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A STOCHASTIC MODEL FOR OPERATING ROOMS PLANNING WITH ELECTIVE AND EMERGENCY SURGERY DEMANDS

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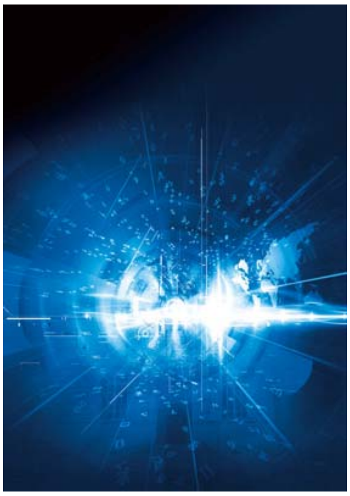

MINISTÈRE DE L'ÉCONOMIE
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A stochastic model for operating rooms planning with elective and emergency surgery demands

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Abstract: This paper proposes a stochastic model for Operating Rooms (ORs) planning with two types of surgery demands: elective surgery and emergency surgery. Elective cases can be planned starting from an earliest date with a patient related cost depending on the surgery date. Emergency cases arrive randomly and have to be performed on the day of arrival. The planning problem consists of assigning elective cases to different periods over a planning horizon in order to minimize the sum of elective patients related costs and overtime costs of operating rooms. A new stochastic mathematical programming model is proposed. More specific, we propose a Monte Carlo optimisation method combining Monte Carlo simulation and Mixed Integer Programming. The solution of this method is proved to converge to a real optimum as the computation budget increases. The optimization method is implemented and numerical results are presented.

Key words: Operating rooms, Surgery planning, Emergency, Stochastic programming, Monte Carlo optimization

1. Introduction

Facing ever increasing health care demands, limited government support and increasing competition, hospitals are more and more aware of the need of using their resources as efficiently as possible. Operating rooms (ORs) are among the most critical resources that generate most costs of a hospital. For these reasons, planning and scheduling ORs' activities has become one of the major priorities of hospitals.

In this paper, we focus on the planning of elective surgery when the ORs' capacity is shared between two competing patient classes: emergency patients and elective patients. These two patients groups have different characteristics. Emergency cases arrive randomly and must be served immediately on the same day. Electives cases can be delayed and planned for future dates.

The planning of surgical activities in ORs has been extensively addressed over the past three decades. Magerlein and Martin (1978) presented a review of surgical suite scheduling and discussed procedures for planning patients in advance of their surgical dates and techniques for the assignment of patients to operating rooms at specific times of a day. Dexter et al. (1999a, 1999b) used on-line and off-line bin-packing techniques to plan elective cases and evaluated their performances using simulation. Marcon et al. (2003) presented a tool to assist in the planning negotiation between the different actors of the surgical suite. Linear programming models were proposed for the planning and the scheduling of ORs’ activities (Jebali et al., 2005; Guinet and Chaabane, 2003). A column generation approach was proposed in Fei et al. (2004) to plan elective surgeries in identical ORs. Ozkarahan (2000) proposed a goal-programming model to allocate surgeries to ORs.

Though a substantial work on planning ORs has appeared in the literature, most of these papers assume that the total ORs capacity is devoted to a single patient class. One exception is the work of Gerchak et al. (1996), which addressed the problem of reservation planning for elective patients when the capacity is shared between elective and emergency surgeries. The focus of their work is on the characterization of the optimal policy that determines at the start of each day how many additional requests for elective surgery to assign for that day. Though similar to our problem, their model is mono-period and does not specify the intervention date for each elective case.

The goal of this paper is to develop an optimization model and algorithms for elective surgery planning in ORs with uncertain demand for emergency surgery. The problem consists of determining a plan that specifies the set of elective cases that would be performed in each period over a planning horizon (one or two weeks). The surgery plan should minimize costs related to the over-utilization of ORs and costs related to performing elective surgeries.

The remainder of this paper is organized as follows. Section 2 presents the planning model, proposes a stochastic programming model of the problem and investigates its complexity.

Section 3 proposes a Monte Carlo optimization method combining the Monte Carlo simulation and Mixed Integer Programming. Numerical results of the optimization method are presented in Section 4. Section 5 concludes the paper and discusses possible extensions of this work.

2. Problem setting

2.1. A stochastic programming model

This work concerns the planning of elective surgery at a hospital surgical suite over a planning horizon H. The surgical suite capacity is shared among two competing patient classes: elective cases, that are to be planned in advance; and emergency cases, that must be served on the day of arrival.

At the beginning of the horizon, there are N requests for elective surgery. Each elective case i (i=1..N) has the following characteristics:

- p_i, time needed for performing elective case *i* that we call operating time, which include surgery time, set-up time, cleaning, etc,
- B_i, release period.

Accurate estimates of operating times are necessary to have an efficient ORs planning. Shukla et al. (1990) recommend using historical information to estimate the operating time of elective cases. Zhou and Dexter (1998) advocate the use of log-normal distributions to approximate surgery durations. Surgeons and ORs managers can also provide good estimations of operating times. In this work, we assume that operating times of all elective cases are known and deterministic.

The release period B_i (i=1..N) is the earliest period that elective case *i* can be performed, it may represent hospitalization date, date of medical test delivery, etc.

