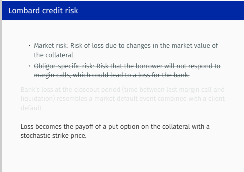
Example Lombard loan

Imagine a client has assets worth 100,000 CHF and wants to take out a Lombard loan.

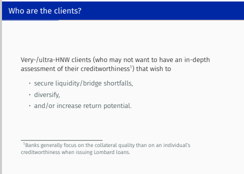
- Lending value (80%): 80,000 CHF
- Initial haircut (20%): 20,000 CHF
- Asset value drops to: 96,000 CHF
- Running haircut: 16,000 CHF
- Haircut erosion (20%): 4,000 CHF

Warning Stage: If the haircut erosion lies btw. 0 - 25% of the req. margin, the client's positions enter a monitoring stage, but no immediate action is taken.

- The bank assesses the risk and quality of the assets and decides it is willing to lend 80% of the asset's market value (i.e. the lending value equals 80%).
- The haircut is the remainder of the asset's value that isn't lent out. In this case, since the bank is lending 80%, the haircut is 20% or 20,000 CHF. This is the bank's safety margin.
- Suppose after some time, the value of the assets drops to 96,000 CHF. The LV based on the original market value is still 80,000 CHF (as it's a percentage of the initial value), but now the haircut is not 20,000 but 16,000 CHF because the assets have depreciated. This new haircut – the current difference between the market value and the loan – is known as the running haircut.



- Stochastic Strike Price is analogous to the lending value or the amount the bank is willing to loan against the collateral.
  Since the market value of the collateral can fluctuate, the point at which the bank would need to exercise its 'option' to liquidate the collateral (analogous to the put option being in the money) is not fixed—it depends on how the market value changes relative to the loan amount.
- When the market value of the collateral falls to a point where the lender would incur losses (the value falls below the loan amount or a certain threshold above it), the lender 'exercises the put' by liquidating the collateral to recover the loan amount, similar to how a put option is exercised when the market price falls below the strike price.



- Don't need to sell assets with high return potential. Can also help avoid realizing taxable capital gains/transaction costs, while still providing liquidity. Proceeds can be used for any purpose. Repayment is also more flexible in general than for many mortgage products. The Lombard loan requires only the payment of interest, and does not have to be amortized.
- Borrowing against concentrated illiquid assets can fund a diversifying portfolio. Entrepreneurs or high-level executives may find their wealth can be highly focused prior to selling a business or the vesting of restricted company stock (e.g. Amazon).
- Holding excess cash (e.g. for future investments) leads to high opportunity cost from keeping funds out of risk assets. No need to sell assets as being out of the market causes investors to sacrifice returns. Provides quick access to capital (e.g. meeting capital calls in private equity stake commitments).

Back

Bank's exposure to the client

Back

The bank sets a policy that involves margin calls[5] to control the loan.

- Margin call trigger, for fixed threshold $\alpha \in (0,1)$:

$$\beta := 1 - (1-\lambda)\alpha > \lambda.$$

- Stopping times (when calls occur) are given by $(\eta_n)_{n\geq 1}$, $n_1 := \inf\{t > 0 \mid V_t/X_t < \beta/\lambda\}$ and, for $n > 1$:

$$\eta_{2n} := \inf\{t > \eta_{2n} \mid V_t/X_t > \beta/\lambda\},$$
$$\eta_{2n+1} := \inf\{t > \eta_{2n} \mid V_t/X_t < \beta/\lambda\}.$$

[5] Occurring when $\frac{X}{V} > \frac{\lambda}{\beta}$.

- The first stopping time η_1 is defined as the first time t when the running haircut $\frac{X_t}{V_t} < \frac{\beta}{\lambda}$.
- η_{2n} denotes the times when a margin call is made due to the collateral's value dropping.
- η_{2n+1} denotes the times when the value of collateral recovers sufficiently that the conditions for a margin call are no longer met.

Back

Default triggers

Default time τ is given as

$$\tau := \inf\left\{t \geq 0 \mid \widetilde{N}_t \geq 1\right\} = \inf\left\{\widetilde{\tau}_n \mid \widetilde{\tau}_n \geq \tau_C n \geq 1\right\}.$$

We assume that immediate liquidation is possible, so incurred loss L becomes

$$L = (X_\tau - V_\tau)^+ 1_{\{\tau \leq T+\delta\}} = \left(\lambda\beta^{-1} V_{\tau-\delta} - V_\tau\right)^+ 1_{\{\tau \leq T+\delta\}}.$$

- $\tau - \delta$ is the last critical margin call time prior to closeout (liquidation).
- We defined default time $\tau = \widetilde{\tau}_n$ for some n and that exposure cannot increase $\in (t-\delta, \tau)$.

$$\begin{aligned} X_\tau &= X_{\tau-\delta} \\ &= \lambda\, V^*_{\widetilde{\tau}_{n-1},\tau-\delta} \\ &= \lambda\beta^{-1}\beta\, V^*_{\widetilde{\tau}_{n-1},\tau-\delta} \\ &= \lambda\beta^{-1}\, V_{\tau-\delta}. \end{aligned}$$

Back

Portfolio value determination

Back

Given supply curve[8]

$$S(t,x) = S(t,0)[1 + \alpha_c 1_c x + \alpha_n (1 - 1_c) x],$$

the value at the position at time T including liquidity costs, denoted V_T^L, is:

$$V * L_T \equiv Y_T + X_T S(T,0) = Y_0 + X_0 S(0, X_0) + \int_0^T X_{u-}\, dS(u,0) - L_T.$$

where

$$L_T = \sum_{0 \leq u \leq T} \Delta X_u [S(u, \Delta X_u) - S(u,0)] + \int_0^T \frac{\partial S}{\partial x}(u,0)\, d[X,X]_u^c.$$

Due to crisis at time T, we assume we liquidate $\theta \in [0,1]$ percent of holdings, so liquidity costs are

$$L_T = -\theta X_T [S(T, -\theta X_T) - S(T,0)].$$

[8] Where slope coefficients $\alpha_c \geq \alpha_n \geq 0$ are constants and 1_c is an indicator function.

- If $X_T > 0$, then liquidation implies that shares are sold and $L_T > 0$.
- If $X_T < 0$, then liquidation implies that shares are purchased and $L_T > 0$.
- L_T represents the total dollars generated $-\theta X_T S(T, -\theta X_T)$ due to liquidation, less the total dollars gen. if there were no quantity/size impace on the price $-\theta X_T S(T,0)$.