Lecture 9 & 10: Stereo Vision

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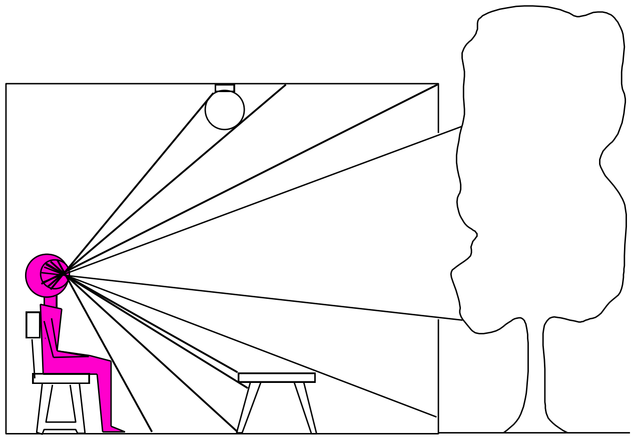
Stanford Vision Lab

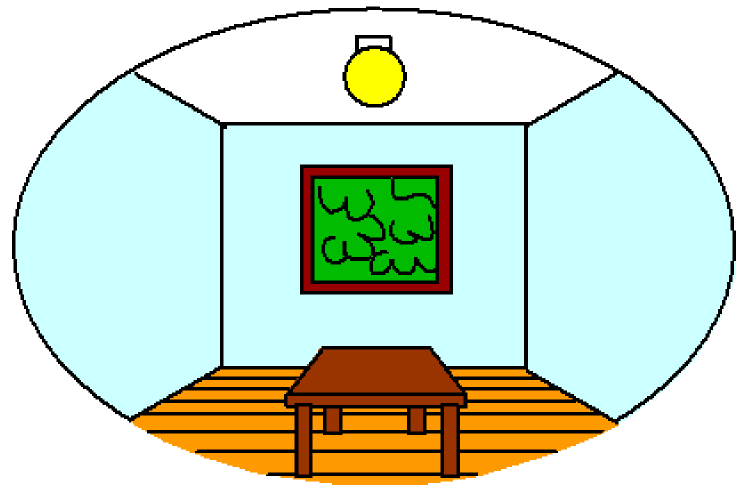
Dimensionality Reduction Machine (3D to 2D)

3D world



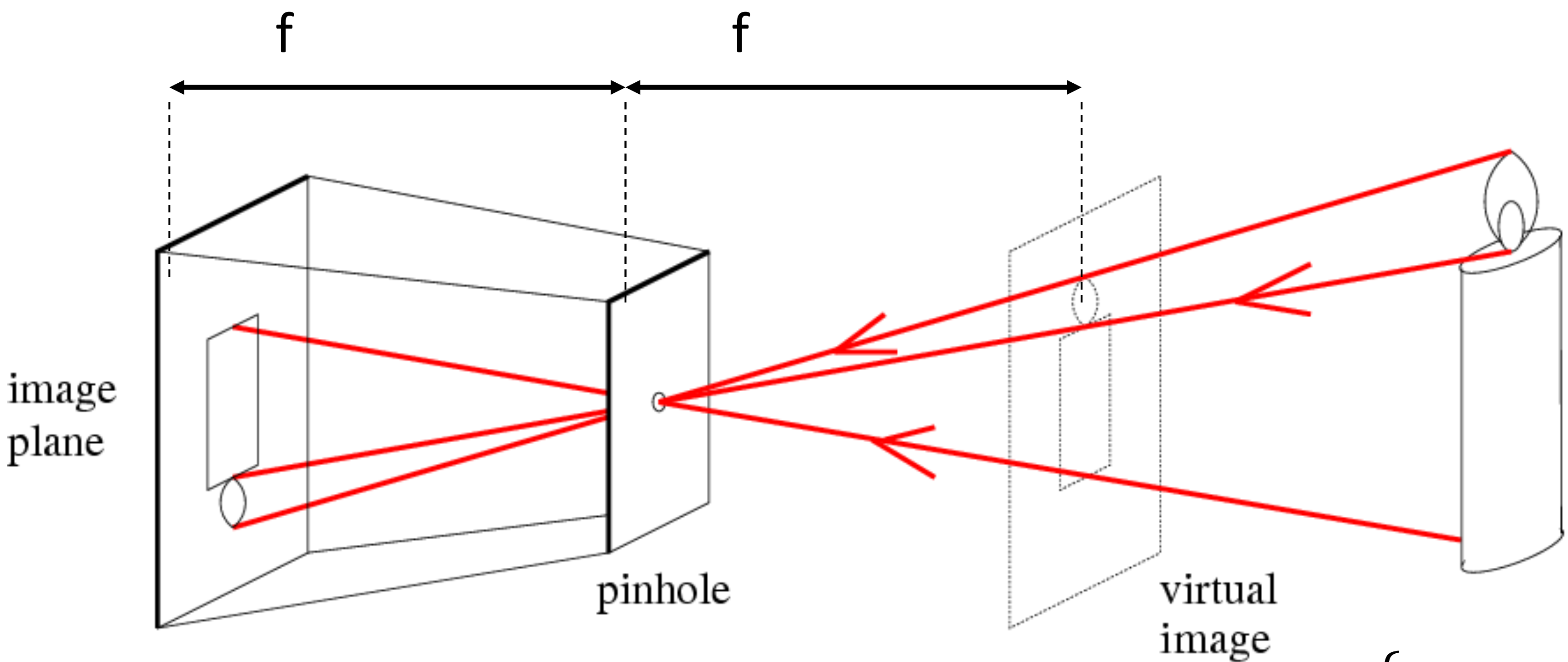
Point of observation

2D image



Figures © Stephen E. Palmer, 2002

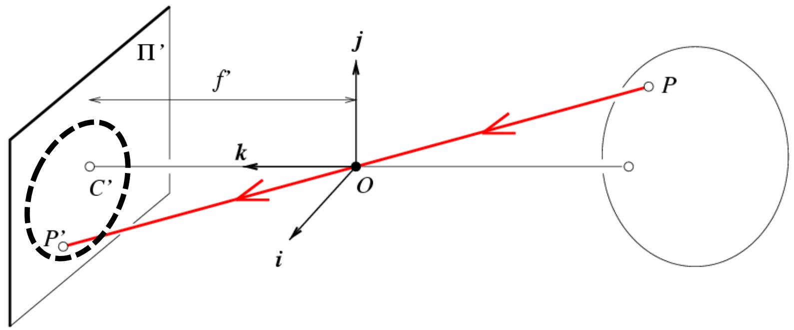
Pinhole camera



$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

- Common to draw image plane *in front* of the focal point
- Moving the image plane merely scales the image.

Relating a real-world point to a point on the camera



In homogeneous coordinates:

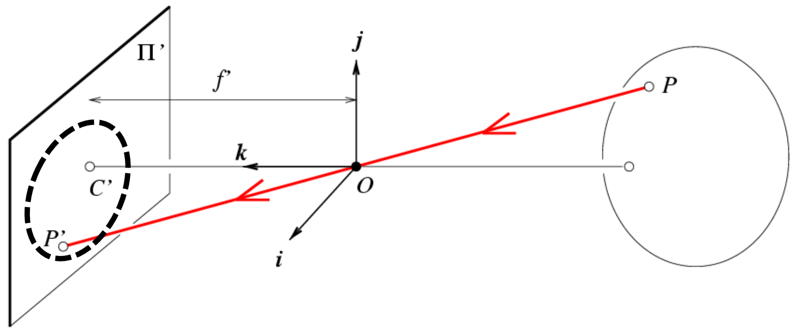
$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

ideal world

Intrinsic Assumptions
- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Relating a real-world point to a point on the camera



In homogeneous coordinates:

$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = K\begin{bmatrix} I & 0 \end{bmatrix} P$$

K

Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Real-world camera

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

Intrinsic parameters

$$P' = K \begin{bmatrix} I & 0 \end{bmatrix} P \Rightarrow w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

Real-world camera + Real-world transformation

Intrinsic Assumptions
- Optical center at (u_0, v_0)
- Rectangular pixels
- Small skew

Extrinsic Assumptions
- Allow rotation
- Camera at (t_x,t_y,t_z)

$$P' = K\begin{bmatrix} R & \bar{t} \end{bmatrix} P \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspiration: S. Savarese

What we will learn today?

- Introduction to stereo vision
- Epipolar geometry: a gentle intro
- Parallel images & image rectification
- Solving the correspondence problem
- Homographic transformation
- Active stereo vision system

Reading:
[HZ] Chapters: 4, 9, 11
[FP] Chapters: 10

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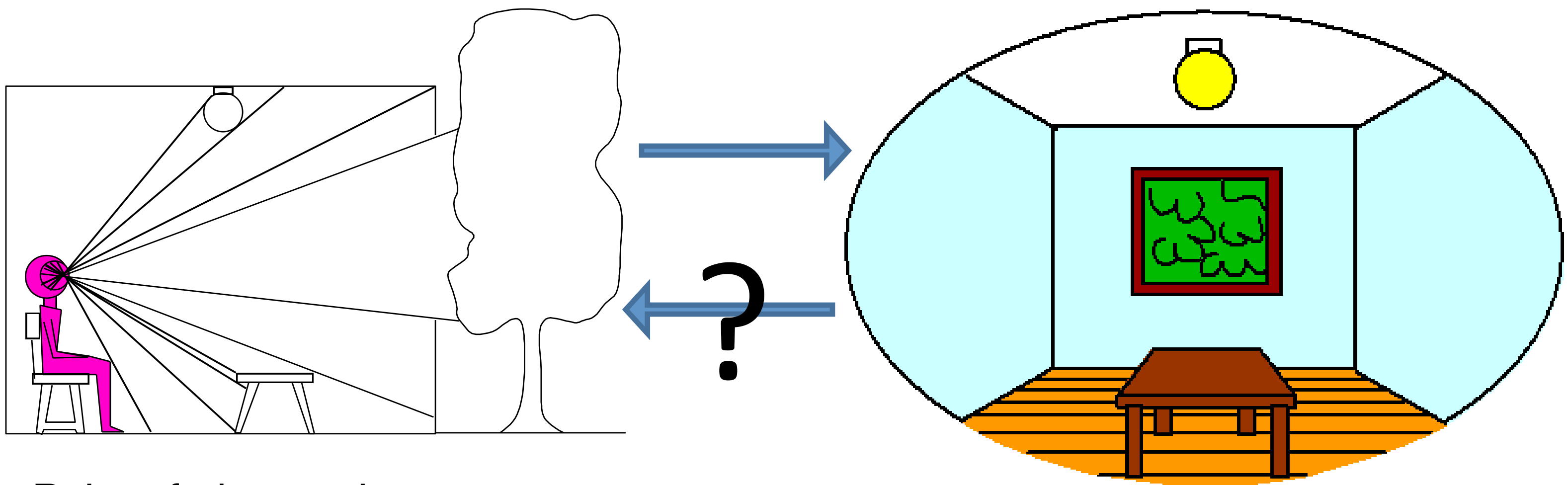
Recovering 3D from Images

- How can we automatically compute 3D geometry from images?
  - What cues in the image provide 3D information?

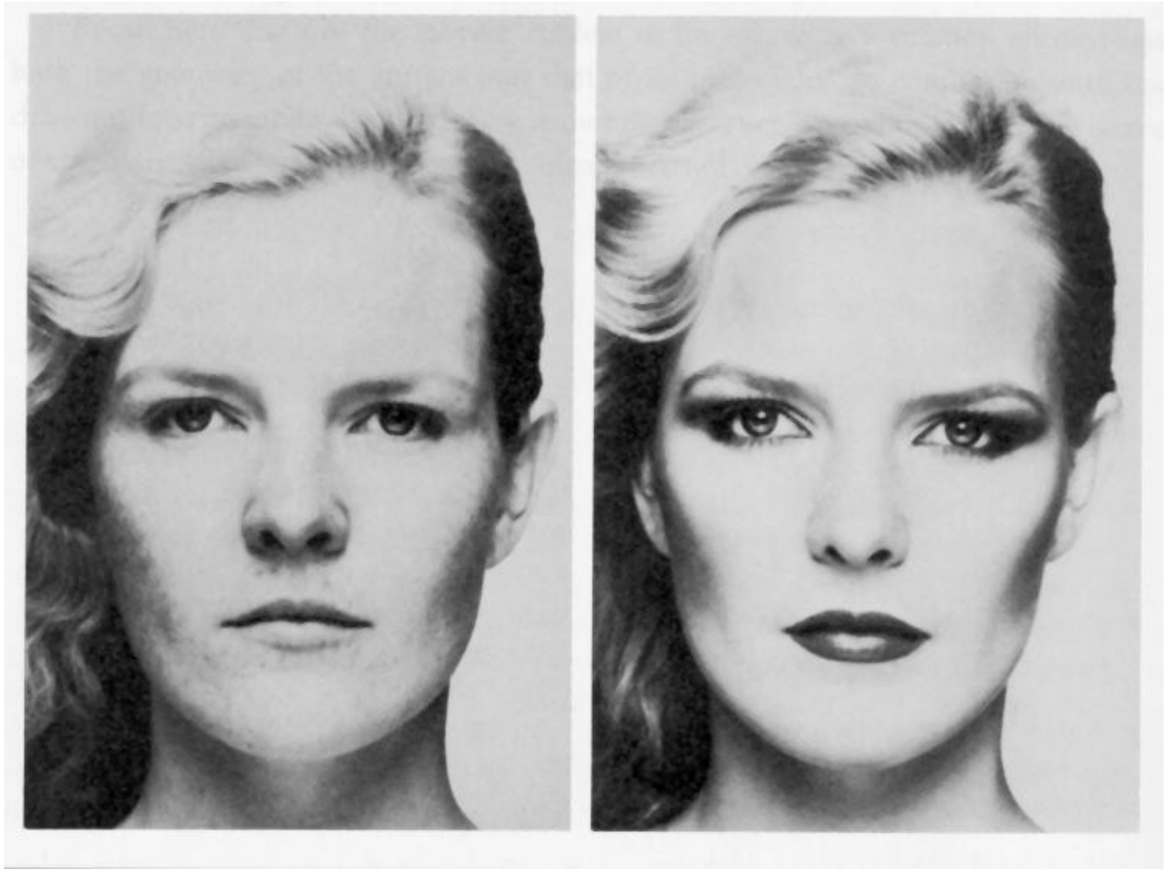
Real 3D world *2D image*



Point of observation

Visual Cues for 3D

- Shading

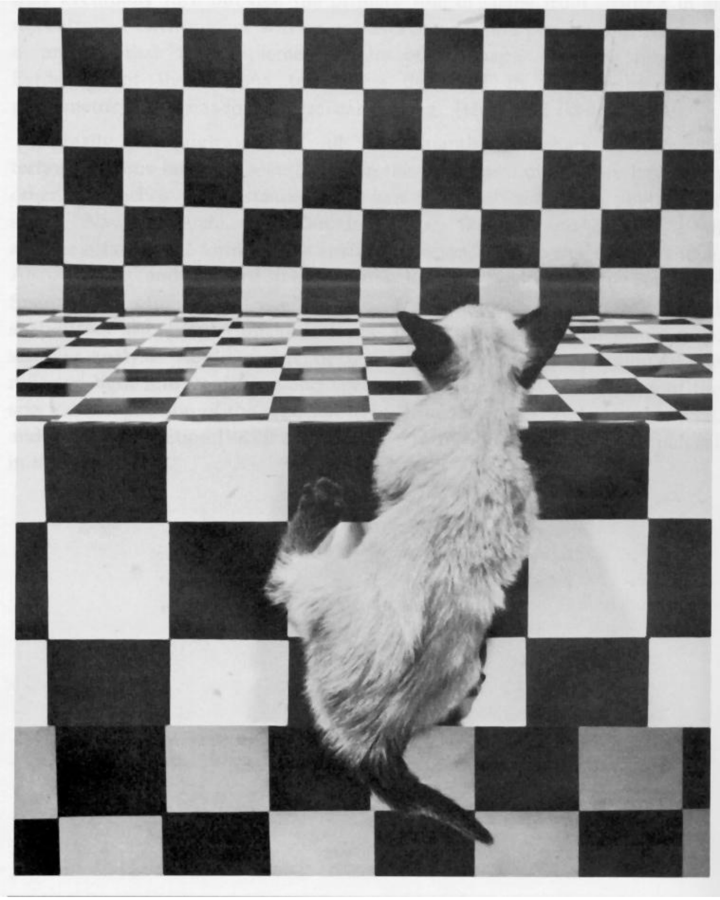


Merle Norman Cosmetics, Los Angeles

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture



***The Visual Cliff*, by William Vandivert, 1960**

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture
- Focus

From *The Art of Photography*, Canon

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture
- Focus
- Motion

Slide credit: J. Hayes

Visual Cues for 3D

- Shading
- Texture
- Focus
- Motion

- Others:
  - Highlights
  - Shadows
  - Silhouettes
  - Inter-reflections
  - Symmetry
  - Light Polarization
  - ...

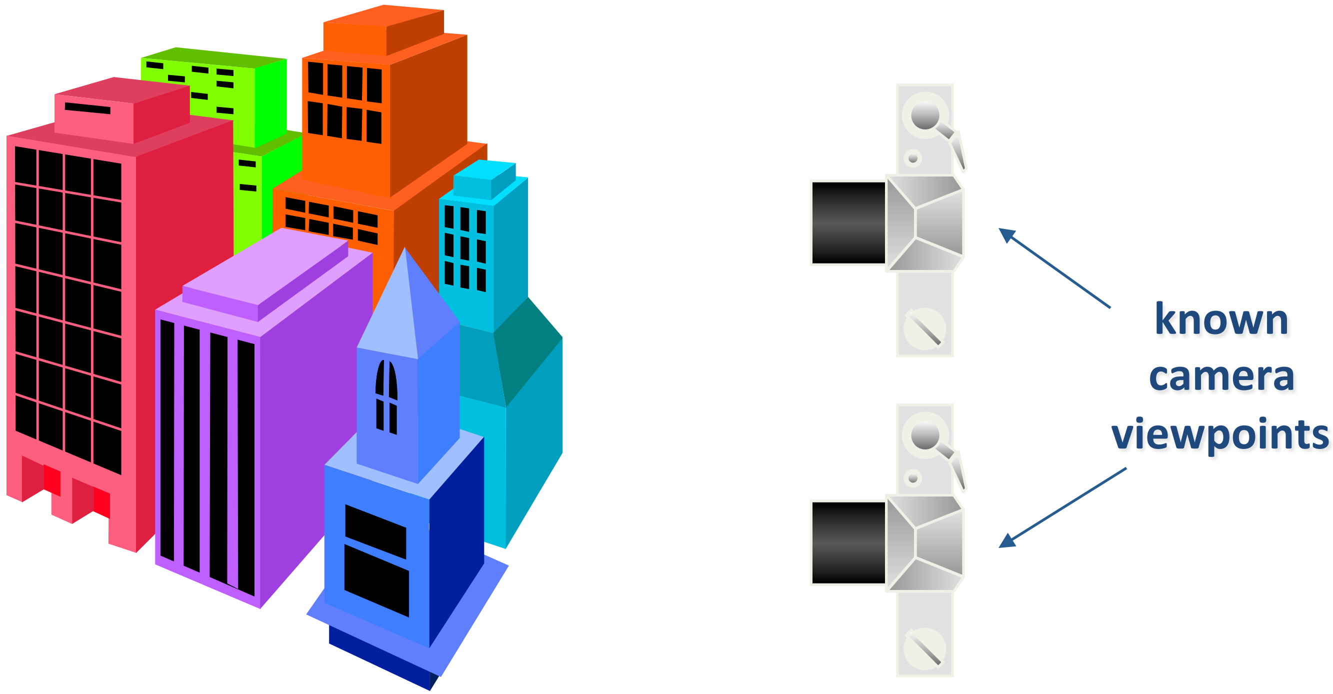
Shape From X

- X = shading, texture, focus, motion, ...
- We'll focus on the motion cue

Slide credit: J. Hayes

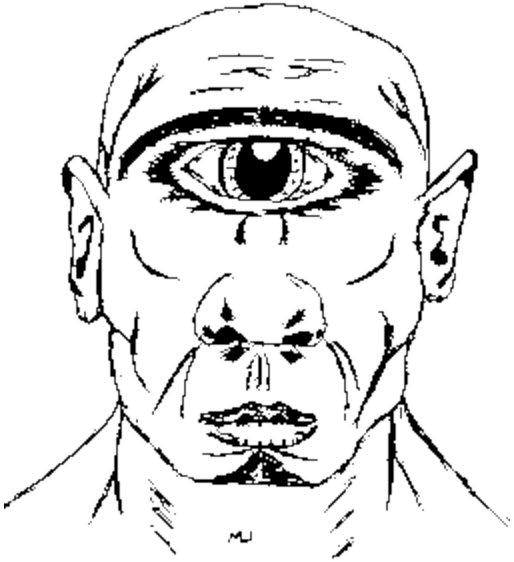
Stereo Reconstruction

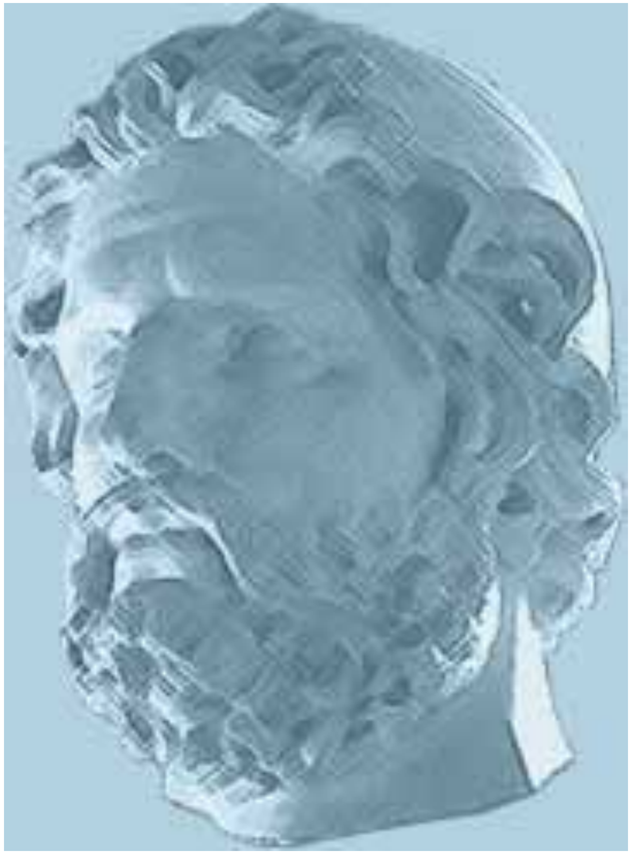
- The Stereo Problem
  - Shape from two (or more) images
  - Biological motivation



Slide credit: J. Hayes

Why do we have two eyes?

