Homework 1 – DSC 240
Winter 2026
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Matrix Computations

(a) $(3B)^T$

First multiply B by 3:

$$3B = \begin{bmatrix} 0 & 12 \\ 21 & 18 \\ 15 & 24 \\ 9 & 33 \end{bmatrix}$$

Now take the transpose:

$$(3B)^T = \begin{bmatrix} 0 & 21 & 15 & 9 \\ 12 & 18 & 24 & 33 \end{bmatrix}$$

(b) $(A - B)^T$

Matrix subtraction is only defined when the two matrices have the same dimensions.
Here,

$$A \text{ is } 2 \times 4, \qquad B \text{ is } 4 \times 2.$$

Since the dimensions do not match, the subtraction $A - B$ is not defined. Therefore,

$$(A - B)^T \text{ is not possible.}$$

(c) $(2B^T - A)^T$

First compute the transpose of B:

$$B^T = \begin{bmatrix} 0 & 7 & 5 & 3 \\ 4 & 6 & 8 & 11 \end{bmatrix}$$

Multiply by 2:

$$2B^T = \begin{bmatrix} 0 & 14 & 10 & 6 \\ 8 & 12 & 16 & 22 \end{bmatrix}$$

Now subtract A:

$$2B^T - A = \begin{bmatrix} -4 & 13 & 7 & 0 \\ 6 & 5 & 11 & 19 \end{bmatrix}$$

Finally, take the transpose:

$$(2B^T - A)^T = \begin{bmatrix} -4 & 6 \\ 13 & 5 \\ 7 & 11 \\ 0 & 19 \end{bmatrix}$$

(d) $(C + 2D^T + E)^T$

We check the dimensions of each matrix:

$$C \text{ is } 3 \times 3, \qquad 2D^T \text{ is } 3 \times 3, \qquad E \text{ is } 2 \times 2.$$

Since a 2×2 matrix cannot be added to a 3×3 matrix, the expression

$$C + 2D^T + E$$

is not defined. Therefore, $(C + 2D^T + E)^T$ is not possible.

(e) $(-A)^T E$

First compute the transpose of A and negate it:

$$(-A)^T = \begin{bmatrix} -4 & -2 \\ -1 & -7 \\ -3 & -5 \\ -6 & -3 \end{bmatrix}$$

Now multiply $(-A)^T$ by E.
Row 1:

$$(-4)(-4) + (-2)(12) = -8, \qquad (-4)(5) + (-2)(7) = -34$$

Row 2:

$$(-1)(-4) + (-7)(12) = -80, \qquad (-1)(5) + (-7)(7) = -54$$

Row 3:

$$(-3)(-4) + (-5)(12) = -48, \qquad (-3)(5) + (-5)(7) = -50$$

Row 4:

$$(-6)(-4) + (-3)(12) = -12, \qquad (-6)(5) + (-3)(7) = -51$$

Thus,

$$(-A)^T E = \begin{bmatrix} -8 & -34 \\ -80 & -54 \\ -48 & -50 \\ -12 & -51 \end{bmatrix}$$

Problem 2

No, $AB \neq BA$.
If we compute AB and BA and then compare, we will be able to see why.

Compute AB

The columns of B are

$$c_1 = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}, \quad c_3 = \begin{bmatrix} 3 \\ 7 \\ -3 \end{bmatrix}.$$

The rows of A are

$$r_1 = [2, 7, 3], \quad r_2 = [1, 0, 9], \quad r_3 = [-1, 2, 10].$$

Row 1:

$$\begin{aligned}
(AB)_{11} &= r_1 \cdot c_1 = 2(-2) + 7(2) + 3(6) = -4 + 14 + 18 = 28, \\
(AB)_{12} &= r_1 \cdot c_2 = 2(0) + 7(-1) + 3(4) = 0 - 7 + 12 = 5, \\
(AB)_{13} &= r_1 \cdot c_3 = 2(3) + 7(7) + 3(-3) = 6 + 49 - 9 = 46.
\end{aligned}$$

Row 2:

$$\begin{aligned}
(AB)_{21} &= r_2 \cdot c_1 = 1(-2) + 0(2) + 9(6) = -2 + 0 + 54 = 52, \\
(AB)_{22} &= r_2 \cdot c_2 = 1(0) + 0(-1) + 9(4) = 0 + 0 + 36 = 36, \\
(AB)_{23} &= r_2 \cdot c_3 = 1(3) + 0(7) + 9(-3) = 3 + 0 - 27 = -24.
\end{aligned}$$

