have associated averages durations, but there are uncertainties due to uncontrollable factors such as unforeseen events or the different nature of each body. Due to these uncertainties, in previous works was considered that an acceptable performance is obtained when the ORs are scheduled at about 80 percent of their occupancy. The remaining 20 percent of the time was reserved to absorb possible surgeries delays and the cleaning of OR after each surgery. In order to solve this problem, a mathematical model based on integer programing (MILP) was obtained. The proposed MILP allows planning of the surgeries with a determined occupation rate (e.g., 80%) respecting, as much as possible, the order of patients in the waiting list.

978-1-5090-1314-2/16$31.00 

However, considering the durations of the surgeries $S_d$ and the cleaning time $C_t$ as random variables with normal probability density function (pdf) ($i \sim N(\mu_i, \sigma_i)$, $\forall i \in \{S_d, C_t\}$) calculated using historical data, it is shown that the solutions obtained by using MILP have high probabilities of overcoming the total time. Time overrun means that staff should lengthen their working day, and this is an uncomfortable situation in a hospital department. In order to obtain a safe planning (with lower probability of overcoming the total time), a new MIQCP model is developed. This model consider the duration of the surgeries ($S_d$) and the cleaning time ($C_t$) as random normal variables. The objective of this model is to obtain the maximum occupation rate of OR respecting as much as much as possible the order of the patients in the waiting list. In addition the model is constrained to ensure a minimal confidence level of not overcoming the total time.

For a set of pathologies (used in the Hospital Department) and a given duration of a working day (e.g., 7 hours), MIQCP model will be used to obtain optimal solutions of operation planning. These results allow us to compute the average occupation rate in order to fulfill with a minimum confidence rate of not overcoming the total time. Using these data as input parameters of MIQCP-P model, the computational time necessary to perform safe schedules of OR decreases.

The operation planning and scheduling of elective patients is a problem studied in literature by many researchers. For a state of the art of the problem we can refer the reader to the survey [1] and their references. Here a literature review that is structured using descriptive fields is proposed. Taking into account this structure, the present paper, can be classified like:

Operation planning of elective inpatients by a Mixed Integer Quadratic Constraint problem with a multicriteria objective: waiting time of patients and utilization of OR. In addition, the surgery durations are stochastic and the problem is based on real data.

Moreover, some works combine the planning and scheduling problem of elective patients with the urgent ones [2], [3] by using stochastic models. The scheduling and planning of resources have been studied for other problems, as for example home care services [4], [5]. Petri net models have been used for modeling and management of healthcare systems, see for example [6]–[9]. The contributions of this paper with respect to the previous results are: (1) the application of the mathematical programming models to the particular problem in the studied Hospital, (2) inclusion of statics concept in a MIQCP problem to ensure a confidence rate not exceeding the total time in the scheduling of ORs; and (3) simulation results using real data from hospital.

The paper is organized as follows. Sec. II describes the organizational structure in the studied Department of the Hospital and a motivation example is showed. In Sec. III the methodology to obtain a safe operation planning with a given occupation rate of OR will be given. Sec. IV shows the models (MILP, MIQCP, and MIQCP-P) to schedule the ORs. In Sec. V some results obtained by implementing the models in a computer software (CPLEX) are analyzed and compared. Finally, in Sec. VI, we provide some conclusions and future works.

## II. Organizational structure of the studied Department and motivation example

The solution proposed here will be applied to a Surgery Department of a Hospital is studied. Despite this, the proposed solution can be extended and used to other departments of the same or other hospital.

### A. Organizational structure of the Department

This hospital department is composed by medical doctors (specialists and residents) divided in five medical teams. Each team has a coordinator (the more experienced doctor), each doctor having his own waiting list of patients waiting for surgery. The patients belonging to the waiting lists of the doctors belonging to a given team compose the patient waiting list of the team.

The Department have available two OR per day for non-urgent surgeries. Each of these OR have an active schedule from 8am to 3pm (7 hours). The Department is organized in such way that during a given day each OR is used by an unique team. The head of the Department (that is a medical doctor as well) is responsible for assigning teams: (a) to the OR, (b) to the external consultations and (c) to the emergency service. The assignment is made with a time horizon of two months, that is, all teams knows two months in advance the days in which they can use the OR.

Actually there no exists an automatic method for operating planning, so each team coordinator is guided by his own intuition and experience to plan the surgeries. During last years, using this manual planning method an occupation rate of ORs of 72 percent has been obtained. This is an excellent rate that can be improved, although not by the manual planning. The organizational structure of the surgical service of the Department Hospital is given in in Fig. 1.

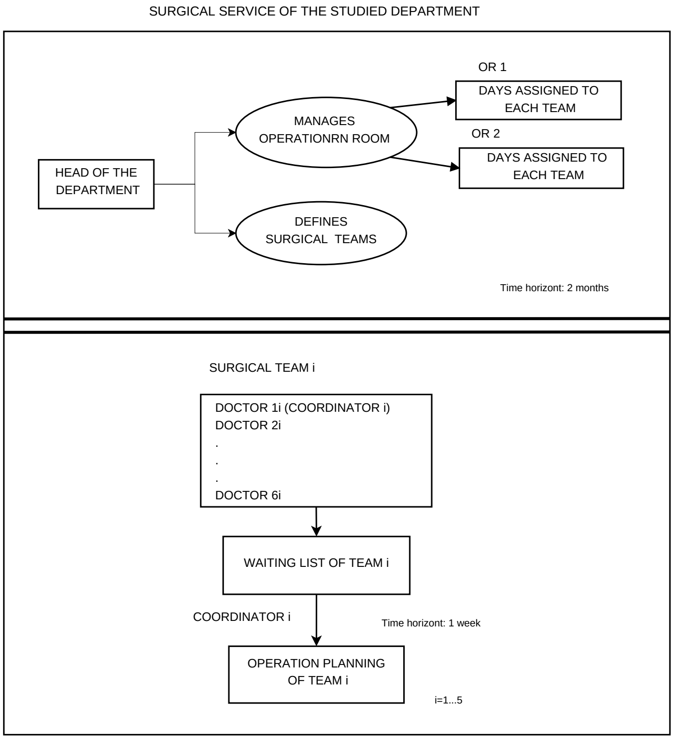


Fig. 1. Organizational structure of the surgical service in Department Hospital.