For each elective case *i* we define a set of costs CE_{it} ($t = B_i,...H, H+1$). The CE_{it} represents the cost of performing elective case *i* in period *t*. The period $H+1$ is added to the planning horizon for cases that are rejected from the current planning horizon and that will be considered in the next horizon. The cost structure proposed in this paper is fairly general. It can represent hospitalization costs (Jebali et al., 2005; Guinet and Chaabane, 2003), penalties for waiting to get on schedule (Gerchak et al., 1996), optimal surgery date, patients' or surgeons' preferences, and eventual deadlines. For example, if case i must be performed before the period L_i, this constraint can be taken in account by choosing large costs CE_{it} for $t > L_i$.

At the planning level, we are interested in determining a plan that specifies the set of elective cases to be performed in each period over the planning horizon. The assignment to specific OR and starting time of each case can be made at a later stage on a period-to-period base (Weiss, 1990; Denton and Gupta 2003). We assumed that ORs are identically equipped, each surgical case can be assigned to any OR, and only the total available capacity of all ORs is taken into account. However, the planning model and the optimization method can be easily extended to plan different types of ORs and to take into account OR assignment of surgical cases.

Let T_t be the total ORs' available regular capacity in period t. If planned elective cases and emergency cases exceed this regular capacity, overtime costs are incurred. Let CO_t be the cost per unit of overtime in period t. While under-utilization costs are not considered in this paper, the results can be easily extended to take into account both costs; this extension is presented in the next section.

Emergency cases arrive randomly and must be served immediately on the day of their arrival. Equivalently, emergency cases arrived in a time period are performed in the same period whatever the available capacity. Let W_t be the capacity, i.e. total OR time, needed for emergency cases arriving in period t. It is assumed to be a random variable. Let $f_{W_t}(x)$, be the density function of W_t.

Notation

H: Planning horizon,

$t = 1, 2, \ldots H$: Time period index,

N: Number of elective cases,

$i = 1, \ldots, N$: Elective case index,

p_i : Time needed for performing elective case i which is assume to be a given constant,

B_i : Earliest period for performing case i,

CE_{it} : Cost of performing elective case i in period t for $t = B_i, B_{i+1}, \ldots, B_{H+1}$,

T_t : Total available regular capacity of all ORs in period t,

CO_t : Cost per unit of overtime in period t,

W_t : Capacity needed for emergency cases of period t,

$f_{W_t}(x)$: The density function of W_t.

Decision variables:

$X_{it} \in \{0,1\}$ with $X_{it} = 1$ if elective case i is performed in period t and 0 otherwise with the obvious convention that $X_{i,H+1} = 1$ implies that elective case i is rejected in the current planning horizon.

Mathematical model

$$\text{(P)} \qquad J^* = \text{Minimize } J\left(X\right) = \sum_{i=1}^{N}\sum_{t=B_i}^{H+1} CE_{it}\ X_{it} + \sum_{t=1}^{H} CO_t\ O_t \qquad (1)$$

Subject to:

$$O_t = E_{W_t}\left[\left(W_t + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^+\right], \forall t = 1, \ldots, H \qquad (2)$$

$$\sum_{t=B_i}^{H+1} X_{it} = 1, \ \forall\, i = 1,\ldots,N \qquad (3)$$

$$X_{it} = \{0,1\}, \ \forall\, i = 1,\ldots,N, \ \ \forall\, t = 1,\ldots,H+1 \qquad (4)$$

where $(x)^+ = \max\{0, x\}$.

The objective function (1) minimizes the expected overtime costs as well as elective cases related costs (waiting time costs, hospitalization costs, etc). Constraints (2) estimate the expected overtime O_t in each period. Constraints (3) ensure that each elective case is assigned once and only once. Constraints (4) are the integrity constraints.

The elective cases planning model (1)-(4) is a stochastic combinatorial problem and its NP-hardness will be established.

Remark: As we have mentioned in the previous section, our planning model can be easily extended to take into account both overtime costs and under-utilization costs. Let CU_t be the cost per unit of under-utilization in period t. To take into account both over-utilization and under-utilization costs, $CO_t\ O_t$ in the objective function (1) is replaced by COR_t, the utilization cost of period *t*, and we replace constraints (2) by the following ones:

$$COR_t = E_{W_t}\left[CO_t\left(W_t + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^+ + CU_t\left(W_t + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^-\right] \ , \ \forall t=1,..,H$$

where $(x)^- = \max\{0, -x\}$.

The solution method proposed in this paper could be easily adapted to take into account this new cost structure.

2.2. Plan evaluation

This subsection addresses the evaluation of a given plan *X*, i.e. the computation of $J(X)$. The computation of $J(X)$ can be carried out analytically or with simulation. Let TP_t be the total planned time in period *t* for elective cases, *i.e.* $TP_t = \sum_{i=1}^{N} p_i X_{it}$, and $RT_t = T_t - TP_t$ the remaining regular capacity. To compute $J(X)$, we can use either of the following methods.

