# Homework 6

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Question 5
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**Question 5:**

Use the definition of $\theta$ in order to show the following:

a. $5n^3 + 2n^2 + 3n = \theta(n^3)$

Proof:

Let $f(n) = 5n^3 + 2n^2 + 3n$ and $g(n) = (n^3)$,
we will prove that $c_2 * g(n) \leq f(n) \leq c_1 * g(n)$ for any $n_0 \geq 1$.

For any $n \geq 1$, we know that $5n^3 \leq 5n^3 + 2n^2 + 3n$.
So if we take $c_2 = 5$, and $n_0 = 1$, then $5n^3 \leq 5n^3 + 2n^2 + 3n$.
**Therefore,** $\mathbf{5 * g(n) \leq f(n)}$ **and** $\mathbf{f = \Omega(g)}$

For any $n \geq 1$, we know that $n^2$ and $n^3$ are larger than n.
So, we know that $5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$.
So if we take $c_1 = 10$ and $n_0 = 1$, then $5n^3 + 2n^2 + 3n \leq 10n^3$.
**Therefore,** $\mathbf{f(n) \leq 10 * g(n)}$ **and** $\mathbf{f = O(g)}$

There for if we take $c_1 = 10$, $c_2 = 5$ and $n_0 = 1$,
then for all $n \geq n_0$, $f = O(g)$ and $f = \Omega(g)$,
**Therefore,** $\mathbf{f = \theta(g) = \theta(n^3)}$

**b.** $\sqrt{7n^2+2n-8} = \theta(n)$

Proof:

Let $f(n) = \sqrt{7n^2+2n-8}$ and $g(n) = (n)$,
we will prove that $c_2 * g(n) \leq f(n) \leq c_1 * g(n)$ for any $n_0 \geq ?$.

Using the expression $7n^2+2n-8$, we can see that $7n^2 \leq 7n^2+2n-8$,
when $2n-8 \geq 0$. This gives us $n_0 = \sqrt{4} = 2$. So we take $c_2 = \sqrt{7n^2}$ .
and round $\sqrt{7}$ down to 2 and take $c_2 = 2$. So, for all $n \geq 2$, $2n \leq \sqrt{7n^2+2n-8}$.
$\mathbf{Therefore,\ \ 2 * g(n) \leq f\ \ and\ \ f = \Omega(g)}$

Using the expression $7n^2+2n-8$, we know that $7n^2+2n-8 \leq 7n^2+2n^2$.
So, we know that $7n^2+2n-8 \leq 9n^2$. $\sqrt{9n^2}$ gives us $3n$.
So if we take $c_1 = 3$ and $n_0 = 2$, then $\sqrt{7n^2+2n-8} \leq 3n$.
$\mathbf{Therefore,\ f(n) \leq 3 * g(n)\ and\ f = O(g)}$

There for if we take $c_1 = 3$, $c_2 = 2$ and $n_0 = 2$,
then for all $n \geq n_0$, $f = O(g)$ and $f = \Omega(g)$,
$\mathbf{Therefore,\ 2 * g(n) \leq f \leq 3 * g(n)}$ and $\mathbf{f} = \theta(\mathbf{g}) = \theta(\mathbf{n})$