A Petri Net Model for Performance Evaluation and Management of Emergency Cardiology Departments

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Abstract: The increased demand of visits in Emergency Department (ED) has saturated the capacity of EDs that require suitable tools in order to properly manage and organize the flow of work and people. This paper proposes a model to describe in a concise and detailed way the structure and the dynamics of a critical emergency department of the general hospital of Bari (Italy): an Emergency Cardiology Department (ECD). The model describes in a Petri Net framework the complete workflow and management of patients starting from their arrival to the ED to the discharge from hospital or to the admission in a cardiology department of the Chest Pain Unit. Moreover, suitable performance indices are single out in order to detect anomalies such as bottlenecks and waiting times. Some simulation studies show that the model is able to efficiently describe the ECD as well as to foresee the impact of variations in the medical staff and department facilities on the selected performances.

1 Introduction

Emergency department (ED) visits have increased over the past decade (Brewster and Felland 2004). This increased demand has saturated the capacity of EDs and has forced hospital executives to run their organizations in a more business-like manner (Belien and Demeuleneester 2008) In particular, the operational resource allocation decisions are a daily necessity for hospital functionality (Konrad et al. 2008). Typically, these decisions involve how allocate resources, dimensioning resources, i.e., determining the type and the number of beds, doctors, nurses, detecting anomalies such us bottlenecks and waiting times. In particular, such aspects are critical issues in an ED where quickness and reactivity with respect to an increased workload are crucial. Hence, researchers and practitioners investigate new approaches that can able to apply in health care systems the methodologies and tools that are straightened in different industrial fields. In particular, formal models and simulation are useful and effective instruments for capacity planning and efficiency improvement.

A hospital system may be effectively described as a Discrete Event System (DES) (Cassandras and Lafortune 2008) whose dynamics depends on the interaction of discrete events exhibiting a high degree of concurrency and parallelism. The DES model is the starting point to study the system dynamics and to perform discrete event simulations (Gunal and Pidd 2007, Kumar and Shim 2007).

Among the DES models, Petri Nets (PN) (Peterson 1981) offer significant advantages because of their twofold representation: graphical and mathematical. Hence, PN may be employed to model emergency medical services and hospitals. In the related literature, extended Petri nets are applied for modeling workflows in hospital operating theatres (Kotb and Baumgart 2005). In (S.S. Choi et al. 2005) PN are used to verify and simulate an ubiquitous RFID healthcare system. Moreover, ED are modeled and simulated by Petri nets in (Xiong, et al., 1994) and (Criswell et al., 2007) where a mixed integer linear programming is solved to determine the optimal workflow.

This paper is about a model which describes in detail the structure and the dynamics of a critical ED of the general hospital in Bari (Italy): the Emergency Cardiology Department (ECD). The model describes in a Timed Petri Net (TPN) framework the complete workflow and management of patients starting from their arrival to the ED up to their discharge or hospitalization in the Chest Pain Unit of the ECD. In particular, places with finite capacities model the medical and nursing staff as well as resources such as the available beds and medical devices. Instead, transitions describe the flow of patients and all that is needed to carry out hospital procedures. Moreover, indications of suitable performance are single out in order to detect and solve anomalies such as bottlenecks and waiting times. Some simulation studies show that the model is able to describe, efficiently, the ECD as well as to foresee the impact of variations in the medical staff and department facilities on the selected performances.

The paper is organized as follows. Section 2 recalls the basics of TPNs. Section 3 describes the ED and presents its TPN model. Section 4 presents the system validation and some simulation results. A conclusion section closes the paper.