<u>Analytical method:</u>

For each period *t*, the expected overtime O_t is determined exactly based on the distribution of W_t with the following analytical expression.

$$O_t = E_{W_t}\left[\left(W_t - RT_t\right)^+\right] = \int_{RT_t}^{\infty} \left(w - RT_t\right) f_{W_t}\left(w\right) dw$$

$J(X)$ is computed using the expression (1) by replacing O_t s by their expectations.

<u>Monte Carlo Simulation:</u>

Monte Carlo simulation is the second alternative that we use to compute the total cost $J(X)$. This method is implemented with the following algorithm.

Monte Carlo simulation algorithm for plan evaluation:

Step 1. Randomly generate *K* samples of emergency capacity requirement for period *t*: $W_{t1}, W_{t2}, \ldots, W_{tk}$.

Step 2. Compute $\hat{O}_{tK} = \dfrac{\sum_{k=1}^{K}\left(W_{tk} - RT_t\right)^+}{K}$

Step 3. Compute $J(X)$ using (1) with O_t replaced by $\hat{O}_{tK}$.

Remark: By law of large numbers, $\hat{O}_{tK}$ converges with probability 1 to O_t as K increases.

2.3. Problem complexity

This subsection establishes the NP-hardness of the planning optimization problem (P), using the 3-Partition decision problem. The 3-Partition problem, known to be strongly NP-hard, can be defined as follows (Garey and Johnson, 1979): Given a set of 3z integers $A = \{a_i\}_{1 \le i \le 3z}$ such that $\sum_{i=1}^{3z} a_i = zB$ and $a_i > B/4$, where z and B are integers, the question is whether there exists a partition of the set A into z triplets $A_1, ..., A_z$ such that $\sum_{i \in A_t} a_i = B$ for $1 \le t \le z$. A positive answer to this question implies that the 3-Partition problem has a solution.

In order to prove the NP-hardness of the stochastic planning problem, we define a polynomial transformation of the 3-Partition problem to the following decision problem: Is there a feasible plan X^* for the planning problem such that $J(X^*) = 0$?

To each instance of the 3-Partition Problem, we associate the following instance (P1) of the planning problem. The planning horizon H is set at value z, and the number of elective cases N is equal to $|A|$. For each period t, the total available regular capacity, the capacity needed for emergency cases and the cost per unit of overtime are given as follows:

$$T_t = B, \; W_t = 0 \text{ and } CO_t = 1, \; \forall \; t \in \{1, ..., H\}.$$

The cost structure of each elective case i ($i = 1, ..., |A|$) is the following one:

$$CE_{it} = 0, \; \forall \; t \in \{1, ..., H\} \text{ and } CE_{i(H+1)} = 1.$$

The release period and the time needed for performing elective case i are respectively equal to $B_i = 1$ and $p_i = a_i$, $\forall i$ for $i = 1, ..., |A|$.

We show that, for the planning problem (P1), a plan X^* such that $J(X^*) = 0$ exists if and only if the related 3-Partition problem has a solution.

Theorem 1. The stochastic planning problem is strongly NP-hard.

Proof. We polynomially transform the 3-Partition problem into the following decision problem: Is there a feasible plan X^* for the planning problem (P1) such that $J(X^*)=0$? Note that $H = z$.

Sufficiency: Assume that the 3-Partition problem has a solution $A = \bigcup_{1 \le t \le z} A_t$ and $\sum_{i \in A_t} a_i = B$ for $1 \le t \le z$. Consider the following plan X^*: cases in subset A_t are planned for period t, $\forall t \in \{1,...z\}$. Since $\sum_{i \in A_t} a_i = B$ ($\forall t \in \{1,...z\}$), it is obvious that the total cost of the plan is $TC(X^*) = 0$. So we have a feasible plan X^* which satisfies the constraint $J(X^*) = 0$.

Necessity: assume that a feasible plan X^* such that $J(X^*) = 0$ exists. Let A_t the set of cases planned for period t ($\forall t \in \{1,...z\}$). Since $a_i > B/4$ for all i, each set A_t contains exactly 3 elements. Since $J(X^*) = 0$, we have $A = \bigcup_{1 \le t \le z} A_t$ and $\sum_{i \in A_t} a_i = B$ for $1 \le t \le z$, which imply that the 3-Partition problem has a solution. Q.E.D.

Having established the strong NP-hardness of the stochastic planning problem, we shall prove that the NP-hardness remains true even for a fixed planning horizon H ($H = 2$). The 2-Partition decision problem is used for this purpose.

The NP-hard 2-Partition problem can be defined as follows (Garey and Johnson, 1979): Given a set of integers $\{a_i\}_{i \in A}$ such that $\sum_{i \in A} a_i = 2B$, the question is whether there exists a partition of the set A into 2 subsets A_1 and A_2 such that $\sum_{i \in A_1} a_i = \sum_{i \in A_2} a_i = B$.

Theorem 2. The two periods stochastic planning problem i.e. $H=2$ is NP-hard.

Proof. The proof of this theorem is based on the polynomial transformation of the 2-Partition problem to the following problem (P2) (instance of the planning problem (P) with a two periods horizon). The problem instance (P2) is the same as the problem instance (P1) except that $H = 2$ and $N = |A|$. More precisely, (P2) can be derived from the set A of the 2-Partition problem as (P1) was derived from the set A of the 3-Partition problem. That is

Instance (P2): Given $H = 2$, we set $T_t = B$, $W_t = 0$ and $CO_t = 1$, $\forall\ t \in \{1,...H\}$.

