Lab 4: Doppler Measurement of Solar Rotation

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Abstract

In order to effectively measure the Earth-Sun distance, roughly 1 Astronomical Unit (1 AU), we have to carefully measure a sub-pixel Doppler effect in the solar spectra. Using a CCD detector, we were able to (i) apply a wavelength calibration for key bands in the 2D image, (ii) determine an Eddington approximation accounting for limb darkening for the transit of the Sun, (iii) apply the cross-correlation method to compute the sub-pixel shifts, (iv) figure out the velocity shift using the Doppler Effect in order to compute the radius of the Sun, in order to ultimately, (v) combine this information with the apparent angular diameter of the Sun to find the Sun-Earth distance. I focused on the 34^{th} order of the solar spectra on the day of November 11, 2015 (11/11/15) for the beginning part of this lab, finding that the total transit time is 136.4±1.4s. When I started working on cross correlation, I was taking the Doppler shifts of 4 observations, 2 on 11/12 and 2 on 11/16, to find, after averaging, that the Earth-Sun distance is 1.09184±.067 AU.

Introduction

The main goal of this lab is to calculate the Sun-Earth distance, which should be roughly 1 AU (not exactly 1 AU considering Earth's orbit is not perfectly circular). We obtain solar spectra from the CCD detector, whose resolving power alone can not differentiate wavelength shifts, so we have to utilize other tactics to observe these sub-pixel shifts. Basically, once we get the pixel shifts, we can convert the pixels to wavelengths in order to apply the Doppler Effect,

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_0}; \qquad v_r = radial\ velocity \tag{1}$$

We also know that the radial velocity relates to the radius of the Sun by,

$$\omega = \frac{v_r}{R_\odot}; \qquad where\ R_\odot = \frac{vP}{2\pi} \tag{2}$$

where P is the period of the Sun and that is already defined as approximately 25 days. Equation 2(a) tells us that we need to use the shift from the limb in order to find the radius of the sun. Continuing, we also have a precise

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measurement for the Sun's angular diameter ($\theta \simeq \frac{1}{2}$) and so, using basic trigonometry, we can relate θ to the radius,

$$sin\theta = \frac{2R_{\odot}}{1AU} \tag{3}$$

So we just practically need to follow these steps.

Analysis

Our data is taken by a spectrometer that uses diffraction grating (discussed more in section 1.1). For each calibration data, we took 20 observations and averaged them out; we did not take any "dark" data considering that the CCD detector is already sitting in a very dark room such that we remotely have to take the data to ensure no stray light or dark currents enters the fiber optic cable[1]. Our solar data ranges from 60 to 100 observations taken with a .5s exposure time in the month of November during the day (around noon).

1 Spectrometer

1.1 Equipment

The spectrometer we are using is quite precise and has two successive diffraction gratings, an echelle grating and a cross disperser. Light enters the $50\mu m$ fiber optic cable, goes through a collimator, then enters another dispersion factor which is angled at α, and depending on the wavelength of the light, the light is dispersed at an angle β. Therefore, the light is dispersed onto a 1024 x 1024 CCD detector, which is kept in a *very* dark room. The resolving power is essential to know since we want to figure out if we would be able to differentiate two objects from each other. A higher resolving power means that it is easier to distinguish two objects right next to each other, whereas it becomes harder to do so with a lower resolving power. The resolving power is defined by,

$$R = \frac{\lambda}{\Delta\lambda} = \left(\frac{sin\alpha + sin\beta}{cos\alpha}\right)\left(\frac{f_{col}}{\Delta x}\right) \tag{4}$$

where α and β are the angles of incidence and diffraction, f_{col} is the focal length of the collimator, and Δx is the size of the object in question. The resolving power can also be,

$$R = \frac{\lambda_0}{\delta\lambda} = \frac{m\lambda}{\sigma cos\alpha\delta\alpha} \tag{5}$$

where λ_0 is the rest wavelength, $\delta\lambda$ is the non-relativistic Doppler shift, m is the echelle order, α is the angle of incidence and $\delta\alpha$ is the angular diameter of the source illuminating the spectrometer. The echelle order, m, and $\delta\alpha$ are defined as

$$m = 2\left(\frac{\sigma}{\lambda}\right) sin\delta cos\theta; \qquad \delta\alpha = \frac{d_{fiber}}{f_{col}} \tag{6}$$