2 Basics on Timed Petri Nets

A TPN (Peterson 1981) is a bipartite digraph described by the five-tuple TPN=(P, T, **Pre**, **Post**, **F,** RS), where P is a set of places, T is a set of transitions partitioned into the set T_I of immediate transitions (represented by bars), the set T_E of exponential transitions (represented by boxes) and the set T_D of deterministic timed transitions (represented by black boxes). Matrices **Pre** and **Post** are the pre-incidence and the post-incidence matrices, respectively, of dimension $|P|\times|T|$. Note that we use symbol $|A|$ to denote the cardinality of the generic set A. Moreover, F is a firing time vector. The firing time of transition $t_j \in T_E$ is an exponentially distributed random variable with mean $F_j=1/\lambda_j$ (i.e., the j-th element of vector **F**), where λ_j is the average firing rate of the exponential transition. Each $t_j \in T_I$ has zero firing time, i.e., $F_j=0$ and the generic transition $t_j \in T_D$ is associated with the constant firing delay $F_j=\delta_j$.

The state of a TPN is given by its current marking, which is a mapping **M**: P→N, where N is the set of non-negative integers. **M** is described by a |P|-vector and the i-th component of **M**, indicated with $M(p_i)$, represents the number of tokens in the i-th place $p_i \in P$. A TPN system $<PN,\mathbf{M_0}>$ is a TPN with initial marking $\mathbf{M_0}$.
Given a TPN and a transition $t \in T$, the following sets of places may be defined: $\bullet t=\{p \in P: \mathbf{Pre}(p,t)>0\}$, named pre-set of t; $t\bullet=\{p \in P: \mathbf{Post}(p,t)>0\}$, named post-set of t. A transition $t_j \in T$ is enabled at a marking **M** if and only if for each $p_i \in \bullet t_j$, $M(p_i) \geq \mathbf{Pre}(p_i,t_j)$. When fired, t_j produces a new marking **M'**, denoted as $\mathbf{M}[t_j>\mathbf{M'}$, where for each $p_i \in P$ it holds $M'(p_i)=M(p_i)+\mathbf{Post}(p_i,t_j)-\mathbf{Pre}(p_i,t_j)$.

Finally, we define a set RS of elements called *random switches*, which associate uniformly probability distributions to subsets of conflicting transitions.

3 Modelling the Cardiology Emergency Department

3.1 The System Description

Figure 1 shows the scheme of the workflow organization of the case study represented by the ECD of the general hospital. The workflow can be divided in two phases and the management of patients is performed according to the of the European Society of Cardiology / American College of Cardiology guidelines. In the first phase the patients arrive at the ED at random time instants. In general, the ED serves various patient categories: we consider here the patients presenting chest pain, dyspnoea, palpitations and syncope. An early anamnesis based on a physical examination and an ECG are urgently completed in the ED. The ECG is sent immediately to the ECD where it is evaluated in real time and the evaluation results are sent to the ED. If the patient is considered urgent, for instance a myocardial infarct is ensured, then he is hospitalized in a suitable department before entering the ECD. On the contrary, the patient continues his path in the ECD where it is

successively submitted to a complete cardiac examination. If by an accurate visit and evaluation of the ECG, the doctor decides that the patient is in a serious health condition, then the patient is admitted in a hospital department else he is submitted to a series of proofs in the ECD. In particular, the patient is submitted to a blood sample and an echodoppler examination so that the doctor can have a complete clinical picture.
At this point the second phase of the workflow begins and the doctor decides whether the patient can be discharged or it is necessary a follow-up of 24 hours in the Chest Pain Unit (CPU) of the ECD. During the short hospitalization of at the most 48 hours, the patient conditions may worsen and he is admitted in an hospital department. Otherwise the patient is submitted to an echo-stress examination. On the basis of such examination result, the patient may be urgently admitted in a cardiology department or he may be discharged by the ECD.