The number of elective cases N is equal to $|A|$. For each elective case i ($i = 1...N$), the operating time and the release period are respectively equal to $p_i = a_i$ and $B_i = 1$. The related costs of performing elective case i ($i = 1...N$), are given as follows:

$$CE_{it} = 0,\ \forall t \in \{1,...H\} \text{ and } CE_{i(H+1)} = 1.$$

Decision problem: Is there a plan X^* for the planning problem (P2) such that $J(X^*)=0$?

Sufficiency and necessity conditions: It is obvious that a plan for which $J(X^*)=0$ exists for the proposed instance if and only if the 2-Partition problem has a solution. These conditions can be proved using the same arguments as in the proof of theorem 1. Hence the two period planning problem is NP-hard. Q.E.D.

To sum up, the stochastic planning problem is strongly NP-hard, and the NP-hardness remains true even for the two periods problem. In the following section we propose a solution method.

3. Monte Carlo simulation and Mixed Integer Programming

In this section we present the Monte Carlo optimisation method. The basic idea of this method is the use of the Monte Carlo approach to formulate the planning problem as a mixed integer program (P').

$$\text{(P')} \qquad J^*_{W,K} = \text{Minimize } J_{W,K}(X) = \sum_{i=1}^{N}\sum_{t=B_i}^{H+1} CE_{it}\ X_{it} + \sum_{t=1}^{H} CO_t\ O_t \qquad (1')$$

Subject to:

$$O_{tk} \geq W_{tk} + \sum_{i=1}^{N} p_i X_{it} - T_t,\ \forall\, t=1,..,H,\ \forall\, k=1,...,K \qquad (2')$$

$$O_t = \frac{\sum_{k=1}^{K} OT_{tk}}{K},\ \forall\, t=1,..,H \qquad (3')$$

$$\sum_{t=B_i}^{H+1} X_{it} = 1,\ \forall\, i=1,...,N \qquad (4')$$

$$\text{(Binary variables) } X_{it} = \{0,1\},\ \forall\, i = 1,\dots,N,\ \forall\, t = 1,\dots,H+1 \qquad (5')$$

$$\text{(Real variables) } O_{tk} \geq 0,\ \forall\, t=1,..,H,\ \forall\, k=1,...,K \qquad (6')$$

where $W_{t1}, W_{t2}, \dots, W_{tk}$ are independent random samples of W_t.

The objective function (1') minimizes the sum of patient related costs and overtime costs using K samples of W_t ($\forall\, t$=1,..,H). Constraints (2') define the overtimes O_{tk} related to each sample or scenario W_{tk} in each period. Constraints (3') estimate the required overtime O_t in each period. Constraints (4') ensure that each elective case is assigned exactly once. Constraints (5') and (6') are variables' type related constraints.

Remark: By optimisation, for any optimal solution of problem (P'), $O_{tk} = \left(W_{tk} + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^+$ and $O_t = \hat{O}_{tK}$ where $\hat{O}_{tK}$ was defined in section 2.2. The variable O_t is exactly the overtime $\hat{O}_{tK}$ estimated by the Monte Carlo simulation method of the previous section. Further, variables O_t can be easily removed from the formulation by replacing it directly into criterion (1').

The planning optimisation can be carried out with the following Algorithm:

Step 1. Generate for each time period K samples of W_t .

Step 2. Solve the mixed integer program (P') and let $X^*_{W,K}$ be the resulting optimal solution.

Step 3. Evaluate the exact cost of the solution (the plan) $J\left(X^*_{W,K}\right)$, using one of the methods presented in section 2.2.

Note that $J_{W,K}(X)$ and $X^*_{W,K}$ are both functions of K and W, i.e. the number of samples and the random sequence of emergency capacity requirement. For simplicity, we will omit the W index in the rest of this paper, and denote $J_{W,K}(X)$ (respectively $X^*_{W,K}$) by $J_K(X)$ (respectively X^*_K).

In the following, we prove that the optimal solution of problem (P') converges to an optimal solution of problem (P), as K increases. For this purpose, we use an equivalent formulation (P'') of problem (P'):

(P") $$J_K^* = \text{Minimize } J_K(X) = \sum_{i=1}^{N}\sum_{t=B_i}^{H+1} CE_{it}\ X_{it} + \sum_{t=1}^{H} CO_t\ O_{t,K} \quad (1")$$

Subject to:

$$O_{t,K} = \frac{\sum_{k=1}^{K}\left(W_{tk} + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^+}{K}, \ \forall\, t=1,..,H \quad (2")$$

$$\sum_{t=B_i}^{H+1} X_{it} = 1, \ \forall\, i=1,...,N \quad (3")$$

$$\text{(Binary variables) } X_{it} = \{0,1\}, \ \forall\, i=1,\ldots,N, \ \ \forall\, t=1,\ldots,H+1 \quad (4")$$

where $W_{t1}, W_{t2}, \ldots, W_{tk}$ are independent random samples of W_t.

It can be easily proved that problem (P") is exactly equivalent to (P'). We are now ready to prove the convergence of (P') related optimal solution to an optimal solution of problem (P).

