Towards an Efficient and Economical Surgical Planning Tool

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***Abstract*—In this paper, the scheduling problem of elective patients in the Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza is considered. This problem takes into account two contradictory objectives: obtain a given occupation rate of the Operation Room (OR) and respect as much as possible the preference order of the patients in the waiting list. Three different mathematical models are discussed: 1) Quadratic Assignment Problem (QAP); 2) a Mixed Integer Linear Programing (MILP) model; and 3) Generalized Assignment Problem (GAP). These models solve combinatorial problems with a high computational cost, for this, heuristics methods have been used to solve large instances. In particular, 1) a meta-heuristic Genetic Algorithm (GA) for the QAP; 2) a heuristic Steepest Descent Multiplier Adjustment Method (SDMAM) for the GAP; and 3) a heuristic iterative method for MILP. Finally, the models and the heuristics are compared according to the occupation rate and the preference order criteria.**

I. Introduction

The OR is one of the most expensive material resources of any hospital. Approximately 60% of all patients visit an OR at some point during their stay [1]. Surgical costs typically account for approximately 40% of the hospital resource costs [2], while surgeries typically generate around 67% of hospital revenues [3]. Additionally, the demand, for surgical services is increasing due to the ageing population. So, it is necessary to have good planning and scheduling methods to increase the efficiency of the OR departments; see [4] for an overview of OR planning and scheduling. Interesting contributions have been proposed, see for example [5], [6].

In this paper, we consider the planning of non-urgent surgeries in the Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza. This can be seen as the planning of a production system: (a) there is a waiting list of patients representing the system demand and (b) there exists a limited number of surgeons and a limited number of OR representing the capacity of the production system. For our application, the bottleneck of the resources are the OR. In particular, there exists two OR for non-emergency surgeries in the Orthopedic Department, each one being available only a certain number of hours per day. Therefore, it is extremely important to obtain its maximum performance. If the maximum performance of the OR is not obtained, the staff associated with the OR do not have committed labor but they have nevertheless economic resources consumption. The purpose of this problem is to optimize the use of the OR. The different surgeries have associated averages durations, but there are uncertainties due to uncontrollable factors such as unforeseen events or the different nature of each body. For this reason, it is considered that an acceptable performance is obtained when the ORs are scheduled near 78 percent of occupancy. A defect or an excess in the occupation rate of OR with respect to the objective programming is a system defect. Defects imply the consumption of human resources without their utilization, and excess means that staff could lengthen their working day. In order to solve this problem, in [7] we propose a mathematical model based on integer programing that allows doctors to plan the surgeries with a determined occupation rate respecting, as much as possible, the order of patients in the waiting list. This model, due to its computational complexity, is solved iteratively (similar with receiding horizon control strategy [8]). In this paper, we propose other two different models of the scheduling problem solved by heuristics methods. The solutions obtained using those methods are analyzed and compared from a computational efficiency point of view and according to the quality of the scheduling. So the principal objective is to find an efficient and economical method to be used as a core in a surgical planning tool. The analysis performed in this paper are motivated by the implementation of a software tool for surgical planning to be used by the hospital. The paper is organized as follows. In Section II we formally introduce the scheduling problem, establish the evaluation criteria to compare the planning obtained and provide a motivation example. In Section III the three proposed models are described. The heuristics used to solve the models are given in Sec. IV. The computational efficiency of the different methods, as well as the quality of the scheduling, are analyzed and compared in Section V. In Section VI we provide some conclusions.

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II. Problem description

In Section II-A, we formally introduce the scheduling problem and in Section II-B, we show a motivation example of the problem.

A. Problem formulation

The scheduling problem focuses on planning elective surgeries to a given number of ORs. A set of elective surgeries is given as $Po = \{\sigma_1, ..., \sigma_n\}$ that have been ordered according to two factors: the waiting time in the queue, and the priority of the surgery. In particular, surgery $\sigma_1 \in Po$ should be preferably scheduled first while surgery $\sigma_n \in Po$ should be preferably scheduled last.

Moreover, each surgery $\sigma_i \in Po$ has an expected duration μ_i and we denote by $\boldsymbol{\mu} = [\mu_i, ..., \mu_n]^T$ the expected durations of all surgeries in the waiting list. We denote by $S = \{\boldsymbol{S}_1, ..., \boldsymbol{S}_m\}$ the set of $\boldsymbol{S}_i \in \{0,1\}^n$ $i = 1, ..., m$ column vectors representing m OR working days. In particular, the OR working day representing by the vector $\boldsymbol{S}_1$ will be performed first and OR working day that represents the vector $\boldsymbol{S}_m$ will be performed last. If $\boldsymbol{S}_i[j]$ is equal to one, then the surgery with a preference order j should be operated the OR working day i.

The daily time available for the OR is given by the set $\boldsymbol{b} = [b_1, ..., b_m]^T$ where each b_i is the total time in minutes of day i.

The objective is to schedule elective surgeries from the waiting list Po in the set of working days S achieving a certain occupancy rate of the OR and respecting as much as possible the order of the patients in the waiting list. Of course, each patient should only be assigned to one day.