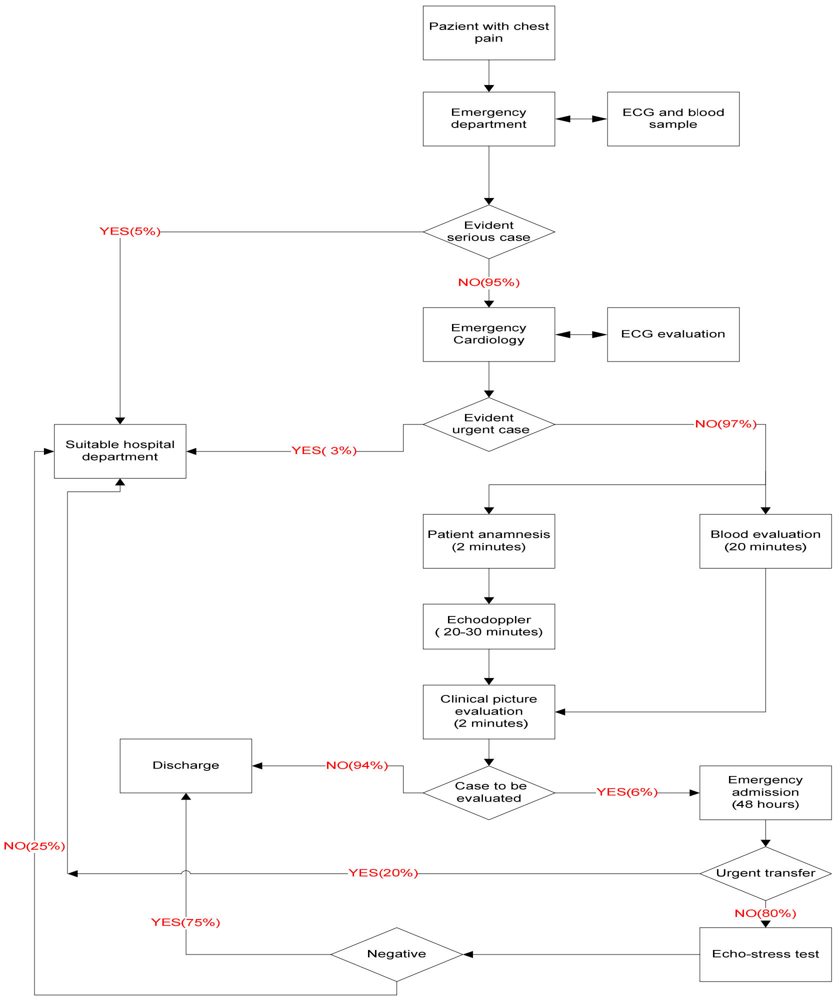

Figure 1: *The workflow organization in the EC*

3.2 The System Model by TPN

We model the hospital department system in the TPN framework, where places with finite capacities model the staff of doctors and nurses as well as the available beds and analysis devices. In addition, transitions describe the flow of patients and the actions of doctors and nurses.

