Example Lombard loan

Imagine a client has assets worth 100,000 CHF and wants to take out a Lombard loan.

- Lending value (80%): 80,000 CHF
- Initial haircut (20%): 20,000 CHF
- Asset value drops to: 96,000 CHF
- Running haircut: 16,000 CHF
- Haircut erosion (20%): 4,000 CHF

Warning Stage: If the haircut erosion lies btw. 0 - 25% of the req. margin, the client's positions enter a monitoring stage, but no immediate action is taken.

- The bank assesses the risk and quality of the assets and decides it is willing to lend 80% of the asset's market value (i.e. the lending value equals 80%).
- Haircut is the remainder of the asset's value that isn't lent out: bank's safety margin.
- Suppose after some time, the value of the assets drops. The LV based on the original market value is same (as it's a percentage of the initial value), but now the haircut has decreased.
- New haircut – the current difference between the market value and the loan – is known as the running haircut.

Lombard credit risk

- Market risk: Risk of loss due to changes in the market value of the collateral.
- ~~Obligor-specific risk: Risk that the borrower will not respond to margin calls, which could lead to a loss for the bank.~~

Bank's loss at the closeout period (time between last margin call and liquidation) resembles a market default event combined with a client default.

Loss becomes the payoff of a put option on the collateral with a stochastic strike price.

- Stochastic strike price is analogous to the LV.
- Since the market value of the collateral can fluctuate, the point at which the bank would need to exercise its 'option' to liquidate the collateral (analogous to the put option being in the money) is not fixed.
- When the market value of the collateral falls to a point where the lender would incur losses (the value falls below the loan amount or a certain threshold above it), the lender 'exercises the put' by liquidating the collateral to recover the loan amount, similar to how a put option is exercised when the market price falls below the strike price.

Who are the clients?

Very-/ultra-HNW clients (who may not want to have an in-depth assessment of their creditworthiness[1]) that wish to

- secure liquidity/bridge shortfalls,
- diversify,
- and/or increase return potential.

[1]Banks generally focus on the collateral quality than on an individual's creditworthiness when issuing Lombard loans.

- Don't need to sell assets with high return potential. Can also help avoid realizing taxable capital gains/transaction costs, while still providing liquidity. Proceeds can be used for any purpose. Repayment is also more flexible in general than for many mortgage products. The Lombard loan requires only the payment of interest, and does not have to be amortized.
- Borrowing against concentrated illiquid assets can fund a diversifying portfolio. Entrepreneurs or high-level executives may find their wealth can be highly focused prior to selling a business or the vesting of restricted company stock (e.g. Amazon).
- Holding excess cash (e.g. for future investments) leads to high opportunity cost from keeping funds out of risk assets. No need to sell assets as being out of the market causes investors to sacrifice returns. Provides quick access to capital (e.g. meeting capital calls in private equity stake commitments).

Critical margin call times

- $\widetilde{\tau}_n$: Critical times for margin calls, defined for $n \geq 1$.
- Exposure process (X_t): Exposure at time t, dependent on $\widetilde{\tau}_n$ and client's cooperation,

$$X_t = \lambda \sum_{n=1}^{\infty} V^*_{\widetilde{\tau}_{n-1}} 1_{\{\widetilde{\tau}_{n-1} \leq t < \widetilde{\tau}_n\}} \text{ on } \{\tau_C > t\}.$$

- Critical Margin Call Times: These are defined as stopping times when the margin call trigger is reached.
- Exposure Process (X_t): Describes the amount of money the client has drawn against the collateral over time. This process is adjusted based on the client's reaction to margin calls.
- The main takeaway is that the Lombard loan value and associated risks are not static but evolve over time with market conditions and the client's actions in response to margin calls.