In this paper, three different models with three different cost functions are proposed. So, in order to compare the different operation planning obtained with each model two important aspects will be analyzed:

1) *Occupation rate*: a static analysis of the occupation rate of the OR including: the average, the standard deviation and the extreme values. A good scheduling should have an average value of occupation rate close to the target and a small standard deviation.
2) *Order of the patients from the waiting list*. Two parameters related with the order of the patients are considered. Observing that normally 3 patients are scheduled per day, a patient with a preference order j should be operated the day $\lceil \frac{j}{3} \rceil$, where $\lceil \cdot \rceil$ is the rounding to the next integer. All patients scheduled in a working day different with respect to the working day in which they should be operated, sum the square of the numbers of days advanced or delayed to the indicator Π. In addition, the number of patients advanced or delayed (P_d) is counted. For example, if a patient with a preference order j is scheduled the day k and $k \neq \lceil \frac{j}{3} \rceil$ then,

$$\begin{cases} \Pi := \Pi + \left[k - \lceil \frac{j}{3} \rceil\right]^2 \\ P_d := P_d + 1 \end{cases} \tag{1}$$

Against lower are Pd and Π, the scheduling respect more the order of the patients in the waiting list.

B. Motivation Example

The problem particularized to the Orthopedic Department in "Lozano Blesa" Hospital has the next values:

- The surgeries $\sigma_i \in Po$ belong to the set of pathologies of the Orthopedic department.
- The total duration $b_i = 390$ for all $\boldsymbol{S}_i \in S$
- The target occupation rate is about 78 percent.

The problem to solve is as follows: Given an ordered waiting list of patients Po that should be scheduled for surgeries, the expected duration of these surgeries $\boldsymbol{\mu}$ and a duration of OR working day equal to b, schedule the next m working days of ORs with an occupancy rate equal to p, respecting as much as possible the order of the patients in the waiting list.

Let us assume the ordered list containing 22 patients given in Tab. I, 5 days to schedule (i.e., $m = 5$), a duration of the OR working day of 6.5 hours (i.e., $d = 6.5$[hours]$= 390$[minutes]) and an objective occupancy rate of 78 (i.e., $p = 0.78$).

TABLE I
AN ORDER LIST OF PATIENTS WAITING FOR SURGERY GIVEN AS $\boldsymbol{Po}$ AND $\boldsymbol{\mu}$.

Po	1	2	3	4	5	6	7	8	9
$\boldsymbol{\mu}$	97	97	81	133	45	81	121	137	74
Po	10	11	12	13	14	15	16	17	18
$\boldsymbol{\mu}$	97	150	86	94	45	81	64	97	155
Po	19	20	21	22					
$\boldsymbol{\mu}$	150	111	104	133					

Tab. II shows a possible solution of this particular problem. Each row of this table represents the operation planning of a working day. The first column represents the number of OR working day, the next three columns indicate the preference order of surgeries that should be operated in the corresponding working day. Finally, the last column is the percentage of the occupancy rate of the OR in the corresponding working day.

TABLE II
OPERATION PLANNING OF THE LIST OF PATIENTS GIVEN IN TAB. I FOR AN OBJECTIVE OCCUPATION RATE OF 78% AND A TIMING HORIZON OF 5 DAYS.

Day	Surgery 1	Surgery 2	Surgery 3	Occupancy
1	1	3	4	79,74
2	2	6	7	76,67
3	8	9	12	76,15
4	11	19	0	76,92
5	5	10	18	76,15

The scheduling showed in Tab. II has an average value of the occupation rate equal to 77.12 and a standard deviation of 1.49. Additionally, the values of the indicators Pd and π are 8 and 24 respectively.

III. MODELS PROPOSED

In this section, three different models for the proposed Scheduling Problem are discussed. These models use a similar penalty in the objective function according to the preference order of the patients. However, each model takes into account in a different way the penalty related with the deviation of the occupation rate with respect to the target. A model with a quadratic cost function (QAP) is explained in subsection III-A. The MILP model proposed in [7] is recalled in subsection III-B and the the scheduling problem modeled as a GAP is discussed in subsection III-C

A. Scheduling problem modeled by QAP

In order to respect the preference order of the patients in the surgery planning we assign a cost to each patient depending on the OR working day in which they are scheduled. This cost is given by the matrix $\boldsymbol{P}$, where the element $\boldsymbol{P}[i,j]$ represents the cost of schedule patient j in day i. The following value is used: $\boldsymbol{P}[i,j] = j \cdot (m - i + 1)$. For example, assuming $n = 9$ patients in the waiting list and $m = 3$ OR workings days, matrix $\boldsymbol{P}$ is given in (2). The first patients in the waiting list have a lower cost than the the patients with higher preference order $\boldsymbol{P}[\cdot,1] < \boldsymbol{P}[\cdot,2] < \cdots < \boldsymbol{P}[\cdot,n]$. Moreover, the first OR working days to perform have a higher cost than the last ones $\boldsymbol{P}[1,\cdot] > \boldsymbol{P}[2,\cdot] > \cdots > \boldsymbol{P}[m,\cdot]$. In this way we are giving preference to schedule the first patients of the waiting list in the first days. In this paper the following matrix notation is assumed: $\boldsymbol{P}[i,\cdot]$ represents row i of the matrix $\boldsymbol{P}$ and $\boldsymbol{P}[\cdot,i]$ represents the column i of matrix $\boldsymbol{P}$.