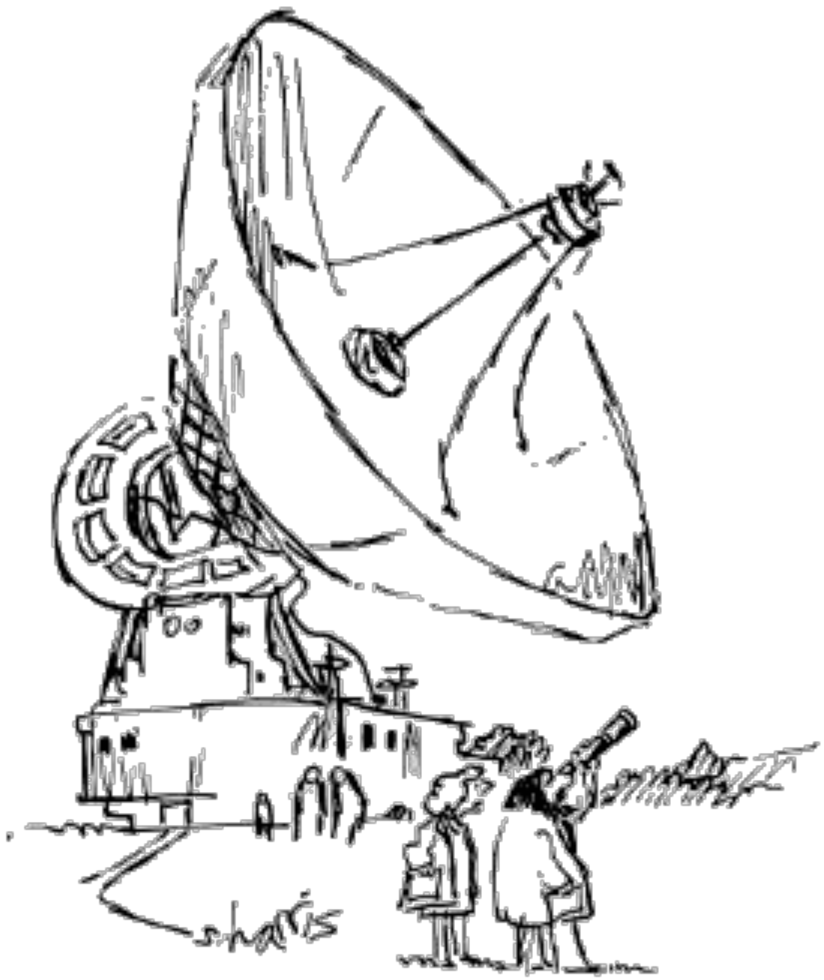




Cyclope **vs.** **Odysseus**

Slide credit: J. Hayes

1. Two is better than one

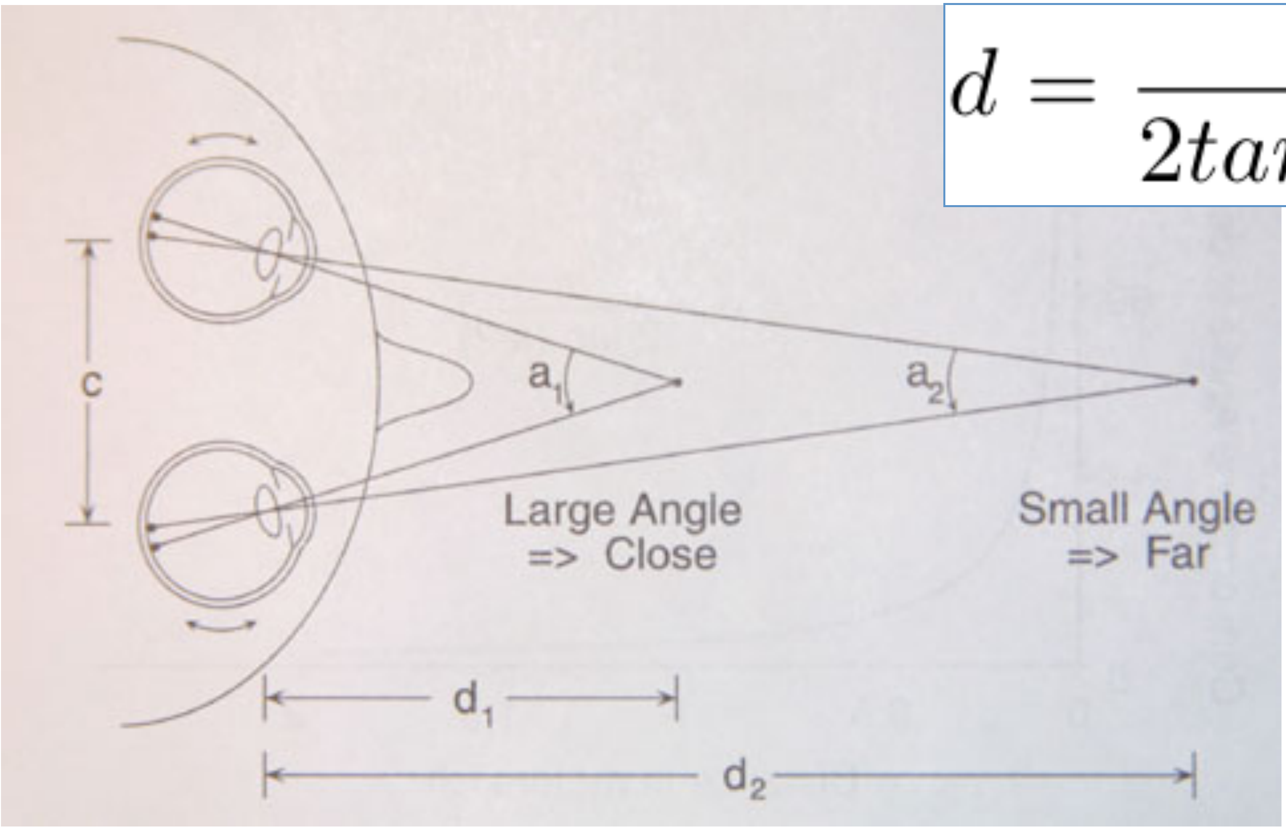


"Just checking."

Slide credit: J. Hayes

2. Depth from Convergence

$$d = \frac{c}{2tan(a/2)}$$



Human performance: up to 6-8 feet

Slide credit: J. Hayes

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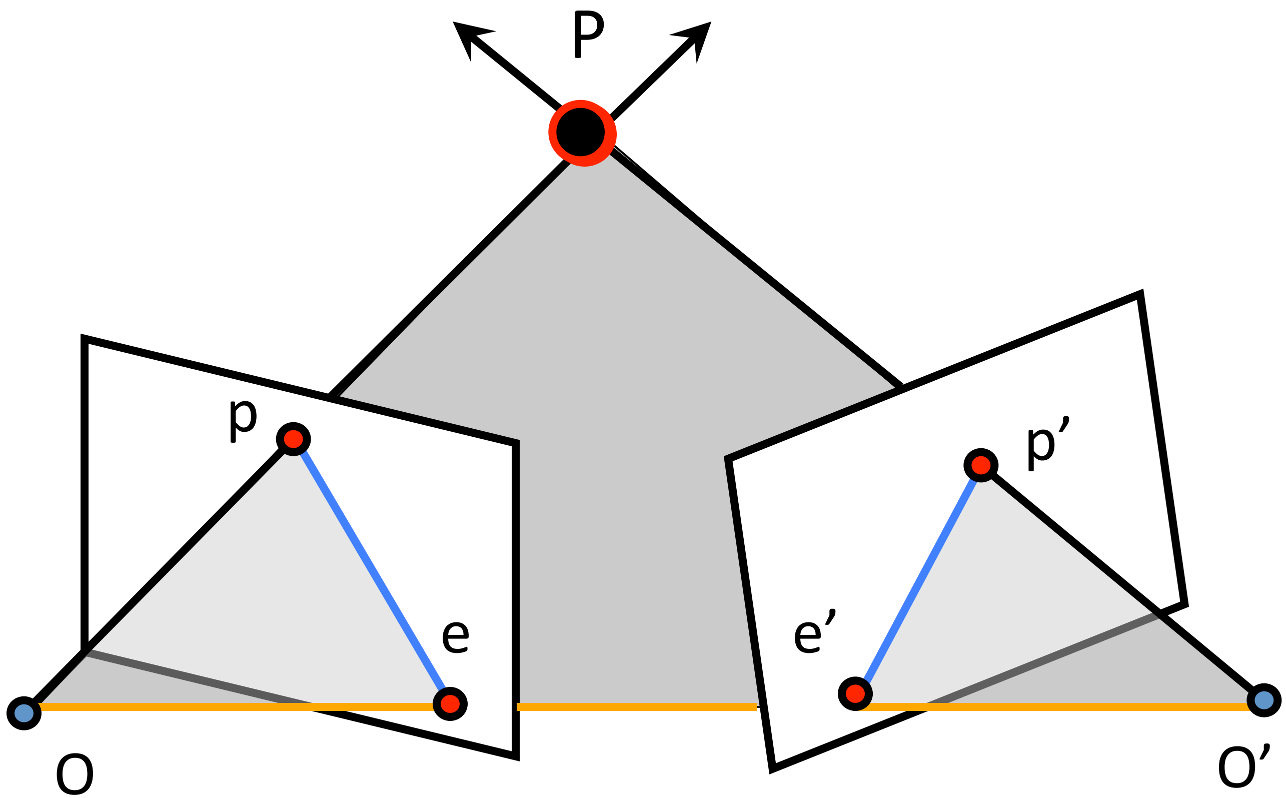
Epipolar geometry

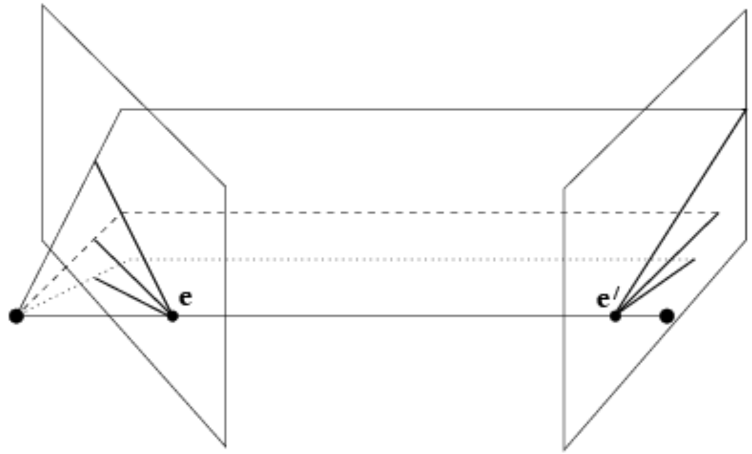


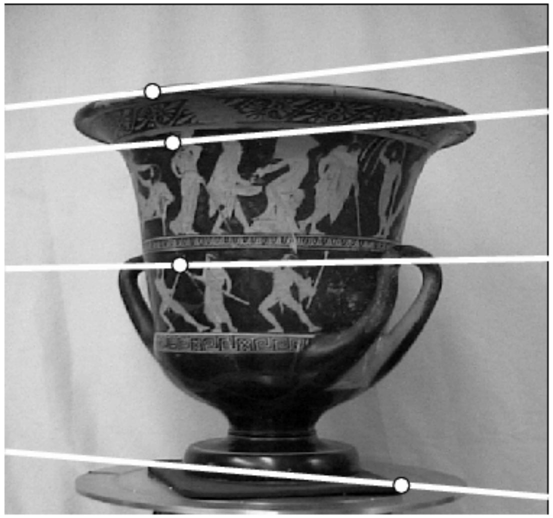
- Epipolar Plane
- Baseline
- Epipolar Lines

- Epipoles e, e’

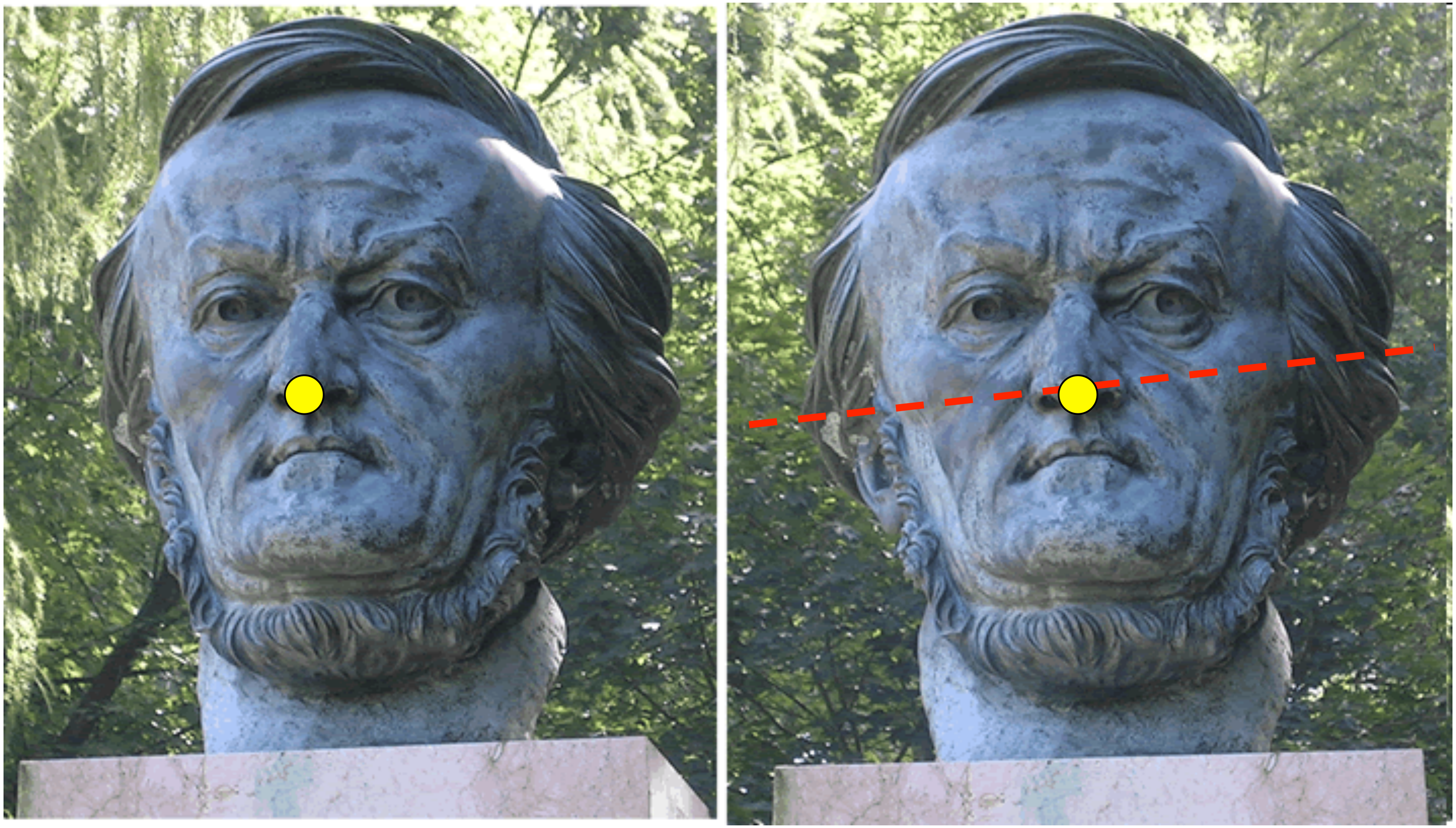
  = intersections of baseline with image planes
  = projections of the other camera center
  = vanishing points of camera motion direction

Example: Converging image planes



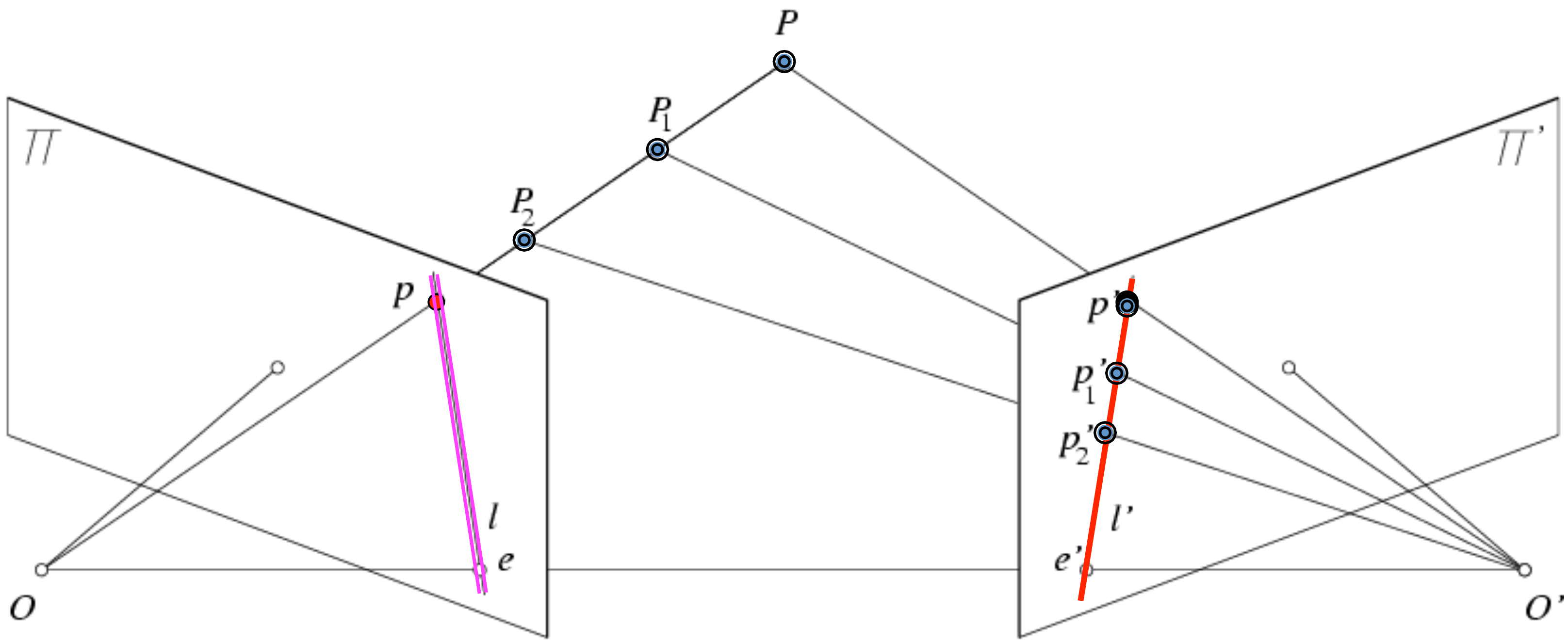


Epipolar Constraint



- Two views of the same object
- Suppose I know the camera positions and camera matrices
- Given a point on left image, how can I find the corresponding point on right image?