Row 3:

$$\begin{aligned}
(AB)_{31} &= r_3 \cdot c_1 = (-1)(-2) + 2(2) + 10(6) = 2 + 4 + 60 = 66, \\
(AB)_{32} &= r_3 \cdot c_2 = (-1)(0) + 2(-1) + 10(4) = 0 - 2 + 40 = 38, \\
(AB)_{33} &= r_3 \cdot c_3 = (-1)(3) + 2(7) + 10(-3) = -3 + 14 - 30 = -19.
\end{aligned}$$

Therefore,

$$AB = \begin{bmatrix} 28 & 5 & 46 \\ 52 & 36 & -24 \\ 66 & 38 & -19 \end{bmatrix}.$$

Compute BA

The columns of A are

$$a_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 7 \\ 0 \\ 2 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 3 \\ 9 \\ 10 \end{bmatrix}.$$

The rows of B are

$$s_1 = [-2, 0, 3], \quad s_2 = [2, -1, 7], \quad s_3 = [6, 4, -3].$$

Row 1:

$$\begin{aligned}
(BA)_{11} &= s_1 \cdot a_1 = (-2)(2) + 0(1) + 3(-1) = -4 + 0 - 3 = -7, \\
(BA)_{12} &= s_1 \cdot a_2 = (-2)(7) + 0(0) + 3(2) = -14 + 0 + 6 = -8, \\
(BA)_{13} &= s_1 \cdot a_3 = (-2)(3) + 0(9) + 3(10) = -6 + 0 + 30 = 24.
\end{aligned}$$

Row 2:

$$\begin{aligned}
(BA)_{21} &= s_2 \cdot a_1 = 2(2) + (-1)(1) + 7(-1) = 4 - 1 - 7 = -4,\\
(BA)_{22} &= s_2 \cdot a_2 = 2(7) + (-1)(0) + 7(2) = 14 + 0 + 14 = 28,\\
(BA)_{23} &= s_2 \cdot a_3 = 2(3) + (-1)(9) + 7(10) = 6 - 9 + 70 = 67.
\end{aligned}$$

Row 3:

$$\begin{aligned}
(BA)_{31} &= s_3 \cdot a_1 = 6(2) + 4(1) + (-3)(-1) = 12 + 4 + 3 = 19,\\
(BA)_{32} &= s_3 \cdot a_2 = 6(7) + 4(0) + (-3)(2) = 42 + 0 - 6 = 36,\\
(BA)_{33} &= s_3 \cdot a_3 = 6(3) + 4(9) + (-3)(10) = 18 + 36 - 30 = 24.
\end{aligned}$$

Therefore,

$$BA = \begin{bmatrix} -7 & -8 & 24 \\ -4 & 28 & 67 \\ 19 & 36 & 24 \end{bmatrix}.$$

Since

$$(AB)_{11} = 28 \neq -7 = (BA)_{11},$$

we have $AB \neq BA$. Hence, the matrices do *not* commute.

Problem 3

For a vector $x = (x_1, x_2, x_3)$, the norms are defined by

$$\|x\|_1 = |x_1| + |x_2| + |x_3|, \qquad \|x\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2}, \qquad \|x\|_\infty = \max\{|x_1|, |x_2|, |x_3|\}.$$

1) $x = [0, 0, 0]$

$$\|x\|_1 = |0| + |0| + |0| = 0, \qquad \|x\|_2 = \sqrt{0^2 + 0^2 + 0^2} = 0, \qquad \|x\|_\infty = \max\{0, 0, 0\} = 0.$$

2) $x = [1, 2, 3]$

$$\begin{aligned}
\|x\|_1 &= |1| + |2| + |3| = 1 + 2 + 3 = 6,\\
\|x\|_2 &= \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14},\\
\|x\|_\infty &= \max\{1, 2, 3\} = 3.
\end{aligned}$$