It is important to ensure that the utilization of the OR does not exceed 3pm because in this case medical staff could lengthen their working day. With the propose of preventing these situations, the medical managers consider appropriated an occupancy rate of about 80 percent, being the remaining 20 percent use for cleaning of operating rooms after each surgery and, on the other hand, absorb possible surgeries delays.

### B. Problem statement and motivation example

Taking into account the organizational structure of the Surgery Department, the solved problem is the following:

*Problem 1:* Given a list of patients (belonging to the medical doctors of one team) that should be scheduled for surgeries and a duration $d$ of daily time of using an operating room, schedule the next $m$ working days of ORs with an occupancy rate of $p$ percent, respecting as far as possible the order of the patients in the waiting list.

The following example illustrates the problem assuming a given waiting lists of patients. The patients in the list are ordered according to the preference of their surgeries. Tab. IV gives an example of two patients of a waiting list. Notice that, each entry in the table has four elements:

- *Preference order* - indicates the preference order of the surgery;
- *Patient name* - the patient name;
- *Pathology* - indicates the type of surgery;
- *Medical Doctor* - surgeon who will perform the surgery.

TABLE I
EXAMPLE OF ORDERED WAITING LIST FOR SURGERY.

| Order | Patient | Pathology | Medical Doctor |
|---|---|---|---|
| 1 | Perez, Juan | KNEE ARTHROPLASTY | Pedro Suarez |
| 2 | Garcia, Maria | HALLUX VALGUS | Raquel Arrellano |
| ... | ... | ... | ... |
| n | ... | ... | ... |

On the other hand, by using the historical data from the last two years, for each pathology we compute its average duration and the standard deviation assuming the average duration be a random variable with normal distribution $S_d = N(\mu_{S_d}, \sigma_{S_d})$. This duration is the time from the moment when the patient enters to OR until she leaves the OR. In addition, using the historical data, the time spent between two operations ($C_t$) would be updated. This time includes the waiting time until a cleaning team is available and the cleaning time of the OR. It is assumed also $C_t$ random variable with normal distribution $C_t = N(\mu_{C_t}, \sigma_{C_t})$. Initially, these values do not depend on the pathology.

Let us define the input data of the problem by $\boldsymbol{V}_e$ as a matrix of dimension $5 \times n$, where $n$ is the size of the waiting list. The first row of $\boldsymbol{V}_e$ (denoted also as $\boldsymbol{P}_o$) is the preference order, the second/third row is the average/standard deviation of the durations associated with the corresponding pathology ($\boldsymbol{\mu}_{S_d}/\boldsymbol{\sigma}_{S_d}$) and the fourth/fifth row is the average/standard deviation of the duration between two surgeries associated with the performed pathology ($\boldsymbol{\mu}_{C_t}/\boldsymbol{\sigma}_{C_t}$). It is assumed that these values are independent by the pathologies ($\mu_{C_t} = 20$ and $\sigma_{C_t} = 10$). For example, if the knee arthroplasty has an average duration of 133 minutes and a standard deviation of 24 minutes while the hallux valgus has an average duration of 83 minutes and a standard deviation of 22, then the first two columns of $\boldsymbol{V}_e$ matrix for the list in table IV is:

$$\boldsymbol{V_e} = \begin{array}{cl} \begin{bmatrix} 1 & 2 & \dots & n \\ 133 & 83 & \dots & \dots \\ 24 & 22 & \dots & \dots \\ 20 & 20 & \dots & \dots \\ 10 & 10 & \dots & \dots \end{bmatrix} & \begin{array}{l} \to \boldsymbol{P}_o \\ \to \boldsymbol{\mu_{S_d}} \\ \to \boldsymbol{\sigma_{S_d}} \\ \to \boldsymbol{\mu_{C_t}} \\ \to \boldsymbol{\sigma_{C_t}} \end{array} \end{array} \quad (1)$$

Furthermore, other three input parameters will allow us to generalize the model:

- $m$: number of days to plan;
- $p$: desired occupancy rate (by default $p = 80$);
- $d$: duration of the OR working day (by default $d =$ 7[hours]).

Let us assume now the ordered list containing 10 patients given as matrix $\boldsymbol{V_e}$ in (2), 3 days to schedule (i.e., $m = 3$), a duration of the OR working day of 7 hours (i.e., $d =$ 7[hours]$=420$[minutes]) and an objective occupancy rate of 80 (i.e., $p = 80$).

$$\boldsymbol{V_e} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 75 & 153 & 90 & 75 & 202 & 45 & 97 & 85 & 111 & 133 \\ 23 & 23 & 19 & 23 & 45 & 12 & 21 & 24 & 23 & 24 \\ 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \end{bmatrix} \quad (2)$$

Tab. II shows a possible solution of this particular problem. Each row of this table represents the operation planning of a working day. The first column represents the number of OR working day, the next four columns indicate the preference order of surgeries that should be operated in this working day (0 means that there is no surgery scheduled). The sixth column is the occupancy rate of the OR obtained the corresponding working day and finally, the seventh column indicates the confidence percentage of not overcoming the total time per day.

TABLE II
OPERATION PLANNING OF THE LIST OF PATIENTS GIVEN IN (2) FOR AN OBJECTIVE OCCUPATION RATE OF 80% AND A TIMING HORIZON OF 3 DAYS.

| Day | Sur. 1 | Sur. 2 | Sur. 3 | Sur. 4 | Occu. | Conf. |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 9 | 0 | 80.71 | 68.56 |
| 2 | 5 | 10 | 0 | 0 | 79.76 | 80.24 |
| 3 | 3 | 4 | 7 | 8 | 82.61 | 44.21 |

This solution has been computed by solving the mixed integer linear programing (MILP) problem described in subsection IV-A. This MILP does not take into account the percentage of confidence rate and consequently very low percentage of confidence could be obtained. This probability of the confidence rate is calculated using some static concepts of the normal distribution:

1) Let $x \sim N(\mu, \sigma)$ be a random variable with mean $\mu$ and standard deviation $\sigma$ and let $z \sim N(1, 0)$ be a random variable with mean 1 and standard deviation 0. Then,
$$P(x \leq X_i) = P(z \leq Z_i) \quad (3)$$
where $Z_i = \frac{X_i - \mu}{\sigma}$ [10].
2) Let $a, b, ..., z$ be independent random variables such that: $i \sim N(\mu_i, \sigma_i), \ \forall i = \{a, b, ...z\}$ and let $U = a+b+...+z$ be the sum of these variables, then $U$ is a random variable with normal distribution $U \sim N(\mu_U, \sigma_U)$ [10] where:
   - $\mu_U = \mu_a + \mu_b + ... + \mu_z$
   - $\sigma_U = \sqrt{\sigma_a^2 + \sigma_b^2 + ... + \sigma_z^2}$

For example, analyzing the proposed solution obtained in the working day 3 in Tab. II, it can be seen that the total working-time of the working day 3 $T_{d3}$ is the sum of 1) *the individual durations of each surgery scheduled in this day* ($S_{d3}, S_{d4}, S_{d7}$ and $S_{d8}$) and 2) *the corresponding cleaning time after each surgery in the day 3* ($C_{t3}, C_{t4}, C_{t7}$ and $C_{t8}$).

Since theses variables ($C_t$ and $S_d$) are considered with normal pdf, then $T_{d3} \sim N(\mu_{T_{d3}}, \sigma_{T_{d3}})$ where:

- $\mu_{T_{d3}} = \sum_{i=\{3,4,7,8\}} (\mu_{S_{di}} + \mu_{C_{ti}}) = 427$
- $\sigma_{T_{d3}} = \sqrt{\sum_{i=\{3,4,7,8\}} (\sigma^2_{S_{di}} + \sigma^2_{C_{ti}})} = 48.03$

So the total working-time of day 3 has a normal pdf such that: $T_{d3} \sim N(427, 48.03)$. As the total time available of an OR working day is 7 [hours] $\sim$ 420 [minutes], it is interesting to know the probability that the working-time is lower than 420 [minutes]. According to (3) $P(T_{d3} \leq 420) = P(z \leq Z_i)$ where $z \sim N(0,1)$. Since $X_i = 420$ then $Z_i = \frac{420-427}{48.03} = -0.145$. Therefore, $P(T_{d3} \leq 420) = P(z \leq -0.145)$ and this probability is tabulated: $P(z \leq -0.145) = 0.4421 \simeq 44.21\%$.

Two mathematical models (MIQCP and MIQCP-P) with quadratic constraints are proposed taking into account these probabilities. They are used to impose a confidence level of not overcoming the total available time $X_i$.

## III. Methedology

In this section the steps necessary to obtain a safety operation planning with a given occupation rate are discussed. An operation planning is considered safer than other, if the confidence level of not exceeding the total time is greater. Three model are proposed:

- MILP: (1) schedule the operations assuming a *desired occupation rate* ($DesOr$) of OR and (2) respect as much as possible the order of the patients in the waiting list.
- MIQCP: (1) schedule the operations by maximizing the occupation rate of the OR (2) respect as much as possible the order of the patients in the waiting list and (3) ensure a *minimum confidence level* ($Cl$) of not exceeding the total time in each working day.
- MICQP-P: (1) schedule the operations with a *desired occupation rate* ($DesOr$) of OR, (2) respect as much as possible the order of the patients in the waiting list and (3) ensure a *minimum confidence level* ($Cl$) of not exceeding the total time in each working day.

TABLE III
Comparison of the models

| | Objectives | | | Constraints |
|---|---|---|---|---|
| | Order | $DesOr$ | Max. Occ | Min $Cl$ |
| MILP | ✓ | ✓ | | |
| MIQCP | ✓ | | ✓ | ✓ |
| MIQCP-P | ✓ | ✓ | | ✓ |

Tab. III shows a comparison of the three proposed mathematical models. Each one of these models is represented by a row and their characteristics by columns. If there is a tick in one column of one row, means that the mathematical model corresponding to this row have the objective/constraint of this column. The first column labeled as Order represents the objective of respecting as much as possible the order of the patients in the waiting list. The second column labeled as $DesOr$ represents the objective of obtaining a desired occupation rate of OR. The third column labeled as Max. Occ represents the objective of obtaining the maximum occupation rate of OR. Finally, the fourth column labeled as $Cl$ represents the objective of ensuring a minimum confidence level of not overcoming the total time.

Fig. 2 shows the steps to follow in order to obtain a safety operation planning with a $DesOr$ and respeting as much as possible the order of the patients in the waiting list. These steps are described in the following:

1) Assuming a large list of patients (generated randomly) composed by a given set of pathologies (e.g., from studied Surgery Department) and a duration of a working day (e.g., 7 hours), MIQCP model is used to obtain the optimal solution of operation planning with a large number of OR working days. Each solution is obtained using a different minimum confidence level of not overcoming the total time $Cl$. The average occupation rate of OR obtained in each optimal solution is computed (Avg. Occ). Tab. IV shows the results obtained.

TABLE IV
Avgerage Occupation (Avg. Occ) depending on the minimum level of confidence of not overcoming the total time ($Cl$) using MIQCP

| $Cl$ | Avg. Occ |
|---|---|
| 51 | 83.39 |
| 55 | 82.33 |
| 60 | 81.04 |
| 65 | 79.91 |
| 70 | 77.92 |
| 75 | 75.94 |
| 80 | 74.38 |

2) Using the values obtained in the previous step a graph that represent the Avg. Occ VS $Cl$ is obtained.
3) In this graph, for a given desired occupation rate ($DesOr$), it is possible to check the minimum confidence level ($Cl$) of not overcoming the reachable total time.
4) Using as input parameter the $C_l$ and $DesOr$ of OR (obtained from the graph), the MIQCP-P problem is solved. The computational time necessary to solve this problem is lower than solving MIQCP, and the solution obtained is safer than the solution obtained by using MILP.

Fig. 3 shows the average occupation rate obtained using MIQCP depending on the minimal percentage of confidence required. The solutions have been computed by using a random list of patients composed by a set of pathologies from the studied Surgery Department. In addition, a duration of an OR working day of 7 hours has been considered. For example, if the objective is to obtain a $DesOr = 80\%$, we can observe that it is possible to ensure a minimum confidence level equal to $64,3\%$. So $Cl = 64.3$ and $DesOr = 80$ are the input parameters to solve MIQCP-P model in order to obtain a safe operation planning, respecting as much as possible the order of the patient in the waiting list and with and occupation rate of OR equal to $80\%$.