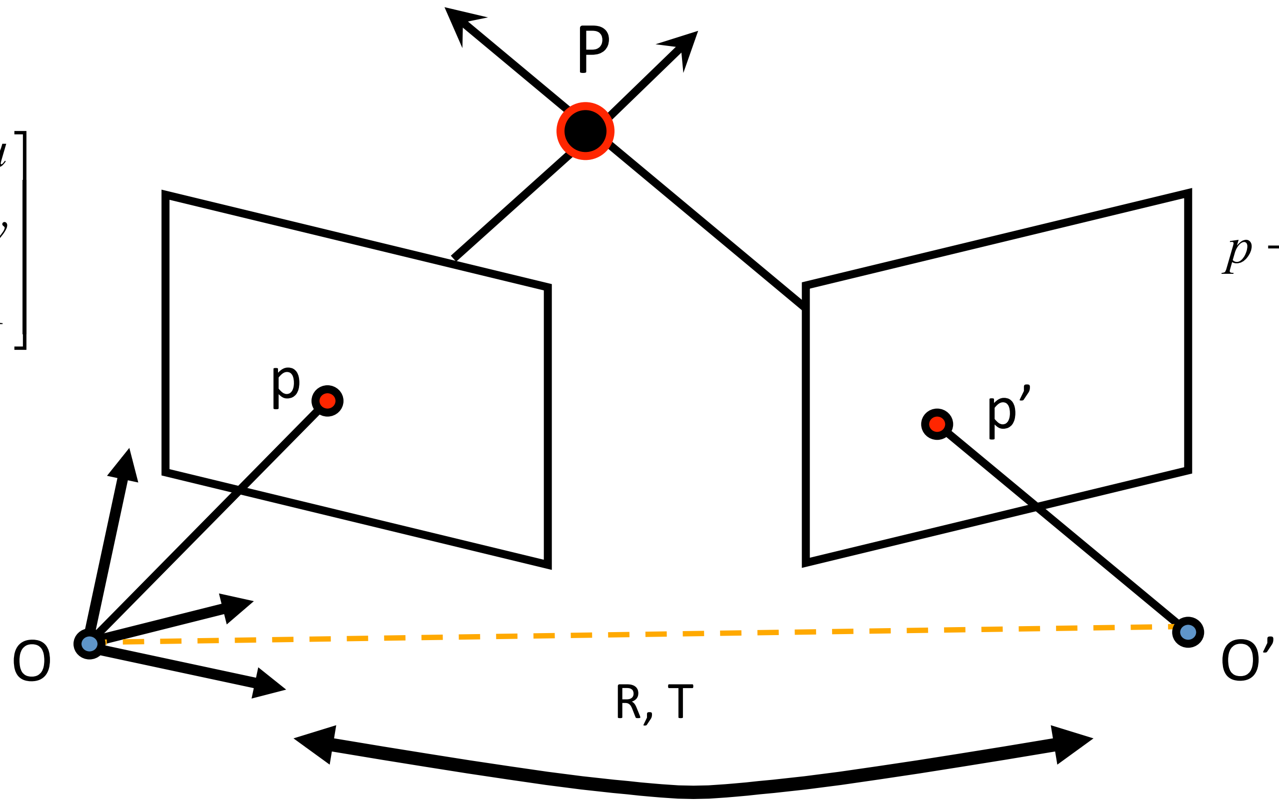
Epipolar Constraint



- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

Epipolar Constraint

$$p \rightarrow M\ P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

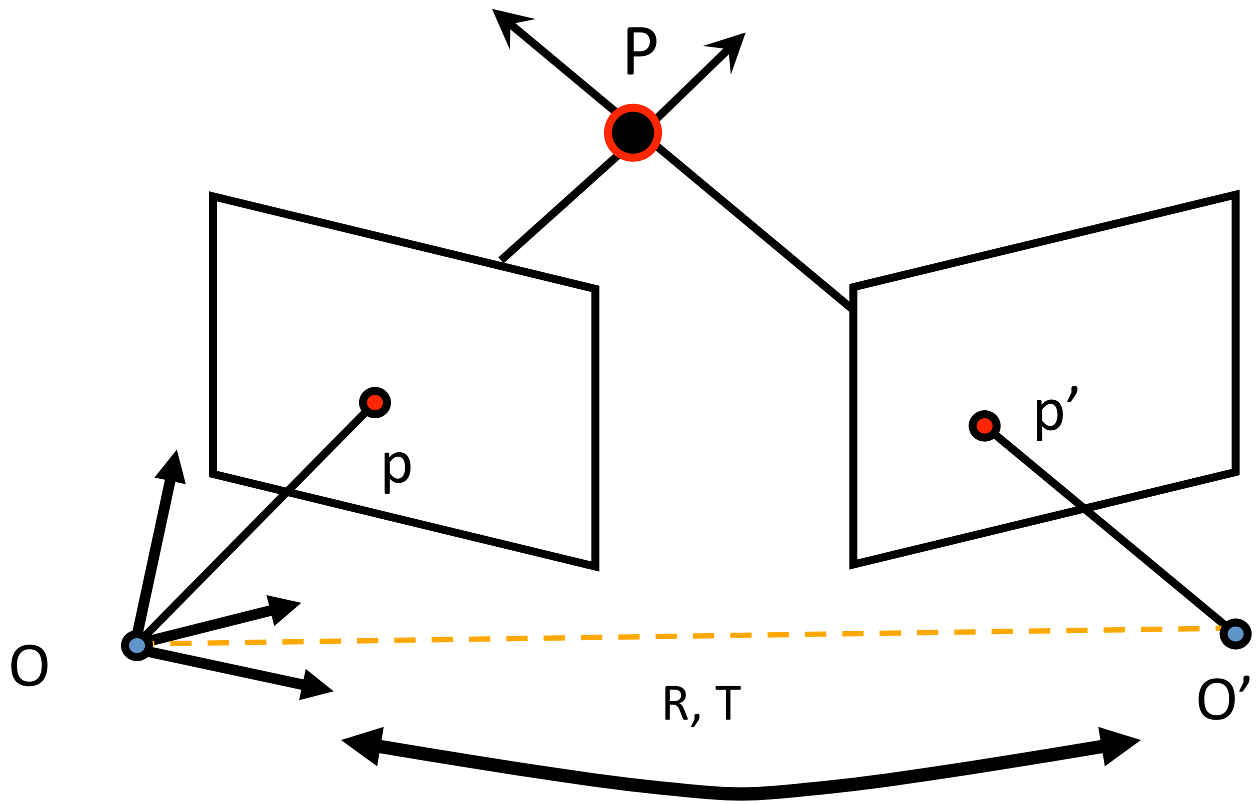


$$p \rightarrow M'\ P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

$$M = K\begin{bmatrix} I & 0 \end{bmatrix}$$

$$M' = K'\begin{bmatrix} R & T \end{bmatrix}$$

Epipolar Constraint



$$M = K\begin{bmatrix} I & 0 \end{bmatrix}$$

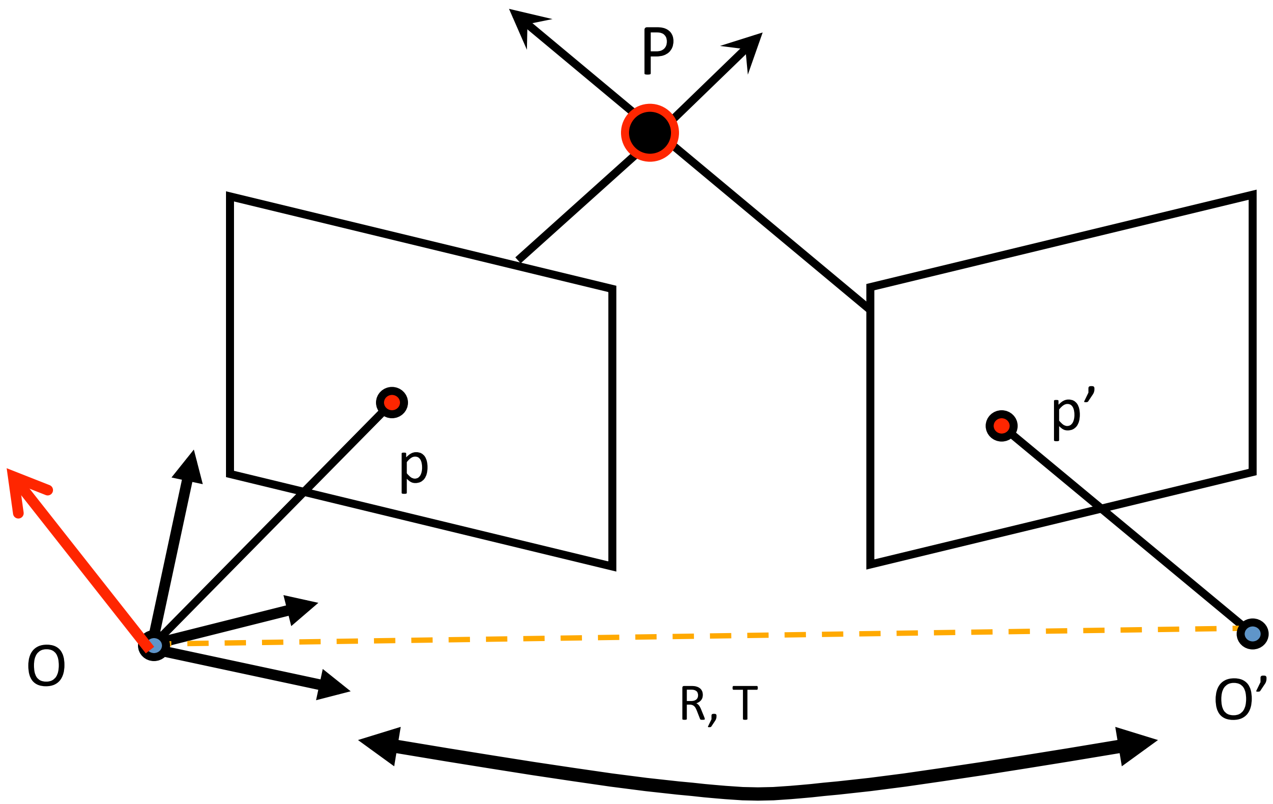
K_1 and K_2 are known
(calibrated cameras)

$$M' = K'\begin{bmatrix} R & T \end{bmatrix}$$

$$M = \begin{bmatrix} I & 0 \end{bmatrix}$$

$$M' = \begin{bmatrix} R & T \end{bmatrix}$$

Epipolar Constraint



$$T \times (R\, p')$$

Perpendicular to epipolar plane

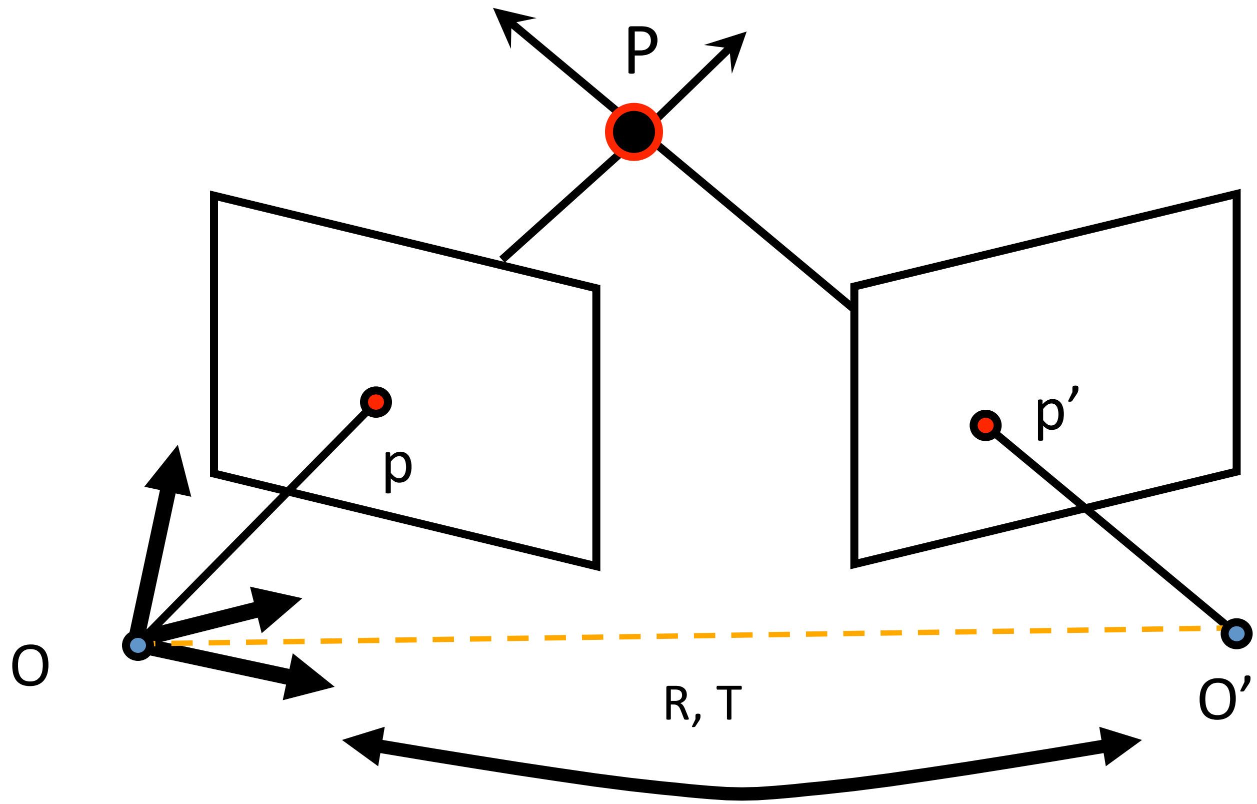
$$p^T \cdot [T \times (R\, p')] = 0$$

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

"skew symmetric matrix"

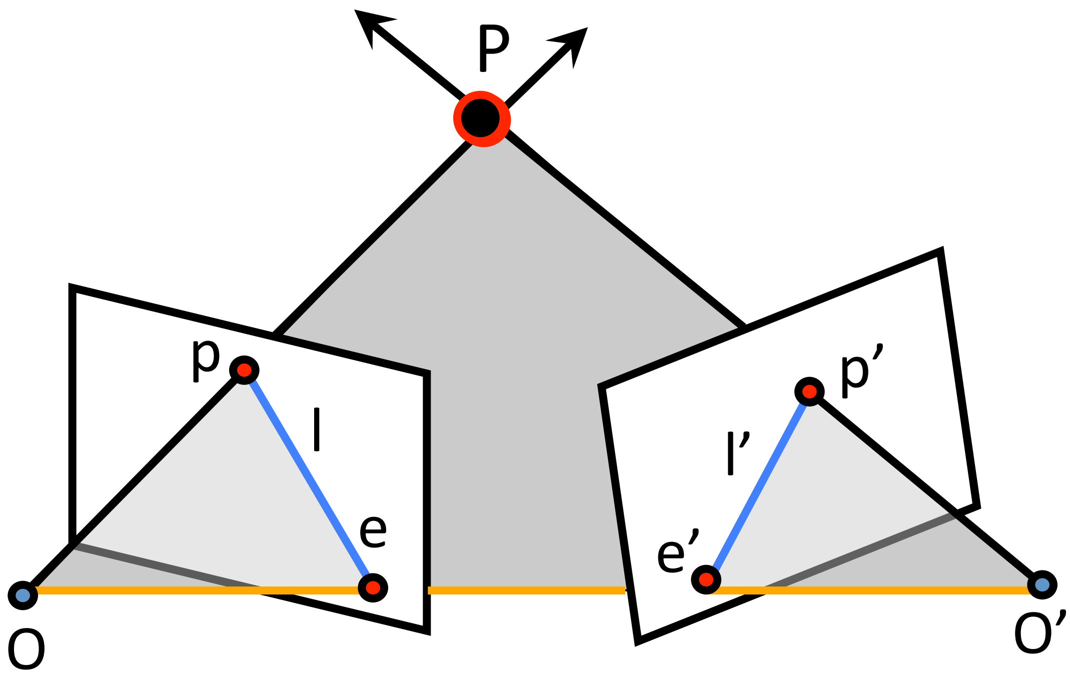
Epipolar Constraint



$$p^T \cdot [T \times (R\,p')] = 0 \rightarrow p^T \cdot \underbrace{[T_\times] \cdot R}_{E}\, p' = 0$$

(Longuet-Higgins, 1981) E = essential matrix

Epipolar Constraint



- $E\,p'$ is the epipolar line associated with p' ($l = E\,p'$)
- $E^T p$ is the epipolar line associated with p ($l' = E^T p$)
- E is singular (rank two)
- $E\,e' = 0$ and $E^T e = 0$
- E is 3x3 matrix; 5 DOF

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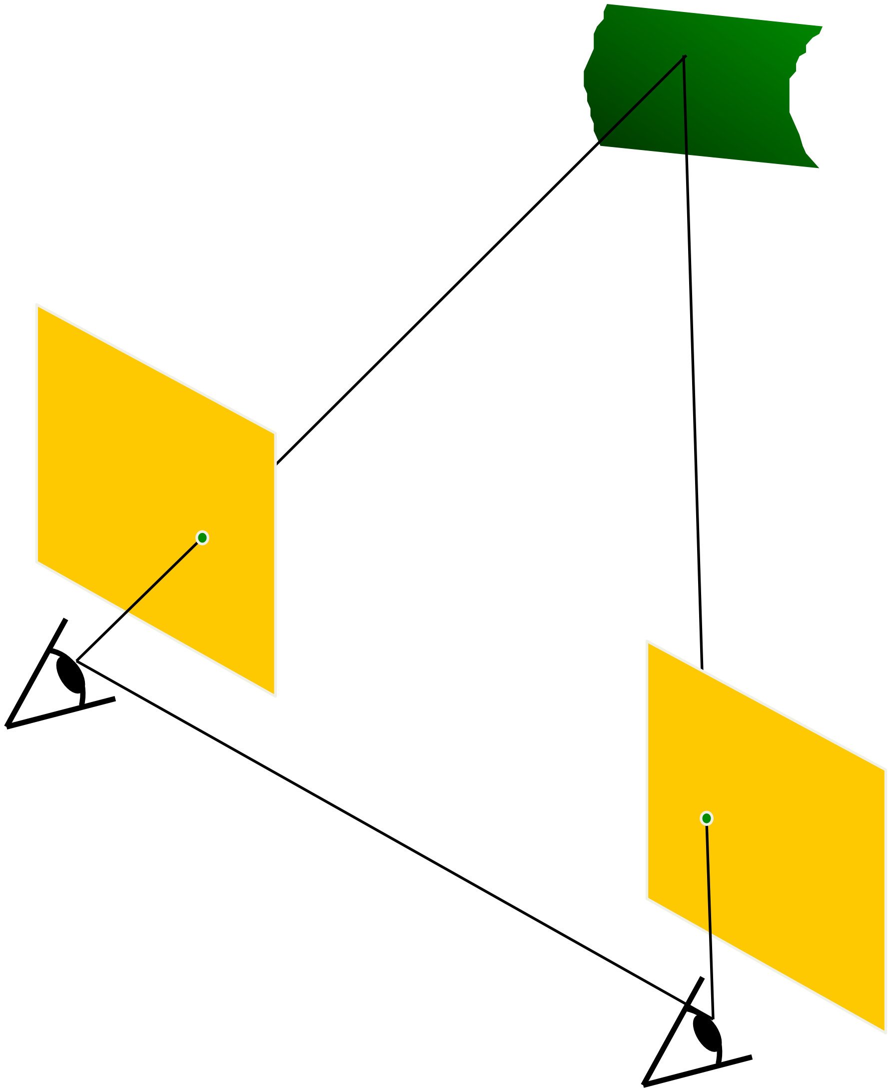
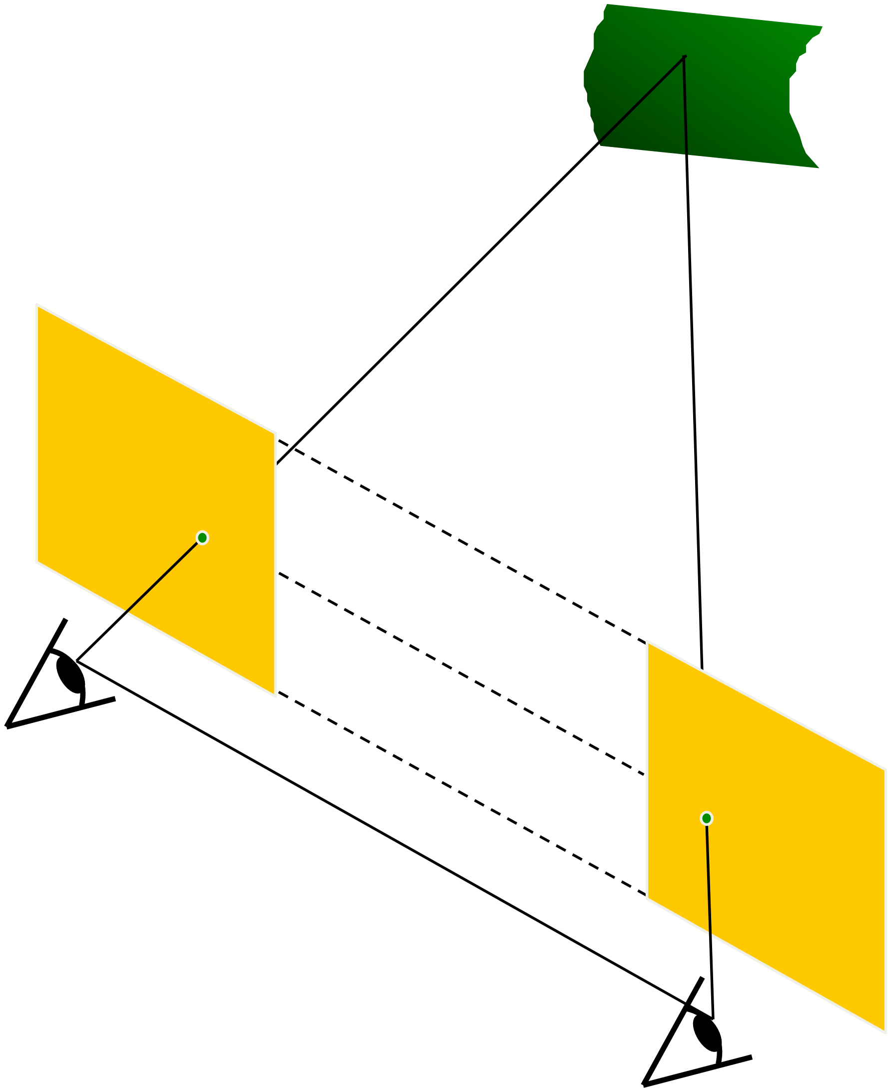
Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same

Slide credit: J. Hayes

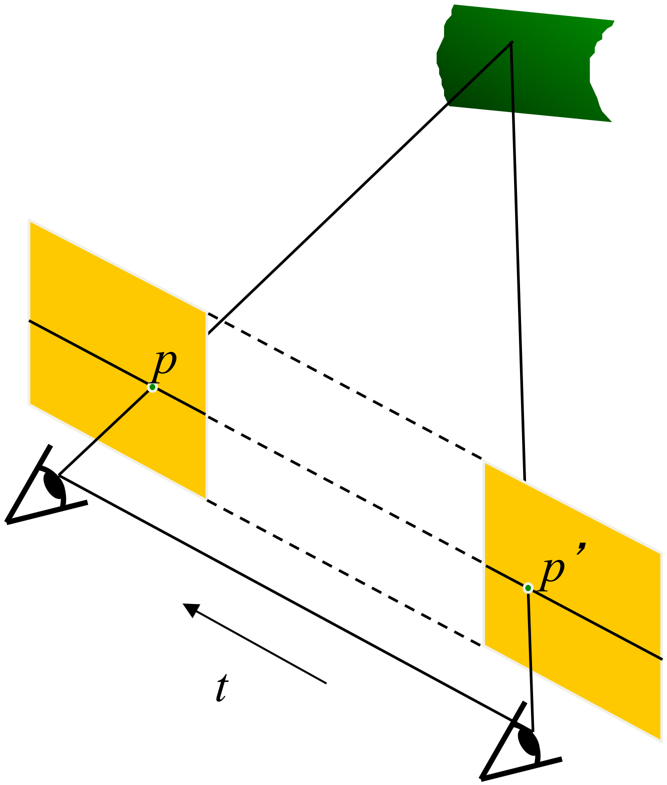
Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same
- Then, epipolar lines fall along the horizontal scan lines of the images

Slide credit: J. Hayes

Essential matrix for parallel images



Epipolar constraint:

$$R = I \qquad t = (T, 0, 0)$$

$$p^T E p' = 0, \quad E = [t_\times]R$$

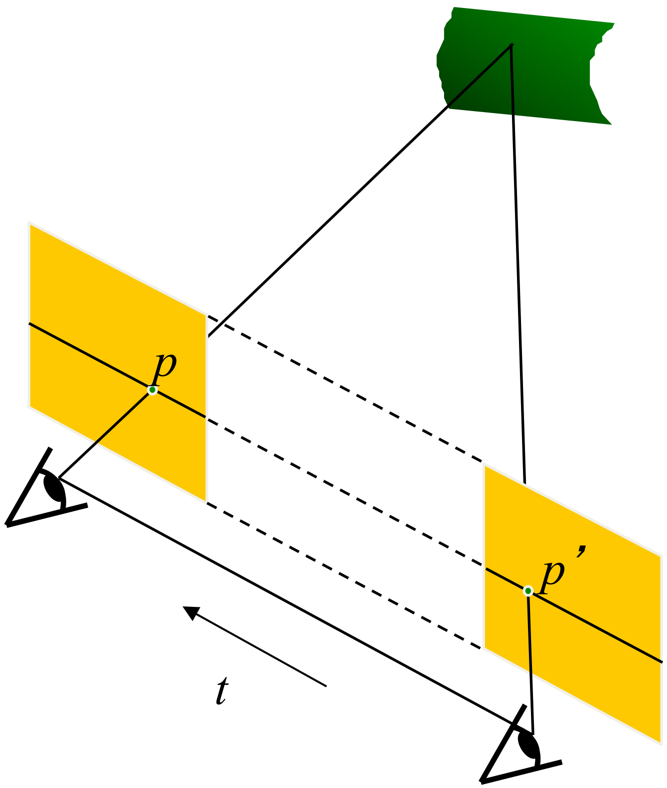
$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

Reminder: skew symmetric matrix

$$[a_\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Slide credit: J. Hayes

Essential matrix for parallel images



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$$p^T E p' = 0, \quad E = [t_\times]R$$

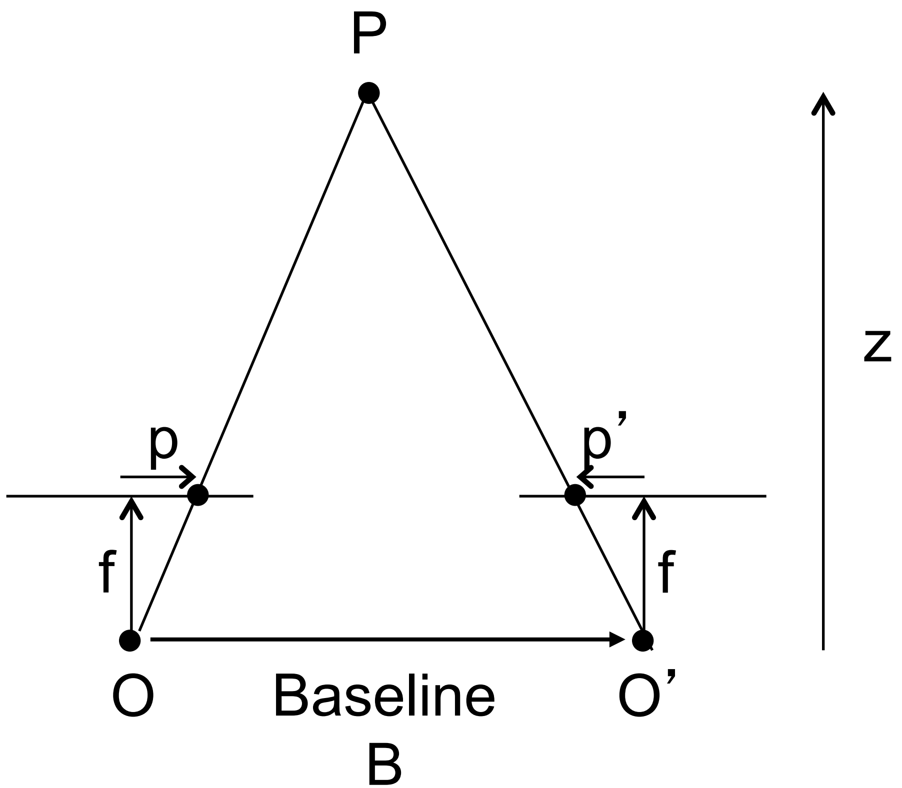
$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \qquad Tv = Tv'$$

The y-coordinates of corresponding points are the same!

