

Interference Graph

Summary

The **interference graph** is the basis of the main technique for performing register allocation

Register allocation consists in choosing the **processor registers** where each value will reside during program execution

As this depends on the target processor, the decision is made **after** code generation

However, as the foundation of the technique is independent from the target architecture, it is presented here applied to the **intermediate representation**

Live ranges (1)

Example (12, repeated)

Liveness analysis for block B_3 resulted in

		UEVAR	VAR KILL	LIVEOUT	LIVEIN
1	$t_4 \leftarrow i_load \ @r$	$@r$	t_4	$t_4, @n$	$@r, @n$
2	$t_5 \leftarrow i_load \ @n$	$@n$	t_5	$t_4, t_5, @n$	$t_4, @n$
3	$t_6 \leftarrow i_mul \ t_4, t_5$	t_4, t_5	t_6	$t_6, @n$	$t_4, t_5, @n$
4	$@r \leftarrow i_store \ t_6$	t_6	$@r$	$@n$	$t_6, @n$
5	$t_7 \leftarrow i_load \ @n$	$@n$	t_7	t_7	$@n$
6	$t_8 \leftarrow i_value \ 1$	—	t_8	t_7, t_8	t_7
7	$t_9 \leftarrow i_sub \ t_7, t_8$	t_7, t_8	t_9	t_9	t_7, t_8
8	$@n \leftarrow i_astore \ t_9$	t_9	$@n$	—	t_9
9	jump B_2	—	—	—	—

Live ranges (2)

Example (12, cont.)

From the results of liveness analysis for block B_3 , we see that

$@r$ is only **live on entry** to instruction 1, and is never used again in the block

$@n$ is **live on entry** to instructions 1, 2, 3, 4 e 5

t_4 is **live** from after instruction 1 until the entry to instruction 3

t_5 is **live** from after instruction 2 until the entry to instruction 3

...

The control flow graph edges where a value is live on exit from the **source node** and live on entry to the **destination node** constitute the **live range** of the value

The live range of a value corresponds to its **lifetime**

Live ranges (3)

Example (12, cont.)

The **live ranges** of the several values of block B_3 are shown on the right

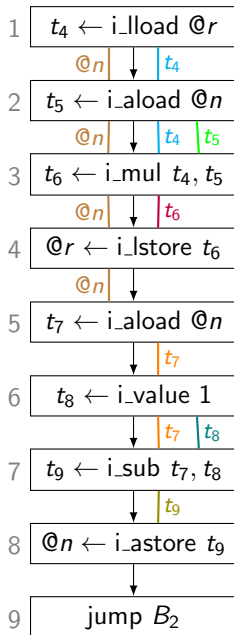
$@n$ is live on edges $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5$,
its live range is **1–5**

t_4 is live on edges $1 \rightarrow 2 \rightarrow 3$, its live range is **1–3**

t_5 is live on edge $2 \rightarrow 3$, its live range is **2–3**

...

In this block, the live range of $@r$ is empty



Interference graph (1)

Interference

Two values **interfere** if their live ranges overlap

Two values **do not interfere** if one is a **copy** of the other everywhere their live ranges overlap

Example (12, cont.)

In B_3 , values t_4 and t_5 interfere along edge $2 \rightarrow 3$

Values $@n$ and t_4 interfere along edges $1 \rightarrow 2 \rightarrow 3$

Value t_9 does not interfere with any other value

The live ranges of values $@n$ and t_5 overlap on edge $2 \rightarrow 3$, but since one is a copy of the other along this edge, they **do not** interfere

Interference graph (2)

The lifetimes of non-interfering values are **disjoint** and they may reside in the same **location**

Example (12, cont.)

