

# Nichols Diagrams

also referred as nichols charts

```
k = 0.05; %proportional controller (gain)
tm = 0.015; %delay time
H = tf(9e4,[1 135 0],'IODelay',tm) %process transfer function
```

H =

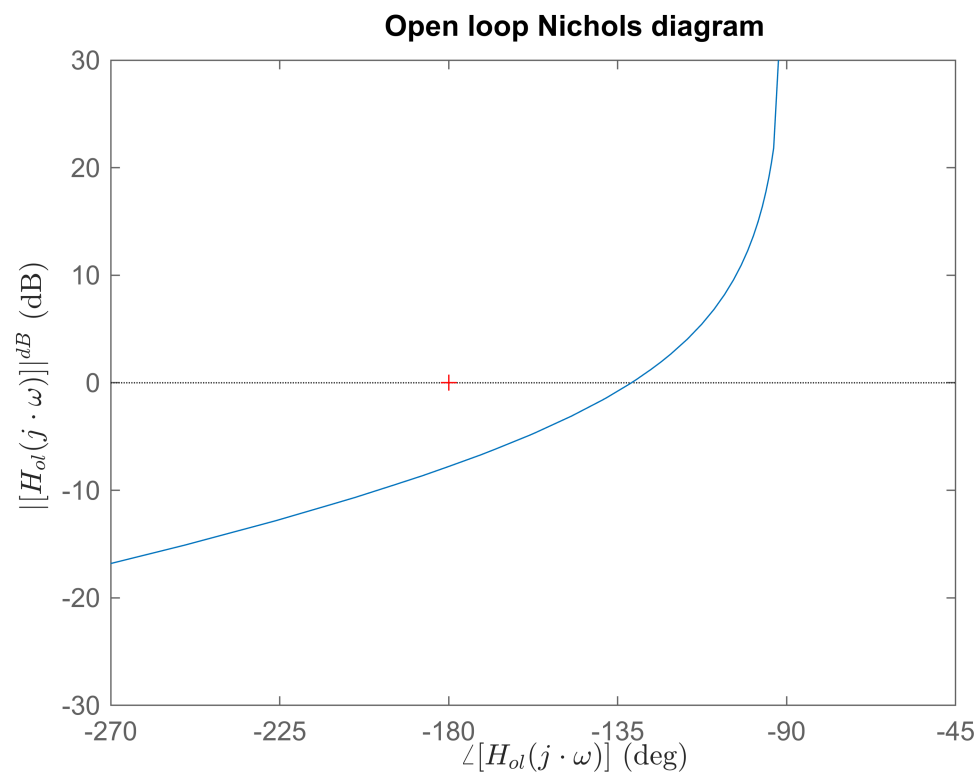
$$\exp(-0.015s) * \frac{90000}{s^2 + 135 s}$$

Continuous-time transfer function.  
Model Properties

## Plot the nichols diagram

- recognize what is represented on nichols

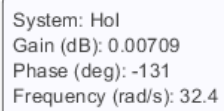
```
Hol = k*H; %open loop transfer function
nichols(Hol);
title('Open loop Nichols diagram');
xlabel('$\angle [H_{ol}(j\cdot \omega)]$', 'interpreter','latex');
ylabel('$ |[H_{ol}(j\cdot \omega)]|^{\text{dB}}$', 'interpreter','latex'); shg
% zoom the diagram in order to center the "critical point from Nyquist (-1, 0j)"
% point (-180 degrees 0dB)
axis ([-270,-45 -30 30]); shg
```



## Stability analysis of closed loop

```
% draw the gain and phase margin (Simplified Nyquist criterion)
% for the phase margin, the cutoff frequency and the phase at the cutoff
% frequency is needed
```