Slide credit: J. Hayes

Triangulation -- depth from disparity

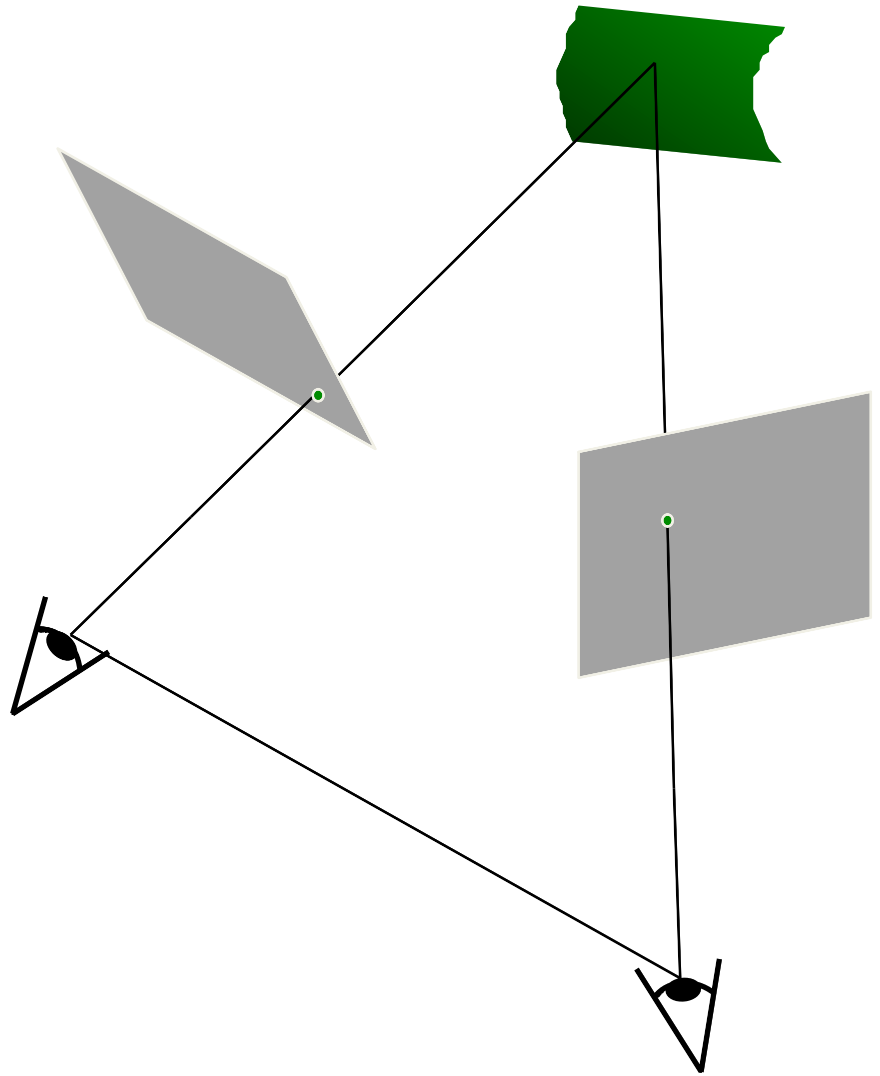


$$disparity = u - u' = \frac{B \cdot f}{z}$$

Disparity is inversely proportional to depth!

Slide credit: J. Hayes

Stereo image rectification



Slide credit: J. Hayes

Stereo image rectification

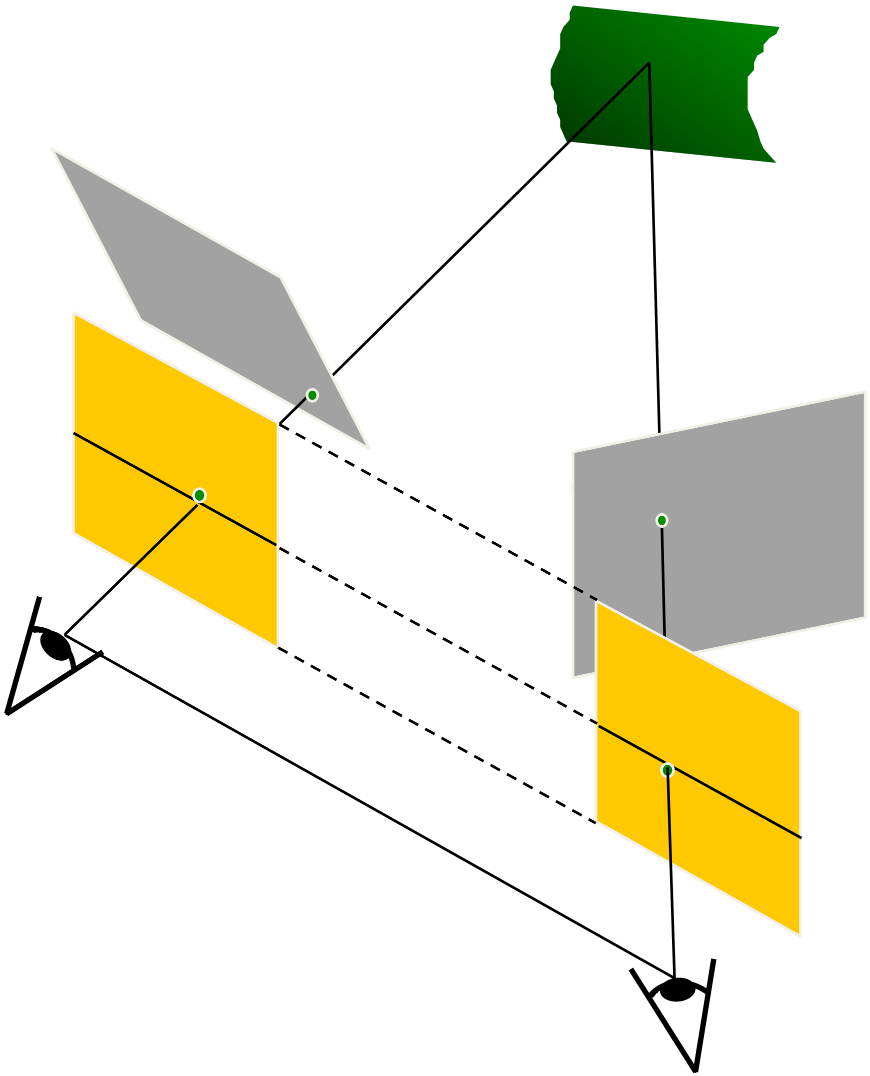
Algorithm:

- Re-project image planes onto a common plane parallel to the line between optical centers

- Pixel motion is horizontal after this transformation

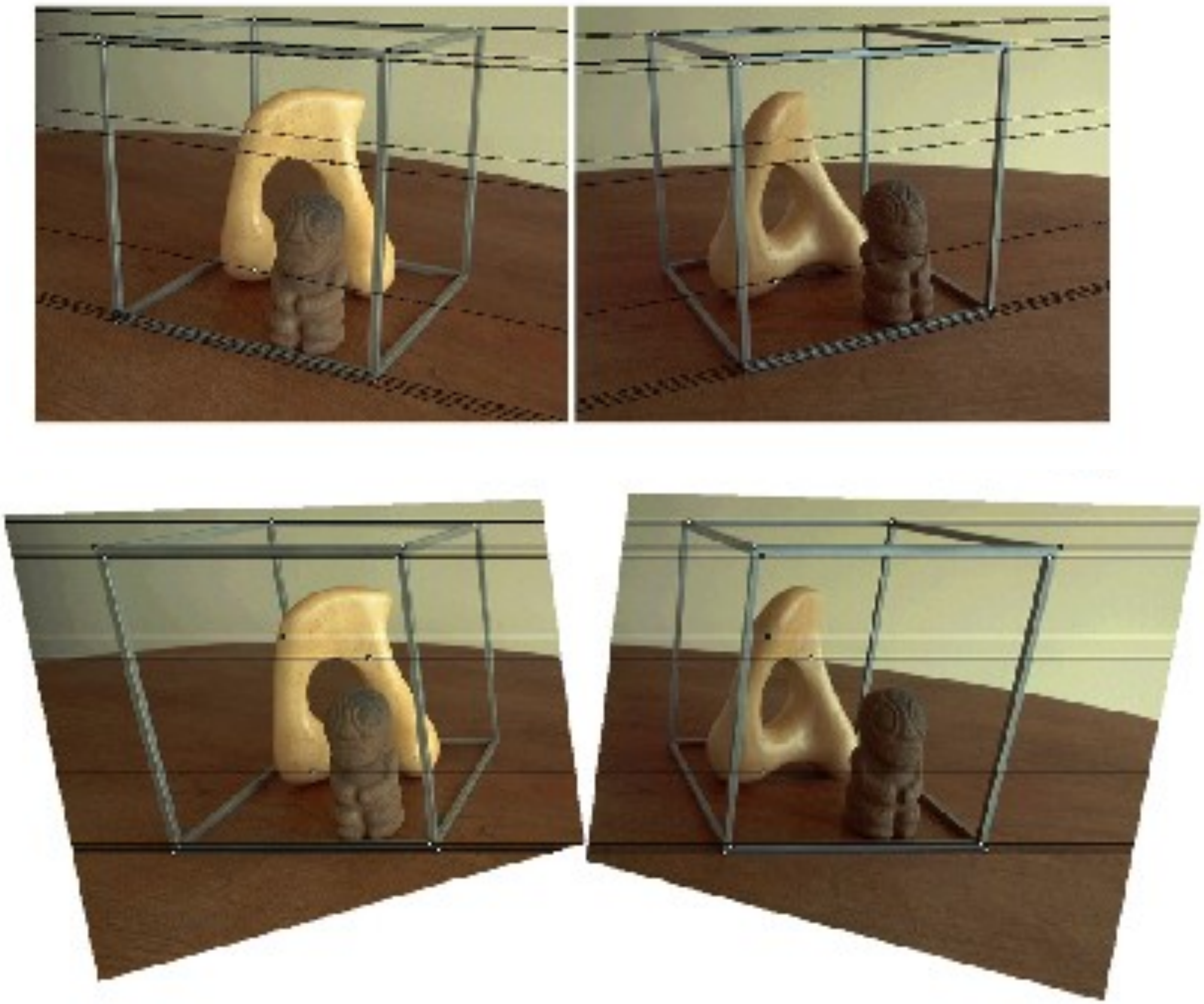
- Two transformation matrices, one for each input image reprojection

➢ C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). IEEE Conf. Computer Vision and Pattern Recognition, 1999.



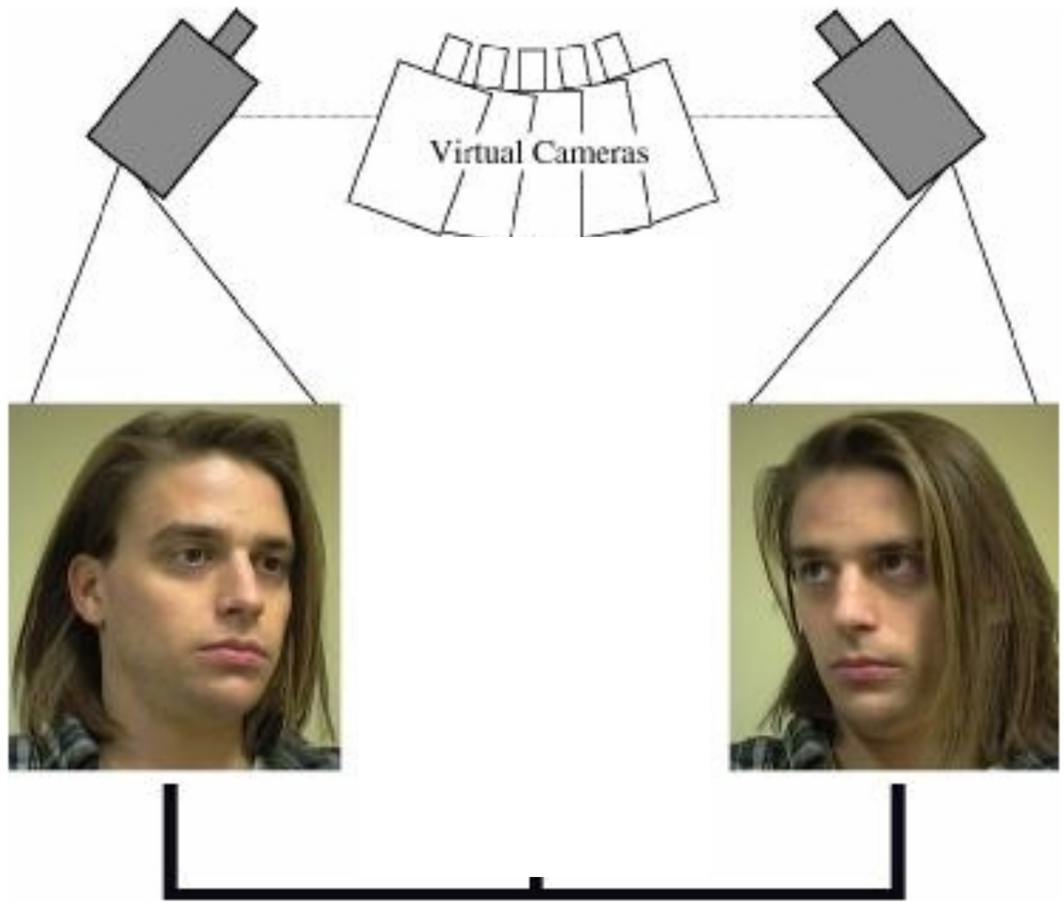
Slide credit: J. Hayes

Rectification example

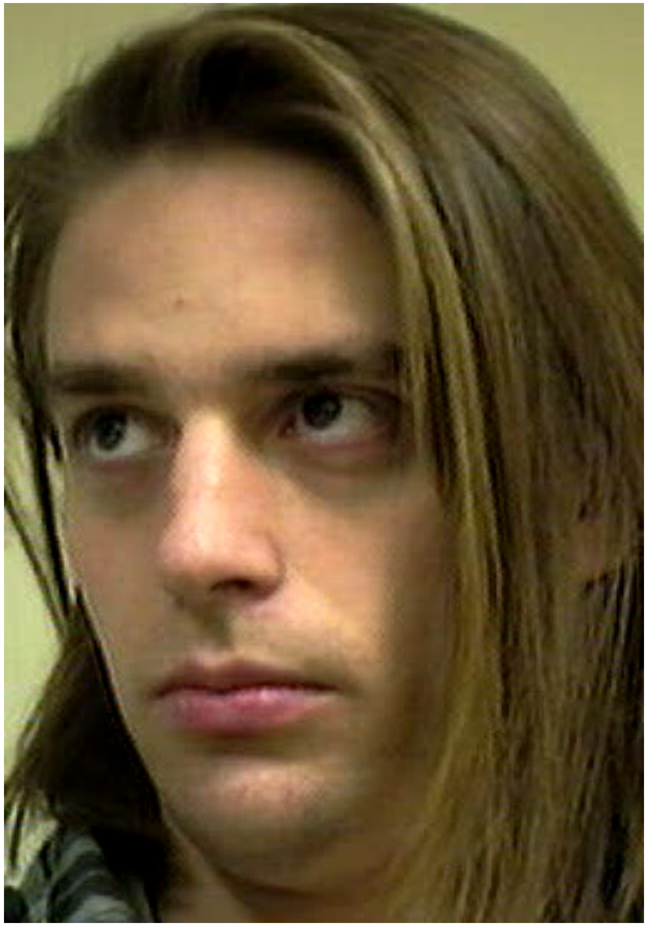


Application: view morphing

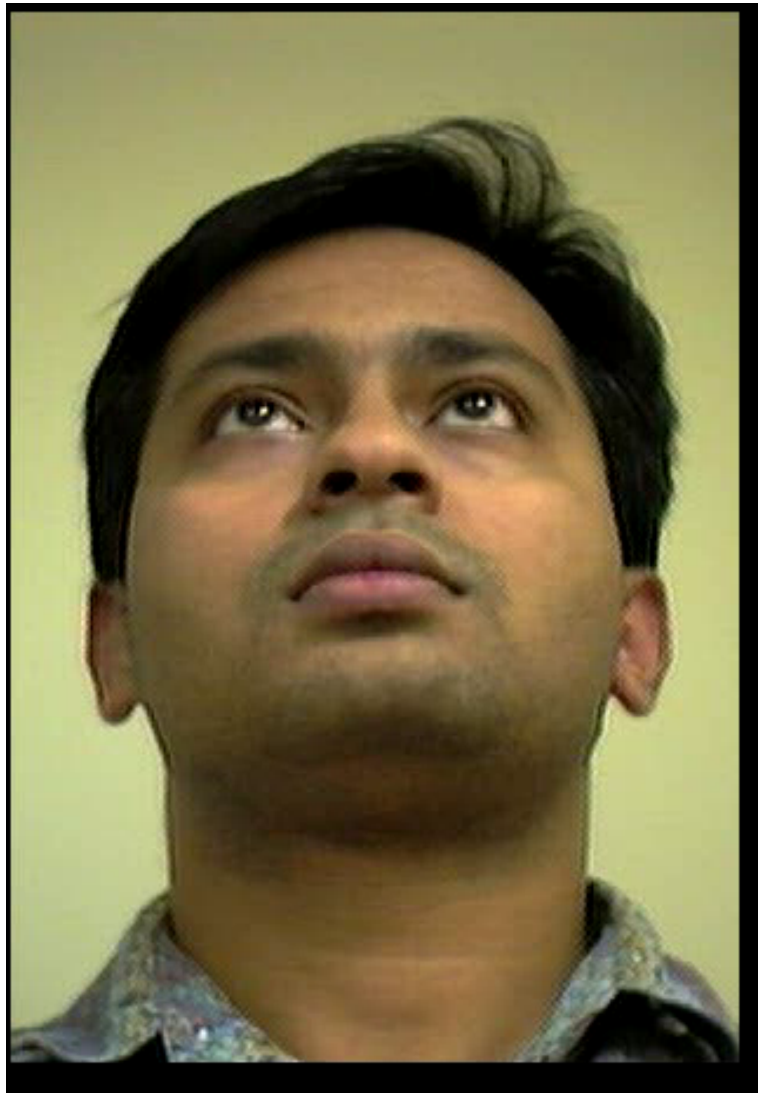
S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30



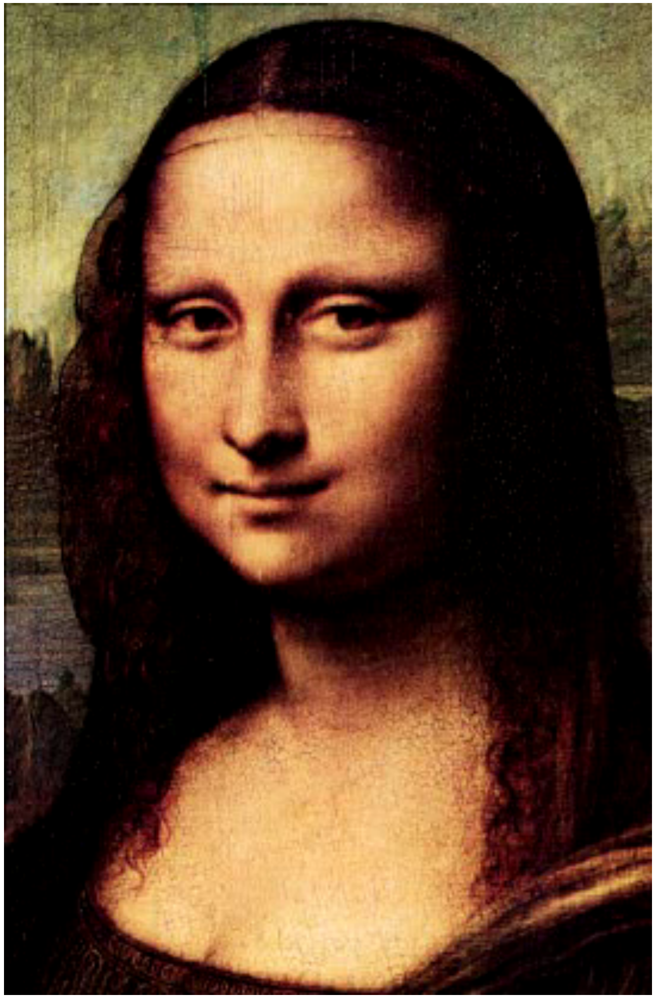
Application: view morphing



Application: view morphing



Application: view morphing



Removing perspective distortion

(rectification)