Since t_4 and t_6 do not interfere, we could replace

$t_4 \leftarrow i_load \ @r$		$t_4 \leftarrow i_load \ @r$
$t_5 \leftarrow i_load \ @n$		$t_5 \leftarrow i_load \ @n$
$t_6 \leftarrow i_mul \ t_4, t_5$	by	$t_4 \leftarrow i_mul \ t_4, t_5$
$@r \leftarrow i_store \ t_6$		$@r \leftarrow i_store \ t_4$

without changing the meaning of the code

Interference graph (3)

Interference graph

- ▶ The **interference graph** is an **undirected** graph
- ▶ The **nodes** of the graph are the **names** (variables and temporaries) appearing in the code
- ▶ The **edges** of the graph correspond to the **interferences** found in the code

Nodes *a* and *b* are connected by an edge if and only if *a* and *b* interfere

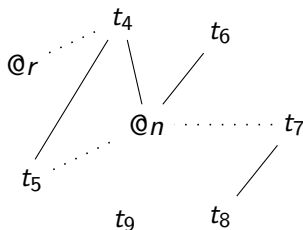
Remarks:

1. Two nodes of an undirected graph are said to be **adjacent** or **neighbours** when they are connected by an edge
2. The **degree** of a node is the number of edges connecting it to other nodes

Interference graph (4)

Example (12, cont.)

Interference graph for block B_3



Values which are **at some point** a copy of one another are said to be **move related**

Dotted edges in the graph connect move related nodes

Register allocation (1)

Graph colouring

Register allocation may be made based on the interference graph

If it is possible to colour the graph

- ▶ using k colours
- ▶ such that adjacent nodes have different colours

then k registers are enough to hold all the values used

Register allocation (2)

Graph colouring

Algorithm

Let k be the number of registers available

1. **Simplify** While there are nodes with degree less than k in the graph, remove them from the graph (along with their edges)
2. **Spill** If there is still some node remaining in the graph
 - i. Remove one from the graph (it is a candidate for **spilling**)
 - ii. Go back to 1.
3. **Select** Rebuild the graph, inserting nodes in the **reverse** order from which they were removed
If possible, colour each inserted node with a colour **different** from those of its neighbours
4. **Restart** If it was not possible to colour some node(s) in step 3., change the code so that it stores and loads the corresponding value(s) to/from memory, and restart from **liveness analysis**

Register allocation (3)

Graph colouring

Remarks

- (a) A node removed from the graph in step 1. has less than k neighbours, hence it and its neighbours can all have a different colour

When it is reinserted into the graph (in step 3.), it is always possible to find a colour for it different from those of its neighbours

- (b) A node removed from the graph in step 2. has at least k neighbours, and it may not be possible to find a colour for it which is different from those of its neighbours

However, if its neighbours do not use all k colours, it will be possible to colour it in step 3. (optimistic colouring)

- (c) Removing nodes in steps 1. and 2., may cause other nodes to be left with less than k neighbours

Register allocation (4)

Graph colouring

Remarks

- (d) If, in step 3., it is not possible to colour a node with a colour different from those of its neighbours, there are, at some point in the code, more than k live values

Since there are only k available registers, at least one of those values will have to be spilled to memory

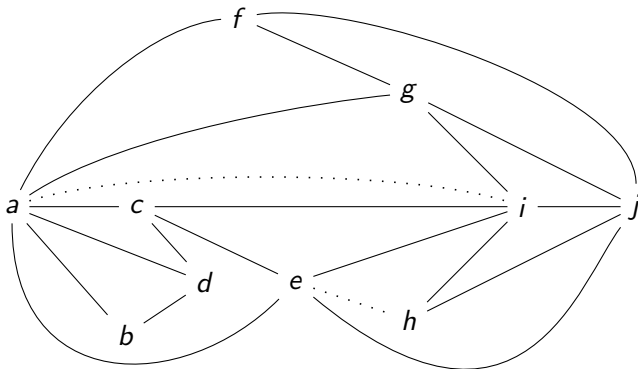
- (e) Spilling a value consists in introducing, into the code, instructions to store it to memory when it is created, and to load it from memory just before it is needed
- (f) The knowledge that two values are move related may be used in step 3. as a hint for colouring both nodes with the same colour, which will allow deleting the instruction which copies one value to the other

Register allocation (5)

Graph colouring

Example (16)