3) $x = [2, 4, 6]$

$$\begin{aligned}
\|x\|_1 &= |2| + |4| + |6| = 2 + 4 + 6 = 12,\\
\|x\|_2 &= \sqrt{2^2 + 4^2 + 6^2} = \sqrt{4 + 16 + 36} = \sqrt{56} = 2\sqrt{14},\\
\|x\|_\infty &= \max\{2, 4, 6\} = 6.
\end{aligned}$$

Relationship between norms of $[2, 4, 6]$ and $[1, 2, 3]$

Observe that

$$[2, 4, 6] = 2[1, 2, 3].$$

A key property of norms is the scaling rule:

$$\|cx\| = |c|\,\|x\| \quad \text{for any scalar } c.$$

Therefore, each norm doubles when the vector is multiplied by 2:

$$\|[2, 4, 6]\|_1 = 2\|[1, 2, 3]\|_1 = 2 \cdot 6 = 12,$$

$$\|[2, 4, 6]\|_2 = 2\|[1, 2, 3]\|_2 = 2\sqrt{14},$$

$$\|[2, 4, 6]\|_\infty = 2\|[1, 2, 3]\|_\infty = 2 \cdot 3 = 6.$$

Can a norm be negative?

No. Norms cannot be negative because they are defined using absolute values and (for ℓ_2) squares, so

$$\|x\| \geq 0 \quad \text{for all vectors } x,$$

and

$$\|x\| = 0 \iff x = 0.$$

Problem 4

Proof

Since $X \in \mathbb{R}^{m\times n}$ and $Y^T \in \mathbb{R}^{p\times n}$, we have $Y \in \mathbb{R}^{n\times p}$ and therefore $XY \in \mathbb{R}^{m\times p}$.

Let $(XY)_{ab}$ denote the (a, b) entry of XY (where $1 \leq a \leq m$ and $1 \leq b \leq p$). By the definition of matrix multiplication,

$$(XY)_{ab} = \sum_{i=1}^{n} X_{ai}\, Y_{ib}.$$

Because x_i is the i-th column of X, its a-th entry satisfies

$$(x_i)_a = X_{ai}.$$

Also, since $Y^T = [y_1, \ldots, y_n]$, the i-th row of Y is y_i^T, so the (i, b) entry of Y satisfies

$$(y_i)_b = Y_{ib}.$$

Substituting these into the multiplication formula gives

$$(XY)_{ab} = \sum_{i=1}^{n} (x_i)_a (y_i)_b.$$

Now consider the outer product $x_i y_i^T \in \mathbb{R}^{m\times p}$. Its (a,b) entry is

$$(x_i y_i^T)_{ab} = (x_i)_a (y_i)_b.$$

Therefore,

$$(XY)_{ab} = \sum_{i=1}^{n} (x_i y_i^T)_{ab}.$$

Since this holds for every a and b, the matrices are equal:

$$XY = \sum_{i=1}^{n} x_i y_i^T.$$

Problem 5

Let $X \in \mathbb{R}^{m\times n}$. Show that $X^T X$ is symmetric and positive semidefinite. Determine when it is positive definite.

(i) Symmetry

Using $(AB)^T = B^T A^T$ and $(X^T)^T = X$,

$$(X^T X)^T = X^T (X^T)^T = X^T X.$$

Hence $X^T X$ is symmetric.

(ii) Positive semidefinite

Let $v \in \mathbb{R}^n$ be arbitrary. Then

$$v^T (X^T X) v = (v^T X^T)(Xv).$$

Since $(Xv)^T = v^T X^T$, we have

$$v^T (X^T X) v = (Xv)^T (Xv) = \|Xv\|_2^2 \geq 0.$$

Therefore, $X^T X$ is positive semidefinite.

(iii) When is $X^T X$ positive definite?

The matrix $X^T X$ is positive definite if and only if

$$v \neq 0 \implies v^T (X^T X) v > 0.$$

But

$$v^T (X^T X) v = \|Xv\|_2^2,$$

so $v^T (X^T X) v > 0$ for all $v \neq 0$ if and only if $Xv \neq 0$ for all $v \neq 0$. Equivalently,

$$\mathrm{Null}(X) = \{0\}.$$

This holds exactly when the columns of X are linearly independent, i.e. when X has full column rank:

$$\mathrm{rank}(X) = n \quad \text{(in particular, this requires } m \geq n\text{)}.$$

Hence, $X^T X$ is positive definite if and only if X has full column rank.

