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9º Gram-Schmidt para construir una base ortonormal

a) $(1,1,1), (2,0,1), (2,4,5)$

$u_1 = (1,1,1)$

$u_2 = (2,0,1) - \frac{(2,0,1)(1,1,1)}{(1,1,1)(1,1,1)}(1,1,1) = (2,0,1) - \frac{2+0+1}{1+1+1}(1,1,1) = (2,0,1) - (1,1,1) = \underbrace{(1,-1,0)}_{u_2}$

$u_3 = (2,4,5) - \frac{(2,4,5)(1,1,1)}{(1,1,1)(1,1,1)}(1,1,1) - \left(\frac{(2,4,5)\cdot(1,-1,0)}{(1,-1,0)\cdot(1,-1,0)}(1,-1,0)\right)$

$u_3 = (2,4,5) - \left(\left(\frac{2+4+5}{1+1+1}\right)\cdot(1,1,1)\right) - \left(\left(\frac{2-4+0}{1+1}\right)\cdot(1,-1,0)\right)$

$u_3 = (2,4,5) - (11/3, 11/3, 11/3) - (-1,1,0)$

$u_3 = (-2/3, -2/3, 4/3)$

$\lambda_{u_1} = \frac{\vec{u_1}}{|\vec{u_1}|} = \frac{(1,1,1)}{\sqrt{1^2+1^2+1^2}} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

$\lambda_{u_2} = \frac{\vec{u_2}}{|\vec{u_2}|} = \frac{(1,-1,0)}{\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$

$\lambda_{u_3} = \frac{\vec{u_3}}{|\vec{u_3}|} = \frac{(-2/3, -2/3, 4/3)}{\sqrt{4/9+4/9+16/9}} = \left\langle -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$

b) $(3,2,0), (1,5,-1), (5,-1,2)$

$u_1 = 3,2,0$

$u_2 = (1,5,-1) - \frac{(1,5,-1)\cdot(3,2,0)}{(3,2,0)\cdot(3,2,0)}(3,2,0) = (1,5,-1) - (3,2,0) = \boxed{(-2,3,-1)}$

$u_3 = (5,-1,2) - \left(\frac{(5,-1,2)\cdot(3,2,0)}{(3,2,0)\cdot(3,2,0)}(3,2,0)\right) - \left(\frac{(5,-1,2)\cdot(-2,3,-1)}{(-2,3,-1)\cdot(-2,3,-1)}(-2,3,-1)\right) =$

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$u_3 = (5,-1,2) - \left(\left(\frac{15-2+0}{13} \cdot (3,2,0)\right) - \left(\frac{-10-3-2}{4+9+1}\right) \cdot (-2,3,-1)\right)$

$u_3 = (5,-1,2) - (3,2,0) - (-15/14 \cdot (-2,3,-1))$

$u_3 = (5,-1,2) - (3,2,0) - (15/7, -45/14, 15/14))$

$u_3 = \left(\frac{-1}{7}, \frac{3}{14}, \frac{13}{14}\right)$

$\lambda_{u_1} = \frac{(3,2,0)}{\sqrt{9+4}} = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}, 0 \right\rangle$

$\lambda_{u_2} = \frac{(-2,3,-1)}{\sqrt{4+9+1}} = \left\langle -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}} \right\rangle$

$\lambda_{u_3} = \frac{(-1/7, 3/14, 13/14)}{\sqrt{\frac{1}{49} + \frac{9}{196} + \frac{169}{16}}} = \left\langle -\frac{\sqrt{182}}{91}, \frac{3}{\sqrt{182}}, \frac{13}{\sqrt{182}} \right\rangle$

6º Determine la proyección del vector v sobre el vector u para los vectores siguientes

a) $v = (7,4)$, $u = (1,2)$

$\text{Proy}_{\vec{u}} v = \frac{(7,4)\cdot(1,2)}{(1,2)\cdot(1,2)} \cdot (1,2) = \frac{7+8}{1+4}(1,2) = \frac{15}{5}(1,2) = \underline{(3,6)}$ //

b) $v = (-1,5)$, $u = (3,-2)$

$\frac{-13}{13}$

$\text{Proy}_u v = \frac{(-1,5)\cdot(3,-2)}{(3,-2)\cdot(3,-2)} \cdot (3,-2) = \frac{-3+(-10)}{(9+4)}(3,-2) = \underline{(-3,2)}$ //

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c) $v = (4,6,4)$, $u = (1,2,3)$