Interference graph with 10 nodes, to be coloured with 4 colours



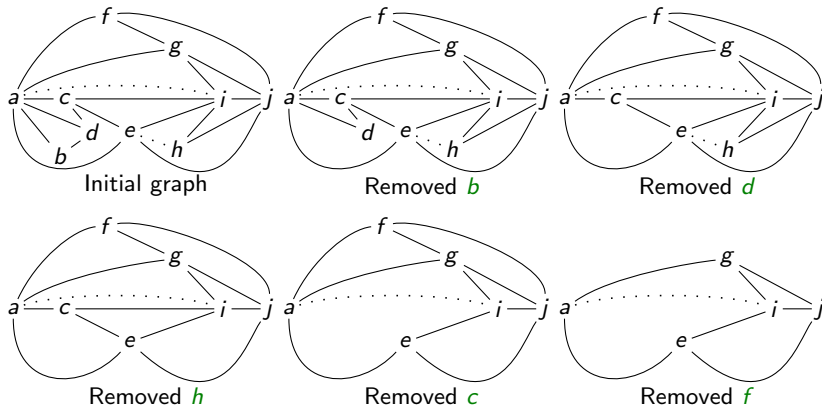
Node *a* has degree 6, nodes *i* and *j* have degree 5, nodes *c*, *e* and *g* have degree 4, *d* and *f* have degree 3, and *b* and *h* have degree 2

Register allocation (6)

Graph colouring

Example (16, cont.)

Simplify

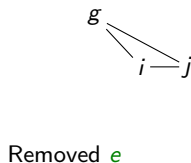
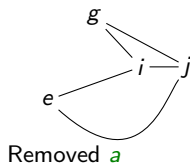


Register allocation (7)

Graph colouring

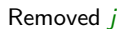
Example (16, cont.)

Simplify (cont.)



After removing node j ,
the graph becomes
empty

The selection phase
initiates at this point

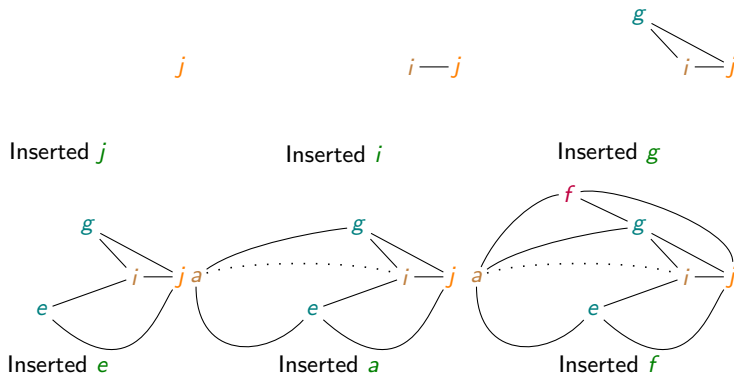


Register allocation (8)

Graph colouring

Example (16, cont.)

Select, using the 4 colours ● ● ● ●

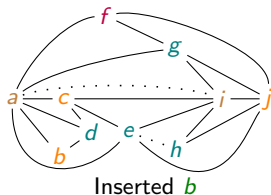
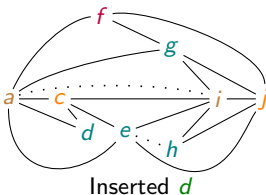
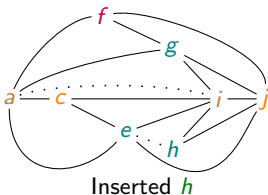
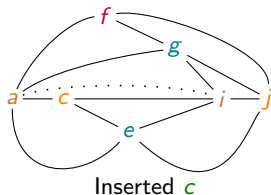


Register allocation (9)

Graph colouring

Example (16, cont.)

Select (cont.), using the 4 colours ● ● ● ●



Every node has now been reinserted into the graph and each one was coloured with one of 4 colours, and all neighbours have different colours

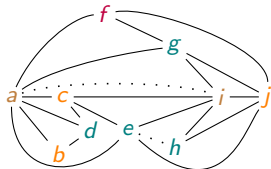
This was possible even though there are nodes in the graph with more than 4 neighbours

Register allocation (10)

Graph colouring

Example (16, cont.)