Theorem 3. With probability 1, as K goes to infinity, $J_K(X_K^*)$ converges to $J(X^*)$ and the optimal solution X_K^* of problem (P') converges to an optimal solution X^* of problem (P).

Proof. Let variables $O_t(X)$ and $O_{t,K}(X)$ be respectively the actual and the estimated overtime incurred in period t if solution X is chosen. Then we have

$$O_t(X) = E_{W_t}\left[\left(W_t + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^+\right] \text{ and } O_{t,K}(X) = \frac{\sum_{k=1}^{K}\left(\left(W_{tk} + \sum_{i=1}^{N} p_i X_{it} - T_t\right)^+\right)}{K}$$

Since $\left(W_{tk}+\sum_{i=1}^{N} p_i X_{it} - T_t\right)^+$ are i.i.d samples of random variable $\left(W_t+\sum_{i=1}^{N} p_i X_{it} - T_t\right)^+$, by law of large numbers, $O_{t,K}(X)$ converges with probability 1 to $O_t(X)$ as K increases. As a result, for any $\varepsilon \geq 0$, there exists an integer $\overline{K}_{t,X} > 0$ such that

$$\left| O_{t,K}(X) - O_t(X) \right| < \varepsilon \qquad \forall K \geq \overline{K}_{t,X}.$$

Let $\overline{K} = \max_{t,X} \overline{K}_{t,X}$. From (1") and (2"),

$$\left| J_K(X) - J(X) \right| \leq \sum_{t=1}^{H} CO_t\ \varepsilon, \ \forall K \geq \overline{K}. \quad (5)$$

For every solution X, the estimated cost converges with probability 1 to the expected (exact) cost as K increases. Hence,

$$\left| J_K\left(X_K^*\right) - J\left(X_K^*\right) \right| \leq \sum_{t=1}^{H} CO_t \ \varepsilon, \qquad \forall K \geq \overline{K}.$$

Further, for every $K > \overline{K}$,

$$\left| J_K\left(X_K^*\right) - J_K\left(X^*\right) \right|$$

$$= J_K\left(X^*\right) - J_K\left(X_K^*\right) \text{ (From the definition of } X_K^* \text{)}$$

$$\leq J_K\left(X^*\right) - J_K\left(X_K^*\right) + J\left(X_K^*\right) - J\left(X^*\right) \text{ (From the definition of } X^* \text{)}$$

$$\leq \left| -J_K\left(X_K^*\right) + J\left(X_K^*\right) \right| + \left| J_K\left(X^*\right) - J\left(X^*\right) \right|$$

$$\leq 2 \sum_{t=1}^{H} CO_t \ \varepsilon \qquad (6)$$

As a result,

$$\left| J_K\left(X_K^*\right) - J\left(X^*\right) \right|$$
$$= \left| J_K\left(X_K^*\right) - J_K\left(X^*\right) + J_K\left(X^*\right) - J\left(X^*\right) \right|$$
$$\leq \left| J_K\left(X_K^*\right) - J_K\left(X^*\right) \right| + \left| J_K\left(X^*\right) - J\left(X^*\right) \right|$$

$$\leq 3 \sum_{t=1}^{H} CO_t \ \varepsilon \text{ (From (5) and (6))}$$

Further,

$$\left| J\left(X_K^*\right) - J\left(X^*\right) \right|$$

$$= J\left(X_K^*\right) - J\left(X^*\right) \text{ (From the definition of } X^* \text{)}$$

$$\leq J\left(X_K^*\right) - J\left(X^*\right) - J_K\left(X_K^*\right) + J_K\left(X^*\right) \text{ (From the definition of } X_K^* \text{)}$$

$$\leq \left| J\left(X_K^*\right) - J_K\left(X_K^*\right) \right| + \left| -J\left(X^*\right) + J_K\left(X^*\right) \right|$$

$$\leq 2 \sum_{t=1}^{H} CO_t \ \varepsilon \quad \text{(From (5))}$$

To summarize, for all ε > 0, there exist a finite integer $\overline{K}$ such that for every $K > \overline{K}$,

$$\left| J_K\left(X_K^*\right) - J\left(X^*\right) \right| \leq 3 \sum_{t=1}^{H} CO_t \ \varepsilon , \tag{7}$$

$$\left| J\left(X_K^*\right) - J\left(X^*\right) \right| \leq 2 \sum_{t=1}^{H} CO_t \ \varepsilon . \tag{8}$$

Inequality (7) implies that as *K* increases, the estimated cost $J_K\left(X_K^*\right)$ of the resulting optimal solution converges with probability 1 to the exact optimal cost $J\left(X^*\right)$. While inequality (8) implies that the exact cost of the resulting optimal solution converges with probability 1 to the exact optimal cost, as *K* increases. Hence X_K^* converges to an optimal solution X^* of problem (P). Q.E.D.

4. Computational experiments

This section presents numerical results of the Monte Carlo optimization method. For each *K*, the related linear programming problem (P') is solved using the commercial software “ILOG CPLEX” version 8.0. The different experiments are realized on a PC with processor Pentium IV 3.0 Ghz.