using the blaze angle ($\delta = 64.4°$), the full angle between incident and diffracted beam ($2\theta = 22°$), the echelle grating groove density ($\frac{1}{\sigma} = 80mm^{-1}$); and the diameter of the optical fiber ($d_{fiber} = 50\mu m$) and the focal length of the collimator ($f_{col} = 100m$). With this in mind, we can substitute for m and get an equation for R,

$$R = \frac{2sin\delta cos\theta}{cos\alpha\delta\alpha} \tag{7}$$

which using our values above gives us $R = 14047$. If we take R and divide it by the speed of light, we get a velocity resolution, $\delta v = 21km/s$. This unfortunately is about 10 times greater than the speed of the Sun, so we would not be able to derive the speed of the Sun with this resolution by using basic Doppler shift of spectra. To improve the velocity resolution, we can consider the signal-to-noise (SNR) ratio,

$$\delta v \approx \frac{1}{SNR}\frac{1}{\sqrt{M}}\frac{c}{R} \tag{8}$$

[1] James Graham. https://drive.google.com/file/d/0B40Ynk22SiBpMFg0Y2o2S3k5aW8/view

where SNR is the signal to noise, R is the resolving power, and M is the number of lines measured. Therefore, the more we increase the SNR or M, the better velocity resolution we will have.

1.2 Wavelength Calibration

We used a quartz-halogen lamp to obtain where the echelle bands are diffracted onto the CCD detector; a 653 nm wavelength red laser to match that wavelength with a pixel location on the detector to further figure out which pixels corresponds to which wavelength; and a neon lamp for the final wavelength calibration.

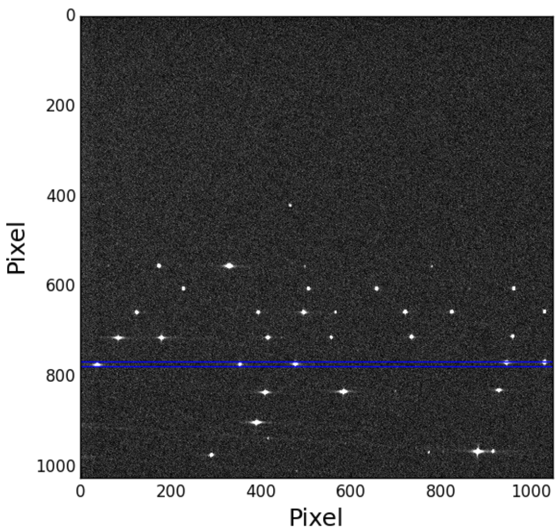


Figure 1: The averaged neon spectrum of 20 FITS file, with exposure time of .5 seconds. The bottom left corner has the reddest wavelengths, and the top right corner has the bluest wavelengths.

Using the image of the halogen lamp, I found the location of the diffraction bands by seeing where the "brightness" of each band starts and ends. It is interesting to note how the bands are not perfectly parallel to the horizontal axis. As well, lower orders were much thicker (20pix) and farther apart than higher orders that were much thinner (10pix) and closer together. Once we determined where the bands are, we used this information to collect and average all the rows within a band to create a 1D plot of pixel against intensity for a given band in the 2D spectrum of neon (Figure 1.2.1). Using equation 6, we obtained that the first and lowest order in Figure 1 correlates to $m = 31$.

1.2.1 Working with 1D Spectra

We then applied a 1D centroid method assuming that the peaks are around 10 pixels wide and found errors on the order of 10^{-5} (exactly like in Lab 2[2]). Looking at the image of the red laser (not shown), we see that 653 nm corresponds to a certain pixel in the 34^{th} order ($m = 34$). From there, we figured out which pixels correspond to which neon wavelengths. When we plotted all the neon band spectra onto the same plot, we find that the reddest wavelengths are in the lower, left corner and the bluest wavelengths are in the upper, right corner (Figure 1). *Note:* Plotting all the band spectra onto the same graph is not fully complete and correct considering that emitted wavelengths at the end of one band are repeated on the higher band; however, the distance between two points on the lower band is smaller than the distance on the higher band (because of diffraction), so it does not overlap quite well. It does give a good general sense that as pixels are increasing, the wavelengths are getting shorter (bluer).