1. This graph is 4-colourable, even though there are nodes with 4 or more neighbours
2. If the 4 registers are r_1 , r_2 , r_3 and r_4
Values b , c and j may use register r_1
Values a and i may use register r_2
Values d , e , g and h may use register r_3
Value f will be the only one to use register r_4
3. Since values e and h occupy the same register, the instruction that copies one to the other may be eliminated
The same is true for values a and i
4. This is not the only possible 4-colouring (not considering colour renaming)



Register allocation (11)

Graph colouring

Example (17)

Spilling a value

Let t_i stand for the value which could not be coloured and let the instructions on the right be those where it occurs

$t_i \leftarrow \text{i_add } t_3, t_2$
...
 $t_5 \leftarrow \text{i_lt } t_i, t_1$

To **spill** t_i to memory, those instructions are replaced by the ones below, where

- ▶ $@s_1$ will correspond to a temporary memory location within the function activation record
- ▶ t_{i1} and t_{i2} are two new temporaries which replace t_i and whose **live ranges** are shorter than that of t_i

$t_{i1} \leftarrow \text{i_add } t_3, t_2$
 $@s_1 \leftarrow \text{i_lstore } t_{i1}$
...
 $t_{i2} \leftarrow \text{i_lload } @s_1$
 $t_5 \leftarrow \text{i_lt } t_{i2}, t_1$

Exercise (1)

For the IR on the **next** slide:

1. Compute the live ranges of all the temporaries
(*Do not peek at slide 22 before finishing*)
2. Draw the interference graph
(*Do not peek at slide 23 before finishing*)
3. Apply the graph colouring algorithm with $k = 4$
4. Modify the IR to include the spilling of t_0
5. Apply the graph colouring algorithm with $k = 3$

Exercise (2)

IR

```
function @facx
1.      t0 <- i_aload @n
2.      t1 <- i_value 1
3.      t2 <- i_value 0
4.      t2 <- i_lt t0, t2
5.      cjump t2, 11, 10
6.  10:   t3 <- i_copy t1
7.  13:   t4 <- i_copy t0
8.      t5 <- i_value 0
9.      t6 <- i_lt t5, t4
10.     cjump t6, 14, 15
11.  14:   t3 <- i_mul t3, t0
12.     t0 <- i_sub t0, t1
13.     jump 13
14.  15:   jump 12
15.  11:   t3 <- i_value 1
16.     t3 <- i_inv t3
17.  12:   i_return t3
```

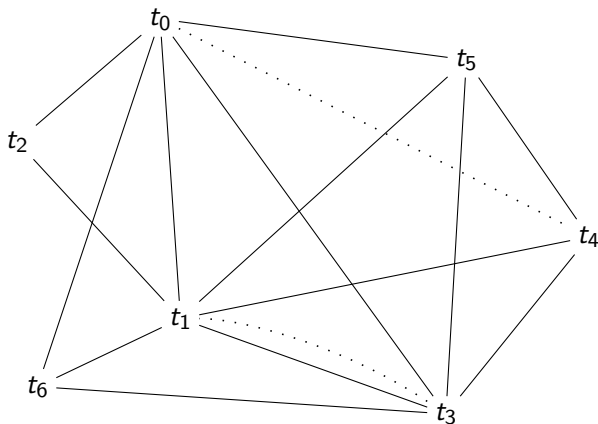
Exercise (3)

Live ranges

Temporary	Live range
t_0	1–13
t_1	2–13
t_2	3–5
t_3	6–17
t_4	7–9
t_5	8–9
t_6	9–10

Exercise (4)

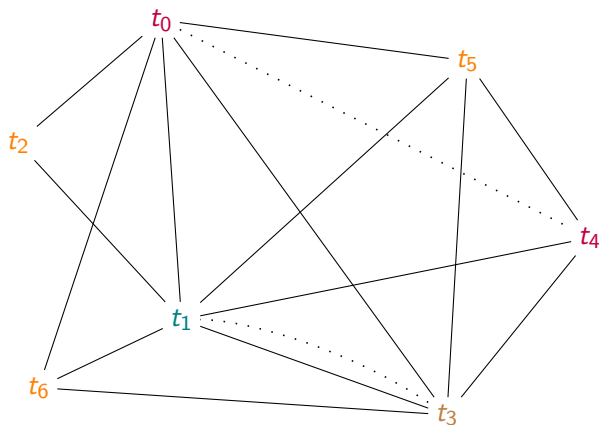
Interference graph



The live ranges of t_0 and t_4 overlap on edges $7 \rightarrow 8$ and $8 \rightarrow 9$, but since one is a copy the other along those edges, they do not interfere