$$\boldsymbol{P} = \begin{bmatrix} 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix} \begin{array}{l} \to OR1 \\ \to OR2 \\ \to OR3 \end{array} \tag{2}$$

$$\boldsymbol{P}[i,j] = j \cdot (m - 1 + i) \quad \forall j \in \{1,...,n\}; i \in \{1,...,m\}$$

The cost associated with the order of the patients scheduled in the OR working day i is showed in (3)

$$\boldsymbol{P}[i,\cdot] \cdot \boldsymbol{S}_i \quad \forall i \in \{1,...,m\} \tag{3}$$

The occupation rate p of an OR is defined as the time in which a patient is within the OR divided by the total time available.

In order to obtain a given occupation rate of the OR working day, we define the following cost. Let us denote by $Obj_i = b_i \cdot p$ the target occupation in minutes of the OR i, where b_i is the daily time available in OR i and p the desired occupation rate. The penalty associated with the occupation rate of the OR working day i is defined as.

$$(\boldsymbol{\mu} \cdot \boldsymbol{S}_i - Obj_i)^2 \quad \forall i \in \{1,...,m\} \tag{4}$$

In (4) the term $\boldsymbol{\mu} \cdot \boldsymbol{S}_i$ is the sum of the durations of the surgeries scheduled in day i. So, the objective will be to minimize the square of the difference between $\boldsymbol{\mu} \cdot \boldsymbol{S_i}$ and Obj_i.

Putting together both costs ((3) and (4)), the objective is:

$$\min \quad \sum_{i=1}^{m} \left(\beta \cdot \boldsymbol{P}[i,\cdot] \cdot \boldsymbol{S_i} + (\boldsymbol{\mu} \cdot \boldsymbol{S_i} - Obj)^2\right) \tag{5}$$

Where β is a parameter to establish a compromise between the cost associated with respect to the order of the patients in the waiting list and the cost associated with the objective of obtaining a target occupation rate. The full model is showed in (6), where the set of constraints $\sum_{i=1}^{m} \boldsymbol{S_i}[j] \leq 1$ impose that each patient j is schedule no more than one.

$$\begin{array}{l} \min \quad \sum_{i=1}^{m} \left(\beta \cdot \boldsymbol{P}[i,\cdot] \cdot \boldsymbol{S_i} + (\boldsymbol{\mu} \cdot \boldsymbol{S_i} - Obj)^2\right) \\ \text{Subject to:} \\ \begin{cases} \sum_{i=1}^{m} \boldsymbol{S_i}[j] \leq 1, & j \in N = \{1,\ldots,N\}, \\ \boldsymbol{S_i}[j] \in \{0,1\}, & i \in M, j \in N, \end{cases} \end{array} \tag{6}$$

Proposition 1. *The proposed scheduling problem* (6) *is NP-hard.*

Proof. We prove this theorem by modeling it as a Quadratic Assignment Problem (QAP). That is proved to be NP-hard [9]. Hence our problem will be NP-hard.

QAP can be described, using the terminology of knapsack problems, as follows: Given n items and m knapsacks, let $\boldsymbol{A}_i$ be the *cost matrix* of assigning items to knapsack i, $\forall i \in \{1,...,m\}$, defined as follows:

$$\boldsymbol{A_i}[j,k] = \begin{cases} \triangleright \text{ cost of assign item j to knapsack i} & \text{if } j = k \\ \triangleright\ \frac{1}{2} \text{ of cost of assign items j and k} & \\ \quad \text{simultaneously to knapsack i} & \text{if } j \neq k \end{cases}$$

The problem is to assign each item at most to one knapsack so as to minimize the total cost assigned.

In order to model our scheduling problem as an QAP we consider that:

1) The ORs working days are the knapsacks.
2) The patients are the items.
3) The cost matrices $\boldsymbol{A}_i$ is obtained form (5), in particular from term i of the sum. Since in the second part of the term there is a quadratic term, the following manipulations can be done:

$$(\boldsymbol{\mu} \cdot \boldsymbol{S_i} - Obj_i)^2 = (\boldsymbol{\mu} \cdot \boldsymbol{S_i})^2 + Obj_i^2 - 2 \cdot Obj_i \cdot \boldsymbol{\Omega} \cdot \boldsymbol{S_i}$$