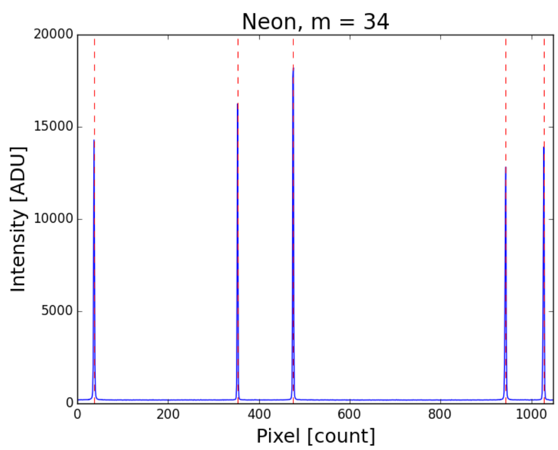
Like in the previous lab, we apply a linear least squares fit[3] and find that the quadratic

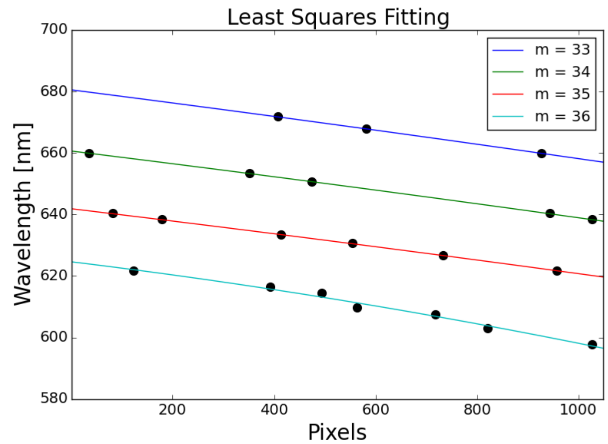
[2] https://sites.google.com/site/ay120fall2015/home/lab-handout
[3] Least-Squares Fitting, J. Graham: https://drive.google.com/file/d/0B40Ynk22SiBpVlRKZFdhTnZMQkU/view

fit is the best (Figure 2). The table below shows the coefficients for the quadratic fit.

Order	a_0 $[nm]$	a_1 $[\frac{nm}{pixel}]$	a_2 $[\frac{nm}{pixel^2}]$
33	680.5510	-.00239	-1.2893e-06
34	660.6514	-.00239	-1.3285e-06
35	641.8970	-.00198	-1.2974e-06
36	624.6417	-.00201	-6.4164e-06

To continue on with this lab, we only need to consider one order, so I will be focusing on the 34^{th} order, for the reason that the dominantly visual H-alpha line (656.3nm) is in this order which will prove to be helpful in section 3.





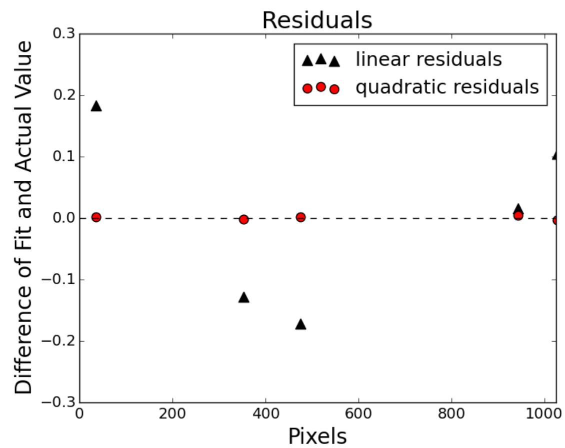


Figure 2: These three plots show the 34^{th} order and its residuals; the other one shows the fits of different orders.

2 Lightcurve

Now that we have our calibration, let's take some solar data. When we take solar observations, we need to make sure that the telescope is facing the declination of the center of the solar disk and we begin each observation a little to the west so that when we plot the light curve (a flux vs time graph), we see a "bell" type of curve with straight lines on both sides (Figure 3). The telescope automatically corrects itself for the Earth's motion.

To create the time series, we need to get the time of each observation by referencing the Julian Day (JD) and converting it to seconds. To get the flux series, we added up the fluxes of each band (15 bands in total) from the image of each file as opposed to just summing up the entire image since that might record false positive flux readings, which would create more error. To further work on the flux series, I found out which files are non-transit and took the average flux of them and subtracted the average from the entire flux series. This accounts for the scattered light that enters the optical fiber when the light from the Sun is not directly hitting it.