Exercise (5)

Possible 4-colouring of the graph



Order in which the nodes were removed from the graph in the Simplify step: t_2 t_4 t_6 t_0 t_1 t_3 t_5

Exercise (6)

Spilling t0

```
function @facx
1 .          t01 <- i_aload @n
1' .         @s1 <- i_lstore t01
...
3' .         t02 <- i_lload @s1
4 .          t2  <- i_lt t02, t2
...
6' .  13:     t03 <- i_lload @s1
7 .          t4  <- i_copy t03
...
10' .  14:     t04 <- i_lload @s1
11 .         t3  <- i_mul t3, t04
11' .        t05 <- i_lload @s1
12 .         t06 <- i_sub t05, t1
12' .        @s1 <- i_lstore t06
...
```

Register allocation — Take 2 (1)

With coalescing and pre-coloured nodes

Node coalescing

Coalescing two **move related** nodes is fusing them together into a **single** node, which will inherit the edges of **both** nodes

When two nodes are **coalesced**, the degree of the **new** node will be less than or equal to the **sum** of the degrees of the coalesced nodes and the degree of **other** nodes may **decrease**

Two interference graph nodes are **move related** if, at some point, one value is the **copy** of the other

If **move related** nodes are **coalesced**, both values will be assigned to the same register and the copy instruction may be eliminated

Coalescing is only done when **safe**, i.e., if it will not turn a k -colourable graph into a non k -colourable one

Register allocation — Take 2 (2)

With coalescing and pre-coloured nodes

Criteria for safe node coalescing

Briggs

Two nodes may be coalesced if the resulting node will have **less than k** neighbours of **degree greater than or equal to k**

George

Nodes a and b may be coalesced if every neighbour of a

- ▶ already **interferes with b** , or
- ▶ has **degree less than k**

Usually, a is a pre-coloured node

These are **conservative** criteria

Register allocation — Take 2 (3)

With coalescing and pre-coloured nodes

Pre-coloured nodes

Pre-coloured nodes in an interference graph correspond to registers with predefined roles, such as holding function arguments, function results or the return address, or callee-saved registers

All pre-coloured nodes interfere with each other

Register allocation — Take 2 (4)

With coalescing and pre-coloured nodes

Algorithm

Let k be the number of registers available

1. **Simplify** While there are **non move related** and **not pre-coloured** nodes with degree less than k in the graph, remove them from the graph (along with their edges)
2. **Coalesce** If there are nodes that can be **safely** coalesced
 - i. Coalesce **every** possible pair
 - ii. Go back to 1
3. **Freeze** If there is a move-related node of degree less than k
 - i. Since it cannot be coalesced, forget about the copy and **simplify** it
This corresponds to giving up the hope of eliminating the copy instruction, since the nodes may end up with different colours
 - ii. Go back to 1

Register allocation — Take 2 (5)

With coalescing and pre-coloured nodes

Algorithm (cont.)

4. **Spill** If there is still some node remaining in the graph, other than **pre-coloured** nodes
 - i. Remove one node from the graph (it is a candidate for **spilling**, may be a coalesced pre-coloured node)
 - ii. Go back to 1.
5. **Select** Rebuild the graph, inserting nodes in the **reverse** order from which they were removed, if it is possible to colour the node with a colour **different** from those of its neighbours
6. **Restart** If it was not possible to colour some node(s) in step 5., change the code so that the corresponding value(s) are stored to and loaded from memory, and restart from the **liveness analysis**

Register allocation — Take 2 (6)

With coalescing and pre-coloured nodes

Choosing a candidate for spilling

Spilling has a **cost**, since it involves **storing** a value into and **loading** it from memory

- ▶ The more times a spilled value is **used** or **defined**, the greater the cost
- ▶ Spilling a value that is accessed inside a **loop** entails a greater cost than a value only accessed outside loops

When a candidate for spilling is removed from the graph, the degree of its neighbours **decreases**

- ▶ The more neighbours the node has, the greater the chances of one other node becoming **simplifiable**

Preference should be given to the **higher degree** values **least accessed** when choosing a spilling candidate

Register allocation — Take 2 (7)

With coalescing and pre-coloured nodes

Example (18)