Since Obj_i^2 is a constant, it can be removed from the cost function obtaining,

$$(\boldsymbol{\mu} \cdot \boldsymbol{S_i})^2 - 2 \cdot Obj_i \cdot \boldsymbol{\Omega} \cdot \boldsymbol{S_i} = \boldsymbol{S_i}^T \cdot \boldsymbol{D} \cdot \boldsymbol{S_i} - 2 \cdot Obj_i \cdot \boldsymbol{\Omega} \cdot \boldsymbol{S_i} \tag{7}$$

where $\boldsymbol{D}$ is a square and symmetric matrix such that: $\boldsymbol{D}[i,j] = \boldsymbol{\Omega}[i] \cdot \boldsymbol{\Omega}[j], \quad \forall i,j \in \{1,...,n\}$. So the term i of the cost function in (5) can be writen as follow:

$$(\beta \cdot \boldsymbol{P}[i,\cdot] - 2 \cdot Obj_i \cdot \boldsymbol{\Omega}) \cdot \boldsymbol{S_i} + \boldsymbol{S_i}^T \cdot \boldsymbol{D} \cdot \boldsymbol{S_i} \tag{8}$$

Finally, because $\boldsymbol{S}_i$ is a binary vector, we can include the linear term of (8) (i.e., $(\beta \cdot \boldsymbol{P}[i,\cdot] - 2 \cdot Obj_i \cdot \boldsymbol{\Omega}) \cdot \boldsymbol{S}_i$) in the diagonal of the matrix $\boldsymbol{D}$ resulting the next quadratic cost function:

$$\min \quad \sum_{i=1}^{m} \boldsymbol{S}_i^T \cdot \boldsymbol{A}_i \cdot \boldsymbol{S}_i \tag{9}$$

where,

$$\boldsymbol{A}_i[j,k] = \begin{cases} \boldsymbol{D}[j,k] + \beta \cdot \boldsymbol{P}[i,j] - 2 \cdot Obj_i \cdot \boldsymbol{\Omega}[j] & \text{if } j = k \\ \boldsymbol{D}[j,k] & \text{if } j \neq k \end{cases}$$

□

The Scheduling surgery problem can be put has a Binary Quadratic Problem (BQP):

$$\begin{array}{l} \min \sum_{i=1}^{m} \boldsymbol{S_i}^T \cdot \boldsymbol{A_i} \cdot \boldsymbol{S_i} \\ \text{Subject to:} \\ \begin{cases} \sum_{i=1}^{m} \boldsymbol{S_i}[j] & \leq & 1, & j \in N = \{1, \dots, N\}, \\ \boldsymbol{S_i}[j] & \in & \{0,1\}, & i \in M, j \in N, \end{cases} \end{array} \tag{10}$$

B. Scheduling problem modeled by a MILP

The MILP model provided in this subsection has been proposed in [7]. This model is very similar to (6). However, the cost associated to the occupation rate (i.e., cost given by (4)) is replaced by the absolute deviation in minutes between Obj_i and $\boldsymbol{\Omega} \cdot \boldsymbol{S_i}$.

$$||Obj_i - \boldsymbol{\Omega} \cdot \boldsymbol{S_i}||_2 \Rightarrow ||Obj_i - \boldsymbol{\Omega} \cdot \boldsymbol{S_i}|\,|_1$$

In order to change the objective function to replace norm 2 to norm 1, in addition to the decision variables $S_i[j]$ (that indicate if surgery j is schedules in day i), let us define the new variable $\boldsymbol{\alpha} \in \mathbf{R}^m_{\geq 0}$ beeing the absolute deviations (in minutes) the days with respect to the objective, i.e., $\alpha_i = |Obj_i - \boldsymbol{\mu} \cdot \boldsymbol{S}_i|$. That is equivalent with the minimum α_i that fulfill the following constraints

$$\begin{cases} \boldsymbol{\mu} \cdot \boldsymbol{S}_i - Obj_i \leq \alpha_i \\ \boldsymbol{\mu} \cdot \boldsymbol{S}_i - Obj_i \geq -\alpha_i \end{cases} \quad \forall i = 1, 2, \dots, m \tag{11}$$

Two constraints will be necessary to define each variable α_i. Since the number of variable α_i is equal to the timing horizon (i.e., number of days to schedule m), we need $2 \times m$ constraints to define all variables α_i.

Notice that the QAP has $n \cdot m$ binary variables and n constraint while the MILP model has $n \cdot m$ binary variables plus m continuous variables and $n{+}2{\cdot}m$ constraints. Although the new model is larger, it is a MILP model without quadratics term which is in general computationally more efficient. The full MILP is as follows:

$$\begin{array}{l} \min \quad \sum_{i=1}^{m} \left(\beta \cdot \boldsymbol{P}[i,\cdot] \cdot \boldsymbol{S}_i + \alpha_i \cdot (m-i+1)\right) \\ \text{Subject to:} \\ \begin{cases} \boldsymbol{\mu} \cdot \boldsymbol{S}_i - Obj_i & \leq & \alpha_i, & \forall i = 1, 2, \dots, m \\ -\boldsymbol{\mu} \cdot \boldsymbol{S}_i + Obj_i & \leq & \alpha_i, & \forall i = 1, 2, \dots, m \\ \sum_{i=1}^{m} \boldsymbol{S}_i[j] & \leq & 1, & \forall j = 1, 2, \dots, n \end{cases} \end{array} \tag{12}$$

Now the objective function is a *linear cost function* composed by two terms. Variable α_i of the second term penalizes the deviation of the occupancy of the OR with respect to the objective. Since $(m - i + 1)$ is multiplying α_i, it gives more importance to obtain a better occupancy rate in the first workings days. In this way, if there are not enough patients for all working days, the last days remain free.