H_p

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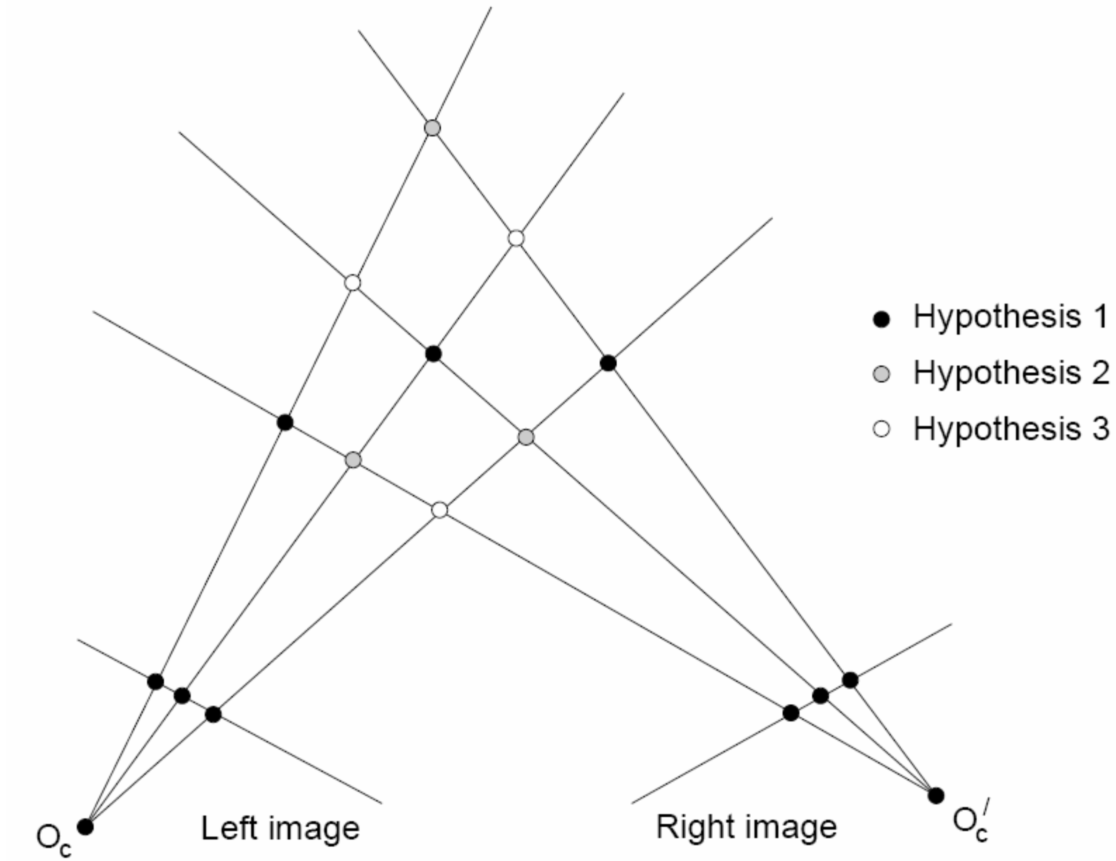
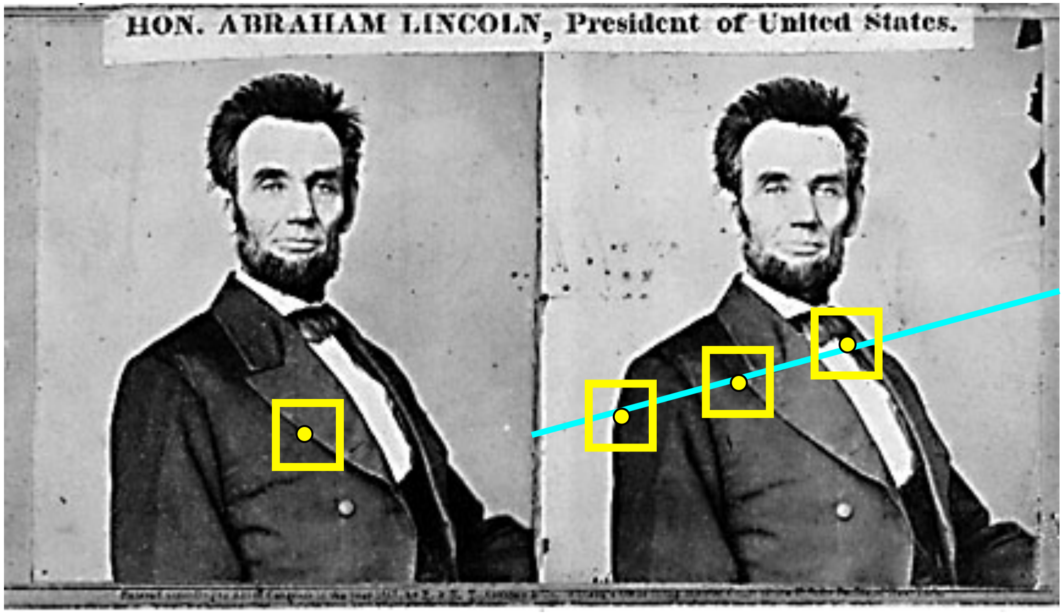
Stereo matching: solving the correspondence problem



- Goal: finding matching points between two images

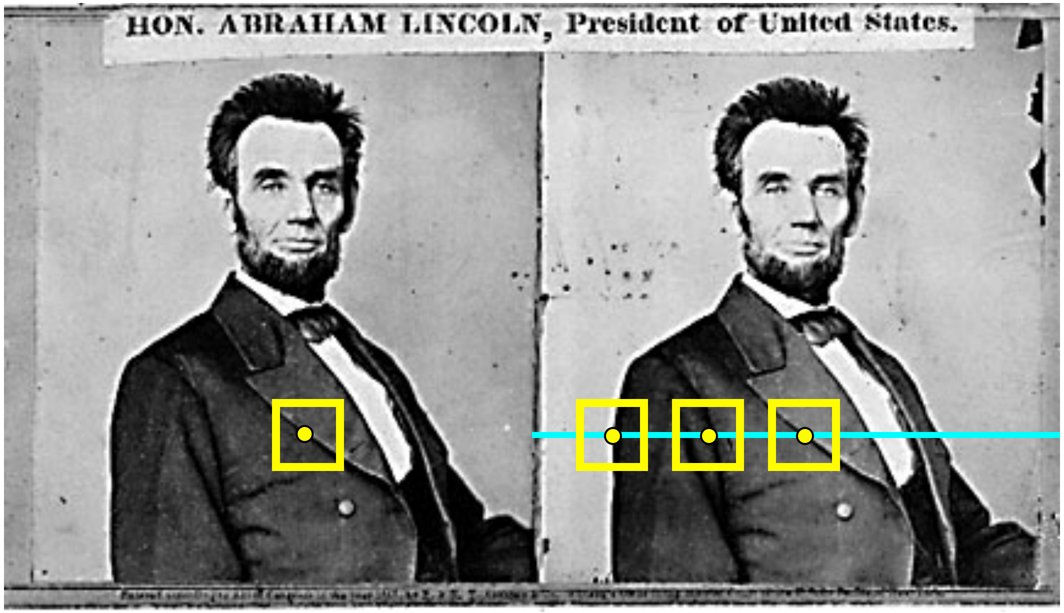
Basic stereo matching algorithm



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines are scanlines
  - When does this happen?

Slide credit: J. Hayes

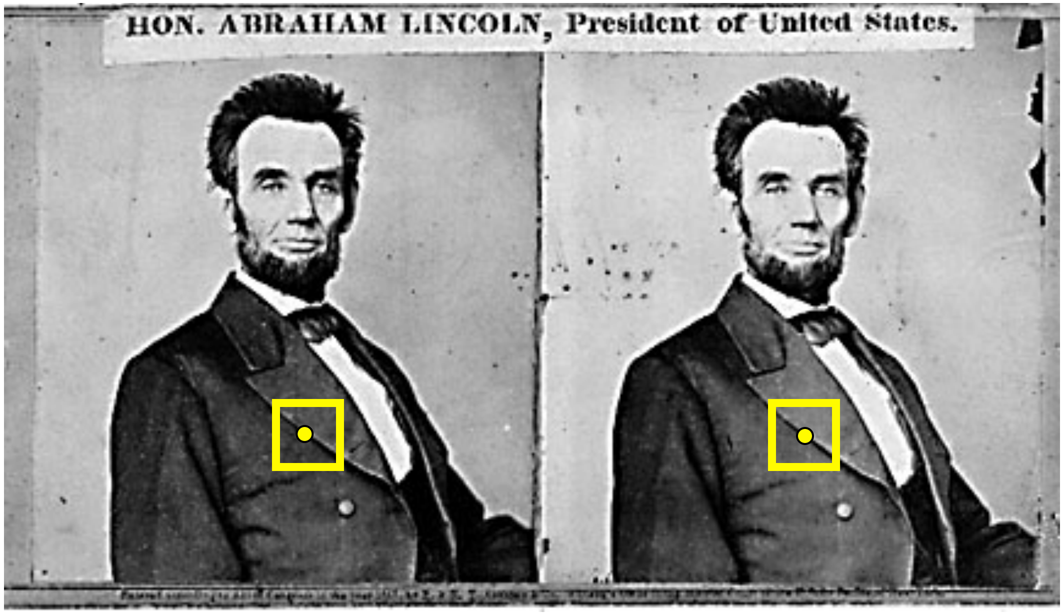
Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match x'
  - Compute disparity x-x' and set depth(x) = 1/(x-x')

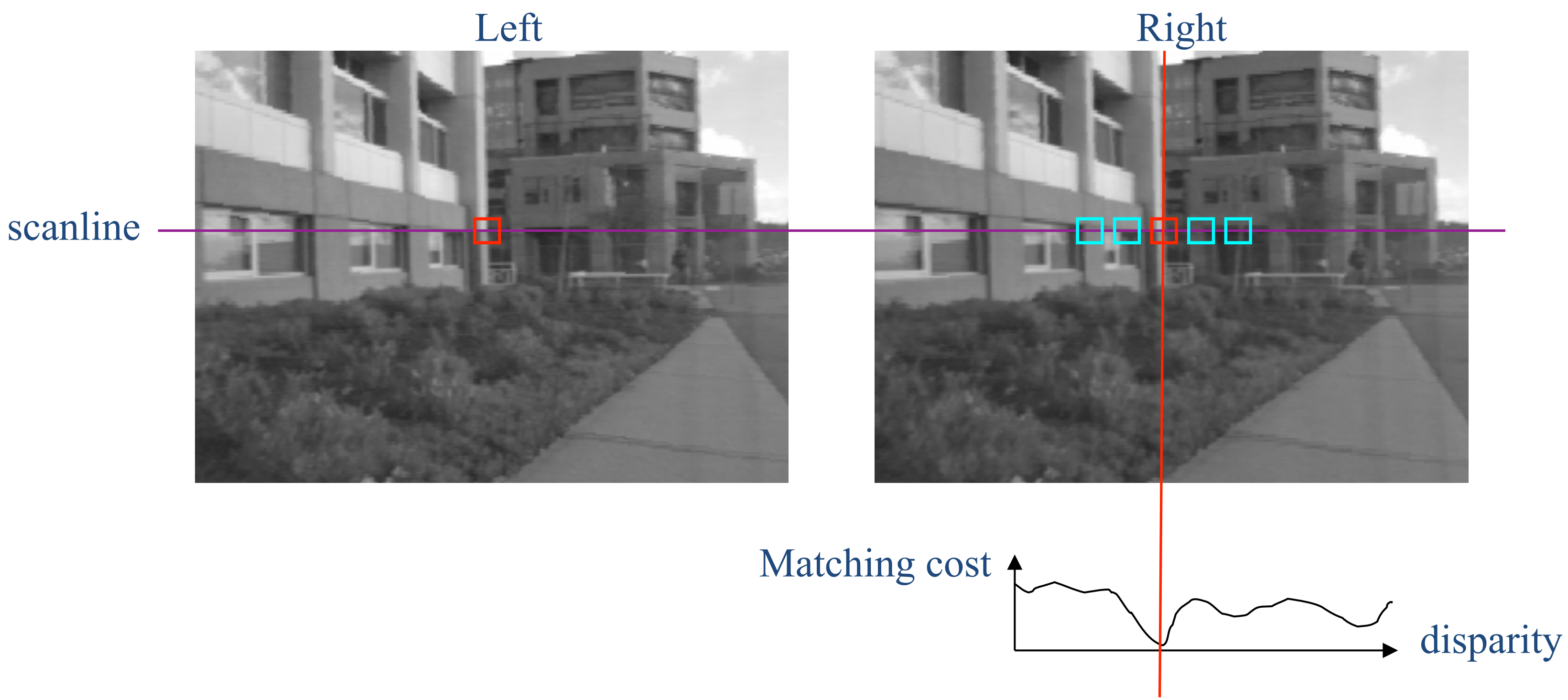
Correspondence problem

- Let's make some assumptions to simplify the matching problem
  - The baseline is relatively small (compared to the depth of scene points)
  - Then most scene points are visible in both views
  - Also, matching regions are similar in appearance



Slide credit: J. Hayes

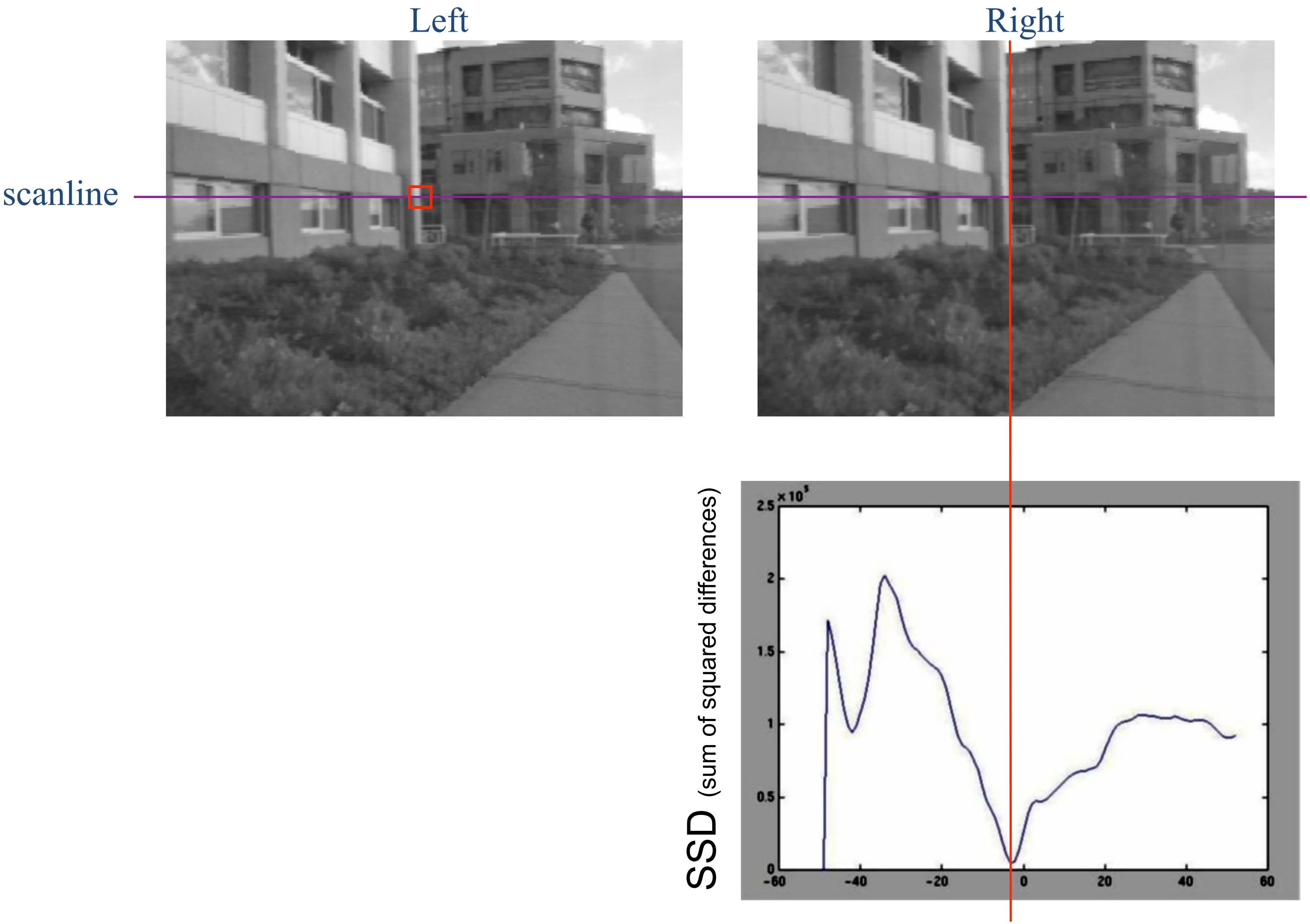
Correspondence search with similarity constraint



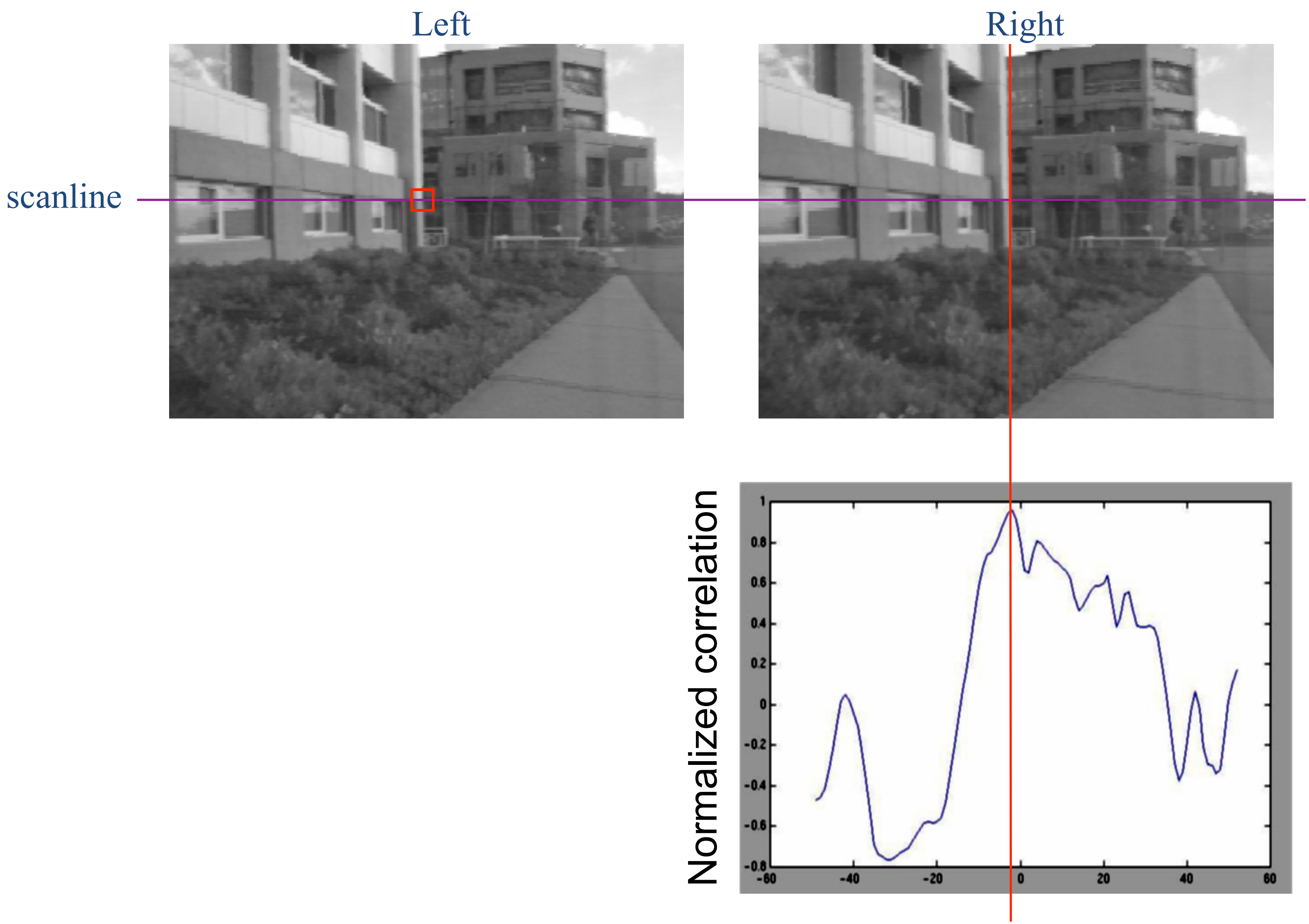
- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

Slide credit: J. Hayes

Correspondence search with similarity constraint

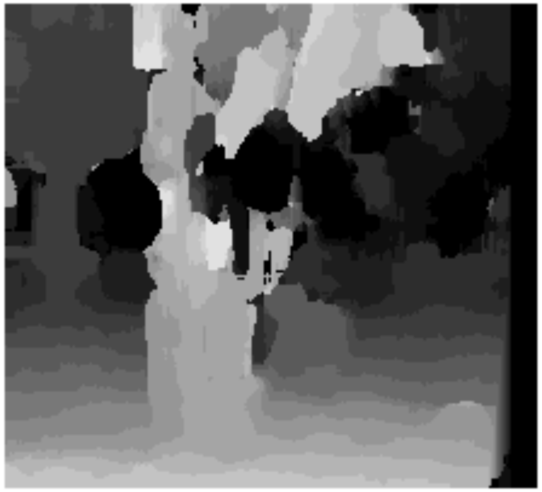


Correspondence search with similarity constraint



Effect of window size

W = 3

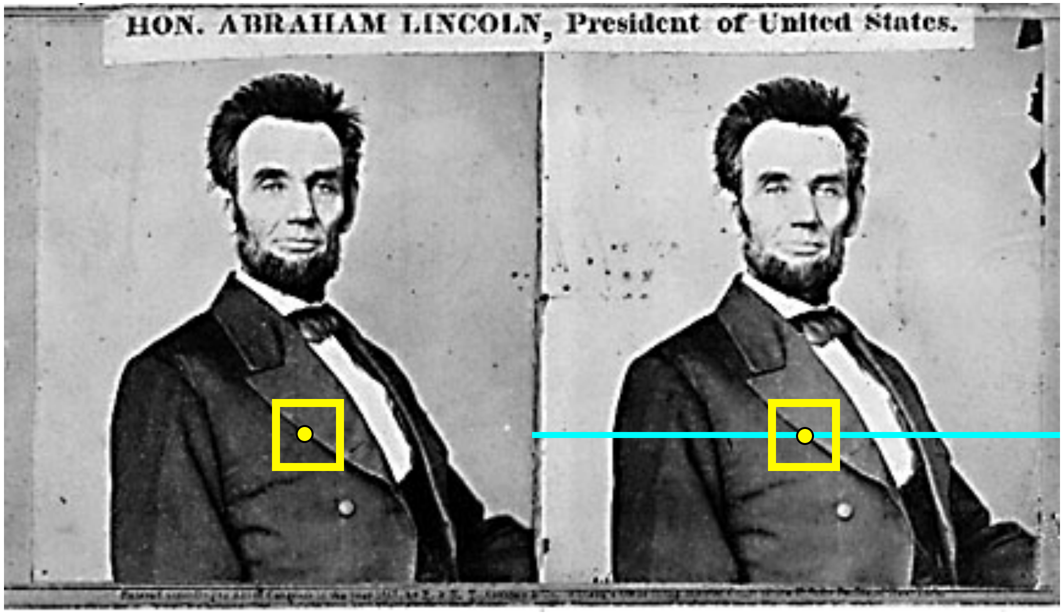


W = 20

- Smaller window
  - + More detail
  - • More noise

- Larger window
  - + Smoother disparity maps
  - • Less detail

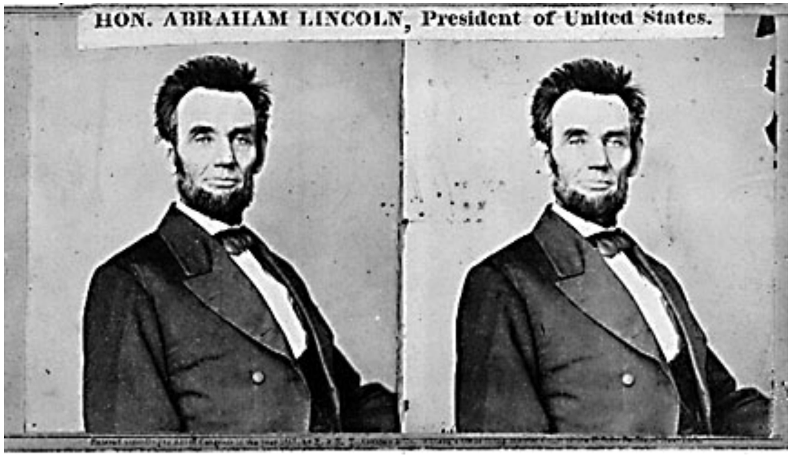
The similarity constraint



- Corresponding regions in two images should be similar in appearance
- …and non-corresponding regions should be different
- When will the similarity constraint fail?