Problem 6

Partial derivative with respect to x

$$\frac{\partial}{\partial x}\left(e^{x+y}\right) = e^{x+y} \cdot \frac{\partial}{\partial x}(x+y) = e^{x+y},$$

$$\frac{\partial}{\partial x}\left(e^{3xy}\right) = e^{3xy} \cdot \frac{\partial}{\partial x}(3xy) = e^{3xy} \cdot 3y = 3y\, e^{3xy},$$

$$\frac{\partial}{\partial x}\left(e^{y^4}\right) = 0.$$

$$\boxed{\frac{\partial g}{\partial x} = e^{x+y} + 3y\, e^{3xy}}.$$

Partial derivative with respect to y

$$\frac{\partial}{\partial y}\left(e^{x+y}\right) = e^{x+y} \cdot \frac{\partial}{\partial y}(x+y) = e^{x+y},$$

$$\frac{\partial}{\partial y}\left(e^{3xy}\right) = e^{3xy} \cdot \frac{\partial}{\partial y}(3xy) = e^{3xy} \cdot 3x = 3x\, e^{3xy},$$

$$\frac{\partial}{\partial y}\left(e^{y^4}\right) = e^{y^4} \cdot \frac{\partial}{\partial y}(y^4) = e^{y^4} \cdot 4y^3 = 4y^3 e^{y^4}.$$

$$\boxed{\frac{\partial g}{\partial y} = e^{x+y} + 3x\, e^{3xy} + 4y^3 e^{y^4}}.$$

Problem 7

(a) Eigenvalues

First start by computing the characteristic polynomial:

$$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda) - 8.$$

Expand:

$$(1-\lambda)(3-\lambda) = 3 - 4\lambda + \lambda^2,$$

so

$$\det(A - \lambda I) = \lambda^2 - 4\lambda - 5.$$

Solve $\lambda^2 - 4\lambda - 5 = 0$:

$$\lambda = \frac{4 \pm \sqrt{16+20}}{2} = \frac{4 \pm 6}{2} \in \{5, -1\}.$$

Thus the eigenvalues are

$$\boxed{\lambda_1 = 5, \qquad \lambda_2 = -1.}$$

Eigenvectors

$$\boxed{v_1 = \begin{bmatrix}1\\1\end{bmatrix}, v_2 = \begin{bmatrix}-2\\1\end{bmatrix}}$$

Python code

```
import numpy as np

A = np.array([[1., 4.],
              [2., 3.]])

# For lambda = 5: y = x  -> choose x=1
v1 = np.array([1., 1.])

# For lambda = -1: x = -2y -> choose y=1
v2 = np.array([-2., 1.])

print("lambda = 5, eigenvector =", v1)
print("lambda = -1, eigenvector =", v2)
```

(b) Eigen-decomposition of A

Using the eigenvectors as columns,

$$V = \begin{bmatrix}1 & -2\\1 & 1\end{bmatrix}, \qquad \Lambda = \begin{bmatrix}5 & 0\\0 & -1\end{bmatrix}.$$

The determinant of V is

$$\det(V) = 1 \cdot 1 - (-2) \cdot 1 = 3 \neq 0,$$

so V is invertible. Its inverse is

$$V^{-1} = \frac{1}{3}\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}.$$

Therefore the eigen-decomposition is

$$\boxed{A = V\Lambda V^{-1} = \begin{bmatrix}1 & -2\\1 & 1\end{bmatrix}\begin{bmatrix}5 & 0\\0 & -1\end{bmatrix}\frac{1}{3}\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}}.$$

(c) Rank of A

Compute $\det(A)$:

$$\det(A) = \det\begin{bmatrix}1 & 4\\2 & 3\end{bmatrix} = 1 \cdot 3 - 4 \cdot 2 = 3 - 8 = -5 \neq 0.$$

Since the determinant is nonzero, A is invertible and has full rank:

$$\boxed{\mathrm{rank}(A) = 2.}$$

(d) Is A positive definite?

Positive definiteness is defined for *symmetric* matrices. But,

$$A^T = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \neq A,$$

so A is not symmetric and therefore is *not* positive definite in the usual sense. Also, A has an eigenvalue $-1 < 0$, which is incompatible with positive definiteness even in the symmetric case.