C. Scheduling problem modeling by a GAP

Using the terminology of scheduling problem, the GAP problem can be described as follow: Given n surgeries and m OR working days, with

- $\bar{\boldsymbol{P}}[i,j]$ = *profit* of surgery j if assigned to OR working day i,
- $\boldsymbol{\mu}[j]$ =*duration* of surgery j,
- w_i = *maximum capacity* allowed in the OR working day i,

assign each surgery to exactly one OR working day and not exceeding the capacity of the working day by maximizing the total profit.

Since the objective in the GAP is to maximize the profit, the new $\bar{\boldsymbol{P}}$ profit matrix is defined as follow:

$$\bar{\boldsymbol{P}}[i,j] = (n{+}1{-}j){\cdot}(m{-}1{+}i) \quad \forall j \in \{1,...,n\}; i \in \{1,...,m\}$$

Assuming $n = 9$ patients in the waiting list and $m = 3$ OR working days, matrix $\bar{\boldsymbol{P}}$ for the GAP problem is given in (13).

$$\bar{\boldsymbol{P}} = \begin{array}{c} \begin{bmatrix} 27 & 24 & 21 & 18 & 15 & 12 & 9 & 6 & 3 \\ 18 & 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix} \end{array} \begin{array}{l} \to OR1 \\ \to OR2 \\ \to OR3 \end{array} \tag{13}$$

The patients with lower preference order have a greater profit than the patients with higher preference order: $\boldsymbol{P}[\cdot,1] > \boldsymbol{P}[\cdot,2] > \cdots > \boldsymbol{P}[\cdot,n]$. Moreover, the first OR working days to perform have a higher profit than the last ones $\boldsymbol{P}[1,\cdot] > \boldsymbol{P}[2,\cdot] > \cdots > \boldsymbol{P}[m,\cdot]$. In this way we are forcing to schedule patients with lower preference order in the first days.

The problem can be put as Binary Integer Lineal Programming (BILP) as follow:

$$\begin{array}{l} \max \sum_{i=1}^{m} \left[\bar{\boldsymbol{P}}[i,\cdot] \cdot \boldsymbol{S}_i\right] \\ \text{Subject to:} \\ \begin{cases} \boldsymbol{\mu} \cdot \boldsymbol{S}_i & \leq & w_i, & \forall i = 1, 2, \dots, m \\ \sum_{i=1}^{m} \boldsymbol{S}_i[j] & = & 1, & \forall j = 1, 2, \dots, n \end{cases} \end{array} \tag{14}$$

Note that GAP model (14) does not control directly the occupation rate obtained because there is no penalty in the cost function related with it. However, in this model all patients in the waiting list should be scheduled. Therefore, for a given waiting list of patients, the model should be solved with an enough number of OR waiting days. If a solution of (14) is obtained, the number of OR working days is decreased and the problem (14) is solved again. This process is repeated until no solution is obtained. The last obtained solution uses the smallest number of OR, and consequently their occupation rate are near to the maximum allowed capacities (w). For this reason, the capacity assigned to each OR is a little greater than the objective (2 %). In this way, the average value of the occupation rate will be near the target.

IV. Heuristics Solution methods

In this section, different heuristics solution methods to solve the models proposed in Section III are discussed. A meta-heuristic Genetic Algorithm (GA) for the QAP is explained in subsection IV-A. A heuristic iteratively method used to solve the MILP model in [7] is recalled in subsection IV-B. Finally, a heuristic Steepest Descent Multiplier Adjustment Method (SDMAM) for the GAP is provided in subsection IV-C

A. Genetic Algorithm for solving QAP

It has been proved that (6) is NP-hard. Moreover, there are no efficient approximation algorithms in polynomial time to solve QAP [9]. Very small instances of problem (10) containing maximum 10 patients and 3 OR working days can be solved optimally using CPLEX in a "reasonable" time. Due to the small size of the resolvable instances it is not appropriated to consider a heuristic iterative method using receding horizon strategy. On the contrary, we propose a meta-hueristc genetic algorithm. The so-called genetic algorithms (GA) are a nature inspired strategy for optimization problems. The basic idea is to adapt the evolutionary mechanisms acting in the selection process in nature to combinatorial optimization problems. The first genetic algorithm for optimization problems was proposed by Holland [10] in 1975. A genetic algorithm starts with a set of initial feasible solutions (generated randomly) called the initial population. The elements of a population are usually termed individuals. The algorithm selects a number of pairs of individuals or parents from the current population and uses so-called cross-over rules to produce some feasible solutions or child out of each pair of individuals. Further, a number of bad solutions, i.e., solutions yielding to high values of the objective function, is thrown out of the current population. This process is repeated until a stop criterion, e.g. a time limit, a limit on the number of iterations, a measure of convergence, is fulfilled. During the algorithm, mutations or immigrations are applied periodically to the current population to improve its overall quality by modifying some of the individuals or replacing them by better ones. We use the function "ga" of the globaloptim toolbox of Matlab for its implementation. The principal parameters of this function are the following:

- *PopulationSize*: Size of the population.
- *EliteCount*: Positive integer specifying how many individuals in the current generation are guaranteed to survive to the next generation.
- *CrossoverFraction*: The fraction of the population at the next generation, not including elite children, that is created by crossover.
- *FunctionTolerance* and *MaxstalGenerations*: The algorithm stops if the average relative change in the best fitness function value over MaxStallGenerations generations is less than or equal to FunctionTolerance.