Slide credit: J. Hayes

Limitations of similarity constraint



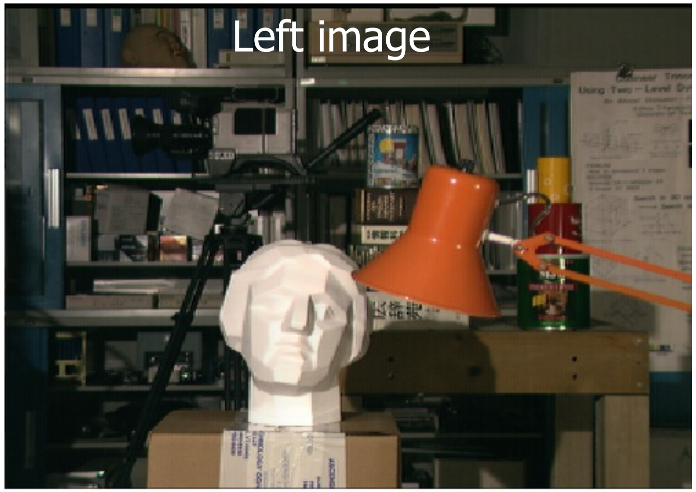
Textureless surfaces

Occlusions, repetition

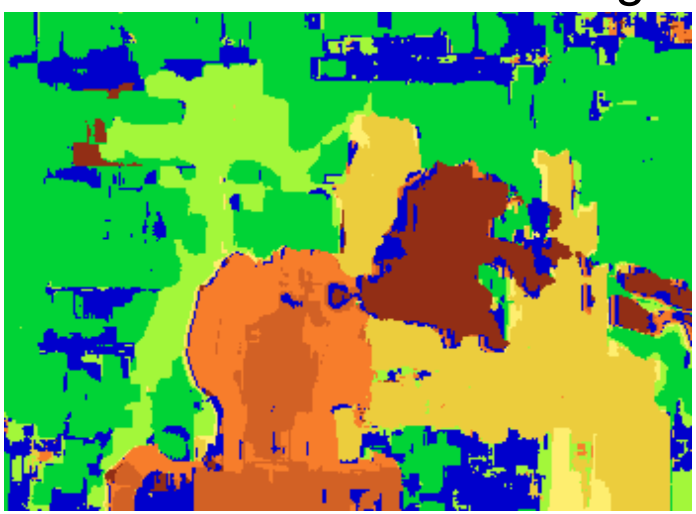
Specular surfaces

Slide credit: J. Hayes

Results with window search

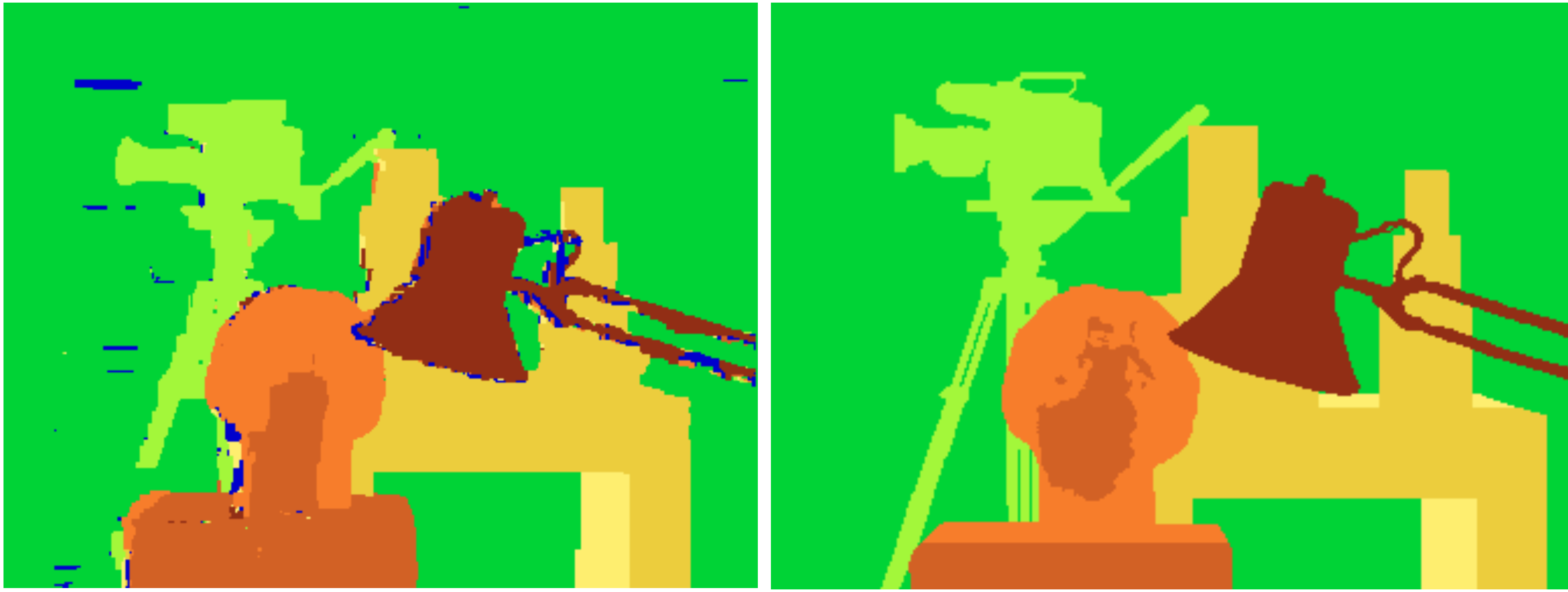


Window-based matching



Ground truth

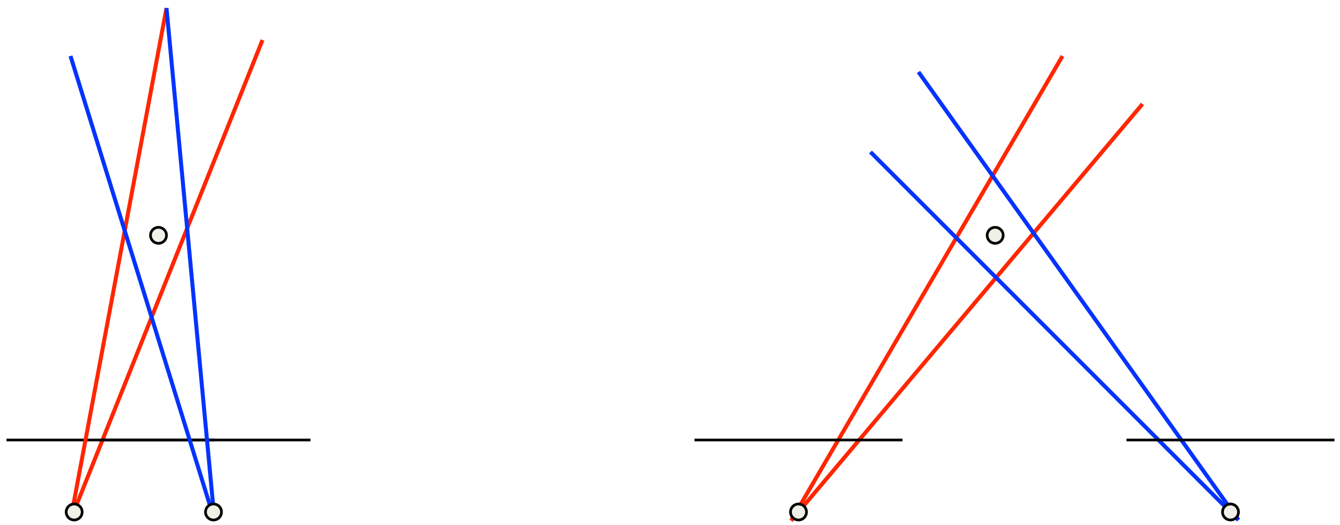
Better methods exist... (CS231a)



Graph cuts Ground truth

Y. Boykov, O. Veksler, and R. Zabih,
[Fast Approximate Energy Minimization via Graph Cuts](), PAMI 2001

The role of the baseline

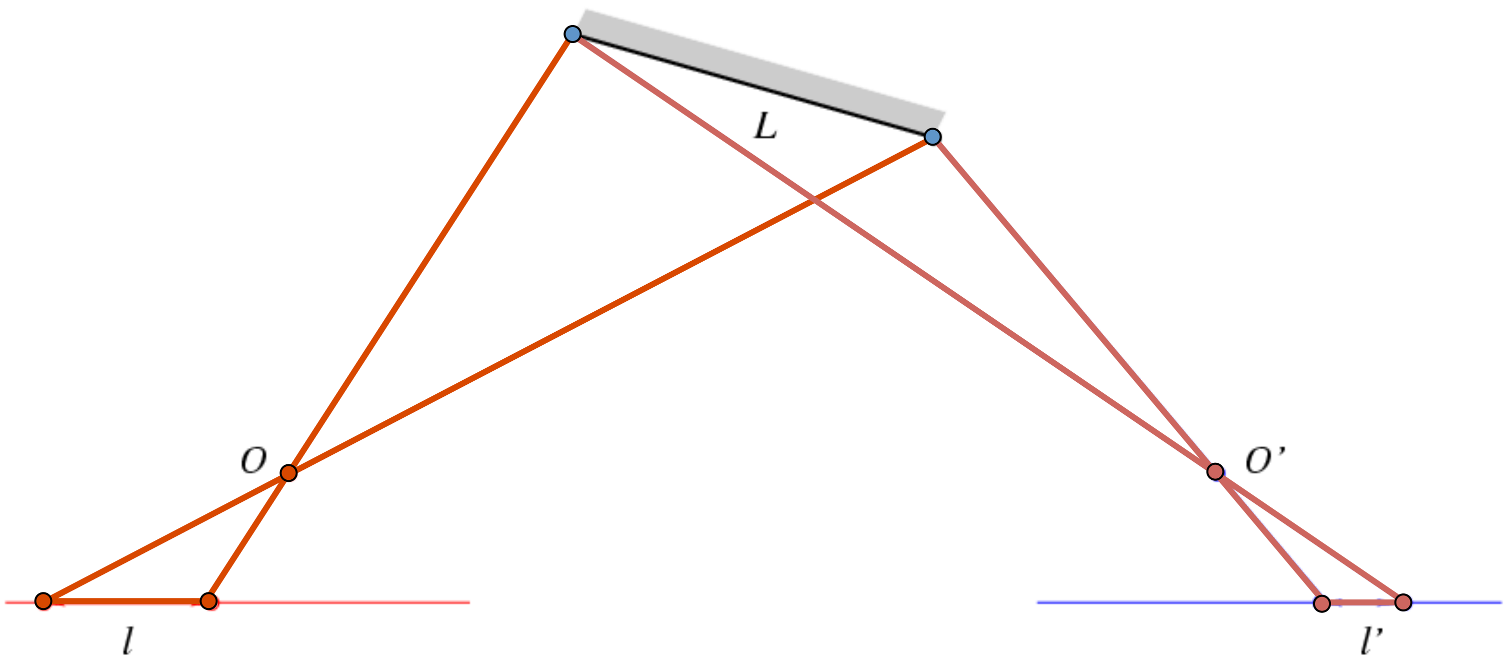


Small Baseline **Large Baseline**

- Small baseline: large depth error
- Large baseline: difficult search problem

Slide credit: S. Seitz

Problem for wide baselines: Foreshortening



- Matching with fixed-size windows will fail!
- Possible solution: adaptively vary window size
- Another solution: *model-based stereo* (CS231a)

Slide credit: J. Hayes

What we will learn today?

- Introduction to stereo vision
- Epipolar geometry: a gentle intro
- Parallel images
- Solving the correspondence problem
- Homographic transformation
- Active stereo vision system

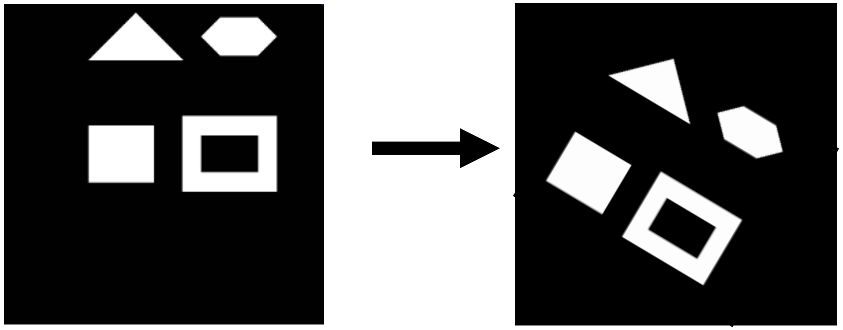
Reading:
[HZ] Chapters: 4, 9, 11
[FP] Chapters: 10

Reminder: transformations in 2D

Special case from lecture 2
(planar rotation & translation)

$$\begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = H_e \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- 3 DOF
- Preserve distance (areas)
- Regulate motion of rigid object

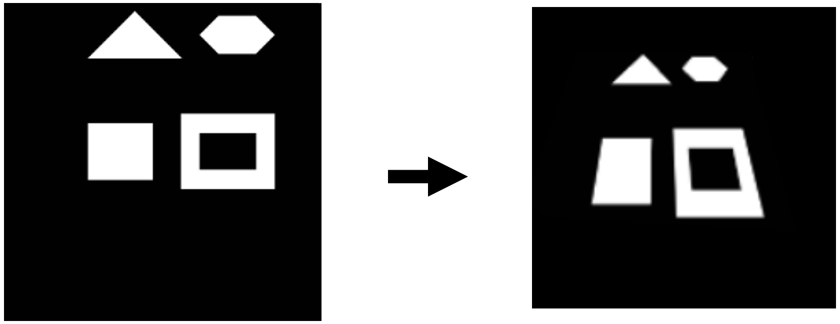


Reminder: transformations in 2D

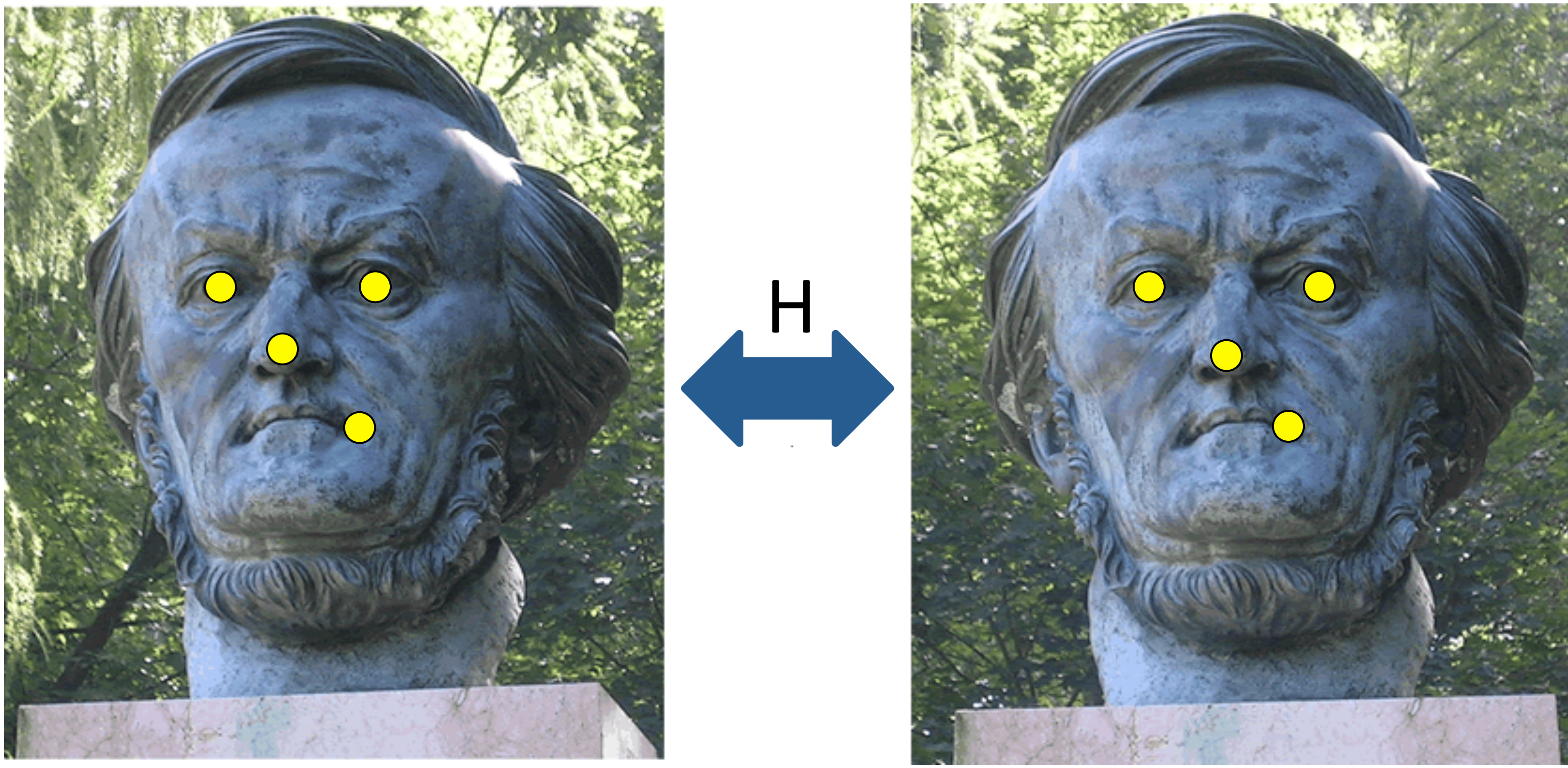
Generic case
(rotation in 3D, scale & translation)

$$\begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = H \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- 8 DOF
- Preserve colinearity

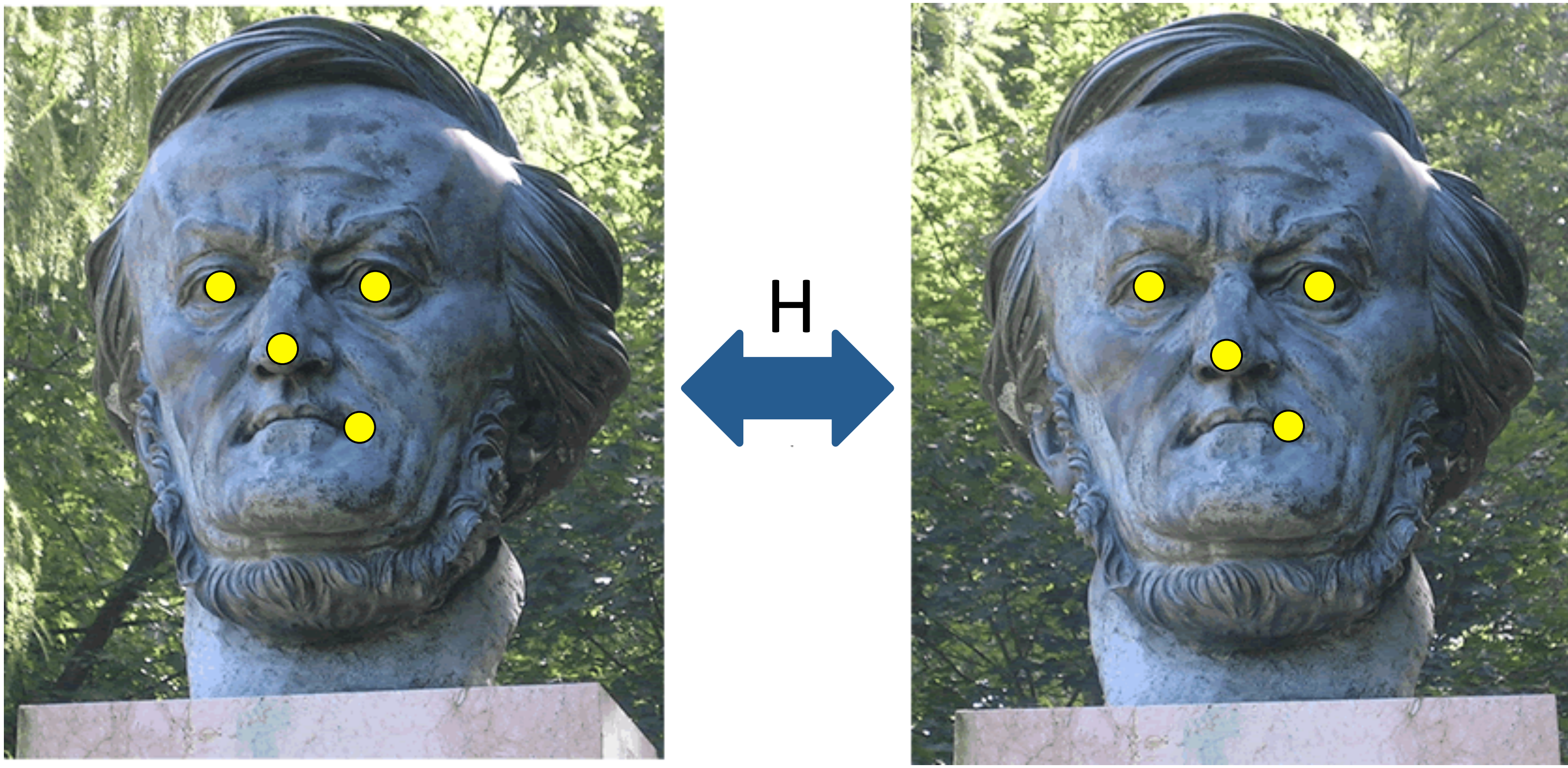


Goal: estimate the homographic transformation between two images



Assumption: Given a set of corresponding points.