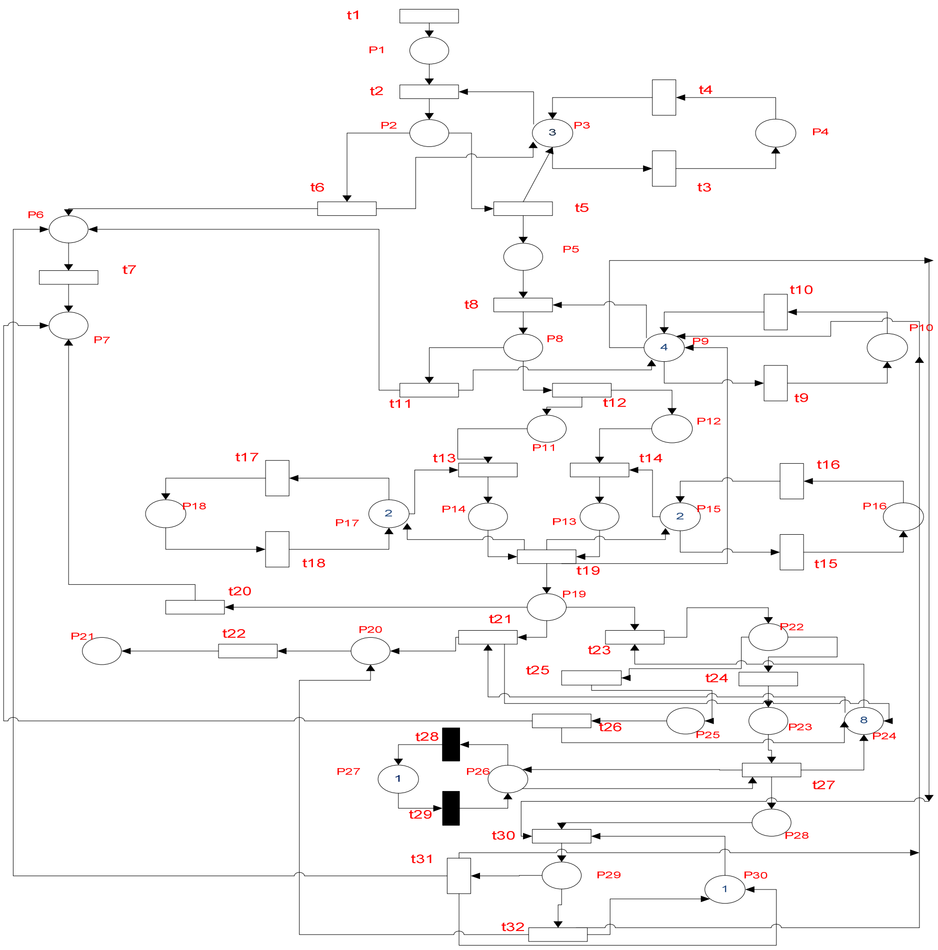

Figure 2: *The TPN model of the patient flow in the ECD*

The TPN represented in Figure 2 models the process steps of the patient flow. The places of the TPN and the corresponding meaning are reported in Table 1. In addition, Table 2 shows the transitions of the TPN, the corresponding description and the associated firing times. Moreover, Table 1 reports the random switches of the conflict transitions. In particular, marking $M(p_1)$ represents the number of patients that enter the ED. After the visit (patients in p_2) the patients (with probability 0.05) are evaluated to be in life threatening situation ($M(p_6)$), or they have to wait ($M(p_5)$) for the visit in the ECD that is represented by the timed transition t_8. This alternative situation is represented by the transitions t_6 and t_5 that are in conflict. The random switches to solve this conflict are $(p_2\ t_6)=0.05$ and $(p_2\ t_5)=0.95$ (see Table 1). After the first visit, the patients (represented by marking $M(p_8)$) that are the life threatening are hospitalized (transition t_7) with probability 0.03 or (with probability 0.97) continue their visit in the ECD (transition t_{12}). Such patients are submitted to a series of proofs: an echodoppler examination (marking $M(p_{14})$) and a blood examination (marking $M(p_{13})$). Moreover, the markings $M(p_{15})$, $M(p_{16})$, $M(p_{17})$ and $M(p_{18})$ represent the availability of the diagnostic devices (see Table 1 for the detailed description). The examination actions are expressed by the exponential transitions t_{15}, t_{16}, t_{17}, and t_{18}. Furthermore, transition t_{19} models the decision of the doctors: patients in p_{19} may be discharged (transition t_{21}) or they may be admitted in the CPU for at the most 48 hours (transition t_{23}) according to the random switches shown in Table 1. Transition t_{20} is enabled when a patient requires the hospitalization in CPU but there aren't available beds. The markings $M(p_{23})$ and $M(p_{24})$ represent the number of hospitalized patients and the available beds, respectively. Moreover during the monitoring in the

CPU, the patients may require a transfer because their health condition worsens (transition t_{25}) with probability 0.2, whereas with probability 0.8 they remain in the CPU (transition t_{24}). At the end of the hospitalization (transition t_{27}), the patients wait for a stress test (transition t_{30}) and with probability 0.25 they require further enquiries (transition t_{31}) otherwise they are discharged (transition t_{32}). The markings $M(p_{26})$ and $M(p_{27})$ are used to enable/disable the flow of patients towards the stress test room, moreover the marking $M(p_{30})$ represents the number of available effort test devices.