2.1 Eddington Approximation

Now, I fit an Eddington approximation given by,

$$I = I_0 \left(\frac{2}{5} + \frac{3}{5} \sqrt{1 - (t - t_0)^2 / \Delta t^2)} \right) \quad (9)$$

where t_0 is the time of the middle of the transit, $2\Delta t$ is the total duration of the transit, and I_0 is the intensity at t_0. To obtain these three parameters, I did a manual iterative process where I first started with an initial guess for Δt by simply taking the difference in time between the start of the transit and the end and dividing it by two. And I found t_0 and I_0 from there. I put these parameters into equation 9 and then plotted a residuals graph (not shown) and calculated the root mean square (rms) error. I was playing around with the parameters until I found the lowest rms error. Table 2.1 shows the initial guess and the final guess after a few tries.

Parameter	Initial	Final 1	Final 2
$2\Delta t$ [s]	134.9999	136.4	136.4 ± 1.4
t_0 [s]	141.4999	142.5	142.5 ± .8
"I_0" $[I/I_0]$	0.99441	0.99441	1.0 ± .006
rms	0.02628	0.02416	0.02372

Table 1: Eddington Approximation for the first solar observation on 11/11/15.

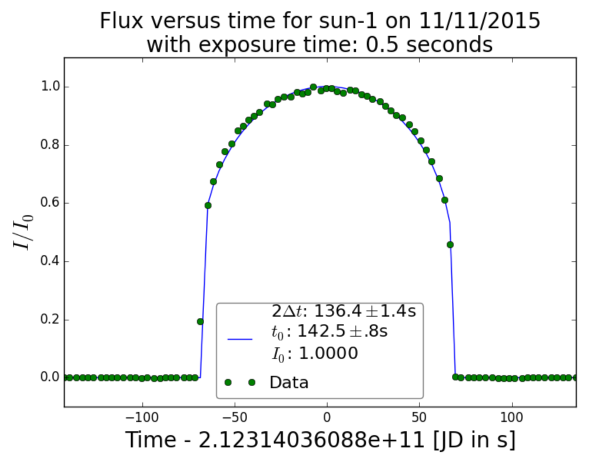


Figure 3: Time Series Plot with an Eddington Approximation (Sun1 on 11/11/15). The limb darkening features are quite visible on both sides of the transit.

3 Cross Correlation

Now that we have a visual representation of what the flux looks like when the Sun transits, we want to figure out the sub-pixel shifts from the spectra at t_0 compared to every other spectra found in the transit, specifically comparing the center to the limb because that is where the shift is most apparent as well as the fact that this shift corresponds most to the radial velocity. The transit starts, peaks, and ends at spectra 23, 44, and 65, respectively. We take two spectra (at t_0 and at the limb) and plot them on top of each other to find that they almost look identical (Figure 5), but zooming in, we find that there are very subtle differences (Figure 4).

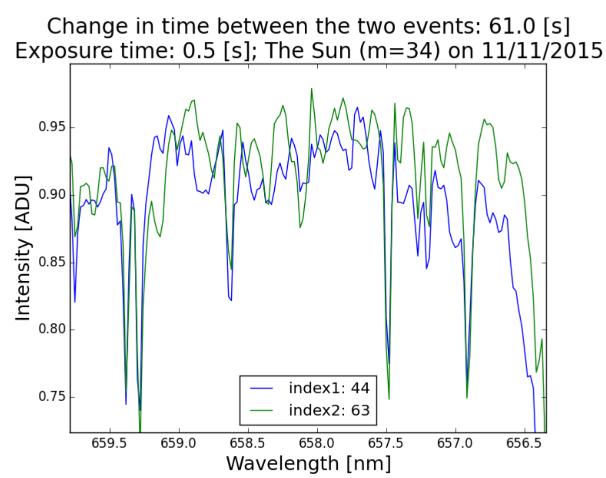


Figure 4: An up close comparison of the spectrum located at t_0 and of the spectrum at the right limb (Sun1 on 11/11/15).