(e) Is A positive semidefinite?

Similarly, positive semidefiniteness is typically defined for symmetric matrices; A is not symmetric. Moreover, the eigenvalue $-1 < 0$ rules out positive semidefiniteness in the symmetric case.

(f) Is A singular?

A matrix is singular iff $\det(A) = 0$. Since $\det(A) = -5 \neq 0$,

Problem 8

(a) Separation Line, Slope, and Intercept

The decision boundary between the class $+1$ and class -1 is defined where the value of the discriminant function is zero:

$$\mathbf{w}^T\mathbf{x} = 0$$

Expanding the dot product using the given vectors:

$$\begin{bmatrix} w_0 & w_1 & w_2 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = 0$$

$$w_0 \cdot 1 + w_1 \cdot x_1 + w_2 \cdot x_2 = 0$$

To find the slope and intercept, we rearrange the equation to solve for x_2 in terms of x_1 (assuming $w_2 \neq 0$):

$$w_2 x_2 = -w_1 x_1 - w_0$$

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$$

Comparing this to the line equation $x_2 = ax_1 + b$, we can identify the slope a and intercept b as:

$$\boxed{a = -\frac{w_1}{w_2}, \quad b = -\frac{w_0}{w_2}}$$

(b) Python Visualization

Analytically, both weight vectors result in the same decision boundary line:

$$x_2 = -\frac{2}{3}x_1 - \frac{1}{3}$$

Although the decision boundary remains identical, the normal vector's direction is inverted, effectively swapping the positive and negative half-spaces. (I used ChatGBT to help me code specifically I wrote my own code and had it correct misstates and add to it)

The following Python code generates the visualization:

```
import numpy as np
import matplotlib.pyplot as plt

def plot_linear_classifier():
    # Define the weights
    w_case1 = np.array([1, 2, 3])   # w = [1, 2, 3]
    w_case2 = -np.array([1, 2, 3])  # w = -[1, 2, 3]

    # Create x1 values for the line range
    x1 = np.linspace(-5, 5, 100)

    # Calculate x2 using formula derived in Part (a)
    slope = -w_case1[1] / w_case1[2]      # -2/3
    intercept = -w_case1[0] / w_case1[2]  # -1/3

    x2 = slope * x1 + intercept

    # Plot setup
    plt.figure(figsize=(8, 6))

    # 1. Plot the decision boundary line
    plt.plot(x1, x2, 'k-', linewidth=2,
             label=f'Boundary: $x_2 = {slope:.2f}x_1 {intercept:.2f}
                $')

    # 2. Visualize the Normal Vectors (w)
    mid_idx = len(x1) // 2
    origin = (x1[mid_idx], x2[mid_idx])

    # Vector for Case 1 (Blue)
    plt.quiver(*origin, w_case1[1], w_case1[2], color='blue', scale
        =20,
               label='w = [1, 2, 3]', width=0.005)

    # Vector for Case 2 (Red)
    plt.quiver(*origin, w_case2[1], w_case2[2], color='red', scale
        =20,
```

```
                 label='w␣=␣-[1,␣2,␣3]', width=0.005)

    # Graph formatting
    plt.axhline(0, color='gray', linestyle='--', alpha=0.5)
    plt.axvline(0, color='gray', linestyle='--', alpha=0.5)
    plt.xlabel('$x_1$')
    plt.ylabel('$x_2$')
    plt.title('Linear␣Classifier␣Boundaries␣and␣Normal␣Vectors')
    plt.legend()
    plt.grid(True)
    plt.axis('equal')
    plt.show()

if __name__ == "__main__":
    plot_linear_classifier()
```

Problem 9

(a) Target Function Identification

From the `GenerateData` function:

```
if point[0] + point[1] - margin > 0:
```

The condition separating the classes relies on the sum of coordinates. The "perfect" boundary lies exactly between the positive and negative regions, which occurs when

$$\texttt{point[0] + point[1]} = 0$$

(ignoring the margin gap). Therefore, the target function f corresponds to the line

$$x_1 + x_2 = 0, \quad \text{or equivalently} \quad x_2 = -x_1.$$

(b) Analysis for $N = 20$ (Run 1)

Closeness of f and g: Since we only have 20 data points, there are big gaps in the data. The perceptron stops updating the moment it finds a line that works, rather than trying to find the 'best' line in the middle. So, even though my hypothesis g gets 100% accuracy on the training data, the line itself is slightly tilted compared to the true target f.