Code for a function, including registers with predefined roles

Registers r_1 and r_2 contain the actual arguments of the function

Register r_1 will hold the return value

Register r_3 is a callee-saved register

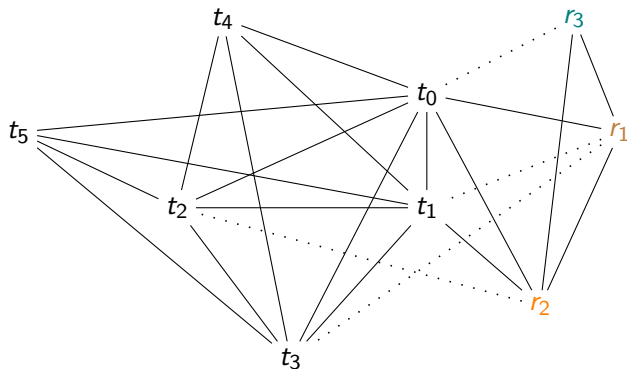
```
 $t_0 \leftarrow \text{i\_copy } r_3$   
 $t_1 \leftarrow \text{i\_copy } r_1$   
 $t_2 \leftarrow \text{i\_copy } r_2$   
 $t_3 \leftarrow \text{i\_value } 0$   
 $l_1 : t_3 \leftarrow \text{i\_add } t_3, t_2$   
 $t_4 \leftarrow \text{i\_value } 1$   
 $t_1 \leftarrow \text{i\_sub } t_1, t_4$   
 $t_5 \leftarrow \text{i\_value } 0$   
 $t_5 \leftarrow \text{i\_lt } t_5, t_1$   
 $\text{cjump } t_5, l_1, l_2$   
 $l_2 : r_1 \leftarrow \text{i\_copy } t_3$   
 $r_3 \leftarrow \text{i\_copy } t_0$   
 $\text{i\_return } r_1$ 
```


Register allocation — Take 2 (8)

With coalescing and pre-coloured nodes

Example (18, cont.)

Interference graph

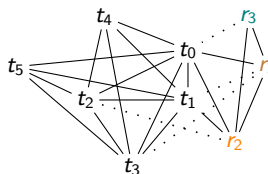


Register allocation — Take 2 (9)

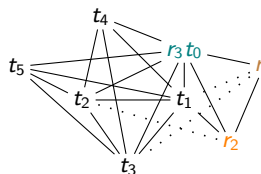
With coalescing and pre-coloured nodes

Example (18, cont.)

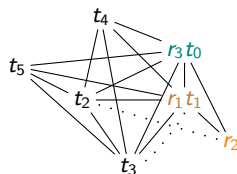
Colouring the graph with $k = 4$ colours



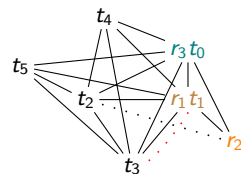
Initial graph



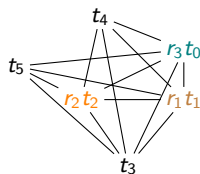
Coalesce r_3 and t_0



Coalesce r_1 and t_1



Constrained move



Coalesce r_2 and t_2

No simplifying,
coalescing or freezing
possible

Must choose a candidate
for spilling

Register allocation — Take 2 (10)

With coalescing and pre-coloured nodes

Example (18, cont.)

Choosing a spilling candidate

Uses and definitions of the candidates

	Outside the loop		Inside the loop		Degree
	Uses	Defs	Uses	Defs	
$r_1 t_1$	2	2	2	1	5
$r_2 t_2$	1	1	1	0	5
$r_3 t_0$	2	2	0	0	5
t_3	1	1	1	1	5
t_4	0	0	1	1	4
t_5	0	0	2	2	4

(Counts for coalesced nodes are the sum of the counts for the individual nodes)

Register allocation — Take 2 (11)

With coalescing and pre-coloured nodes

Example (18, cont.)

Choosing a spilling candidate (cont.)

