## Szymon Czupryński #33

## Numerical Methods (ENUME 2019) – Project Assignment A: Solving linear algebraic equations

1. Design a procedure for generation of the following matrices:

Besign a procedure for generation of the fonowing matrices.							
	$\int x^2$	$-\frac{3x}{2}$	$\frac{3x}{2}$	$-\frac{3x}{2}$	•••	$\frac{3x}{(-1)^{N-2}\cdot 2}$	$\frac{3x}{(-1)^{N-1}\cdot 2}$
	$-\frac{3x}{}$	18	18	18		18	18
	$-{2}$	4	4	4		$\overline{(-1)^{N-3}\cdot 4}$	
	$\frac{3x}{2}$	$-\frac{18}{4}$	$\frac{27}{4}$	$-\frac{27}{4}$	•••	$\frac{27}{(-1)^{N-4}\cdot 4}$	$\frac{27}{\left(-1\right)^{N-3}\cdot 4}$
$\mathbf{A}_{N,x} =$	$-\frac{3x}{2}$	$\frac{18}{4}$	$-\frac{27}{4}$	$\frac{36}{4}$	•••	$\frac{36}{(-1)^{N-5}\cdot 4}$	$\frac{36}{(-1)^{N-4}\cdot 4}$
	$\frac{3x}{2}$	$-\frac{18}{4}$	$\frac{27}{4}$	$-\frac{36}{4}$	•••	÷	÷
	:	:	:	:		$\frac{(N-1)\cdot 9}{4}$	$-\frac{(N-1)\cdot 9}{4}$
	$\frac{3x}{(-1)^{N-1}\cdot 2}$	$\frac{18}{\left(-1\right)^{N-2}\cdot 4}$	$\frac{27}{(-1)^{N-3}\cdot 4}$	$\frac{36}{(-1)^{N-4}\cdot 4}$	•••	$-\frac{(N-1)\cdot 9}{4}$	$\frac{N \cdot 9}{4}$
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- 2. For each matrix  $\mathbf{A}_{N,x}$ , generated for  $N \in \{3, 10, 20\}$  and  $x = \sqrt{\alpha^2 + \frac{1}{2}} 1$ :
  - determine the smallest positive value  $\alpha_N$  of  $\alpha$  which yields  $\det(\mathbf{A}_{N,x}) = 0$ ;
  - draw the dependence of  $\det(\mathbf{A}_{N,x})$  on  $\alpha$  for  $\alpha \in [\alpha_N 0.01, \alpha_N + 0.01]$ ;
  - draw the dependence of cond  $(\mathbf{A}_{N,x})$  on  $\alpha$  for  $\alpha \in [\alpha_N 0.01, \alpha_N + 0.01]$ .
- 3. Design a procedure for inverting the matrix  $\mathbf{A}_{N,x}$  according to the scheme presented on the lecture slide #3-16 in two versions: (a) based on the LU factorisation, (b) based on the LLT factorisation. Check the correctness of this procedure using several low-dimensional positive definite matrices.
- 4. Apply the above procedure for finding the estimates  $\hat{\mathbf{A}}_{N,x}^{-1}$  of the matrices  $\mathbf{A}_{N,x}$  generated for  $N \in \{3, 10, 20\}$  and  $x = \frac{2^k}{300}$  with  $k \in \{0, 1, 2, ..., 21\}$ .
- 5. For each estimate  $\hat{\mathbf{A}}_{N,x}^{-1}$  determine the following indicators of its uncertainty:

$$\delta_2 = \left\| \mathbf{A}_{N,x} \cdot \hat{\mathbf{A}}_{N,x}^{-1} - \mathbf{I}_N \right\|_2$$
 (the root-mean-square error)

$$\delta_{\infty} = \left\| \mathbf{A}_{N,x} \cdot \hat{\mathbf{A}}_{N,x}^{-1} - \mathbf{I}_{N} \right\|_{\infty}$$
 (the maximum error)

Compute the norms of the matrices according to the formulae presented on the lecture slide #1-15 (compare the norms obtained in this way with the corresponding norms computed by means of the operator *norm* implemented in MATLAB). Compare the estimates  $\hat{\mathbf{A}}_{N,x}^{-1}$ , obtained by means of the procedure defined in Section 3, with the estimates obtained by means of the operator of matrix inversion *inv* implemented in MATLAB. Draw the dependence of  $\delta_2$  and  $\delta_\infty$  on x.