Place	*Description*	*Random switches*
p_1	Incoming patients	
p_2	Patients put through ECG + blood sample	$(p_2\ t_6)=0.05$ $(p_2\ t_5)=0.95$
p_3	Available doctors for patient visits in ED	
p_4	Doctors not present in the ED	
p_5	Patients that are waiting for visit in ECD	
p_6	Patients to be hospitalized	
p_7	Patient hospedalized in a hospital department	
p_8	Patients visited in ECD	$(p_8\ t_{11})=0.03$ $(p_8\ t_{12})=0.97$
p_9	Available doctors for the visit in the ECD	
p_{10}	Doctors not present in the ECD	
p_{11}	Serious patients that have to continue the visit in the ECD	
p_{12}	No serious patients	
p_{13}	Patients waiting for the anamnesys and blood analysis	
p_{14}	Patients under Echodoppler proof	
p_{15}	Number of available devices for blood analysis	
p_{16}	Number of devices for blood analysis under repair	
p_{17}	Available Echodoppler devices	
p_{18}	Ecodoppler devices under repair	
p_{19}	Patients waiting for diagnosis	$(p_{19}t_{21})=0.02$ $(p_{19}t_{23})=0.98$
p_{20}	Patients waiting for discharge	
p_{21}	Discharged patients	
p_{22}	Patients waiting for the hospitalization in the CPU	$(p_{22}\ t_{24})=0.8$ $(p_{22}\ t_{25})=0.2$
p_{23}	Patients hospitalized in the CPU	
p_{24}	Available beds in the CPU	
p_{25}	Patients that have to be hospitalized in a hospital department	
p_{26}	Enable the phase of effort test	
p_{27}	Disable the phase of effort test	
p_{28}	Patients waiting for echo-stress examination	
p_{29}	Patients under echo-stress examination	$(p_{29}\ t_{31})=0.25$ $(p_{29}\ t_{32})=0.75$
p_{30}	Available facilities for echo-stress examination	

Table 1*: Place Description*

Transition	*Description*	$F_j[h]$
t_1	Patients arrive to the ED	0.25
t_2	Visit of the ED doctor	0.08
t_3	An ED doctor is absent	2160
t_4	An ED doctor comes back to work	72
t_5	Evaluation time for non serious patient	0.03
t_6	Evaluation time for a serious patient	0.03
t_7	Delay time since the patient is hospitalized in a hospital department	0.08
t_8	Delay time since the doctor of ECD visits the patient	0.03
t_9	An ECD doctor is absent	2160
t_{10}	An ECD doctor comes back to work	72
t_{11}	A serious patient is hospitalized in a hospital department	0.03
t_{12}	A non serious patient has to do the echodoppler proof	0.03
t_{13}	Echodoppler examination	0.20
t_{14}	Anamnesys and blood analysis evaluation	0.16
t_{15}	Failure of the analysis device	1440
t_{16}	Repair of the analysis device	48
t_{17}	Failure of an echodoppler facility	2160
t_{18}	Repair of an echodoppler facility	72
t_{19}	Diagnosis	0.03
t_{20}	The patient is hospitalized in a hospital department	0.08
t_{21}	Patient has to be discharged	0.016
t_{22}	Patient Discharge	0.08
t_{23}	Patient has to be hospitalized	0.016
t_{24}	Actual time to hospitalize the patient	0.16
t_{25}	Effective time to organize the transfer of the patient	0.16
t_{26}	Range of time in which the patient needs transfer during the hospitalization	15
t_{27}	End of the hospitalization - decision to submit the patient to the effort test	0.03
t_{28}	Test disabled	0.07
t_{29}	Test enabled	11.93
t_{30}	Delay time for Physical effort test	0.58
t_{31}	The patient is transferred to a hospital department	0.08
t_{32}	The patient is discharged	0.05

Table 2: *Transition Description*

4 Simulation of the ECD Workflow

4.1 Simulation Specification and Validation

The system dynamics is analyzed via numerical simulation using the data reported in Table 2 that shows the average firing delays of stochastic transitions and the constant firing delay of deterministic transitions. In order to analyze the system behaviour, the following performance indices are selected:

- *APa*: the average number of patients that are waiting for visit in the ECD per hour;
- *LT*: the average lead time i.e., the time spent by patients waiting for visit in ECD;
- *APo*: the average number of patient hospitalized in the CPU per hour.