(c) Analysis for $N = 20$ (Run 2)

Reason: The perceptron algorithm is sensitive to the specific order and location of data points. A different random seed produces different points, some of which might be "harder" (closer to the boundary) to classify, requiring more updates to fine-tune the weights.

(d) Analysis for $N = 100$

I observed an increase in the number of updates compared to when $N = 20$. This makes sense because having more data points adds more constraints, so the algorithm has to work harder to find a line that correctly classifies everyone. Also, since the "gap" between the classes is now filled with more points, there is less room for the decision boundary to vary. This forces the hypothesis g to align much better with the true target f, making them look nearly parallel compared to the previous run.

(e) Analysis for $N = 1{,}000$

When I increased the dataset size to 1,000 points, the number of updates skyrocketed. With so many points, the valid region for the decision boundary becomes narrow. As a result, the final hypothesis g is extremely close to the true target f; visually, the dashed black line and the solid green line are almost indistinguishable.

Conclusion

As N increases, the Perceptron works harder (more updates) but produces a much more accurate estimate of the true target function.

Question 10

(a) Type of Problem

This is a Supervised Learning problem, since the inputs (features like age, weight) and the desired answers (blood pressure numbers) are provided. Furthermore, because the target values (blood pressure) are real-valued numbers (continuous) rather than discrete categories, this is specifically a Regression problem.

(b) Label Space

Since we are estimating two numbers, systolic and diastolic, which are real-valued, the label space consists of pairs of real numbers. So the label space is $\mathcal{Y} = \mathbb{R}^2$ (or more precisely, $\mathbb{R}_+ \times \mathbb{R}_+$ since blood pressure must be positive.

(c) Output Space

The "output space" represents the set of all possible predictions the model can generate. In a standard regression setting, the model attempts to predict the target values directly. Therefore, the output space is identical to the label space: the model outputs a pair of real numbers representing the predicted systolic and diastolic blood pressure.

Problem 11

We are evaluating a spam detection system using a test set of 10,000 emails.

- **Actual Spam (P):** 2,000
- **Actual Non-Spam (N):** 8,000

Based on the problem description, we can identify the confusion matrix components:

- **False Negatives (FN):** 250 (Spam classified as Non-Spam)
- **True Positives (TP):** $2,000 - 250 = 1,750$ (Spam classified as Spam)
- **False Positives (FP):** 250 (Non-Spam classified as Spam)
- **True Negatives (TN):** $8,000 - 250 = 7,750$ (Non-Spam classified as Non-Spam)

(a) Confusion Matrix

Below is the confusion matrix for the binary classification experiment (Positive = Spam, Negative = Non-Spam).

	Predicted Spam	**Predicted Non-Spam**
Actual Spam	TP = 1,750	FN = 250
Actual Non-Spam	FP = 250	TN = 7,750

Table 1: Confusion Matrix

(b) False Positive Rate (FPR)

The False Positive Rate is the proportion of actual negatives that are incorrectly identified as positives.

$$\text{FPR} = \frac{\text{FP}}{N} = \frac{250}{8,000} = \frac{1}{32} = \boxed{0.03125}$$

(c) False Negative Rate (FNR)

The False Negative Rate is the proportion of actual positives that are incorrectly identified as negatives.

$$\text{FNR} = \frac{\text{FN}}{P} = \frac{250}{2,000} = \frac{1}{8} = \boxed{0.125}$$

(d) Error Rate

The Error Rate is the proportion of total predictions that were incorrect.

$$\text{Error Rate} = \frac{\text{FP} + \text{FN}}{\text{Total Test Set}} = \frac{250 + 250}{10,000} = \frac{500}{10,000} = \boxed{0.05}$$

(e) Precision

Precision is the proportion of predicted positives that are actually positive.

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} = \frac{1,750}{1,750 + 250} = \frac{1,750}{2,000} = \boxed{0.875}$$

(f) Sensitivity

Sensitivity or True Positive Rate is the proportion of actual positives that are correctly identified.

$$\text{Sensitivity} = \frac{\text{TP}}{P} = \frac{1,750}{2,000} = \boxed{0.875}$$