$$\gamma_k = \pi + \angle[H_{ol}(j\omega_c)]$$



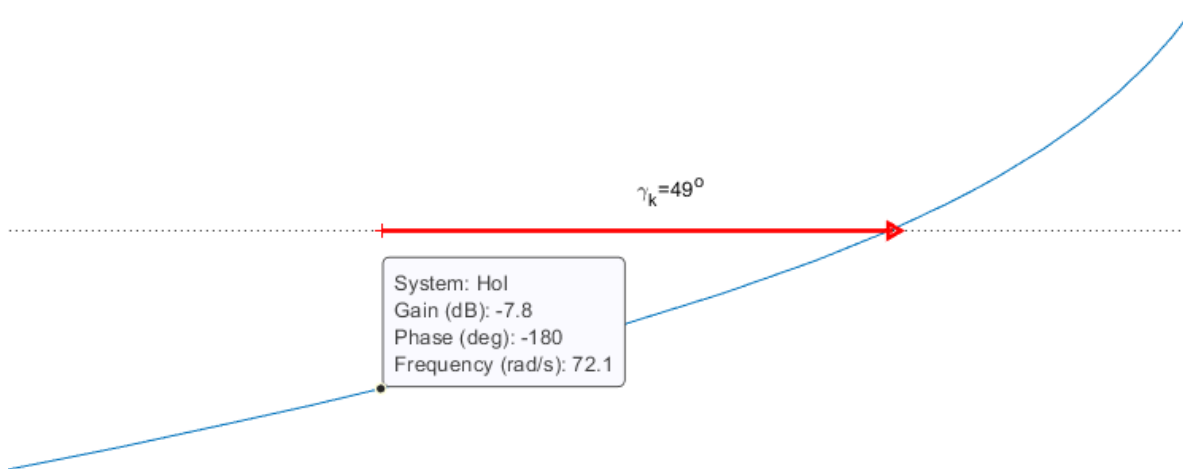
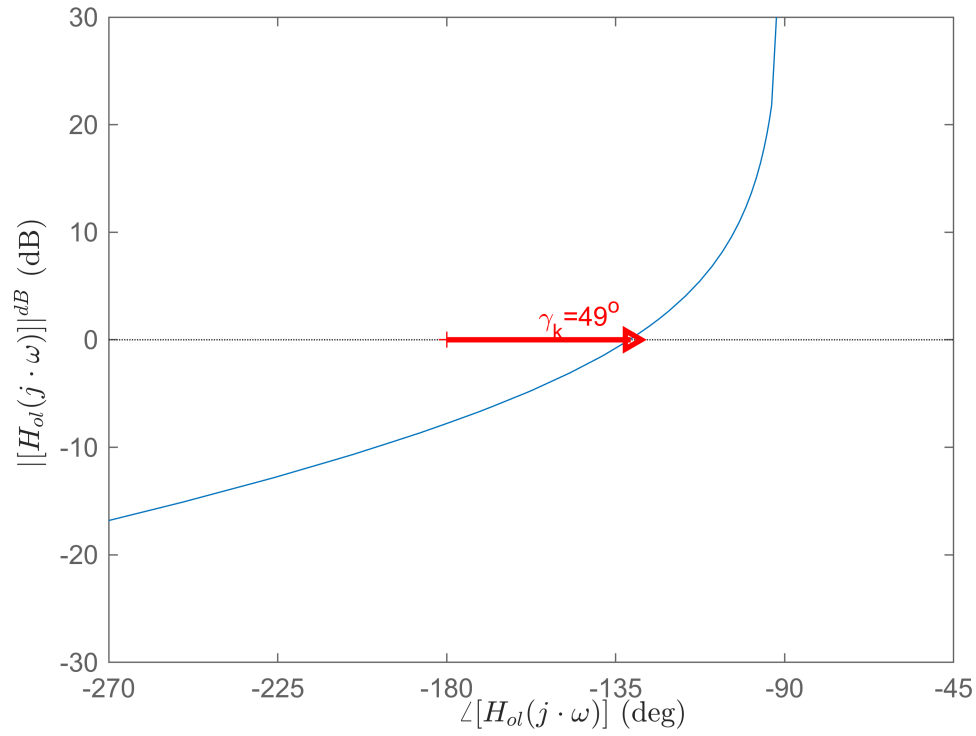
System: Hol  
Gain (dB): 0.00709  
Phase (deg): -131  
Frequency (rad/s): 32.4

```

wc = 32.4; %the cutoff frequency
pwc = -131; %the open loop argument evaluated for the cutoff frequency
hold on;
plot([-180 pwc],[0 0],'r-','LineWidth',2);
plot(pwc,0,'r>','LineWidth',2);
%place a text above the arrow indicating the phase margin with the
%resulting value for the phase margin
pm = 180 + pwc; %the phase margin
text(-180+pm/2,2,['\gamma_k=',num2str(round(pm)),'\^o'],'Color','r')