B. Heuristic iterative method to solve MILP

In this subsection, the iterative method proposed in [7] is recalled. The iterative method consists in solving MILP (12) iteratively. From the waiting list of patients, we are planning the first k working days and we consider the scheduling of the first s days (where $s \leq k$). The patients with surgeries planned in these s workings days are removed from the waiting list and the process is repeated until all desired OR working days have been scheduled.

C. A Steepest Descent Multiplier Adjustment Method for the GAP

The GAP problem is NP-complete [11], however, unlike QAP [9], there exits efficient approximation algorithms in polynomial time [12], [13]. A constructive heuristic method called *Steepest Descent Multiplier Adjustment Method for the Generalized Assignment Problem* [13] is used to solve large instances of model (14). This method use a branch-and-bound algorithm that incorporates an improved Multiplier Adjustment Method (MAM) as a bounding tool. MAMs are heuristic algorithms for solving Lagrangian duals of the Generalized Assignment Problem (GAP) exploiting their special structure. The proposed algorithm uses an improved version of the traditional MAM by incorporating a post-optimality analysis for the 0-1 knapsack subproblems based on a dynamic programming formulation. The algorithm guarantees a steepest descent required by the traditional MAM for calculating a step length.

V. Simulation results

In this Section the different models and their heuristic approximation method solutions are numerically analyzed. In Subsection V-A, the computational efficiency is showed, while in Subsection V-B the results obtained using the proposed heuristics methods are compared according to the criteria of occupation rate and order of the patients.

The numerical input data of the models (waiting list) has been randomly generated using some real data form Orthopedic Department of the "Lozano Blesa" hospital in Zaragoza. In particular, we used a lot of average durations for different pathologies that have been operated in the Orthopedic Department in the last two years. We have generated randomly only the pathologies of the patients but we use some probabilities computed based on the real data. All solution have been obtained using a computer with and Intel Core i5 and 8 GB of memory.

A. Computational Efficiency of the Methods

The purpose of this section is to analyze the computational efficiency of the models and their heuristic approximation methods. For each one, the computational times used to solve instances of different sizes are showed, knowing in this way the limitations (due to computational time) of each model or heuristic approximation method.

Exact solution method of QAP (10)

Using the IBM ILOG CPLEX Optimization Studio which is often referred as CPLEX [14] a large number of solutions of model (10) with different number of patients in the waiting list and different number of days to schedule have been obtained. In these simulations, a duration of a OR working day of 390 [minutes], a desired occupation rate of 78% and a value of parameter β of 20 are considered. After performing some simulations has been observed that this is the best value from the medical point of view in order to balance the two objectives (occupation rate and preference order). Table III shows the average computational time necessary to solve different instances of problem (10). The first column represents the number of days to schedule, the second column indicates the number of patients in the waiting list and the last column is the average computational time necessary to solve the corresponding instance.

TABLE III
COMPUTATIONAL TIME TO SOLVE OPTIMALLY DIFFERENT INSTANCES OF MODEL (10)

# Days	# Patients	Avg. Time [s]
3	10	3,63
	11	11,03
	12	39,56
4	12	463
	13	1933

Note that only small instances composed by 3 OR and 10-12 patients can be solve in reasonable time.

Meta-Heuristics Genetic Algorithm of model (10)

The function “ga” of the globaloptim toolbox of matlab has been used to solve the scheduling problem. In particular we solve the model with quadratic cost function (10) assigning a value of $\beta = 20$. After analyzing the results obtained with different values of the parameters using “ga” function, we have fixed the next values:

- PopulationSize = Nvar $\cdot 10$ where $Nvar$ is the number of variables $(n \cdot m)$
- EliteCount $= 0.3\cdot$ PopulationSize
- CrossoverFraction $= 0.6$
- FunctionTolerance $= 1e^{-15}$
- MaxstalGenerations $= 20$

Table IV shows the average computational time obtained for different instances of the ga.

TABLE IV
COMPUTATIONAL TIME TO SOLVE DIFFERENT INSTANCES OF SCHEDULING PROBLEM (10) USING GA

# Days	# Patients	Avg. Time [s]
5	20	83
10	35	426
15	50	2176
20	65	7629
25	80	21542

In all instances studied in Tab. IV the GA converged to a solution. The computational times necessary are very big and, in addition, as it is shown in Section V-B, the solutions obtained according to the order of the patients are very poor.