3.1 Theory

To figure out how much the spectra are shifted from one another, we must use the method of cross correlation. This measures how similar two functions, $\{x_i\}$ and $\{y_i\}$ are to one another, given by the equation,

$$s_j = \frac{1}{N-1}\sum_i (x_i y_{i+1}) - \frac{N}{N-1}\overline{xy} \qquad (10)$$

Before inputting the two spectra into equation 10, we first subtracted the scattered light noise, ignored the data of the last 10 pixels (they were strange for some reason), and then took a second order degree polynomial fit on each spectrum and subtracted that, in order to flatten out the spectra (Figure 5(a)). Additionally, each spectrum was divided by its maximum value to ensure for the differences in flux readings. Then, we applied a hamming function (equation 11) to each spectra to ensure that the ends go to zero. Figure 5(b) shows how the cross correlation beautifully peaks right around zero and hovers around the y-axis elsewhere. If the two functions are identical, we will find that the lag is at zero. When we zoom in on Figure 5(b), we notice that the lag is a little bit shifted. The shift, shown in Figure 5(c), is determined by finding the x location of the maximum of a quadratic fit on the three highest values of the cross correlation.

$$w(n) = 0.5 - 0.5cos\left(\frac{2\pi n}{M-1}\right) \qquad (11)$$

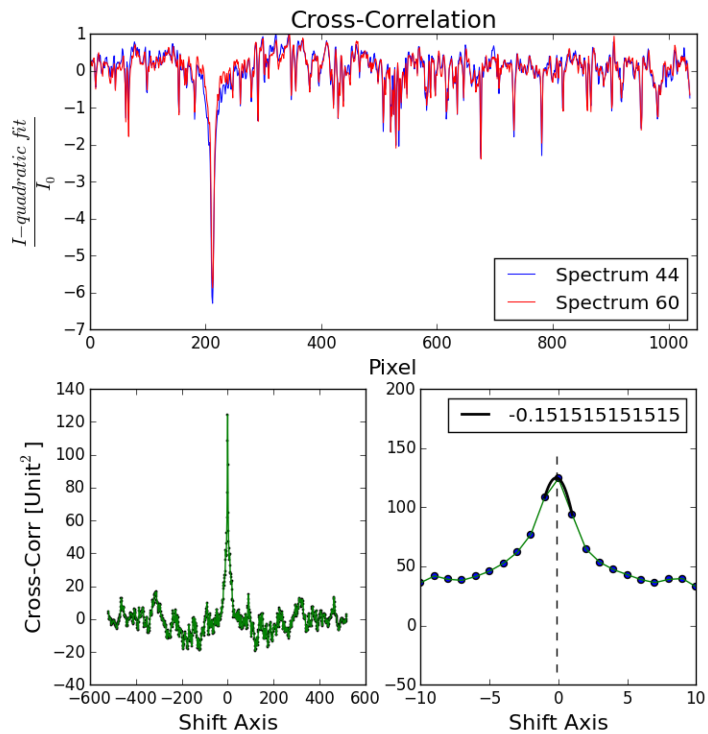


Figure 5: The top is a plot of two spectra (one at t_0 and the other on the limb); the bottom left and right show the cross correlation as a whole and zoomed in, respectively.

3.2 Finding Shifts

Because the Sun has a constant ω value for any point on a shell within or on the surface, and because of the relation of radius to radial velocity given by equation 2(a), we should expect that as the radius from the center of the Sun increases, so must its radial velocity. Based on the fact that the Earth orbits the Sun in a counter-clockwise manner and the Sun rotates counter-clockwise on its axis (from an North bound view), the beginning of the transit should be blue-shifted (coming towards us and negative) and the end should be red-shifted (going away from us and positive). Figure 6 (which is a result of the process described in 3.2.2) shows the beauty of this relation.

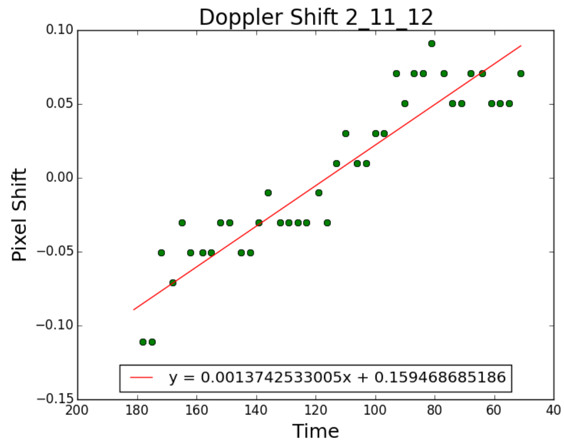


Figure 6: The measured shift from the cross correlation of one spectrum compared to the reference spectrum is plotted as a function of time of Sun2 on 11/12/15. There is a linear trend.