```

Open loop Nichols diagram

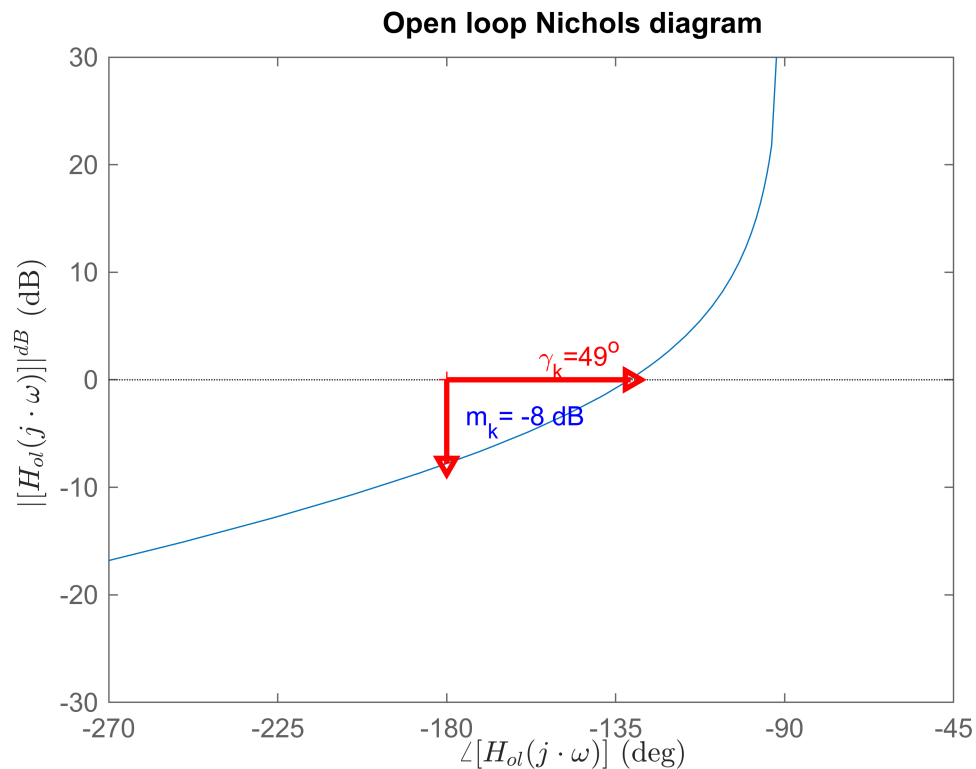


```
wpi = 72.4; mwpi = -7.8;
plot([-180 -180],[0 mwpi],'r-','LineWidth',2);
plot(-180, mwpi, 'rv','LineWidth',2);
text(-175, mwpi/2,['m_k= ',num2str(round(mwpi)), ' dB'],'Color','b');
```

And stability conclusions :

$m_k^{\text{dB}} < 0$  and  $\gamma_k > 0$  are both true, so the closed loop system results asymptotically stable

hold off;