Exact solution method of the MILP model (12)

Let us consider now the model (12). After solving different instances using CPLEX, it can be observed that the most influential variable in the computational time is the number of days to schedule. Considering a duration of a OR working day of 390 [minutes], a desired occupation rate of 78%, a value of parameter β of 2 and waiting lists composed by 70 patients, we have computed the average computational time necessary to solve different number of OR working days. Table V shows the results obtained. The value of β has been chosen from the medical point of view as the better.

TABLE V
COMPUTATIONAL TIME TO SOLVE OPTIMALLY DIFFERENT INSTANCES OF MODEL (12)

# Days	Avg. Time [s]
3	0,1421
4	0,1965
5	0,3953
6	0,8311
7	3,083
8	5,97
9	22,31
10	749
12	-

It can be seen that this model is computationally more efficient than (10). However in order to schedule more than 9 OR working days in a reasonable time is necessary to use the iterative method. Furthermore, for some waiting list, the memory necessary to plan 12 OR working day is not enough and consequently the simulation stops being out of memory after 6 hours of computation.

Iterative method (receding horizon strategy) solving (12)

The expected computational time (ET) necessary to schedule m OR workings days using the iterative method explained in Section IV-B is the following:

$$ET = \left\lceil \frac{m}{s} \right\rceil \cdot Avg(k)$$

where $\lceil \cdot \rceil$ is the rounding to the next integer, $Avg(k)$ is the average computational time to schedule a time horizon of k days, and s is the number of days to select from the k days. For example, assuming $m = 20$ days, $k = 7$ and $s = 5$, the expected computation time to schedule the 20 days is:

$$ET = \left\lceil \frac{20}{5} \right\rceil \cdot Avg(7) = 4 \cdot 3.08 = 12.32[s]$$

Heuristic: Steepest Descent MAM for the GAP

The implementation of the Steepest Descent MAM for the GAP proposed by Nejat Karabakal in [15] is used to solve the scheduling problem. Moreover, we fix the maximum capacity of the OR working day to 80%, that is a 2% lager than the objective occupation rate. Since in the GAP is necessary to plan all patients in the waiting list, a sufficiently large number of OR are considered. In particular for the solution obtained in this simulation, we consider 22 OR with different number of patients in the waiting list. If there are not enough patients in the waiting list to complete all OR, the last ones remain free. Table VI shows the computational times necessary to plan different number of patients.

TABLE VI
COMPUTATIONAL TIME TO SOLVE DIFFERENT INSTANCES OF SCHEDULING PROBLEM USING STEEPEST DESCENT MAM FOR THE GAP

# Patients	Avg. Time [s]
10	0.27
20	0.71
30	2.07
40	10.24
50	15.69
60	19.32

The heuristic Steepest Descent MAM algorithm for GAP is computationally very efficient. Moreover, as it is shown in Section V-B, the solution obtained according to the capacity and order of patients are very interesting.

B. Comparison of the quality of solutions

In this subsection the quality of the schedules obtained using the different heuristic approximation methods of the models are compared. In particular, we use the *iterative method to solve the MILP*, the heuristic *Steepest Descent MAM for the GAP* and the meta-heuristic *GA for the QAP*. Three different instances showed in Tab. VII are considered. The objective is to obtain a occupation rate of OR working days equal to 78% respecting, as much as possible, the order of the patients in the waiting lists. The iterative method plans 8 OR workings days in each iteration (k=8) and select the first six days (s=6). Moreover, $\beta = 2$ is fixed in the MILP problem (12) for each iteration. Using the heuristic Steepest Descent MAM for the GAP a maximum capacity of OR of 80% (2% larger than the objective) is imposed. Finally, the solution for the meta-heuristic GA has been obtained with the same values of the parameters as those used in the computational time simulations.

TABLE VII
INSTANCES TO EVALUATE

INSTANCE	PATIENTS	ORs WORKING DAYS
1	30	11
2	45	18
3	60	22

In order to compare different operation planning two aspects have been taken into account:

1) *Occupation rate*: a static analysis of the occupation rate.
2) *Order of the patients from the waiting list*: The parameters related with the order of the patients Π and Pd (given in eqs. (1).

Table VIII shows a comparison of the solution obtained by using the three proposed heuristic approximation methods for the 3 instances considered. The first column indicates the instance to solve, the second column represents the heuristic approximation method (model) used to obtain the scheduling, the next fourth columns show a statics analysis of the occupation rate including the average, the extreme values and the standard deviation. Finally, the last two columns evaluate the order of the patients with the indicators Pd and Π respectively.

Using the iterative method and the heuristic Steepest Descent MAM for the GAP all patients in the waiting list have been planned in every instance. However, using the Meta-heuristic GA, some patients remain in the waiting list after the planning. In particular for instance 1, the patients with preference order 9 and 20 remain in the waiting list. For instance 2 patients 3, 8, 17, 24, 30, 33, 36, 37 and 40 are not planned. Finally for instance 3 patients 5, 7, 10, 13, 31, 37 and 41 are not selected in the scheduling. From the point of view of patient's order, these patients should have been planned, so in order to compare the indicators of preference order (Pd and Π) obtained with the GA in an egalitarian way with the other two heuristics solutions, we suppose that those patients are planned in the last working days. For example, for instance 2 (18 OR working days) patients 3, 8 and 17 are planned in OR working day 19, patients 24, 30 and 33 in OR 20 and patients 36, 37 and 40 in OR 21. In this way the Pd value increase from 33 to 42 and the Π value increase from 619 to 1887. In Tab. VIII the new values of indicators *Pd* and Π appear in parentheses.