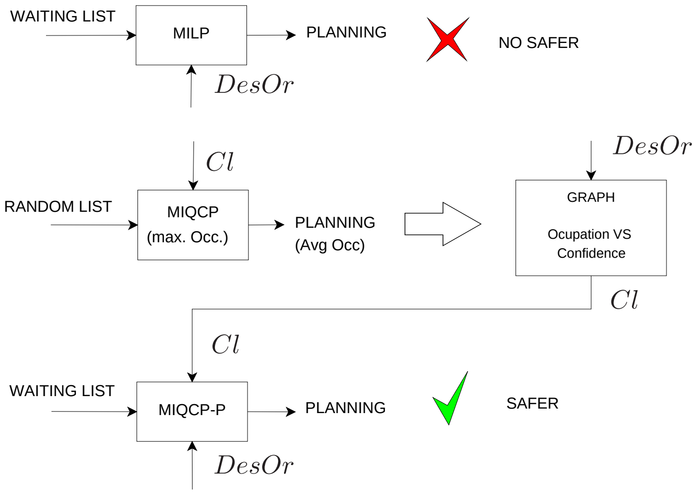


Fig. 2. Methodology to obtain a safe operation planning.

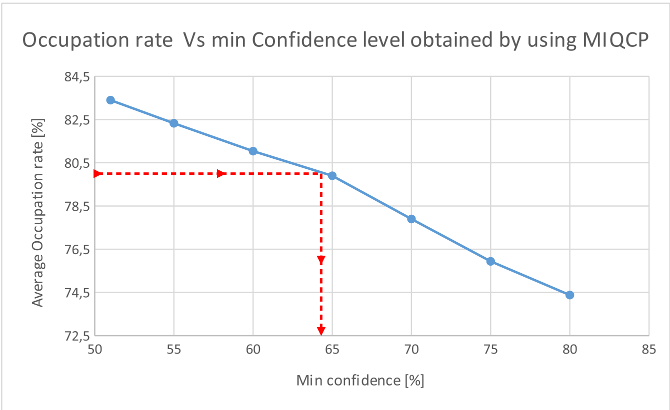


Fig. 3. Average occupation rate obtained using MIQCP depending on the minimal percentage of confidence required (blue). Input list composing by a set of pathologies of the studied Surgery Department and a duration of OR working day of 7 hours.

## IV. Mathematical Programing Models

In this section the proposed programming models are described.

### A. MILP

Let us define the following variables:

- $\boldsymbol{S}_1, \boldsymbol{S}_2, \boldsymbol{S}_3, \ldots, \boldsymbol{S}_m$ vectors of binary variables where $m$ is the number of working days to schedule. Hence $\boldsymbol{S}_i \in \{0,1\}^n$ where $n$ is the size of the waiting list. $\boldsymbol{S}_i[j] = 1$ means that surgery $j$ should be operated in the $i^{th}$ working day;
- $\boldsymbol{\alpha} \in \mathbf{R}^m_{\geq 0}$ is a vector of absolute deviations (in minutes) of each day with respect to the objective. Let us denoted by $Obj = d \cdot \frac{p}{100}$ the target objective where d is the numbers of minutes available in one OR working day and p the desired occupation rate.

Notice that the total number of variables is $n \times m + m = (n+1) \times m$.

There exist two sets of constraints in this problem. The first constraint set is related to the definition of variables $\alpha_i$, while the second set imposes that each surgery is performed only once.

Since $\alpha_i$ is the absolute deviation (in minutes) of day $i$ with respect to the objective occupation, assuming an occupancy rate of $p = 80\%$ and a duration of a working day of $d = 7$ hours, we can write:

$$\begin{aligned}\alpha_i &= \left|\sum_{j=1}^{k_i} \tau_{ij} - \left(d \cdot \tfrac{p}{100}\right)\right| = \left|\sum_{j=1}^{k_i} \tau_{ij} - Obj\right| \\ &= \left|\sum_{j=1}^{k_i} \tau_{ij} - \left(7 \cdot 60 \cdot \tfrac{80}{100}\right)\right| = \left|\sum_{j=1}^{k_i} \tau_{ij} - 336\right|\end{aligned} \quad (4)$$

where $k_i$ is the total number of surgeries scheduled in the working day $i$ and $\tau_{ij}$ is the average duration of the $j^{th}$ surgery scheduled in the working day $i$. Using the input array of theoretical durations (second row in matrix $V_e$: $\boldsymbol{\mu}_{S_d}$, see (**??**)), we can equivalently rewrite (4) as

$$\alpha_i = \left|\sum_{j=1}^{k_i} \tau_{ij} - Obj\right| = |\boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i - Obj| \quad (5)$$

That is equivalent with the minimum $\alpha_i$ that fulfill the following constraints

$$\begin{cases} \boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i - Obj \leq \alpha_i \\ \boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i - Obj \geq -\alpha_i \end{cases} \quad \forall i = 1, 2, \ldots, m \quad (6)$$

Two constraints will be necessary to define each variable $\alpha_i$. Since the number of variable $\alpha_i$ is equal to the timing horizon (i.e., number of days to schedule $m$), we need $2 \times m$ constraints to define all variables $\alpha_i$.

The other set of constraints ensures that each surgery will be planned at most once:

$$\sum_{i=1}^{m} \boldsymbol{S}_i[j] \leq 1 \quad \forall j = 1, 2, \ldots, n \quad (7)$$

Finally, the MILP has the following size

- number of variables $(n+1) \times m$;
- number of constrains $2 \times m + n$.

Like in many optimization problems there are two contradictory objectives: a) obtain a occupation rate of ORs b) and respect the order of patients in the waiting list. We solve this problem by a linear cost function composed by two balanced terms: the first one is related to the objective occupancy rate while the second one with the preference order of patients in the list. The two objectives are balanced by a parameter $\beta$. In particular, the objective is to minimize,

$$\sum_{i=1}^{m} [\alpha_i \cdot (m - i + 1) + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m - i + 1)] \quad (8)$$

Variable $\alpha_i$ of the first term penalizes the deviation of the occupancy of the OR with respect to the objective $p$

(e.g., $80\%$). Since $(m-i+1)$ is multiplying $\alpha_i$, it gives more importance to obtain a better occupancy rate in the first workings days. In this way, if there are not enough patients for all working days, the last days remain free.