The performance of our optimization method was examined by solving a given problem for various values of *K* i.e. number of samples. The different values of *K* are [2, 5, 10, 20, 50, 100, 200, 500, 700, 1000]. For each *K*, we perform 7 independent runs of the optimization method to provide 7 Monte Carlo optimum solutions X_K^*. The “exact” cost $J(X_K^*)$ of each resulting optimal solution X_K^* is estimated using the “Monte Carlo simulation algorithm”, presented in section 2.2, with large number of samples (10^6 samples), i.e $J(X_K^*) \simeq J_{K'}(X_K^*)$ with $K' = 10^6$.

4.1. Testing problem

The number of periods *H* is equal to 5 (one week). The ORs' regular capacities T_t (t=1..H) are equal to 16 hours. Assuming that an OR’s regular capacity is 8 hours, this is equivalent to have two available ORs. The overtime cost is 500 €/ hour. The daily capacity needed for emergency cases W_t (t=1..H) is assumed to be exponentially distributed with mean of 3 hours.

Durations of elective surgery p_i are randomly and uniformly generated from interval [0.5 hour, 3 hours].

For each case *i* we randomly select its release period B_i from the set $\{1...H\}$. To take into account cases with B_i = 1 that were postponed from the previous plan, we introduce a new variable $B_i^{'}$ the *effective* earliest period of case *i* (or *effective* release period). $B_i^{'}$ can take negative value. Earliest dates B_i are generated in two steps as follows. First, we generate for each case *i* the *effective* earliest period $B_i^{'}$. The $B_i^{'}$s are integer numbers randomly selected from the set {-2, …, 5}. Then, cases with zero or negative $B_i^{'}$ will have B_i equal to one, while the others will have B_i equal to $B_i^{'}$ (B_i=1 if $B_i^{'}$<1; $B_i = B_i^{'}$ otherwise).

The CE_{it} are assumed to be increasing in *t* for every *i*.

$$CE_{it} \;=\; \left(t - B_i^{'}\right) \times c \quad \text{for } t = B_i ... B_{H+1}$$

The constant *c* can be interpreted as a penalty per day of waiting to get on schedule, or as hospitalization cost per day if patient *i* was hospitalized in period $B_i^{'}$. The value of *c* is set equal to 100€.

The number of elective cases is determined such that the workload of ORs is 100% of the regular capacity of the entire planning horizon.

4.2. Numerical results

This subsection presents numerical results of a testing problem of 44 elective cases generated as explained above. For each value of *K*, the resulting optimal solutions of 7 independent runs of the Monte Carlo optimization algorithm lead to 7 Monte Carlo optimums X_K^*. The "exact" criterion value $J(X_K^*)$ of these Monte Carlo optimum solutions are evaluated with extremely long Monte Carlo simulation of 10^6 samples. For each *K*, we further derive: the minimal "exact" optimal cost, the average "exact" optimal cost, the maximal "exact" optimal cost and the standard deviation. Computational results are presented in Table 1.

Using the same runs, we provide in Table 2 (presented in appendix), for each value of K, the minimum, the average, the maximum and the standard deviation of "estimated" optimal costs ($J_K(X_K^*)$). This table presents also information concerning the computation time.

We consider also the deterministic version of the problem with W_t replaced by its expectation, i.e. with $W_t = E[W_t] = 3$ hours. The "exact" cost of the deterministic solution is evaluated as 8213€.

Default solution, i.e. planning elective cases for their releases periods ($X_{iB_i} = 1, \forall i = 1,...,N$) incurs cost equal to 17960€. It is clear that the default solution is a very bad solution.

From results presented in Table 1, we note that solutions provided by the Monte Carlo optimization method are better than those of the deterministic method, even for small values of K (K=20). Further, the proposed method achieves cost reduction of 4% with K=1000.

Number of samples	Minimal optimal cost	Average optimal cost	Maximal optimal cost	Standard deviation
2	8203,49	8861,27	9542,76	476,07
5	7925,29	8154,68	8451,78	204,20
10	7921,76	8091,69	8346,12	146,48
20	7943,49	7986,52	8067,53	49,28
50	7888,69	7925,22	7972,52	27,88
100	7885,24	7891,41	7903,17	5,63
200	7883,87	7913,75	7952,46	26,92
500	7882,26	7890,93	7903,93	8,47
700	7885,83	7892,38	7915,87	10,46
1000	7883,10	7888,91	7904,24	7,27

Table 1. Computational results

In Figure 1, we present the minimal, the average and the maximal "exact" costs of optimal solutions for each value of K. The results show that the proposed method achieves a better performance than the deterministic one, for K greater than 20. Further, it is also clear that the curves converge when the number K of samples is greater than 200, which confirms the convergence property of Theorem 3.