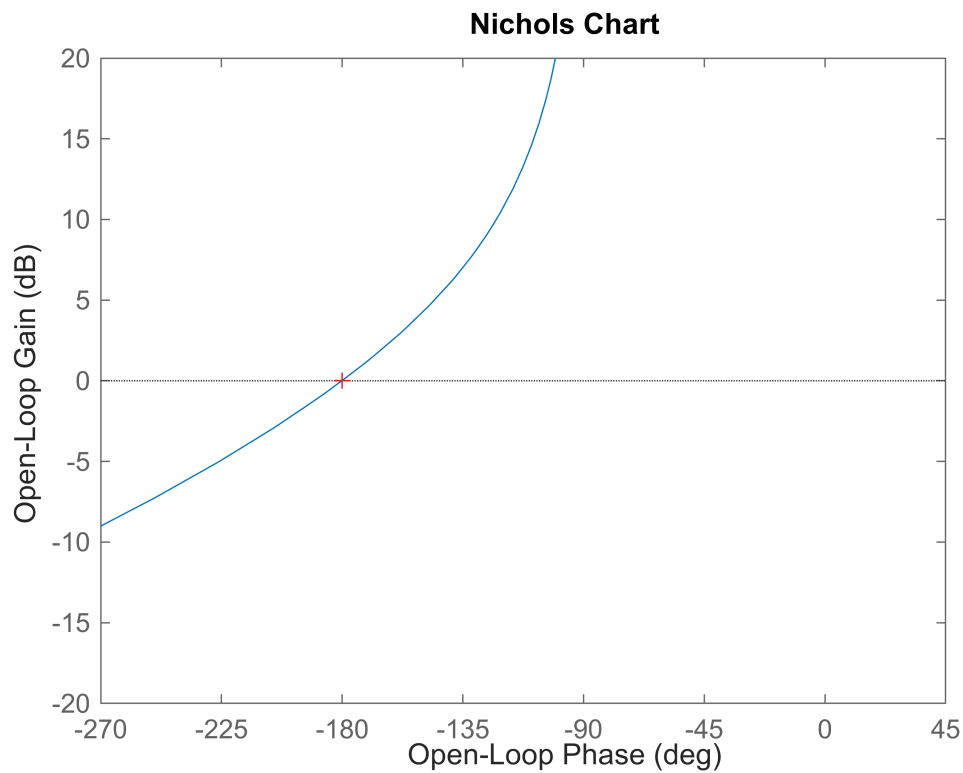


## Stability analysis depending on k, $k \in (0, \infty)$

```
kc = 10^(-mwpi/20) % kc =db2mag(mwpi), k = 0.05
```

```
kc = 2.4547
```

```
%closed loop stable for k in(0,0.05*2.466)
% the max value of k
k = 0.05*2.4547;
nichols(k*H)
axis([-270 45 -20 20])
```



## Stability analysis depending on time delay ( $\tau_m$ )

$$\gamma_k > 0$$

$$\pi + \angle(\text{Hol}) = \pi - \frac{\pi}{2} + \text{atan}\left(\frac{\omega}{135}\right) - (0.015 + \text{delay margin})\omega > 0$$

$$\frac{\pi}{2} - \tau_m \omega > \text{atan}\left(\frac{\omega}{135}\right)$$

$$\frac{\pi}{2} - \text{atan}\left(\frac{\omega}{135}\right) > \tau_m \omega$$

$$\tau_m < \frac{\left[\frac{\pi}{2} - \text{atan}\left(\frac{\omega}{135}\right)\right]}{\omega_c}$$

```
taum = (pi/2-atan(wc/135))/wc
```

```
taum = 0.0412
```

```
%delay margin
```

```
dm = taum - tm
```

```
dm = 0.0150
```

```
k = 0.05; %proportional controller (gain)
```

```
tm = 0.0412; %delay time
```

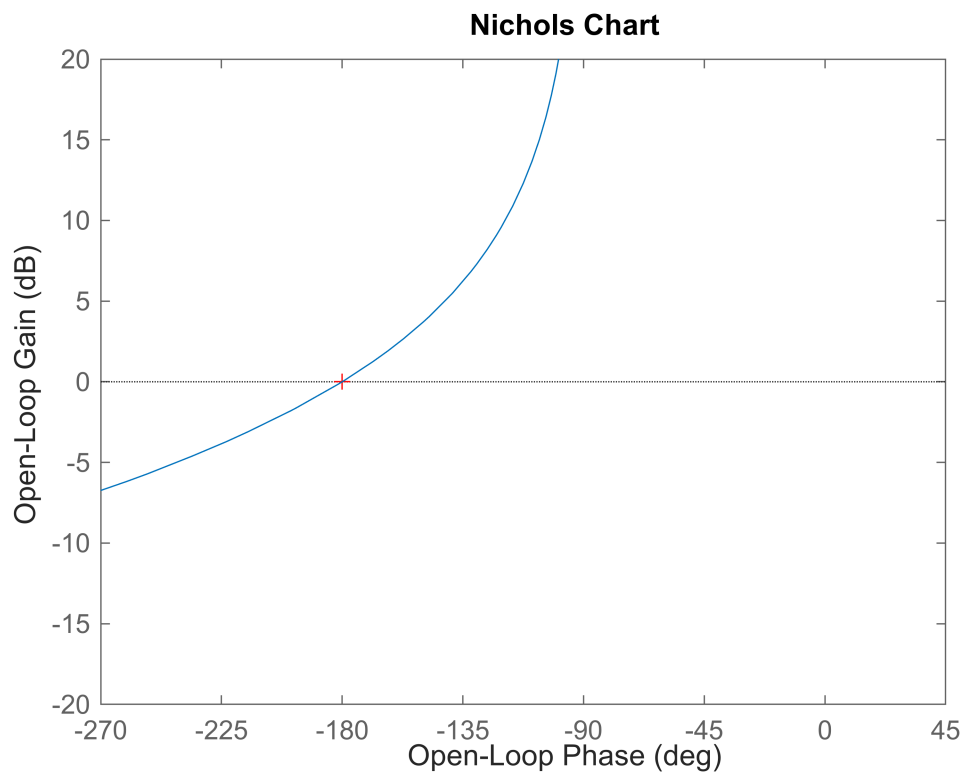
```
H = tf(9e4,[1 135 0],'IODelay',tm) %process transfer function
```

H =

$$\exp(-0.0412s) * \frac{90000}{s^2 + 135s}$$

Continuous-time transfer function.  
Model Properties

```
nichols(k*H);  
axis([-270 45 -20 20])
```



$\tau_m \in (0, 0.0412)$  for stable closed loop