The second term in (8) is related to the order in the waiting list. The result of multiplying $\boldsymbol{P}_o$ by $\boldsymbol{S}_i$ is the sum of the preference order of surgeries scheduled day $i$. In this way, we give preference to the first patients of the waiting list over the patients with higher order number (in general, patients with a lower order number have a longer time in the waiting list). Again we multiply the second term by $(m-i+1)$, this implies that the patients with lower preference order will be scheduled the first days.

Parameter $\beta$ is a design parameter and it is used to give more importance of respecting the order of the patients in the waiting list or to the occupancy rate of the OR.

The full MILP is as follows:

$$\min \quad \sum_{i=1}^{m} \left[\alpha_i \cdot (m-i+1) + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m-i+1)\right]$$

Subject to:

$$\begin{cases} \boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i - Obj & \leq \quad \alpha_i, \quad \forall i = 1,2,\ldots,m \\ -\boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i + Obj & \leq \quad \alpha_i, \quad \forall i = 1,2,\ldots,m \\ \sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq \quad 1, \qquad \forall j = 1,2,\ldots,n \end{cases} \tag{9}$$

### B. MIQCP

This model is similar to (9), begin the objective to obtain the maximum occupation rate of the OR but at the same time to ensure a minimum confidence level of not overcoming the daily total time.

The set of first two constraints of (9) are replaced by the following set of constraints:

$$X - \boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i = \alpha_i \quad \forall i = 1,2,\ldots,m \tag{10}$$

where X is the total time available (e.g., 7 [hours] $\sim$ 420 [minutes]) in a OR working day. In this way, the variable $\alpha_i$ represents the difference in minutes between the total time available in a working day (7[hours] $\sim$ 420[minutes]) and the average duration of the surgeries scheduled the day i. Since the sum of variables $\alpha_i$ is minimized in the objective function, high occupation rates are obtained.

The second change is a set of constraints that ensure the planning of OR working days with a confidence level of not overcoming the total time greater than a threshold $C_l$.

Considering the total time of a OR working day i as a random variable with normal pdf $T_{di} \sim N(\mu_{T_{di}}, \sigma_{T_{di}})$ such that:

- $\mu_{T_{di}} = (\boldsymbol{\mu}_{S_d} + \boldsymbol{\mu}_{C_t}) \cdot \boldsymbol{S}_i$
- $\sigma_{T_{di}} = \sqrt{(\bar{\boldsymbol{\sigma}}^2_{S_d} + \bar{\boldsymbol{\sigma}}^2_{C_t}) \cdot \boldsymbol{S}_i}$ [1]

[1] In this paper $\bar{\boldsymbol{x}}^2$ is a vector of dimension equal to $\boldsymbol{x}$ such that $\bar{\boldsymbol{x}}^2(i) = \boldsymbol{x}(i)^2 = x(i) \cdot x(i)$

and the time available for a OR working day as $X$ (e.g., 420[minutes]). Then it is necessary to ensure that $P(T_{di} \leq X) > C_l$. Where $C_l$ is the minimum confidence level of not overcoming the total time allowed in each OR working day. Using the static concepts explained in Sec. II, this set of constraints is modeled by the next inequality:

$$\frac{X - \mu_{T_{di}}}{\sigma_{T_{di}}} \geq V_{Cl} \quad \forall i = 1,2,\ldots,m \tag{11}$$

where $V_{Cl}$ is the value corresponding a normal variable ($x \sim N(0,1)$) with an accumulative probability $C_l$: ($P(x \leq V_{Cl}) = C_l$).

Developing the inequality (11):

$$\begin{aligned} & \frac{X - \mu_{T_{di}}}{\sigma_{T_{di}}} \geq V_{Cl} \Rightarrow X - \mu_{T_{di}} \geq V_{Cl} \cdot \sigma_{T_{di}} \Rightarrow \\ & X - \underbrace{(\boldsymbol{\mu}_{S_d} + \boldsymbol{\mu}_{C_t})}_{\boldsymbol{A}} \cdot \boldsymbol{S}_i \geq V_{Cl} \cdot \sqrt{\underbrace{(\bar{\boldsymbol{\sigma}}^2_{S_d} + \bar{\boldsymbol{\sigma}}^2_{C_t})}_{\boldsymbol{B}} \cdot \boldsymbol{S}_i} \\ & \text{if} \quad X - \boldsymbol{A} \cdot \boldsymbol{S}_i > 0 \quad \text{then} \\ & [X - \boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \geq \left[V_{Cl} \cdot \sqrt{\boldsymbol{B} \cdot \boldsymbol{S}_i}\ \right]^2 \Rightarrow \\ & X^2 + [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 - 2 \cdot X \cdot \boldsymbol{A} \cdot \boldsymbol{S}_i \geq V_{Cl}^2 \cdot \boldsymbol{B} \cdot \boldsymbol{S}_i \Rightarrow \\ & \underbrace{\left[V_{Cl}^2 \cdot \boldsymbol{B} + 2 \cdot X \cdot \boldsymbol{A}\right]}_{\boldsymbol{K}} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \leq X^2 \Rightarrow \\ & \boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \leq X^2 \end{aligned}$$

The constraint $X - \boldsymbol{A} \cdot \boldsymbol{S}_i > 0$ ensures that the working time of each day $\boldsymbol{A} \cdot \boldsymbol{S}_i$ (sum of the average time of each surgery scheduled and their corresponding cleaning time) is lower than the total time available in one working day $X$. In this way the possible false solutions obtained due to $X - \boldsymbol{A} \cdot \boldsymbol{S}_i{}^2$ are prevented. However, the model can only scheduled working days with a minimum confidence level of not overcoming the total time greater than 50. In order to impose a minimum confidence level of not overcoming the total time lower than 50, the constraint $X - \boldsymbol{A} \cdot \boldsymbol{S}_i > 0$ has to be changed to $X - \boldsymbol{A} \cdot \boldsymbol{S}_i < 0$. In this paper, minimum confidence level lower than 50 are not considered.