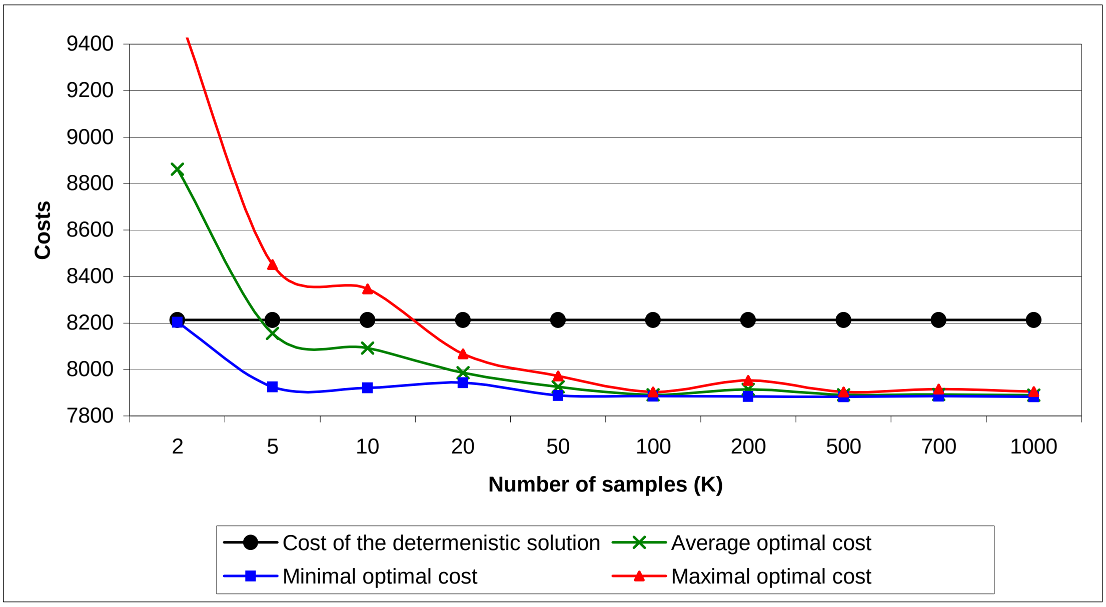

Figure 1. Costs evolution

The standard deviations of “exact” and “estimated” optimal costs, obtained from 7 independent runs for each value of *K*, are presented in Figure 2. Results show that both quantities decrease with increasing *K*, which confirm that the Monte Carlo optimal solution converges to an optimal solution of problem (P). Though the standard deviation are estimated using a small number of runs (7 runs), we think that they can be used as an indicator to show the convergence. We note that, the standard deviations of the “estimated” optimal costs are always grater than those of the “exact” optimal costs. This can be explained by the small number of samples (K) used to estimate the cost of the optimal solution $J_K(X_K^*)$.

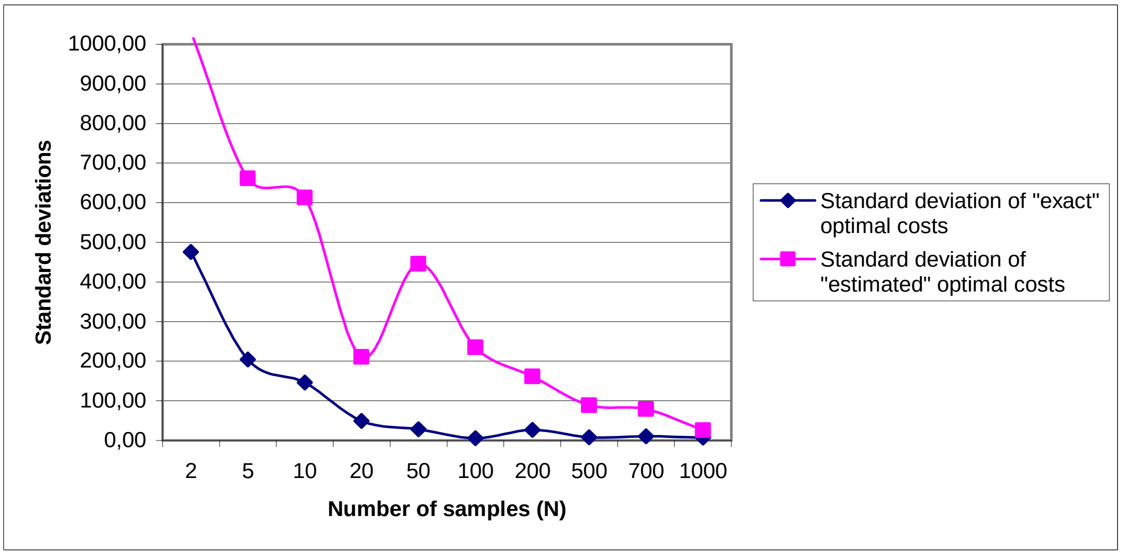

Figure 2. Standard deviations evolution

In Figure 3, we present the minimal, the average and the maximal computation time for each value of *K*. For *K* grater than 100 the computation time is increasing with *K*.