Goal: estimate the homographic transformation between two images

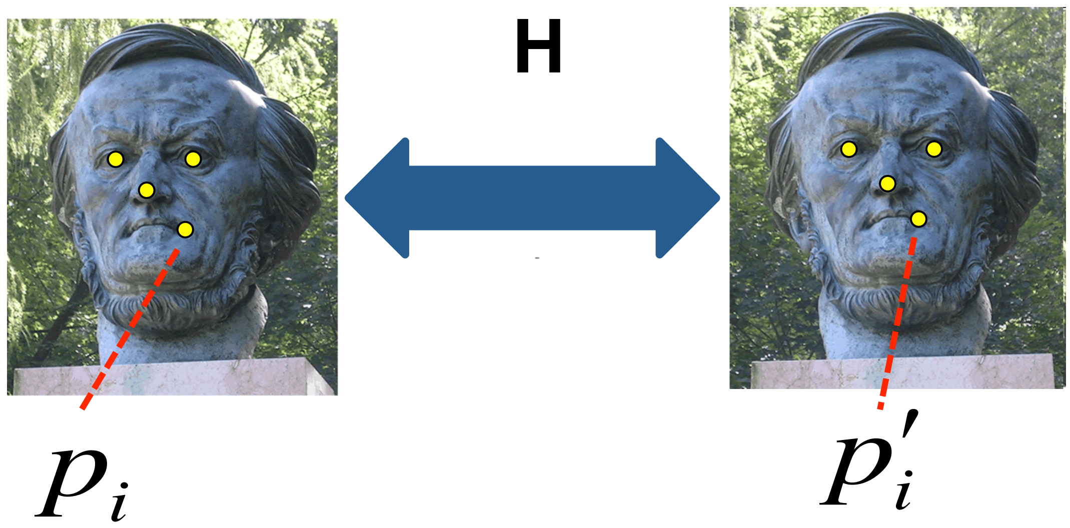


Assumption: Given a set of corresponding points.

Question: How many points are needed? → At least 4 points (8 equations)

Hint: DoF for H? 8!

DLT algorithm (Direct Linear Transformation)



$$p'_i = H \; p_i$$

DLT algorithm (direct Linear Transformation)

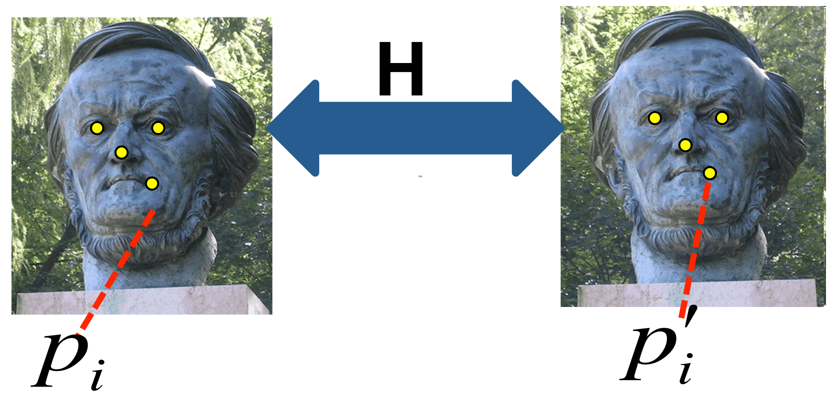
$$p_i' \times H \, p_i = 0 \longrightarrow \underbrace{\mathrm{A}_\mathrm{i}}_{\text{Function of measurements [2x9]}} \overbrace{\mathbf{h}}^{\text{Unknown [9x1]}} = 0$$

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \Rightarrow \mathbf{h} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix}}_{9\text{x}1}$$

2 independent equations

DLT algorithm (direct Linear Transformation)

$$\underbrace{\boxed{A_i}}_{}\!\!{}^{A_{2\times 9}}\ \boxed{h}^{\,h_{9\times 1}} = 0$$



$$\begin{cases} A_1\ h = 0 \\ A_2\ h = 0 \\ \\ A_N\ h = 0 \end{cases} \rightarrow \quad A_{2N\times 9}\ h_{9\times 1} = 0$$

Over determined
Homogenous system

DLT algorithm (direct Linear Transformation)

How to solve $A_{2N\times 9}\, h_{9\times 1} = 0$?

Singular Value Decomposition (SVD)!

DLT algorithm (direct Linear Transformation)

How to solve $\mathrm{A}_{2\mathrm{N}\times 9}\,\mathrm{h}_{9\times 1} = 0$?

Singular Value Decomposition (SVD)!

$$\downarrow$$

$$U_{2n\times 9}\,\Sigma_{9\times 9}\,\boxed{V^{T}{}_{9\times 9}}$$

Last column of V gives h! → H!

Why? See pag 593 of AZ

DLT algorithm (direct Linear Transformation)

How to solve $\mathrm{A}_{2N\times 9}\,\mathrm{h}_{9\times 1} = 0$?

```
[U,D,V] = svd(A,0);
x = V(:,end);
```

Clarification about SVD

$$P_{m\times n} = \boxed{U}_{m\times n} \; D_{n\times n} \; \boxed{V}^T_{n\times n}$$

Has n orthogonal columns

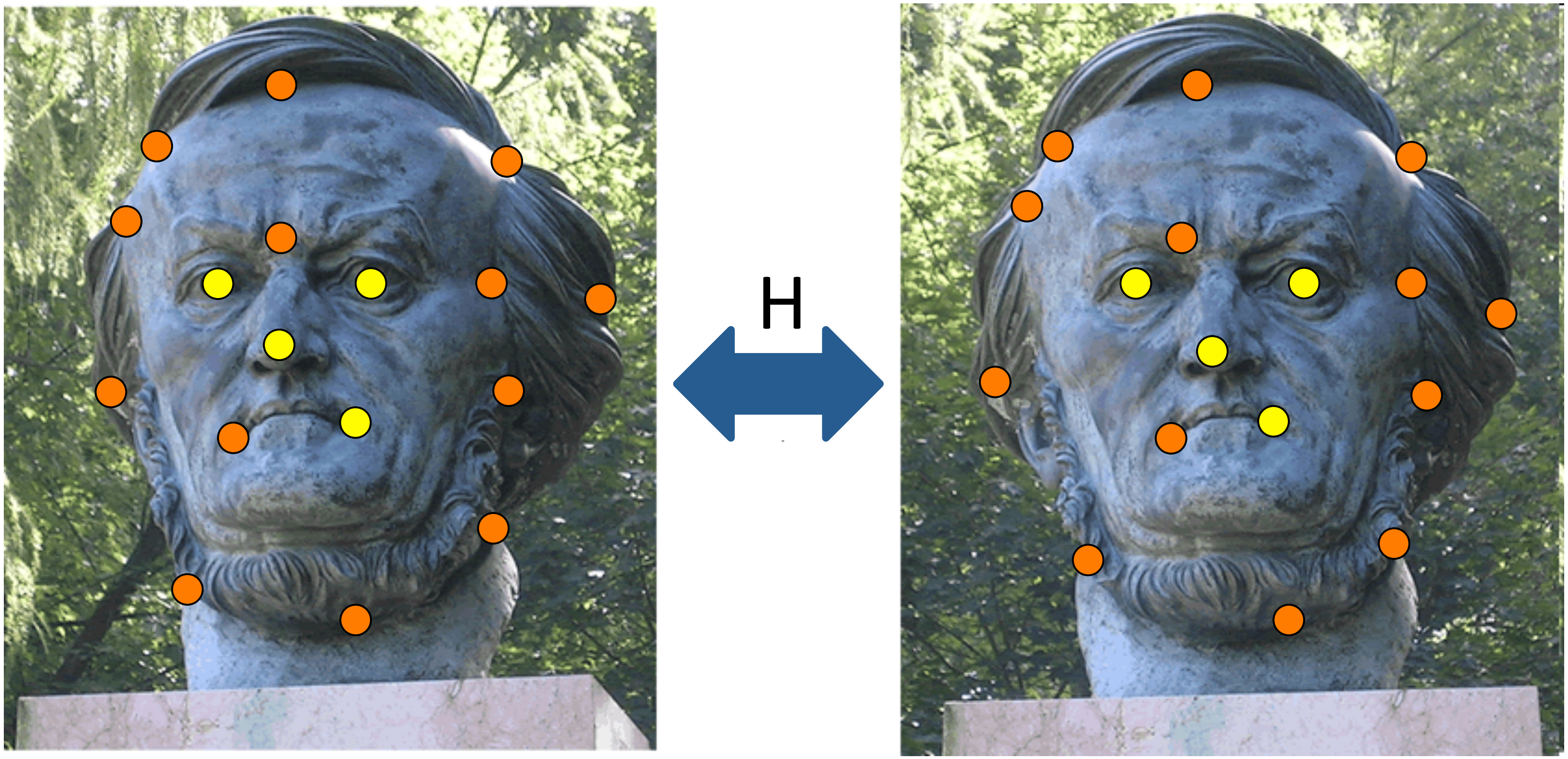
Orthogonal matrix

- This is one of the possible SVD decompositions
- This is typically used for efficiency
- The classic SVD is actually:

$$P_{m\times n} = \boxed{U}_{m\times m} \; D_{m\times n} \; \boxed{V}^T_{n\times n}$$

orthogonal

Orthogonal


H

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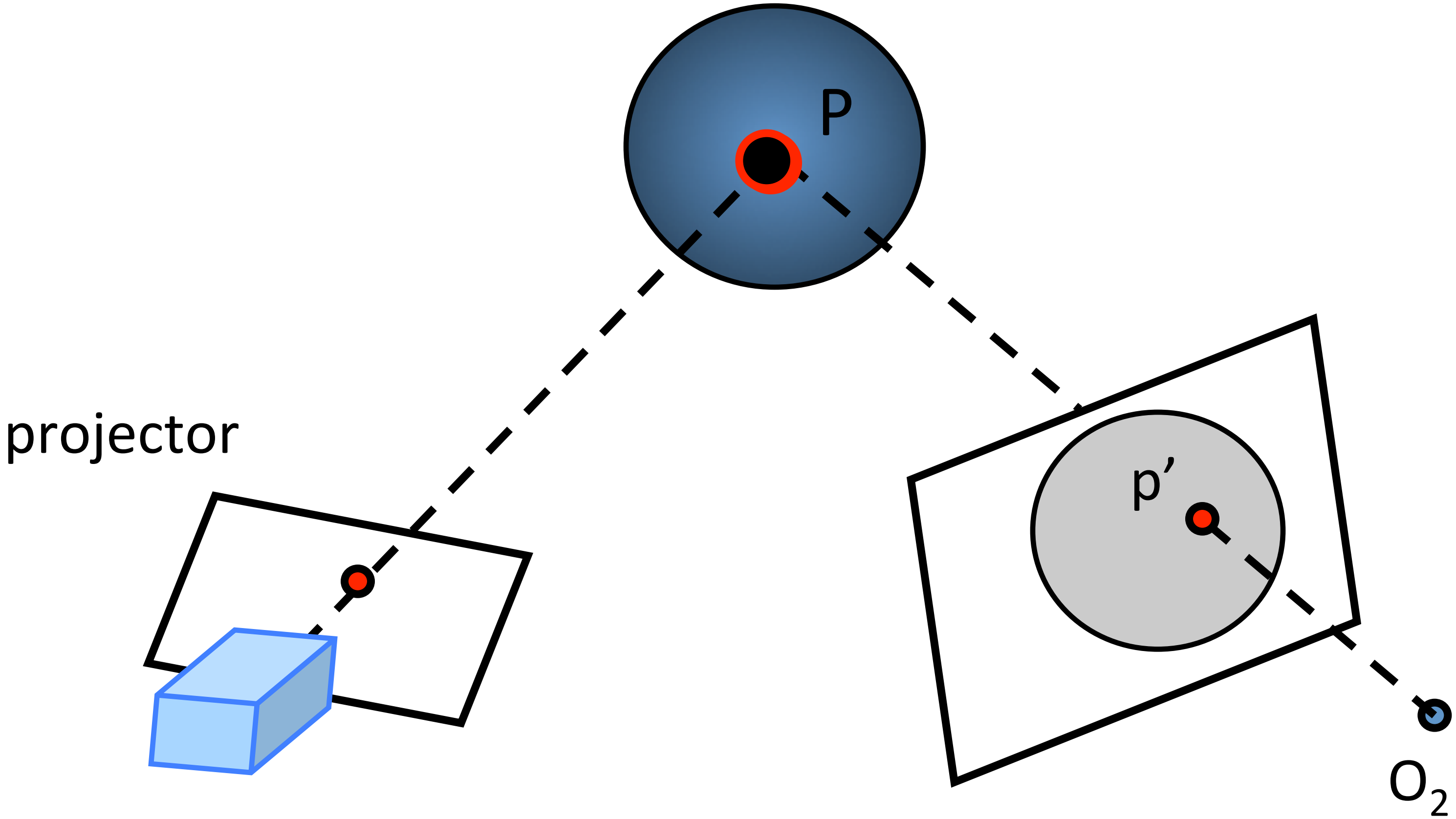
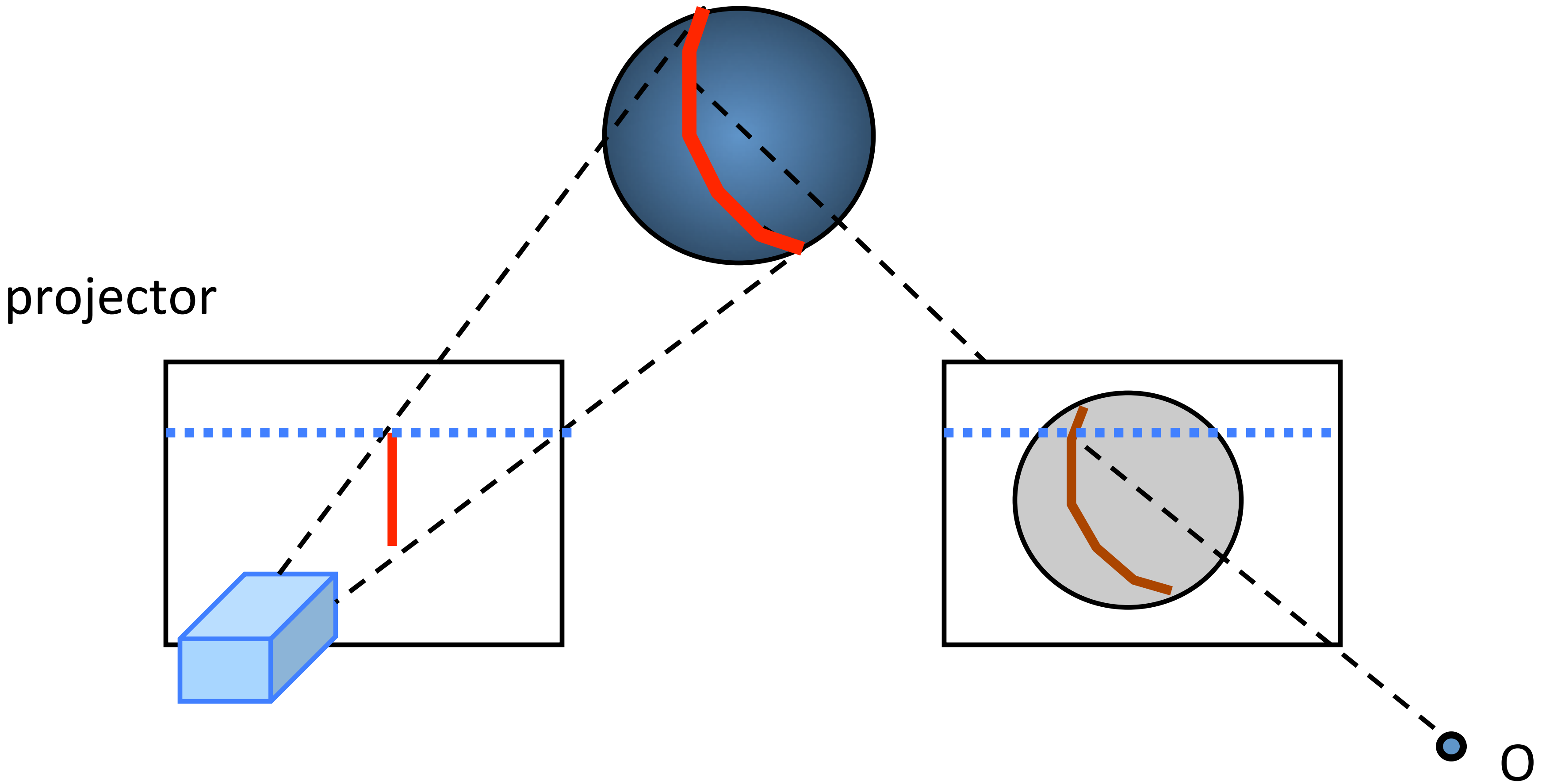
Active stereo (point)



Replace one of the two cameras by a projector

- Single camera
- Projector geometry calibrated
- What's the advantage of having the projector? Correspondence problem solved!

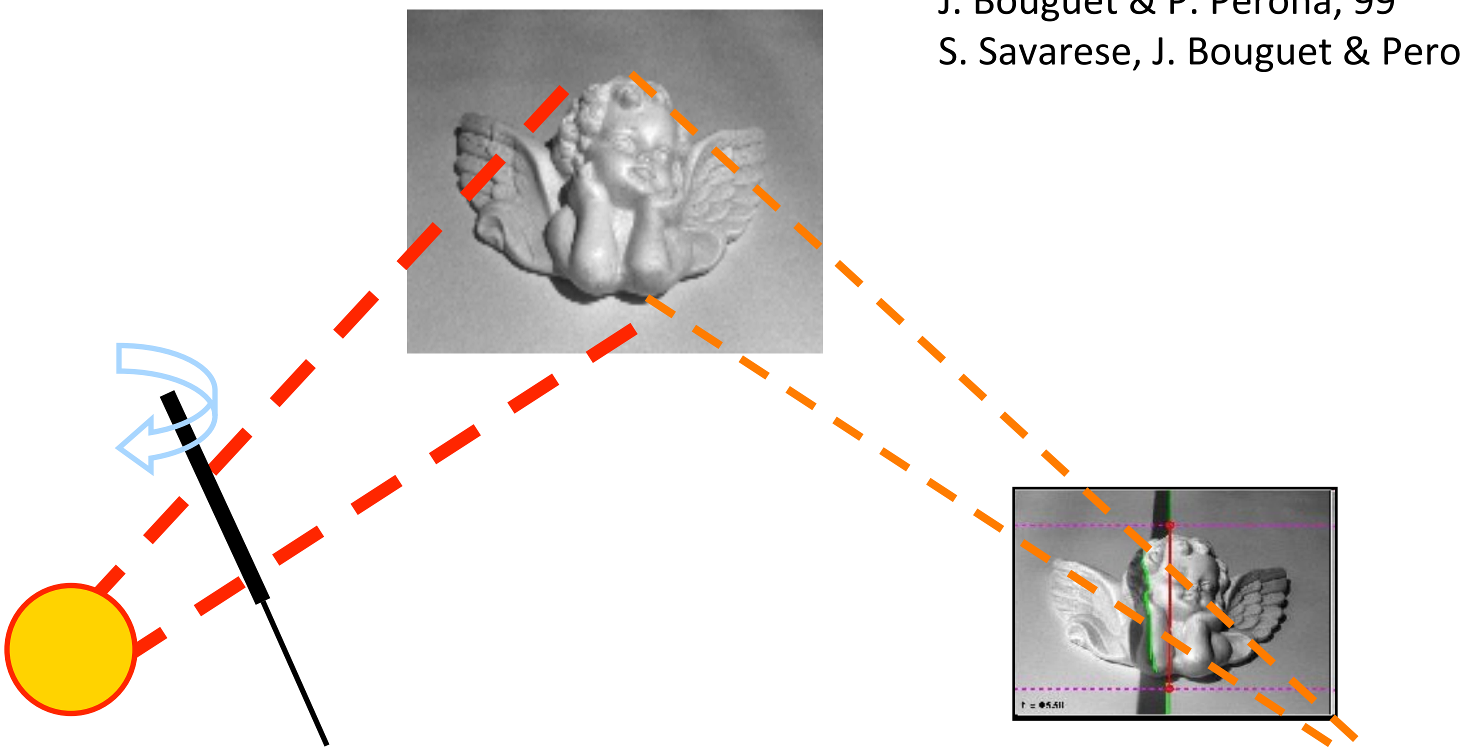
Active stereo (stripe)



-Projector and camera are parallel
- Correspondence problem solved!