The set of constraints that prevents the planning of OR working days with a confidence level lower than $C_l$ are showed in (12).

$$\begin{cases} \boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 \leq X^2 \\ X - \boldsymbol{A} \cdot \boldsymbol{S}_i \ \geq 0 \end{cases} \quad \forall i = 1,2,\ldots,m \tag{12}$$

The first constraint of this set is quadratic, so the new proposed model is a Mixed Integer Quadratic Constraint Programming (MIQCP) problem. The full MIQCP is as follow:

$$
\begin{aligned}
&\min \quad \sum_{i=1}^{m} \left[\alpha_i \cdot (m-i+1) + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m-i+1)\right] \\
&\text{Subject to:} \\
&\begin{cases}
X - \boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i & = \quad \alpha_i, \qquad \forall i = 1,2,\ldots,m \\
\sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq \quad 1, \qquad \forall j = 1,2,\ldots,n \\
\boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 & \leq \quad X^2, \qquad \forall i = 1,2,\ldots,m \\
X - \boldsymbol{A} \cdot \boldsymbol{S}_i & \geq \quad 0, \qquad \forall i = 1,2,\ldots,m
\end{cases}
\end{aligned} \tag{13}
$$

Where $\boldsymbol{K}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are known input data:

$$
\begin{aligned}
\boldsymbol{K} &= V_{Cl}^2 \cdot \boldsymbol{B} + 2 \cdot X \cdot \boldsymbol{A} \\
\boldsymbol{A} &= \boldsymbol{\mu}_{S_d} + \boldsymbol{\mu}_{C_t} \\
\boldsymbol{B} &= \bar{\boldsymbol{\sigma}}_{S_d}^2 + \bar{\boldsymbol{\sigma}}_{C_t}^2
\end{aligned}
$$

### C. MIQCP-P

Finally MIQCP-P is combining the objective of obtaining a given occupation rate of OR and the set of constraint to ensure a minimum confidence level of not overcoming the total time. So, in this model, we add to the MILP the set of constraints that prevent the planning of OR working day with a confidence level of not overcoming the total time lower than $C_l$ (12).

$$
\begin{aligned}
&\min \quad \sum_{i=1}^{m} \left[\alpha_i \cdot (m-i+1) + \beta \cdot \boldsymbol{P}_o \cdot \boldsymbol{S}_i \cdot (m-i+1)\right] \\
&\text{Subject to:} \\
&\begin{cases}
\boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i - Obj & \leq \quad \alpha_i, \qquad \forall i = 1,2,\ldots,m \\
-\boldsymbol{\mu}_{S_d} \cdot \boldsymbol{S}_i + Obj & \leq \quad \alpha_i, \qquad \forall i = 1,2,\ldots,m \\
\sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq \quad 1, \qquad \forall j = 1,2,\ldots,n \\
\boldsymbol{K} \cdot \boldsymbol{S}_i - [\boldsymbol{A} \cdot \boldsymbol{S}_i]^2 & \leq \quad X^2, \qquad \forall i = 1,2,\ldots,m \\
X - \boldsymbol{A} \cdot \boldsymbol{S}_i & \geq \quad 0, \qquad \forall i = 1,2,\ldots,m
\end{cases}
\end{aligned} \tag{14}
$$

The main advantage of this model compared to MIQCP (13) is the computational time which is smaller. In MIQCP the objective is to obtain the maximum occupation rate respecting the established confidence level $Cl$, while, in MIQCP-P the objective is to obtain a desired occupation rate respecting the established confidence level. If for a given set of pathologies, a given duration of an OR working day and a given minimum confidence level, the average occupation rate reachable is known, then it is possible to use MIQCP-P in order to obtain a very similar solution that using MIQCP but with a much smaller computational time.

## V. Simulation results using CPLEX

In this section the behavior of the models (9), (13) and (14) will be tested and the operation planning obtained with them will be compared. In particular the models are validated and compared from the point of view of occupation rate; order of the patients in the waiting list and; the confidence of not overcoming the total time. In addition, the computational time necessary to solve the problems will be given. The simulations have been obtained by using the IBM ILOG CPLEX Optimization Studio which is often referred as CPLEX [11] which is an commercial solver designed to tackle (among others) large scale (mixed integer) linear problems. CPLEX is now actively developed by IBM and it is one of the fastest software solution for MILP problems [12].

All simulations in this section have been performing using real data from a hospital. In order to generate randomly the numeric input data of the models (i.e., matrix $\boldsymbol{V}_e$), we used a lot of average durations for different pathologies that have been operated in the Surgery Department in the last two years. Only the pathologies of the patients have been generated randomly, but some probabilities computed bases on the real data have been used to generate the pathologies. Although CPLEX is one of the most powerful tool, due to the large size of the problems, the computational time and memory usage to solve its is too high. For example, assuming a waiting list composed by 300 patients and 50 days to scheduled, the MIQCP model (13) have the following size:

- number of variables: $(n+1) \times m = (300+1) \times 50 = 15050$;
- number of linear constraints: $2 \times m + n = 2 \times 50 + 300 = 400$;
- number of quadratic constraints: $m = 200$;

After some simulations with different values of number of patients ($n$) and horizon time ($m$), we observed that the variable that influence more the computational time is $m$. Moreover, the computational time depends also on the value of the design parameter $\beta$. Against smaller is beta (that is more importance is the occupation rate), greater is the computational time.

In order to be able to plan all patients in the waiting list, in previous work was proposed to solve MILP (9) iteratively (similar with receding horizon strategy [13], [14]): from the waiting list of patients, first 7 working days are scheduled, and the scheduling of the first 5 days are considered. The patients who surgeries have been planned in these 5 workings days are removed from the waiting list and the process is repeated until the waiting list becomes empty. However, in quadratic constraint problems (13) and (14), due to the greater computational complexity, only the first 3 working days are planned and the scheduling of all of them are considered. The patients planning in these 3 days will be removed from the list and the process will be repeated until the desired number of days will be scheduled.