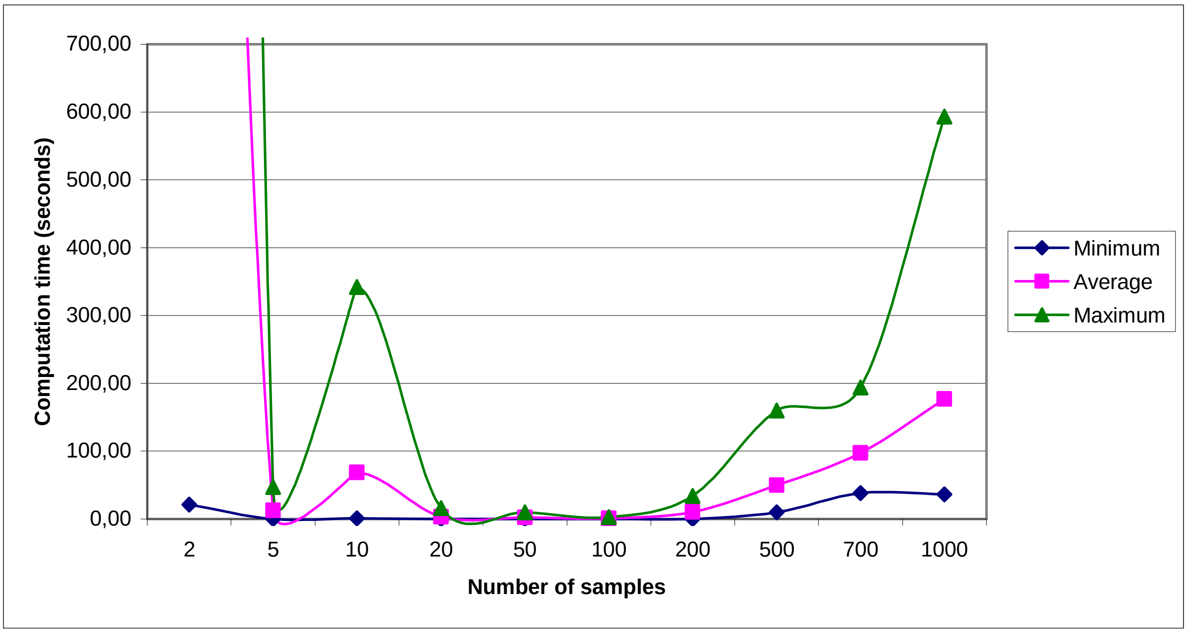

Figure 3. Computation time evolution

We have also experimented the proposed solution method with others problem configurations. Computational results confirm that our optimization method provides solution that converges to an optimal solution of the problem, as *K* increases. Considerable costs reductions can be achieved using stochastic optimisation model.

5. Conclusion

In this paper we proposed a stochastic model for ORs planning with two classes of patients: elective patients and emergency patients. Elective cases can be planned starting from an earliest date with a patient related cost depending on the surgery date. Emergency cases arrive randomly and have to be performed in the day of arrival. The planning problem consists of assigning elective cases to different periods over a planning horizon in order to minimize the sum of elective patients' related costs and overtime costs of operating rooms.

The planning model proposed in this work is useful for hospitals using a "blocked" advance scheduling system, which reserve blocs of ORs time to surgical specialities. Each speciality serving elective and emergency surgery demands can use the proposed model for the planning of the electives cases.

The elective cases planning problem has been formulated as a stochastic mathematical program. A solution method combining the Monte Carlo simulation with Mixed Integer Programming has been proposed. We have successfully proved and experimented the performance of the solution method.

The performance of the Monte Carlo optimization method was experimented by solving random instances of a surgeries planning problem. Numerical results show that important cost reductions can be achieved by this optimisation method. We are presently testing the model with data from a hospital and expect to report results in the near future.

To solve the problem of elective cases planning, we can also use a local search method. This solution method solves the problem in two steps. First, a feasible planning is determined by solving a deterministic program, which is obtained from the stochastic program by replacing the random variables (capacities needed for emergency cases) by their expected values. Second, starting from the feasible planning, we try to iteratively improve the planning by searching through its neighbourhood. The neighborhood structure can be defined in several ways. For example, we can use a neighborhood structure by exchanging the intervention dates, modifying the intervention date of an elective case, etc. At this step, other methods such as simulated annealing, taboo search and genetic algorithms can be used. Further work will include the experimentation of these methods with the elective cases planning problem.

The model proposed in this paper can be extended in several ways to take into account various characteristics such as (i) constraints of overtime capacity, (ii) assignment of patient to ORs, (iii) different types of ORs, (iv) other criteria such as probability of reject of planned cases or emergency cases due to limited overtime capacity. The improvement of the efficiency of the optimisation methods by taking into account the properties of the optimal solutions is another important direction.

Appendix

	Estimated cost				Computation time (seconds)		
Number of samples	Minimal "estimated" optimal cost	Average "estimated" optimal cost	Maximal "estimated" optimal cost	Standard deviation of "estimated" optimal costs	Minimum	Average	Maximum
2	6133,83	7145,68	8827,25	1031,33	21,00	3012,86	8956,00
5	6002,66	6897,07	7988,60	661,68	0,00	12,57	47,00
10	6005,55	7190,72	7870,92	613,48	1,00	68,57	342,00
20	7569,42	7797,23	8082,37	211,14	0,00	3,43	16,00
50	7468,89	7953,54	8770,72	446,11	0,00	2,29	10,00
100	7745,82	7943,03	8422,59	235,27	0,00	1,14	3,00
200	7697,00	7879,34	8120,84	161,70	0,00	10,57	34,00
500	7772,26	7909,08	8040,48	89,10	10,00	50,00	160,00
700	7739,33	7829,44	7963,80	79,26	38,00	97,57	194,00
1000	7829,51	7865,02	7901,29	26,09	36,00	176,86	593,00

Table 2. Computational results

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