Active stereo (shadows)

J. Bouguet & P. Perona, 99
S. Savarese, J. Bouguet & Perona, 00

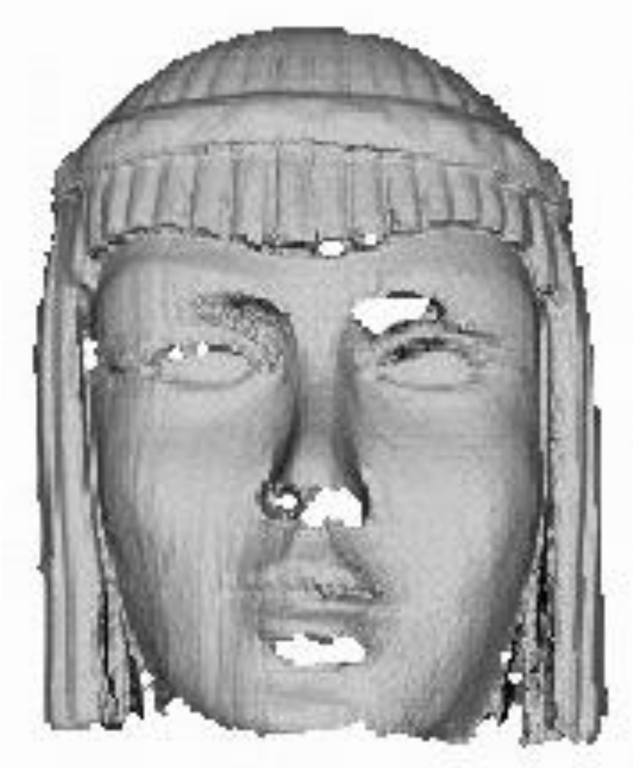


Light source

- 1 camera,1 light source
- very cheap setup
- calibrated light source

Active stereo (shadows)

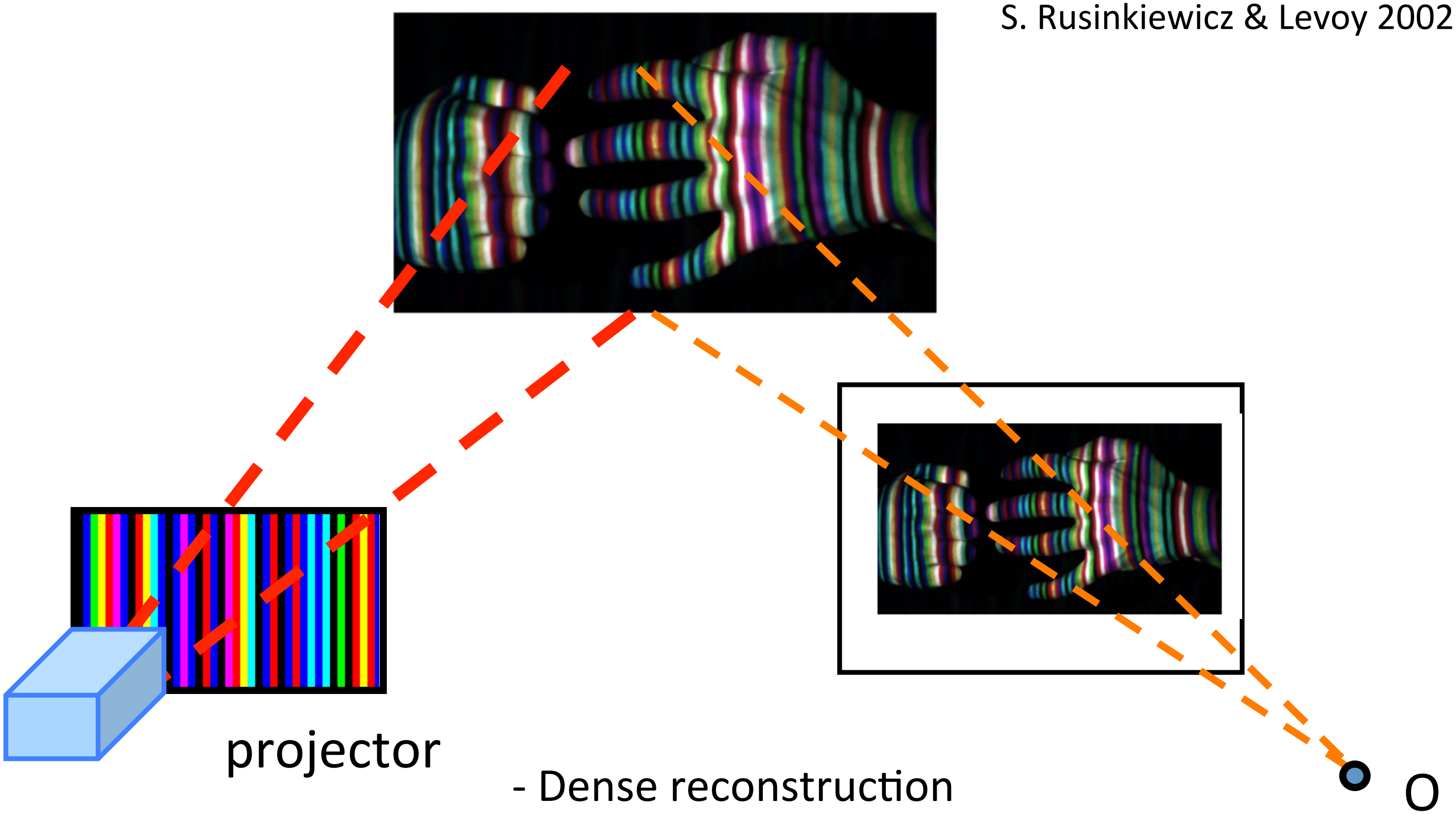
J. Bouguet & P. Perona, 99
S. Savarese, J. Bouguet & Perona, 00



Active stereo (color-coded stripes)

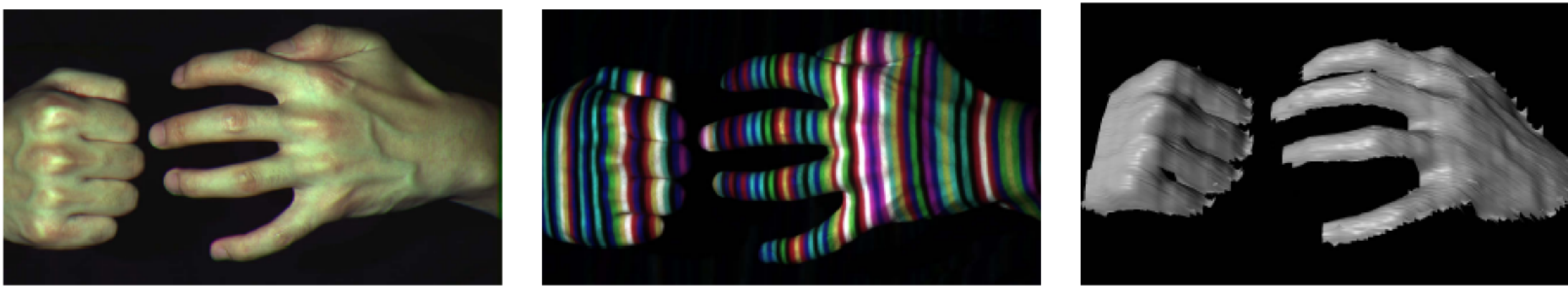
L. Zhang, B. Curless, and S. M. Seitz 2002

S. Rusinkiewicz & Levoy 2002



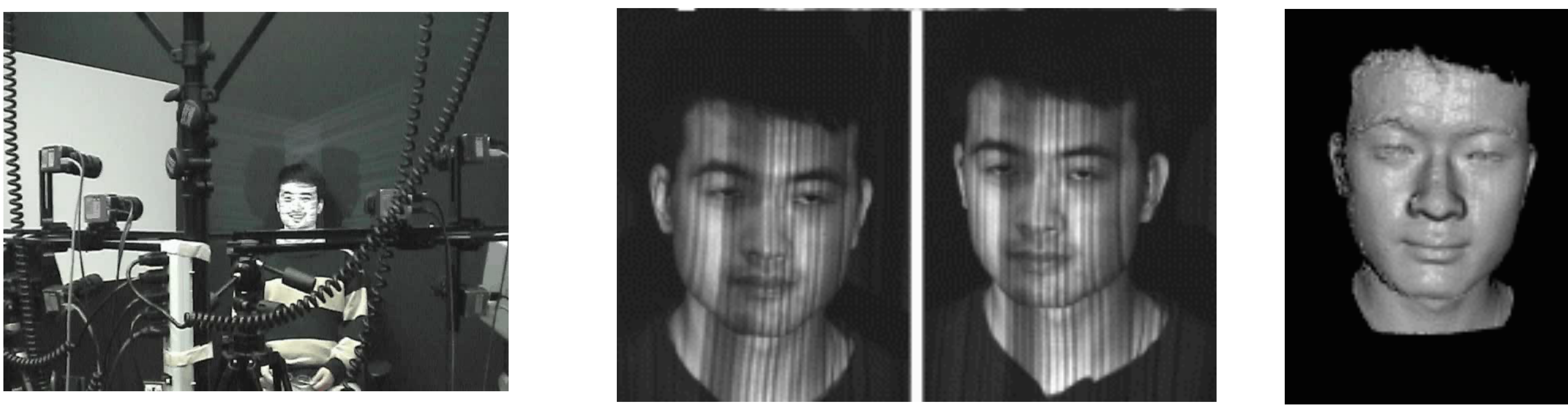
- Dense reconstruction
- Correspondence problem again
- Get around it by using color codes

Active stereo (color-coded stripes)

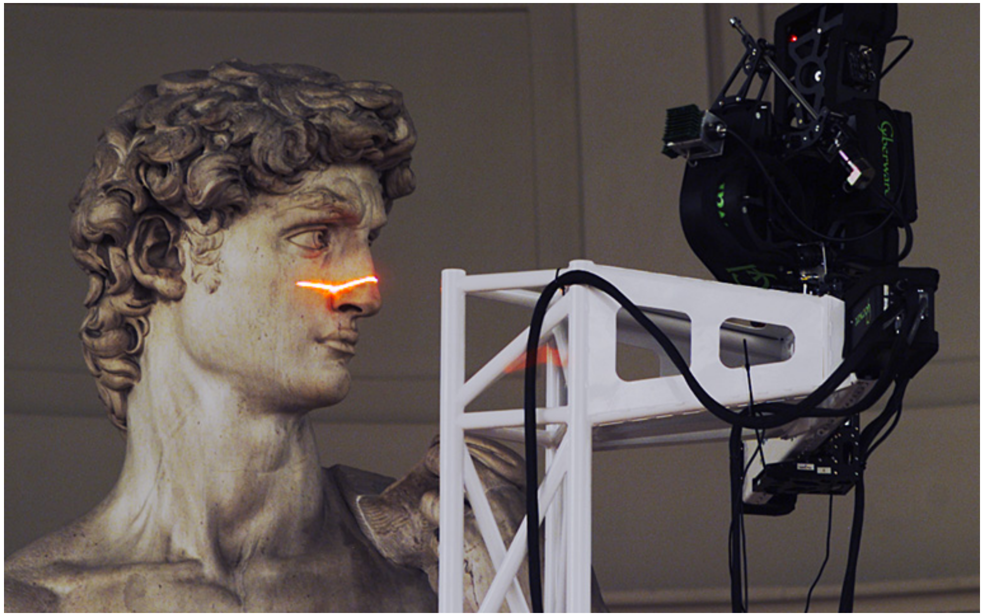


Rapid shape acquisition: Projector + stereo cameras

L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. *3DPVT* 2002



Active stereo (stripe)



Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

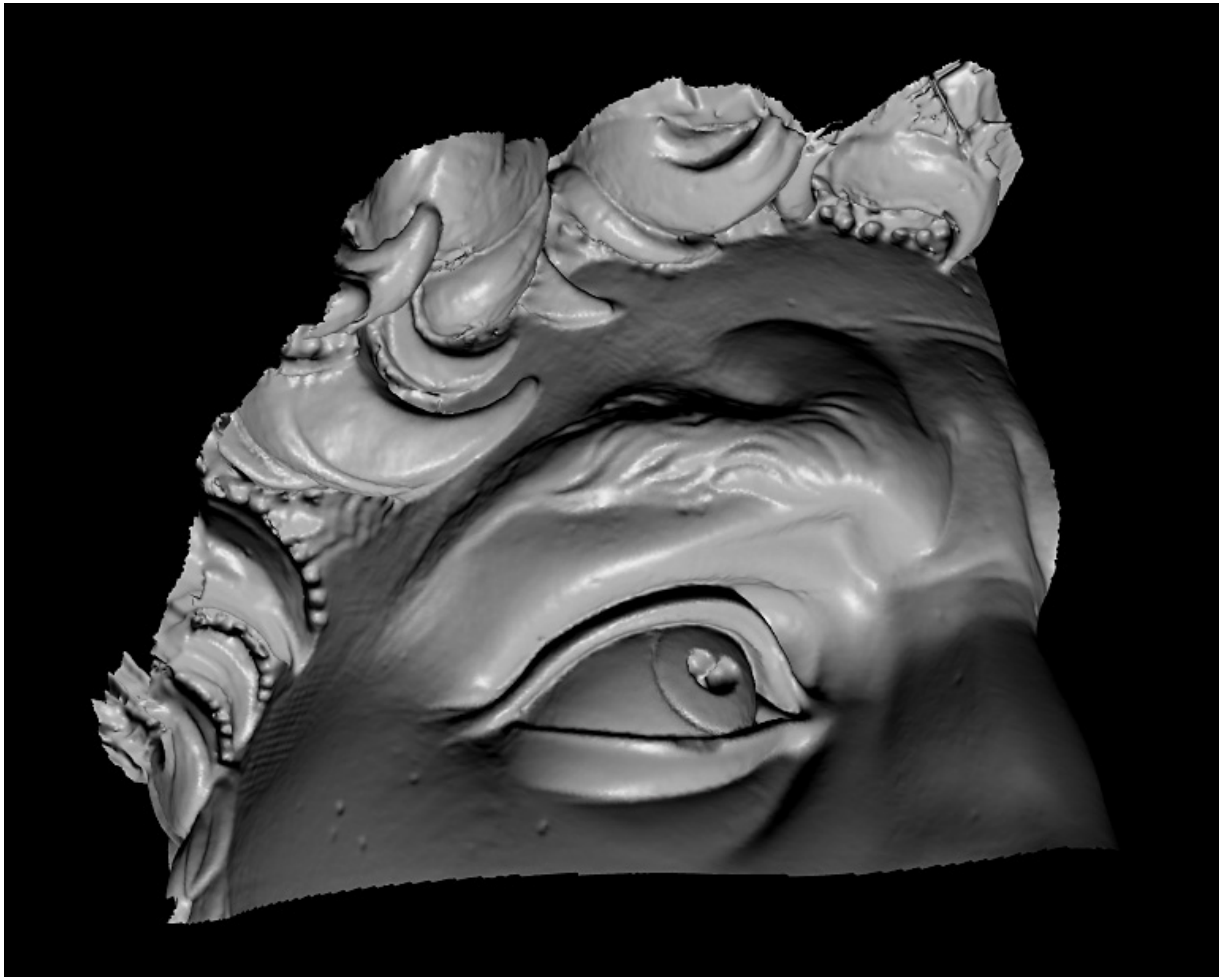
Slide credit: S. Seitz

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

Slide credit: S. Seitz

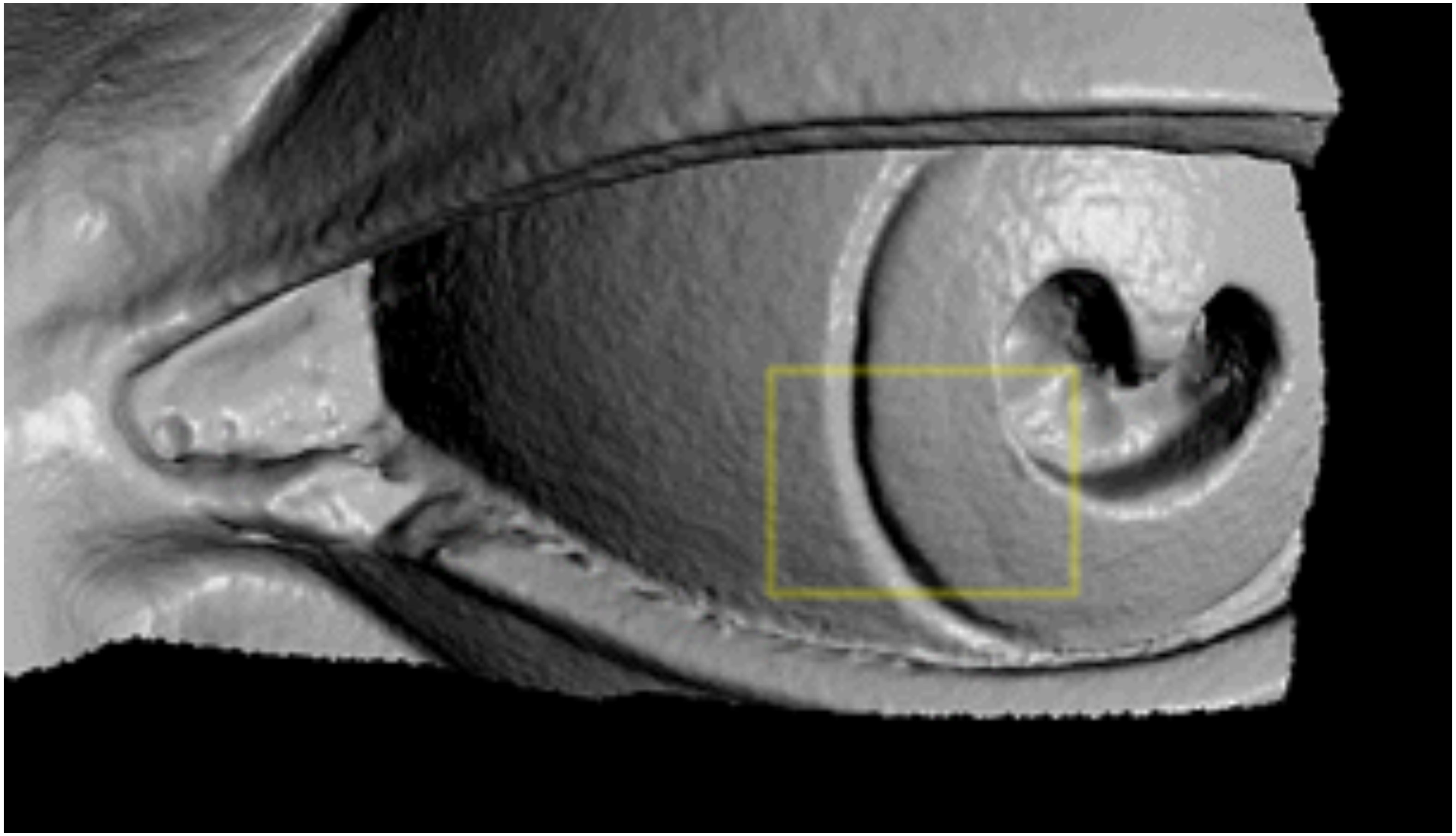
Laser scanned models



The Digital Michelangelo Project, Levoy et al.

Slide credit: S. Seitz

Laser scanned models

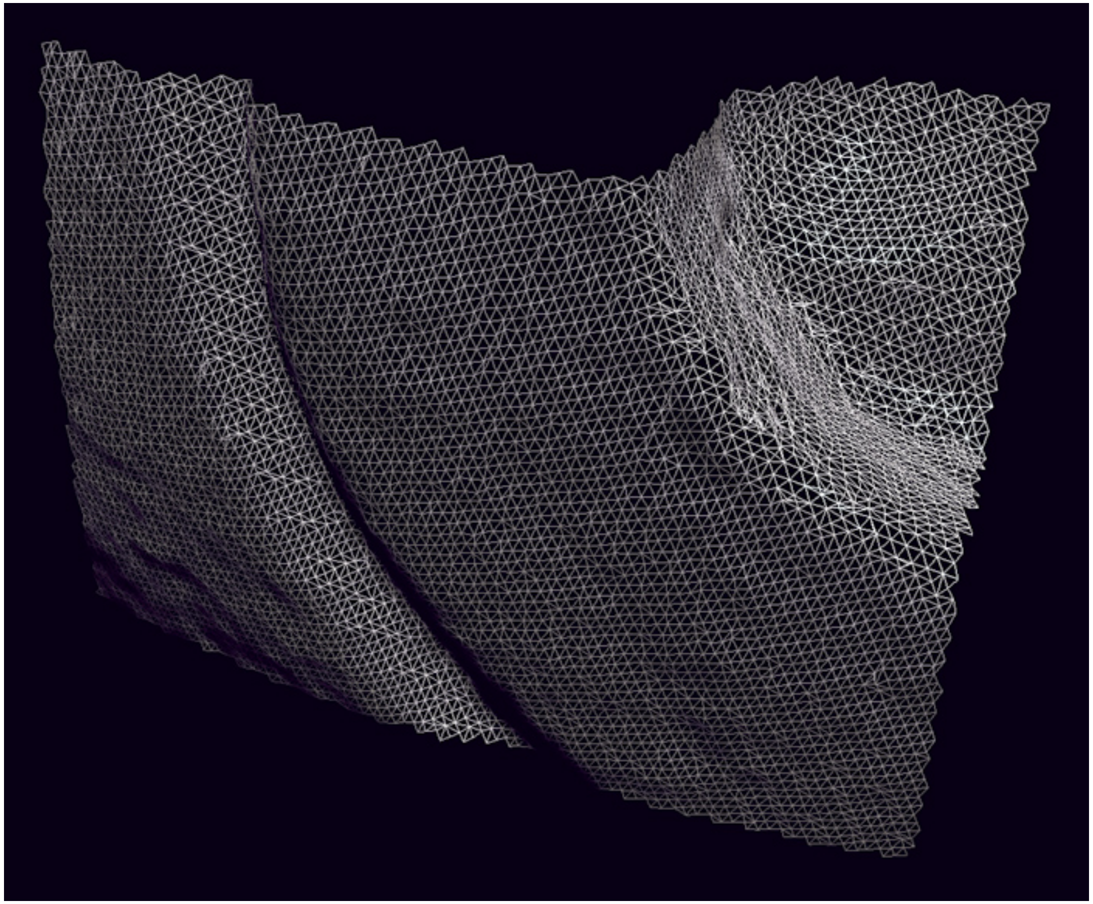


The Digital Michelangelo Project, Levoy et al.

Slide credit: S. Seitz

Laser scanned models

1.0 mm resolution (56 million triangles)



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