Appendix

Default triggers

Appendix

- Default time (τ): The earliest time a default can occur, defined by certain financial conditions. It measures the potential for collateral failure in covering the loan, where

$$\tau := \inf\left\{t \geq 0 \mid \widetilde{N}_t \geq 1\right\} = \inf\left\{\widetilde{\tau}_n \mid \widetilde{\tau}_n \geq \tau_C\right\}.$$

- Incurred loss (L): The loss calculated at default time, reflecting the shortfall in collateral value compared to the loan exposure,

$$L = (X_\tau - V_\tau)^+ 1_{\{\tau \leq T+\delta\}}.$$

- Default times: Defined as the first time when a jump in the process N_i (associated with margin calls) occurs after the time τ_c (the time when the client is willing to respond to margin calls). We simplify it by neglecting obligor-specific risk, where we assume client will never react to calls.
- If default occurs, the bank incurs a Lombard loss (L).
- The positive difference between the exposure at default and the market value of the collateral at the time of liquidation.
- This is contingent on the assumption that assets can be liquidated instantaneously at default.

Lending values

- Lombard risk definition: Define LV as largest number in $(0,1)$ such that the probability of the collateral's value being less than the loan exposure at any default time plus δ remains below a threshold ϵ,

$$P[V_{\tau_n+\delta} \leq X_{\tau_n}] \leq \epsilon.$$

- Lending value: The maximum proportion of the collateral that can be borrowed, considering the risk tolerance level ϵ, drift μ, and volatility σ;

$$\lambda \leq \beta \exp\left((\mu - \frac{\sigma^2}{2})\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon) \right).$$

- The usual practice to control Lombard risk is to limit this value to a certain percentage. We set the lending value to 1% less than the market value of the collateral after a margin call. This percentage serves as a buffer to account for market volatility and potential depreciation of the collateral's value.
- This probability is determined using the GBM model for the asset value.
- The probability is given by the CDF of the standard normal distribution, applied to the logarithm of the ratio of the lending value to the market value, adjusted by the drift and volatility of the process.

Liquidity adjusted lending values

- Price adjustment: Adjusts the price of the collateral to include liquidity costs[2]

$$V_t(x) = e^{\gamma x} V_t.$$

- Liquidity adj. LV: Reflects liquidity costs at the time of liquidation, with dependence on transaction size and market conditions,

$$\lambda \leq \beta \exp\left(\boxed{-\gamma x} + (\mu - \frac{\sigma^2}{2})\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon) \right).$$

[2] Modeled as an exponential function of the transaction size.

- The previous section's assumption that assets can be immediately liquidated at the end of the closeout period without affecting the price might not hold, especially when large quantities are involved.
- The impact of transaction size on price, a key component of liquidity risk, must be considered. This is especially true if liquidating a large position may not be possible or could significantly drive down the price.
- γ is a constant that quantifies the impact of the transaction size on the asset price within the exponential supply curve model. Represents how much the price per share decreases/increases with each additional unit of the asset sold/purchased.
- x represents the order flow, which can be positive (when buying) or negative (when selling). When zero, it corresponds to the marginal trade, meaning there is no impact on the liquidity cost from the transaction size.

Liquidity adjusted lending values cont.

$$\lambda \leq \frac{(1-\alpha)\exp\left(-\gamma x + (\mu - \frac{\sigma^2}{2})\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}{1-\alpha\exp\left(-\gamma x + (\mu - \frac{\sigma^2}{2})\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}$$

where

$$\begin{aligned}
\sigma &\equiv \text{Historical volatility (1-month)}, \\
\mu &\equiv \text{Drift of underlying GBM} := \frac{\sigma^2}{2}, \\
\delta &\equiv \text{Response time period} := \frac{10}{250}, \\
\epsilon &\equiv \text{Risk tolerance level} := 0.01, \\
\alpha &\equiv \text{Margin call threshold} := 0.25.
\end{aligned}$$

- The liquidity-adjusted lending value takes into account the potential decrease in value due to liquidity costs during a rapid liquidation of the collateral.
- The model thus captures the idea that selling a large number of shares might reduce the price per share due to increased supply in the market, which is a liquidity consideration.
- This more conservative approach aims to mitigate the risks associated with the possibility of not being able to liquidate a large position without incurring significant costs or market impact.

