Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2025

Lecture 14 - List of problems

1. How many solutions has the following problem: a) $x'' + t^2x = 0$, $x(0) = 0$? b) $x''+t^2x = 0$, $x(0) = 0$, $x'(0) = 0$? c) $x''+t^2x = 0$; $x(0) = 0$, $x'(0) = 0$, $x''(0) = 1$? $\diamond$

2. Find a range of values for h such that the attractor equilibrium point of $x' = x^2 + 5x + 6$ is also an attractor fixed point of the discrete dynamical system associated to the Euler's numerical formula with stepsize $h > 0$ for the given differential equation. $\diamond$

3. We consider the pray-predator system

$\dot{x} = x(1 - y), \quad \dot{y} = -y(2 - x)$.

(a) Find the expression of a first integral in $(0, \infty) \times (0, \infty)$. Check it using the corresponding first order partial differential equation.

(b) If $(2, 1)$ is an equilibrium point, is it hyperbolic? $\diamond$

4. Let $a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R}$ be such that $a_{12} \neq 0$. Show that the roots of the characteristic equation corresponding to the second-order difference equation obtained by reducing the linear system

$$x_{k+1} = a_{11}x_k + a_{12}y_k, \quad y_{k+1} = a_{21}x_k + a_{22}y_k$$

are the eigenvalues of the matrix associated to this system. $\diamond$

5. Prove that the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = m+\varepsilon \sin x$, where $m > 0$ and $0 < \varepsilon < 1$, has a unique fixed point which is a global attractor. The equation $x = m+\varepsilon \sin x$ is known as *Kepler equation* and arises in the study of planetary motion. $\diamond$