The TPN model of the case study is simulated in the MATLAB environment (The Mathworks 2006): such a matrix-based engineering software appears particularly appropriate for simulating the dynamics of TPN based on the matrix formulation of the marking update, as well as to describe and simulate PN systems with a large number of places and transitions.
The system is studied in five different scenarios that are specified in Table 3. In particular, Case C1 represents the real case and Table 3 reports the real value of the available resources: 4 doctors in each work shift, 2 echodoppler devices and 8 beds in the CPU. On the other hands, the remaining cases are obtained by varying the number of doctors (cases C2, C3 and C5), the number of available echodoppler devices (case C3) and the number of beds (case C4).

In each case the system is simulated by a long simulation run of 4000 time units, so that the total run time equals 1 year, if we associate one time unit to one hour. The performance indices estimates are deduced by 100 independent replications with a 95% confidence interval.

Scenarios	*Doctors*	*Echodoppler devices*	*Beds*
C1 (real)	4	2	8
C2	5	2	8
C3	5	3	8
C4	4	2	9
C5	3	2	8

***Table 3**: Simulation data in the five cases*

The model is validated by simulating case C1 and applying the well known *single mean test* (Law 2007). More precisely, we compare the value of A*Pa* estimated by the simulation with the real value of patients that are waiting for visit in the ECD per hours (denoted *RAPa*). By means of 30 real independent observations, we obtain *RAPa*=3.44 patients/h. The 100 replications of the simulation provide the value *APa*=3.42 patients/h with an half width of the confidence interval equal to ρ=0.03.
Since it holds:

$$APa - \rho \leq RAPa \leq APa + \rho. \quad (1)$$

Hence, the relation (1) validates the system model and confirms the sufficient accuracy of the performance estimation indices.

4.2 Simulation Results and Discussion

The simulation results are depicted in Figs. 3, 4 and 5 that show the values of *APa*, *LT* and *APo* in the different cases, respectively. In the scenario C2 we assess the impact of the number of doctors on the performance indices. By the Figs. 3, 4 and 5, it is apparent that the average number of patients waiting for visit and the average lead time decrease. However, increasing the number of available doctors and the number of echodoppler devices (scenario C3) *APa* decreases of the 48% and *LT* decreases of the 40% with respect to case C1. Moreover, the results show that the value *APo* is slightly affected by the number of the doctors and of the echodoppler devices. In the scenario C4 we assume that just the available number of beds increases. The simulation results point out that this choice does not affect the values of *APa* and *LT* with respect to scenario C1. On the other hand, only the value of *APo* slightly increases showing that the number of beds is well dimensioned in the real case C1.

Finally, we consider scenario C5 where the number of available doctors decreases and, as expected, the performances of the system considerably worsen.

Summing up, the simulation results help in determining the bottleneck of the ECD represented by the stage of the echodoppler proof. Indeed, increasing the number of the doctors is not sufficient to improve the department management. While the selected performance indices reach optimal values if the staff of doctors and the number of echodoppler devices increase of one.