3.2.1 Relative Velocity

Initially, we took the highest shift and use that as our sub-pixel shift in using it for future calculations to determine the Earth-Sun distance. We can use the Doppler equation (equation 1) to find the relative velocity, which is still not the radial velocity we want, but it is a step closer. To obtain the relative velocity, I went through each pixel (1038, again ignoring the last 10) and applied the wavelength calibration for the pixel number plus the shift (referring it as the "calculated" wavelength), and again applied the calibration for just the pixel number (referring it as the "rest" wavelength). Using the Doppler formula (equation 1), I obtained 1038 relative velocities and took an average of them. This was interesting, however, I was not able to continue with the relative velocity (the velocity along our line of sight) because I was unsure of how to transform it into the radial velocity, so I decided to try out a different tactic.

3.2.2 Doppler Slope

I then decided to completely change my approach in applying the cross-correlation method. I believe assuming that the reference spectrum should only consist of the spectrum located in the middle of the Sun's transit was a mistake; instead it would be better to average all solar spectra given an observation to ensure no systematic errors (scattering light effects) are present and use that spectrum as the reference spectrum. This makes sure that there is no Doppler shift at this reference spectrum, so now we can effectively cross-correlate it with all the other spectrum in the Sun's transit and obtain an amazing and expected linear trend (Figure 6). This method makes sense since the points on the limb have very low SNR, and that is why it is important to look at *all* measurements. Each spectrum before cross correlating had a 2^{nd} degree polynomial fit subtracted to flatten the original spectrum, then was divided by the maximum value of the spectrum to ensure consistent intensity between different spectra, and lastly, a window function (equation 11) was applied to ensure the ends of each spectrum go to zero.

We can use the Doppler slope obtained from Figure 6, which is in units of $\frac{\Delta pix}{\Delta t}$ and multiply that by the pixel resolution in units of $\frac{\Delta nm}{\Delta pix}$, which we found when using the wavelength calibration method described in section 1.2. Thus, we can find the Doppler shift in $\frac{nm}{s}$. We essentially have everything we need to now compute the distance, since every other variable can be found using HORIZONS[4].

[4]http://ssd.jpl.nasa.gov/

4 Finding the Earth-Sun Distance

Our job would be too easy if relative velocity is all we needed to plug into the equations and obtain an Earth-Sun distance. We need to take into account other factors, such as the tilted rotating axis of both the Sun and the Earth.

4.1 Coordinate Transformation

If we consider a regular Cartesian coordinate system (x,y,z) and let **R** denote the vector to the Sun, then we can perform two transforming matrices[5] to obtain a (x',y',z') coordinate system (by rotating around the y-axis at an angle ξ), and a (x",y",z") coordinate system (by rotating around the x'-axis at an angle η). Below are the transformation matrices, where (x,y,z) are converted into spherical coordinates.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} cos\xi & 0 & sin\xi \\ 0 & 1 & 0 \\ -sin\xi & 0 & cos\xi \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\eta & cos\eta \\ 0 & -sin\eta & cos\eta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \tag{13}$$

These angles can be found using JPL HORIZONS.

4.2 Results

Now that we have that in mind, we can find the rotational velocity using the expression below,

$$v_{rotational} = \frac{\beta\gamma c 2\Delta t}{\lambda cos\xi cos\eta} \tag{14}$$

where β is the Doppler slope $[\frac{\Delta pix}{\Delta t}]$, γ is the pixel resolution $[\frac{\Delta nm}{\Delta pix}]$, $2\Delta t$ is the total transit time [s], and λ is the center wavelength of the echelle order (for the 34^{th} order, $\lambda \doteq 650$ nm). The center wavelength is more precisely found by applying the linear least squares fit wavelength calibration onto pixel 524 (the center one). I focused on 4 observations and the all the parameters above can be found in the top portion of Table , and the calculated values can be found in the bottom portion.