In previous works, the influence of the parameter $\beta$ in MILP model was shown. Decreasing the parameter $\beta$ better results of occupation rate are obtained: the standard deviation decreases and consequently the data are more concentrated around the average value. Unfortunately, this improvement is achieved by allowing a greater disorder in the operations planning. Here a value of parameter $\beta = 2$ is fixed. In this way the computational time to solve the problems is not very high and the compromise between the occupation rate and the order of the patients is appropriated from a medical point of view. For this reason all simulations in this work have been performed using a value of $\beta$ equal to 2.

In order to compare different operation planning three important aspects will be analyzed:

1) *Occupation rate*: a static analysis of the occupation rate of the OR is calculated. The average, the standard deviation and the extreme values are showed.
2) *Confidence level of not overcoming total time*: a static analysis of the confidence level corresponding to days scheduled is computed. The average, the standard deviation and the extreme values are showed.
3) *Order of the patients from the waiting list*. In this aspect, we are going to consider two parameters related with the delay of the patients. Observing that normally 3 patients are scheduled per day, a patient with a preference order j should be operated the day $\lceil \frac{j}{3} \rceil$, where $\lceil \cdot \rceil$ is the rounding to the next integer. All patients scheduled in a working day delayed with respect to the working day in which she should be operated, sum the square of the numbers of days delayed to the indicator $\Pi$. In addition, the number of patients delayed $(P_d)$ is counted. For example, if a patient with a preference order $j$ is scheduled the day $k$ and $k > \lceil \frac{j}{3} \rceil$ then,

$$\begin{cases} \Pi = \Pi + \left[k - \lceil \frac{j}{3} \rceil\right]^2 \\ P_d = P_d + 1 \end{cases} \tag{15}$$

Tab. V shows an analysis of the solutions obtained using the 3 proposed model in this paper (MILP, MIQCP and MIQCP-P). Considering historical data from the hospital department, the probabilities of performing each pathology are computed. Using these probabilities, three waiting lists composed by 150 patients have been generated randomly. The 3 waiting list have been scheduled using the three models. With the first list a desired occupation rate of 80 percent is required, with the second list a desired occupation rate of 77.9 percent and finally with the third list a desired occupation rate of 75.94 percent. For each one of the three lists and with the 3 models, 42 working days have been scheduled. All solutions have been obtained using a value of $\beta$ equal to 2. The other input parameters of the models are shown in the 3 first columns of the Tab. V (INPUT PARAMETERS). The last 10 columns of Tab. V show the RESULT OBTAINED in the operation planning related with: a) occupation rate, b) confidence level and c) order of the patients.

Let us to analyze the results obtained in the operation planning of the first list. Whit this list the objective is to obtain a desired occupation rate of 80%.

1) Using MILP, the average occupation rate obtained is 79.64% and the standard deviation of the occupation rate is 0.87. The average confidence level of not overcoming the total time is 69.15% with a minimum value of 53.43% and a standard deviation of 5.73. The number of patients delayed is 30 and the parameter $\Pi$ is equal to 134.
2) Using MIQCP the solution obtained from the point of view of the occupation rate is very similar to the solution obtained using MILP: the average occupation rate is 79.9% and the standard deviation is equal to 1.59. However, using MIQCP, the solution obtained from the point of view of the confidence level are improved respect to the obtained using MILP. The average confidence level using MIQCP (68.36%) is similar to the obtained using MILP (69.15%), but the minimum value of confidence level is improved using MIQCP (65.23%) respect the obtained using MILP (53.43%). In addition, the standard deviation of the confidence level using MIQCP (2.84) decreases respect to the MILP (5.73). In this way the solution obtained using MIQCP are safer than using MILP due to the prevention of the operation planing of working days whit low confidence levels. In this solution, the number of patients delayed is lower (26) than using MILP (30), however the parameter $\Pi$ is greater (166) than using MILP (134).
3) The solution obtained using MIQCP-P related with the occupation rate and the confidence level are very similar to the obtained using MIQCP. The average occupation rate is a little lower: from 79.9% with MIQCP to 79.26% using MIQCP-P and the average confidence level is a little greater: from 68.36% using MIQCP to 70.8% using MIQCP-P. In relation with the order of the patients, using MIQCP-P, the number of patients delayed increase until 33 and the parameter $\Pi$ increases until 210.

In Tab. V can be seen that using other waiting list (2 and 3) and with other desired occupation rate (77.9% and 75.94%), the results obtained from the point of view of occupation rate and confidence level using MILP, MIQCP and MIQCP-P, follow the same pattern explained for the first list with a desired occupation rate of 80%. However, in relation with the order of the patients, the models do not follow any pattern. Depending of the list, any model can get the solution with the patients most ordered.

So, the solution obtaining with MIQCP and MIQCP-P are safer than the solution obtaining using MILP. In addition the solution obtaining using MIQCP and MIQCP-P are very similar. However the computational time necessary to solve MIQCP-P is much lower than MIQCP.

Using a computer with an Intel Core i3 and 4 GB of memory, the computational times to scheduled 42 working day (for the 3 list) using MIQCP and MIQCP-P are shown in Tab. VI.

TABLE VI
COMPARING OF COMPUTATIONAL TIME TO SOLVE MIQCP AND MIQCP-P

| | | Computational time (sec) | |
|---|---|---|---|
| Lista | $DesOr$ | MIQC | MIQC-P |
| 1 | 80 | 438 | 68 |
| 2 | 77.9 | 4228 | 30 |
| 3 | 75.94 | 71500 | 838 |

## VI. Conclusions

By modeling and solving MILP (9) it is possible to perform surgical operations planning of elective patients with a given occupation rate of ORs and respecting as much as possible