It can be seen that the *iterative method solving MILP* obtains the best scheduling in terms of occupancy rate and patient order. For each instance, using the iterative method, the lowest standard deviation of the occupancy rate is obtained (0.76, 4.46 and 3.11). Moreover, the average occupation rate is the closest to the target value (77.3, 76.46 and 77). According to the preference order of the patients, the lowest values of Pd (23, 37 and 50) and Π (97, 294 and 302) are obtained for each instance using the iterative method. So, using this method the best scheduling are obtained according to both objectives (occupation rate and preference order).

The solutions obtained using the heuristic *Steepest Descent MAM for the GAP* are good according to the occupation rate. The averages values for instance are 77.2, 76 and 77 respectively , very similar to those obtained using the iterative method. However the standard deviations are a little larger (2.38, 5.91 and 3.19). On the contrary, the scheduling obtained using the heuristic Steepest Descent MAM for the GAP respects less the order of the patients than those obtained with the iterative method. It can be seen in the values of the indicator Pd (24, 40, 53) and Π (128, 473 and 984). This disorder of the scheduling is acceptable from the clinical point of view. So, the scheduling obtained by using this model is not as good as that obtained by using the iterative one, however it is acceptable from the medical point of view.

Finally, using the meta-heuristic *GA*, very bad solutions are obtained according to both criteria of occupation rate and order of the patients. The average values of the occupation rate are far to the objective (72.6, 63.6 and 67.5). Moreover the standard deviations are very high (8.46, 23.33 and 11.7). The scheduling obtained using GA do not respect the order of

TABLE VIII
COMPARING OF OPERATION PLANNING USING THE ITERATIVE METHOD, HEURISTICS ALGORITHM AND META-HEURISTIC GA

		Ocupation Rate				**Order**	
INSTANCE	**MODEL**	**Average**	**Maximun**	**Minimun**	**Std. Deviation**	**Pd**	Π
1	Iterative	77.3	78.2	76.15	0.76	23	97
Days=11	Heuristic	77.2	79.3	72.56	2.38	24	128
Patients=30	GA	72.6	90	60	8.46	26 (28)	287 (489)
2	Iterative	76.46	78.46	58.79	4.46	37	294
Days=18	Heuristic	76	80	58.79	5.91	40	473
Patients=45	GA	63.6	90.25	0	23.33	33 (42)	619 (1887)
3	Iterative	77	78.94	63.3	3.11	50	302
Days=22	Heuristic	77	80	66	3.19	53	984
Patients=60	GA	67.5	90	45.1	11.7	49 (56)	1384 (3358)

the patients in the waiting list and consequently very high values of the indicators Pd (28, 42 and 56) and Π (489, 1887 and 3358) are obtained. The scheduling obtained using the meta-heuristic GA are not acceptable from the medical point of view. A general genetic algorithm implemented in the "globaloptim" toolbox of Matlab has been used. More specific heuristics rules should be considered in order to improve the solutions obtained.

VI. CONCLUSIONS

The scheduling problem of elective patients in the Orthopedic Department of the "Lozano Blesa" Hospital in Zaragoza has been considered. Three different mathematical models have been proposed. However the computational times necessary to solve them optimally are very high and only small instances can be solved in reasonable time. In order to solve larger instances, we have tested and compared three heuristic approximation methods (one for each model): the first one solves a Mixed Integer Linear Programming (MILP) model iteratively, the second one considers the heuristic SDMAM for the GAP and the last one uses the meta-heuristic GA for solving the QAP. The results according to the computational time show that using the iterative method and the heuristic SDMAM it is possible to plan 22 OR workings days in a reasonable time: 15.41 and 19,32 [s] respectively. However, the meta-heuristic GA spends 7629 [s] in the scheduling of 20 OR workings days. Regarding the quality of the solutions, two criteria have been taken into account: occupation rate and preference order of the patients. Using the iterative method and the heuristics SDMAM very similar and good occupation rates have been obtained: the average values are not more than 2 points of percentage away from the target values and the standard deviations are lower than 6. The meta-heuristic GA obtain a very bad solutions according to the occupation rate: the average values are up to 14.4 points away from the target value and standard deviations are up to 23.3. Taking into account the preference order of the patients, the iterative way gets the most ordered scheduling. A little more disordered, although acceptable from the medical point of view, are the schedules obtained by using the heuristic SDMAM for the GAP. Again, the meta-heuristic GA obtains a very bad solution with a great disorder of the patients, resulting unacceptable from the medical point of view. As a future work, specifics heuristics rules should be used to improve the solutions obtained using the GA for the QAP.

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