Bank's exposure to the client

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The bank sets a policy that involves margin calls[5] to control the loan.

- Margin call trigger, for fixed threshold $\alpha \in (0,1)$:

$$\beta := 1 - (1-\lambda)\alpha > \lambda.$$

- Stopping times (when calls occur) are given by $(\eta_n)_{n\geq 1}$, $n_1 := \inf\{t > 0 \mid V_t/X_t < \beta/\lambda\}$ and, for $n > 1$:

$$\eta_{2n} := \inf\{t > \eta_{2n} \mid V_t/X_t > \beta/\lambda\},$$
$$\eta_{2n+1} := \inf\{t > \eta_{2n} \mid V_t/X_t < \beta/\lambda\}.$$

[5] Occurring when $\frac{X}{V} > \frac{\lambda}{\beta}$.

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- The first stopping time η_1 is defined as the first time t when the running haircut $\frac{X_t}{V_t} < \frac{\beta}{\lambda}$.
- η_{2n} denotes the times when a margin call is made due to the collateral's value dropping.
- η_{2n+1} denotes the times when the value of collateral recovers sufficiently that the conditions for a margin call are no longer met.

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Default triggers

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Default time τ is given as

$$\tau := \inf\left\{t \geq 0 \mid \widetilde{N}_t \geq 1\right\} = \inf\left\{\widetilde{\tau}_n \mid \widetilde{\tau}_n \geq \tau_C n \geq 1\right\}.$$

We assume that immediate liquidation is possible, so incurred loss L becomes

$$L = (X_\tau - V_\tau)^+ 1_{\{\tau \leq T+\delta\}} = \left(\lambda\beta^{-1} V_{\tau-\delta} - V_\tau\right)^+ 1_{\{\tau \leq T+\delta\}}.$$

- $\tau - \delta$ is the last critical margin call time prior to closeout (liquidation).
- We defined default time $\tau = \widetilde{\tau}_n$ for some n and that exposure cannot increase $\in (t-\delta, \tau)$.

$$\begin{aligned}
X_\tau &= X_{\tau-\delta} \\
&= \lambda V^*_{\widetilde{\tau}_{n-1}, \tau-\delta} \\
&= \lambda\beta^{-1}\beta V^*_{\widetilde{\tau}_{n-1}, \tau-\delta} \\
&= \lambda\beta^{-1} V_{\tau-\delta}.
\end{aligned}$$

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Portfolio value determination

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Given supply curve[8]

$$S(t,x) = S(t,0)[1 + \alpha_c 1_c x + \alpha_n (1 - 1_c) x],$$

the value at the position at time T including liquidity costs, denoted V_T^L, is:

$$V * L_T \equiv Y_T + X_T S(T,0) = Y_0 + X_0 S(0, X_0) + \int_0^T X_{u-}\, dS(u,0) - L_T.$$

where

$$L_T = \sum_{0 \le u \le T} \Delta X_u [S(u, \Delta X_u) - S(u,0)] + \int_0^T \frac{\partial S}{\partial x}(u,0)\, d[X,X]_u^c.$$

Due to crisis at time T, we assume we liquidate $\theta \in [0,1]$ percent of holdings, so liquidity costs are

$$L_T = -\theta X_T [S(T, -\theta X_T) - S(T,0)].$$

[8]Where slope coefficients $\alpha_c \ge \alpha_n \ge 0$ are constants and 1_c is an indicator function.

- If $X_T > 0$, then liquidation implies that shares are sold and $L_T > 0$.
- If $X_T < 0$, then liquidation implies that shares are purchased and $L_T > 0$.
- L_T represents the total dollars generated $-\theta X_T S(T, -\theta X_T)$ due to liquidation, less the total dollars gen. if there were no quantity/size impace on the price $-\theta X_T S(T, 0)$.