

## PDP Lab 7 (1) Documentation

### (1) Requirement

Given a sequence of  $n$  numbers, compute the sums of the first  $k$  numbers, for each  $k$  between 1 and  $n$ . Parallelize the computations, to optimize for low latency on a large number of processors. Use at most  $2*n$  additions, but no more than  $2*\log(n)$  additions on each computation path from inputs to an output. Example: if the input sequence is 1 5 2 4, then the output should be 1 6 8 12.

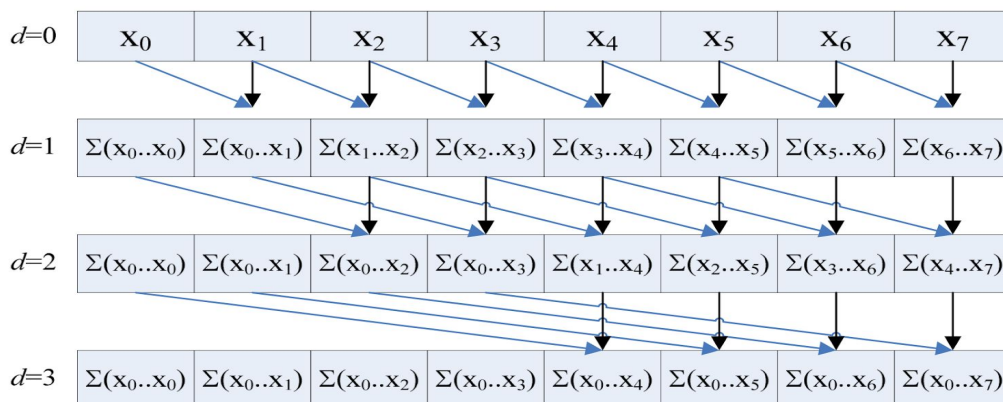
### (2) Algorithm

The problem is very famous, usually named Parallel Prefix Scan, which refers to applying a binary associative operator over an array in a manner similar to reduce in high level languages.

For example, having an array  $A = a_1 a_2 \dots a_n$  and a binary associative operator  $x$ , we want to obtain another array  $B = b_1 b_2 \dots b_n$ , where  $b_1 = a_1$ ,  $b_2 = a_1 x a_2$ ,  $b_3 = a_1 x a_2 x a_3$ ,  $b_i = a_1 x a_2 \dots x a_i$ .

The algorithm has two phases:

1. upSweep which adds the numbers like in a binary tree
2. downSweep which distributes the results



The implementation is the one described in this article:

[https://developer.nvidia.com/gpugems/GPUGems3/gpugems3\\_ch39.htm](https://developer.nvidia.com/gpugems/GPUGems3/gpugems3_ch39.htm)

### (3) Setup

MacBook Pro (Retina, 13-inch, Early 2015)

Processor 2.7 GHz Intel Core i5

Memory 8 GB 1867 MHz DDR3

Graphics Intel Iris Graphics 6100 1536 MB

Language of implementation: C++

### (4) Implementation

#### I Description

For the upSweep phase I've used the following approach:

```
1: for  $d = 0$  to  $\log_2 n - 1$  do  
2:   for all  $k = 0$  to  $n - 1$  by  $2^{d+1}$  in parallel do  
3:      $x[k + 2^{d+1} - 1] = x[k + 2^d - 1] + x[k + 2^d + 1 - 1]$ 
```

As we can see, we can have  $\log_2 n$  threads, each thread doing a level of computation, like in a binary tree, where the *for all*  $k = 0$  to  $n - 1$  by  $2^{d+1}$  in parallel *do* represents the threadWork for the upSweep phase.

For the downSweep phase the following algorithm was used, again distributing the results in a binary tree manner, it is very similar to the upSweep phase but mirrored.

```
1:  $x[n - 1] \leftarrow 0$   
2: for  $d = \log_2 n - 1$  down to 0 do  
3:   for all  $k = 0$  to  $n - 1$  by  $2^d + 1$  in parallel do  
4:      $t = x[k + 2^d - 1]$   
5:      $x[k + 2^d - 1] = x[k + 2^d + 1 - 1]$   
6:      $x[k + 2^d + 1 - 1] = t + x[k + 2^d + 1 - 1]$ 
```

## II Parallelization

It's quite simple to deduce the parallelization techniques I've used, the fors with *in parallel do* represents the thread functions for upSweep, and downSweep phases.

## III Locking

For synchronization I've used locks on every atomic add operation in thread functions

### (5) Performance Measurements

```
1 3 6 10 15 25 45 75 Sequential time: 4.398e-06  
1 3 6 10 15 25 45 75 Parallel time: 4.398e-06
```

Because the algorithm is tested on a CPU rather than on a GPU, the performance metrics are not better in comparison to the sequential algorithm implementation, especially not when ran on such small arrays.

```
1 3 6 10 15 25 45 75 119 164 210 257 305 354 404 455 Sequential time: 4.264e-06  
1 3 6 10 15 25 45 75 119 164 210 257 305 354 404 455 Parallel time: 0.00027101
```

Plus, in certain cases the sequential algorithm outperforms the parallel algorithm, as it doesn't have to wait for any locks to be released.