TABLE V
COMPARING OF OPERATION PLANNING USING MILP, MIQCP AND MIQCP-P

| | INPUT PARAMETERS | | | SIMULATION RESULTS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Objectives | | Constraints | Occupation Rate | | | | Confidence | | | | Order | |
| **MODEL (List)** | $DesOr$ | **Max. Occ** | **Min** $C_l$ | **Ave.** | **Max** | **Min** | **Std.** | **Ave.** | **Max** | **Min** | **Std.** | $\Pi$ | $P_d$ |
| MILP (1) | 80 | | | 79.64 | 81.42 | 78.09 | 0.87 | 69.15 | 78.6 | 53.43 | 5.73 | 134 | 30 |
| MIQCP (1) | | ✓ | 65 | 79.90 | 83.33 | 75.47 | 1.59 | 68.36 | 80.84 | 65.23 | 2.84 | 166 | 26 |
| MIQCP-P (1) | 80 | | 65 | 79.26 | 81.42 | 76.19 | 1.2415 | 70.8 | 82.05 | 65.09 | 4.3 | 210 | 33 |
| MILP (2) | 77.9 | | | 77.58 | 80.23 | 72.85 | 1.29 | 75.79 | 98.15 | 57.93 | 7.15 | 206 | 54 |
| MIQCP (2) | | ✓ | 70 | 77.95 | 80.23 | 73.57 | 1.38 | 74.79 | 87.43 | 70.30 | 3.78 | 192 | 54 |
| MIQCP-P (2) | 77.9 | | 70 | 77.12 | 79.28 | 74.52 | 0.99 | 77.52 | 86.16 | 70.92 | 4.28 | 252 | 61 |
| MILP (3) | 75.94 | | | 75.59 | 77.38 | 71.42 | 1.09 | 79.75 | 96.02 | 60.34 | 7.56 | 171 | 42 |
| MIQCP (3) | | ✓ | 75 | 75.94 | 79.04 | 70 | 2.3 | 78.58 | 90.05 | 75.04 | 3.33 | 394 | 39 |
| MIQCP-P (3) | 75.94 | | 75 | 75.38 | 77.85 | 71.666 | 1.34 | 80.22 | 90.05 | 75.26 | 3.86 | 148 | 33 |

the order of the patients in the waiting list. However, it has been shown that high occupation rates of OR lead to unsafe scheduling from the point of view of not overcoming the total daily time. Considering the duration of the surgeries and the duration associated with the cleaning time random variables with normal pfd and using some statics concepts, MIQCP (13) has been developed. By solving this model it is possible to perform surgical operations planning of elective patients with the maximum occupation rate of the OR but at the same time ensuring a minimum confidence level of not overcoming the total daily time. Of course, this model also respects as much as possible the order of the patients in the waiting list. For the set of pathologies in the studied department and a duration of OR working day of 7 hours, several solutions using MIQCP have been obtained using different values of minimum confidence level $Cl$. For each solution, the resulting average occupation rates (Avg. Occ.) have been computed (Tab.IV) and a graph that represent Avg. Occ VS $Cl$ is showed in Fig. 3.

Finally, in order to reduce the computational time to solve MIQCP, MIQCP-P (14) has been developed. This model try to obtain a given occupation rate of OR $DesOr$ ensuring a minimum confidence level $Cl$ of not overcoming the total daily time and respecting as much as possible the order of the patients in the waiting list. From the graph generated by using MIQCP we obtain the input parameters $DesOr$ and $Cl$ to solve MIQCP-P with a much smaller computational time (Tab. V) than using MIQCP but with a very similar results.

## REFERENCES


[1] B. Cardoen, E. Demeulemeester, and J. Belien, "Operating room planning and scheduling: A literature review," *European Journal of Operational Research*, vol. 201, no. 3, pp. 921 – 932, 2010.

[2] M. Lamiri, X. Xie, A. Dolgui, and F. Grimaud, "A stochastic model for operating room planning with elective and emergency demand for surgery," *European Journal of Operational Research*, vol. 185, no. 3, pp. 1026–1037, 2008.

[3] M. Lamiri, X. Xie, and S. Zhanga, "Column generation approach to operating theater planning with elective and emergency patientsy," *IIE Transactions*, vol. 40, no. 9, pp. 838–852, 2008.

[4] E. Lanzarone, A. Matta, and G. Scaccabarozzi, "A patient stochastic model to support human resource planning in home care," *Production Planning & Control*, vol. 21, no. 1, pp. 3–25, 2010.

[5] E. Lanzarone, A. Matta, and E. Sahin, "Operations management applied to home care services: The problem of assigning human resources to patients," *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, vol. 42, no. 6, pp. 1346–1363, 2012.

[6] G. Amodio, M.-P. Fanti, L. Martino, A. Mangini, and W. Ukovich, "A Petri Net Model for Performance Evaluation and Management of an Emergency Cardiology Departments," in *Proc. of the XXXV ORHAS conference*, Leuven, Belgium, 2009.

[7] M. Dotoli, M.-P. Fanti, A. Mangini, and W. Ukovich, "A continuous Petri net model for the management and design of emergency cardiology departments," in *ADHS'2019: 3rd IFAC Conference on Analysis and Design of Hybrid Systems*, Zaragoza, Spain, 2009.

[8] S. Bernardi, J. Albareda, J.-M. Colom, and C. Mahulea, "A model-based Approach for the Specification and Verification of Clinical Guidelines," in *First Workshop on Models and Methods for Hospital Management and Planning held in conjunction with ETFA'2014: 19th IEEE International Conference on Emerging Technologies and Factory Automation*, Barcelona, Spain, 2014.

[9] C. Mahulea, L. Mahulea, J.-M. Garcia-Soriano, and J.-M. Colom, "Petri Nets with Resources for Modeling Primary Healthcare Systems," in *ICSTCC'2014: 18th International Conference on System Theory, Control and Computing*, Sinaia, Romania, 2014.

[10] C. Walck, *Statistical distributions for experimentalists*, ser. University of Stockholm, 10 September 2007.

[11] IBM. (2016) Ibm ilog cplex optimization studio. software. [Online]. Available: http://www-01.ibm.com/software/integration/optimization/cplex-optimization-studio/

[12] J. Gearhart, K. Adair, R. Detry, J. Durfee, K.Jones, and N. Martin, "Comparison of open-source linear programming solvers," Sandia National Laboratories, Albuquerque, California, Tech. Rep. SAND2013-8847, October 2013. [Online]. Available: http://prod.sandia.gov/techlib/access-control.cgi/2013/138847.pdf

[13] E. Camacho and C. Bordons, *Model Predictive Control*, ser. Advanced Textbooks in Control and Signal Processing. Springer London, 2004.

[14] I. Jurado, J. Maestre, P. Velarde, C. Ocampo-Martinez, I. FernÃ¡ndez, B. I. Tejera, and J. del Prado, "Stock management in hospital pharmacy using chance-constrained model predictive control," *Computers in Biology and Medicine*, vol. 72, pp. 248 – 255, 2016.