Pokropek Ernest #33

Numerical Methods (ENUME 2019) – Project
Assignment B: Approximation of functions

1. Make a graph of the function $f(x)=\left(x+\frac{1}{3}\right)^2+e^{-x-2}$ for $x\in[-1,1]$ and indicate a sequence of its values which will be next used for approximation:

$$\left\{y_n=f(x_n)\middle|\, n=1,2,...,N\right\},\ \text{where}\ x_n=-1+2\frac{n-1}{N-1}$$

Repeat this exercise for $N=10,\ 20\ \text{and}\ 30$.

2. Develop a program for the least-squares approximation of the function $f(x)$ on the basis of the data $\left\{(x_n,\,y_n)\middle|\, n=1,...,N\right\}$, using the operator of pseudoinversion "\" implemented in MATLAB. Use the B-spline functions definedby the formula:

$$Bs_k(x)=B_S\left(2(x-x'_k)+2\right)\qquad B_S(x)=\begin{cases}x^3 & x\in[0,1)\\ -3(x-1)^3+3(x-1)^2+3(x-1)+1 & x\in[1,2)\\ 3(x-2)^3-6(x-2)^2+4 & x\in[2,3)\\ -(x-3)^3+3(x-3)^2-3(x-3)+1 & x\in[3,4]\end{cases}$$

where $x'_k=-1+2\frac{k-1}{K-1}$ for $k=1,2,\ldots,K$

as a basis of linearly independent functions. Check the correctness of the program for several pairs of the values of N and K. Add the results of approximation to the corresponding graphs made according to the instruction provided in Section 1.

3. Carry out a systematic investigation of the dependence of the accuracy of approximation on the values of N and K. Use the following accuracy indicators for this purpose:

$$\delta_2(K,N)=\frac{\left\|\hat{f}(x;K,N)-f(x)\right\|_2}{\left\|f(x)\right\|_2}\quad \text{(the root-mean-square error)}$$

$$\delta_\infty(K,N)=\frac{\left\|\hat{f}(x;K,N)-f(x)\right\|_\infty}{\left\|f(x)\right\|_\infty}\quad \text{(the maximum error)}$$

where $\hat{f}(x;K,N)$ is an approximating function obtained for N and K. Make the three-dimensional graphs of the functions $\delta_2(K,N)$ and $\delta_\infty(K,N)$ for $N\in\{5,...,50\}$ and $K<N$.

4. Carry out a systematic investigation of the dependence of the indicators $\delta_2(K,N)$ and $\delta_\infty(K,N)$ on the standard deviation $\sigma_y\in\left[10^{-5},10^{-1}\right]$ of random errors the data used for approximation are corrupted with. For this purpose:

- Generate the error-corrupted data according to the formula:
  $\tilde{y}_n=y_n+\Delta\tilde{y}_n$ for $n=1,...,N$
  where $\{\Delta\tilde{y}_n\}$ are pseudorandom numbers following the zero-mean normal distribution with the variance σ_y^2, obtained by means of the MATLAB operator ***randn***.
- For each value of the standard deviation σ_y, determine the values $\breve{N}$ and $\breve{K}$ minimising $\delta_2(K,N)$ and compute $\delta_{2,MIN}(\sigma_y)\equiv\delta_2(\breve{K},\breve{N})$.
- Approximate the sequence of pairs $\left\langle\sigma_y,\,\delta_{2,MIN}(\sigma_y)\right\rangle$, determined for several dozen values of σ_y, by means of the MATLAB operator ***polyfit***; present the result of approximation using the logarithmic scale on both axes.