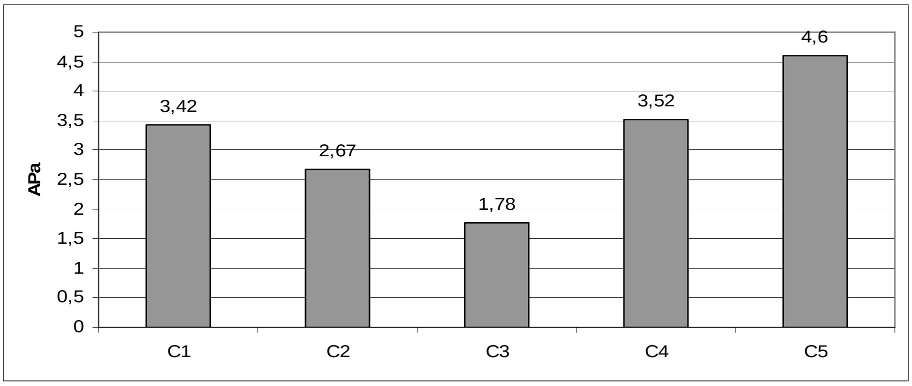

Figure 3: *The average number of patients waiting for visit in the ECD per hour in each scenario.*

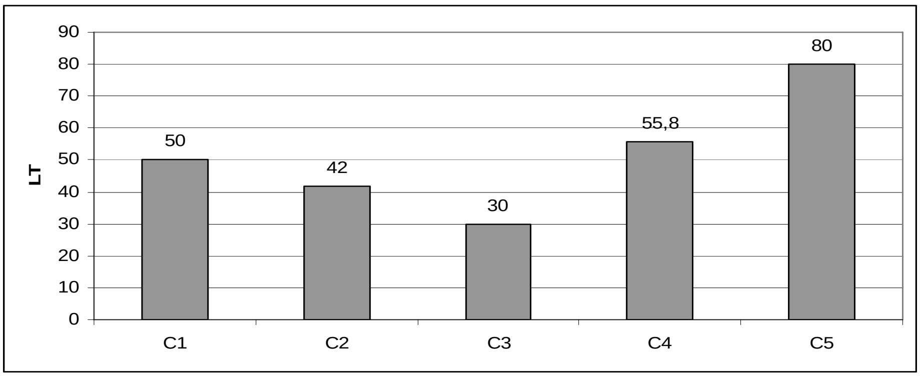

Figure 4: *The average lead time (minutes) in each scenario.*

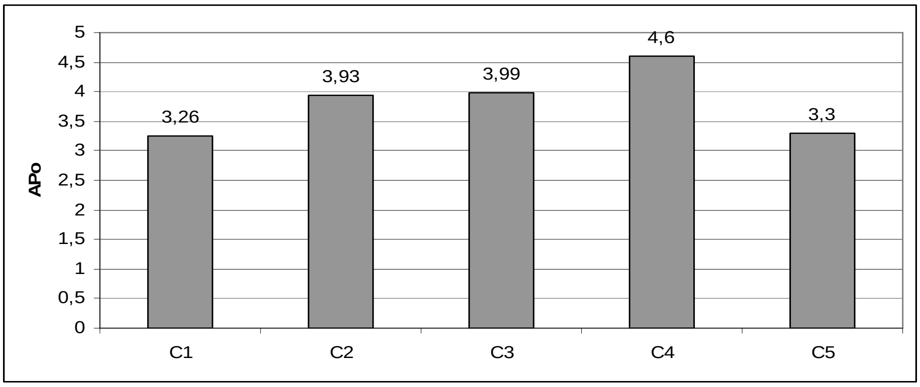

Figure 5: *The average number of patients hospitalized in the CPU per hour in each scenario.*

5 Conclusion

The paper proposes a timed Petri net model for analyzing and simulating the workflow in the Emergency Cardiology Department of the general hospital of Bari (Italy), starting from the arrival of patients to the Emergency Department to their discharge or admission in a suitable hospital department. The model allows us to define and evaluate suitable performance indices in order to determine the optimal value of key hospital parameters. In particular, we consider the planning of the optimal number of beds, doctors and inspection devices that are able to guarantee efficiency and minimize the patient waiting times. After the simulation validation on the basis of the observed data, the proposed model is utilized to determine the bottleneck of the department workflow and to provide the solution that improves the system performances. Simulation results show that the system management improves if the staff of doctors and the number of echodoppler devices increase of one.

Future research will propose suitable models in order to obtain solutions by using formal optimization methods.

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