$\frac{28}{14} = 2$ por x

$$Proy_u v = \frac{(4,6,4)\cdot(1,2,3)}{(1,2,3)\cdot(1,2,3)}(1,2,3) = \frac{4+12+12}{1+4+9}\cdot(1,2,3) = \boxed{(2,4,6)}$$

d) $v = (6,-8,7)$, $u = (-1,3,0)$

$\frac{-30}{10} = -3$

$$Proy_u v = \frac{(6,-8,7)\cdot(-1,3,0)}{(-1,3,0)\cdot(-1,3,0)}(-1,3,0) = \frac{-6,-24}{1+9}(-1,3,0) = \boxed{[3,-9,0]}$$

e) $v = (1,2,3,0)$, $u = (1,-1,2,3)$

$$Proy_u v = \frac{(1,2,3,0)\cdot(1,-1,2,3)}{(1,-1,2,3)\cdot(1,-1,2,3)}(1,-1,2,3) = \frac{1-2+6}{1+1+4+9}(1,-1,2,3)$$

$$= \frac{5}{15}(1,-1,2,3) = \left\langle \frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 1 \right\rangle$$

$\to \frac{1}{3}$

Determine el núcleo y la imagen de:

6. $T(x,y,z) = (2x, y)$ de $\mathbb{R}^3 \to \mathbb{R}^2$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Núcleo:

$x = 0$

$y = 0$

núcleo $\{(x,y,z) \in \mathbb{R} \mid x=0, y=0\}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & a \\ 0 & 1 & 0 & b \end{array}\right) \xrightarrow{f_1/2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & a/2 \\ 0 & 1 & 0 & b \end{array}\right)$$

Imagen $\{(x,y) \in \mathbb{R} \mid x, y \neq 0\}$

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7º $T(x,y) = x-y$ de $\mathbb{R}^2 \to \mathbb{R}$

$x-y=4$

$x-y=0$

$x=y$

Imagen $\{x \in \mathbb{R} \mid x \neq y\}$

Núcleo $\{(x,y \in \mathbb{R} \mid x=y\}$

8º $T(x,y) = (\overset{a}{x}, \overset{b}{y}, \overset{c}{z})$ de $\mathbb{R}^2 \to \mathbb{R}^3$ cuando

a) $z=0$

$$\left(\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & c \end{array}\right)$$

Imagen $\{(x,y,z) \in \mathbb{R} \mid x,y \in \mathbb{R} \wedge z=0\}$

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Núcleo $\{(x,y) \in \mathbb{R} \mid x=0 \wedge y=0\}$

b) $z=1$

$$\left(\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & c \end{array}\right)$$

Imagen $\{(x,y,z) \in \mathbb{R} \mid x,y \in \mathbb{R} \wedge z=1\}$

Núcleo

$x=0 \quad y=0$

$\{x,y,z \in \mathbb{R} \mid x=0 \wedge y=0\}$

9) $T(x) = (x, 2x, 3x)$ de $\mathbb{R} \to \mathbb{R}^3$

$$\left(\begin{array}{c|c} 1 & a \\ 2 & b \\ 3 & c \end{array}\right) \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left(\begin{array}{c|c} 1 & a \\ 0 & b-2a \\ 0 & c-3a \end{array}\right) \xrightarrow{x=0}$$

Imagen $\{x,y,z) \in \mathbb{R} \mid z-3z=0 \wedge y-2x=0\}$

Núcleo: $\{x \in \mathbb{R} \mid x=0\}$

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10º $T(x,y) = (x^2, y)$ de $\mathbb{R}^2 \to \mathbb{R}^2$

$T(v_1+v_2) = T((x_1+x_2, y_1+y_2)) = ((x_1+x_2)^2, y_1+y_2)$

$T(v_1) + T(v_2) = (x_1^2, y_1) + (x_2^2, y_2) = (x_1^2 + x_2^2, y_1 + y_2)$

$T(v_1+v_2) \neq T(v_1) + T(v_2)$

$\therefore$ No es una transformación lineal

11º $T(x,y,z) = (x+2y, x+y+z, 3z)$ de $\mathbb{R}^3 \to \mathbb{R}^3$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 1 & 1 & 1 & b \\ 0 & 0 & 3 & c \end{array}\right) \xrightarrow{-R_1+R_2} \left(\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 0 & -1 & 1 & b-a \\ 0 & 0 & 3 & c \end{array}\right) \xrightarrow[R_3/3]{-R_2} \left(\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 0 & 1 & -1 & -a-b \\ 0 & 0 & 1 & c/3 \end{array}\right) R_2+R_3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 0 & 1 & -1 & a-b \\ 0 & 0 & 0 & \frac{c}{3}+a-b \end{array}\right)$$

Imagen $\{(x,y,z) \in \mathbb{R} \mid \frac{z}{3} + x - b = 0\}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ Núcleo

$y - z = 0$ $x + 2y = 0$

$y = z$ $x = -2z$

$\{(x,y,z) \in \mathbb{R} \mid x + 2y = 0 \wedge y = 0 \wedge z = 0\}$