Then, once we get a rotational velocity, we can plug that into equation 2 to obtain a calculated solar radius where we can plug that into equation 3 to obtain a calculated Earth-Sun distance in terms of kilometers. Again, all the values are computed in the bottom portion of Table .

c $[\frac{km}{s}]$	$R_\odot$ [km]	P [days]	$\theta_\odot$ [deg]
299,792.548	696,300	26.24	.533

Table 2: Constants

Table 4.2 contains all the constants throughout the equations. Choosing the period of the Sun was a little tricky but we decided to go with the synodic period to account for the fact that the Earth is also moving.

[5]Lab 4 Coordinates https://drive.google.com/file/d/0B40Ynk22SiBpX2dvRFZzbEdZQmc/view

Date	ξ $[deg]$	η $[deg]$	Doppler Slope ($\frac{pix}{s}$)	Pixel Res. ($\frac{nm}{pix}$)	$2\Delta t$ (s)
Sun1 11/12	3.25	213.15	0.00122	-0.02188	135
Sun2 11/12	3.25	213.15	0.00137	-0.02188	134
Sun1 11/16	2.79	160.42	0.00125	-0.02188	134
Sun2 11/16	2.79	160.42	0.00165	-0.02188	135
Avg.	3.02	186.79	0.00134	-0.02188	NOOO
Date	Rot. $\mathbf{v}$ ($\frac{km}{s}$)	$R_\odot$ (km)	Measured AU (km)	Actual AU (km)	% Error for AU
Sun1 11/12	1.99472	719749.20	154703354	148103538.536	4.45621
Sun2 11/12	2.22551	803023.07	172602293	148103538.536	16.54164
Sun1 11/16	1.80421	651005.81	139927605	147966546.268	-5.43294
Sun2 11/16	2.39975	865892.07	186115395	147966546.268	25.78207
Avg.	2.10604	759917.55	163337161.75	148035042.4	10.33682

Table 3: This a table of all the results. Note: the rows labeled "Avg." are just the average of the values of the 4 observations. The values in the "Avg." row were NOT inputted into the equations.

Quantity	Measured Average	Known Value	% Error	STD (σ)	SDOM
v_{rot} (km/s)	2.10604	1.995	-5.565	0.26078	0.13039
$R_\odot$ (km)	759,917.55	696,300	-9.1365	94,099.14	47,049.57
dist. (AU)	1.09184	1.0	-9.174	0.13520	0.06760

Table 4: Taking the average results and comparing them to actual ones.

5 Errors

Throughout this lab, there were *many* rooms for error. I feel fairly confident in calibrating the spectrometer for order number 34 because the residuals were so low, but as I was increasing the order, I noticed residuals were increasing as well. When calculating the transit time of the Sun using an Eddington approximation, it was not a very computational guess and check, but rather a manual one where I would plug in numbers and see if they yield better results. That being said, I also feel confident that the transit times are not a major source of error.

The cross correlation part became a little tricky because we were doing multiple things to manipulate the spectra including: taking all of them and averaging them to create a "reference" spectrum, subtracting the average of the non-transit spectra, subtracting a second order polynomial fit to flatten spectra, applying a window function, finding the shift by just fitting the highest three values with a second order polynomial and finding the max. Using a higher order polynomial to flatten the spectra may have yielded better results, or using a Gaussian fit on the cross correlation may have provided a more precise value for the shift.

Lastly, when calculating the rotational velocity, we were inputting all these values with slight errors because of our assumptions, which would give rise to an error bar on the velocity, and perhaps that is why our measured velocity is not *that* accurate to the actual one. Additionally, before I just calculated the rotational velocity, I was calculating the relative velocity (the velocity on our line of sight) and I was applying the max pixel shift (obtained from Figure 5) to each pixel and I was finding that the velocities are not consistent throughout all the pixels. With that type of error on the velocity, it carries throughout the other equations up to when we find the Earth-Sun distance.

Conclusion

Even though it seems like many errors were present, I believe we did an amazing job finding the Earth-Sun distance given the fact that we had to deal with a CCD detector whose resolving power was not strong enough to the point where we had to go on this journey of finding a sub-pixel shift. Honestly, I am very happy and proud of our result. I liked how we applied our knowledge and experience on previous labs, such as calibrating wavelength and finding that a quadratic fit is best, or applying error propagation. It made the process very familiar and now it is embedded in my mind. Overall, I learned the importance of cross correlation and how it can pick up subtle differences, as well as the importance of resolving power and how sometimes two objects may look like one if a low-resolution detector is used.