Pokropek Ernest #33

Numerical Methods (ENUME 2019) – Project
Assignment A: Solving linear algebraic equations

1. Design a procedure for generation of the following matrices:

$$\mathbf{A}_{N,x}=\begin{bmatrix} x^2 & -\frac{3x}{2} & \frac{3x}{2} & -\frac{3x}{2} & \cdots & \frac{3x}{(-1)^{N-2}\cdot 2} & \frac{3x}{(-1)^{N-1}\cdot 2} \\ -\frac{3x}{2} & \frac{18}{4} & -\frac{18}{4} & \frac{18}{4} & \cdots & \frac{18}{(-1)^{N-3}\cdot 4} & \frac{18}{(-1)^{N-2}\cdot 4} \\ \frac{3x}{2} & -\frac{18}{4} & \frac{27}{4} & -\frac{27}{4} & \cdots & \frac{27}{(-1)^{N-4}\cdot 4} & \frac{27}{(-1)^{N-3}\cdot 4} \\ -\frac{3x}{2} & \frac{18}{4} & -\frac{27}{4} & \frac{36}{4} & \cdots & \frac{36}{(-1)^{N-5}\cdot 4} & \frac{36}{(-1)^{N-4}\cdot 4} \\ \frac{3x}{2} & -\frac{18}{4} & \frac{27}{4} & -\frac{36}{4} & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \frac{(N-1)\cdot 9}{4} & -\frac{(N-1)\cdot 9}{4} \\ \frac{3x}{(-1)^{N-1}\cdot 2} & \frac{18}{(-1)^{N-2}\cdot 4} & \frac{27}{(-1)^{N-3}\cdot 4} & \frac{36}{(-1)^{N-4}\cdot 4} & \cdots & -\frac{(N-1)\cdot 9}{4} & \frac{N\cdot 9}{4} \end{bmatrix}$$

2. For each matrix $\mathbf{A}_{N,x}$, generated for $N\in\{3, 10, 20\}$ and $x=\sqrt{\alpha^2+\frac{1}{2}}-1$:
   - determine the smallest positive value α_N of α which yields $\det\left(\mathbf{A}_{N,x}\right)=0$;
   - draw the dependence of $\det\left(\mathbf{A}_{N,x}\right)$ on α for $\alpha\in\left[\alpha_N-0.01, \alpha_N+0.01\right]$;
   - draw the dependence of $\operatorname{cond}\left(\mathbf{A}_{N,x}\right)$ on α for $\alpha\in\left[\alpha_N-0.01, \alpha_N+0.01\right]$.
3. Design a procedure for inverting the matrix $\mathbf{A}_{N,x}$ according to the scheme presented on the lecture slide #3-16 – in two versions: (a) based on the LU factorisation, (b) based on the LLT factorisation. Check the correctness of this procedure using several low-dimensional positive definite matrices.
4. Apply the above procedure for finding the estimates $\hat{\mathbf{A}}_{N,x}^{-1}$ of the matrices $\mathbf{A}_{N,x}$ generated for $N\in\{3, 10, 20\}$ and $x=\frac{2^k}{300}$ with $k\in\{0,1,2,\ldots,21\}$.
5. For each estimate $\hat{\mathbf{A}}_{N,x}^{-1}$ determine the following indicators of its uncertainty:

   $\delta_2=\left\|\mathbf{A}_{N,x}\cdot\hat{\mathbf{A}}_{N,x}^{-1}-\mathbf{I}_N\right\|_2$ (the root-mean-square error)

   $\delta_\infty=\left\|\mathbf{A}_{N,x}\cdot\hat{\mathbf{A}}_{N,x}^{-1}-\mathbf{I}_N\right\|_\infty$ (the maximum error)

   Compute the norms of the matrices according to the formulae presented on the lecture slide #1-15 (compare the norms obtained in this way with the corresponding norms computed by means of the operator ***norm*** implemented in MATLAB). Compare the estimates $\hat{\mathbf{A}}_{N,x}^{-1}$, obtained by means of the procedure defined in Section 3, with the estimates obtained by means of the operator of matrix inversion ***inv*** implemented in MATLAB. Draw the dependence of δ_2 and δ_∞ on x.