Estimating spilling costs

$$r_1 t_1 : (2 + 2 + 10 \times (2 + 1)) / 5 = \frac{34}{5}$$

$$r_2 t_2 : (1 + 1 + 10 \times (1 + 0)) / 5 = \frac{12}{5}$$

$$r_3 t_0 : (2 + 2 + 10 \times (0 + 0)) / 5 = \frac{4}{5} \quad \leftarrow$$

$$t_3 : (1 + 1 + 10 \times (1 + 1)) / 5 = \frac{22}{5}$$

$$t_4 : (0 + 0 + 10 \times (1 + 1)) / 4 = \frac{20}{4}$$

$$t_5 : (0 + 0 + 10 \times (2 + 2)) / 4 = \frac{40}{4}$$

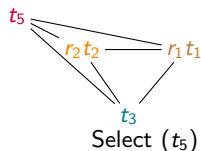
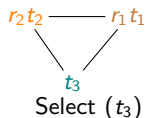
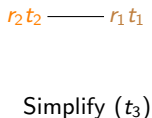
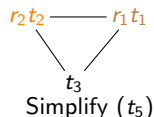
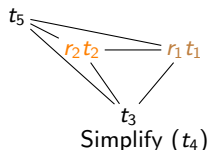
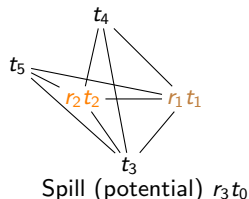
Uses and definitions outside any loop have cost 1, each loop level increases the cost by a factor of 10, and an if would halve the cost

Register allocation — Take 2 (12)

With coalescing and pre-coloured nodes

Example (18, cont.)

Colouring the graph (cont.)

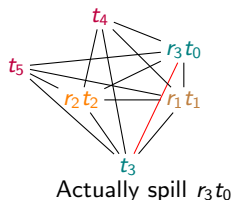
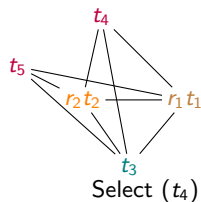


Register allocation — Take 2 (13)

With coalescing and pre-coloured nodes

Example (18, cont.)

Colouring the graph (cont.)



The stack is empty, no more **selecting** is possible

It is time to actually **spill** a candidate

In this case, there is only **one** candidate

Register allocation — Take 2 (14)

With coalescing and pre-coloured nodes

Example (18, cont.)

Spilling $r_3 t_0$

t_0 is replaced by a series of new temporaries which hold its value in transit to and from memory

The new temporaries **must not** be spilled (they should have a spilling cost of ∞)

$@s_1$ represents the location in the current activation record where the value will be stored

(This location must be independently allocated)

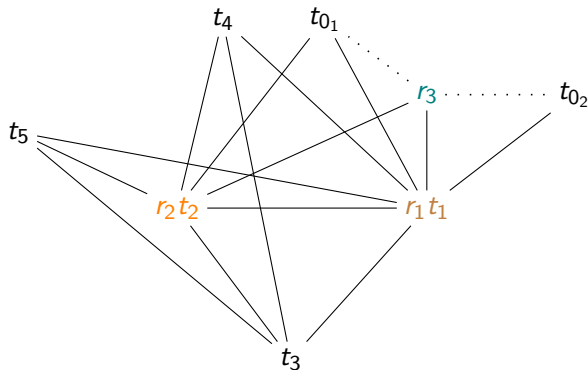
```
 $t_{0_1} \leftarrow i\_copy\ r_3$   
 $@s_1 \leftarrow i\_lstore\ t_{0_1}$   
 $t_1 \leftarrow i\_copy\ r_1$   
 $t_2 \leftarrow i\_copy\ r_2$   
 $t_3 \leftarrow i\_value\ 0$   
 $l_1 :$   $t_3 \leftarrow i\_add\ t_3, t_2$   
 $t_4 \leftarrow i\_value\ 1$   
 $t_1 \leftarrow i\_sub\ t_1, t_4$   
 $t_5 \leftarrow i\_value\ 0$   
 $t_5 \leftarrow i\_lt\ t_5, t_1$   
 $cjump\ t_5, l_1, l_2$   
 $l_2 :$   $r_1 \leftarrow i\_copy\ t_3$   
 $t_{0_2} \leftarrow i\_lload\ @s_1$   
 $r_3 \leftarrow i\_copy\ t_{0_2}$   
 $i\_return\ r_1$ 
```

Register allocation — Take 2 (15)

With coalescing and pre-coloured nodes

Example (18, cont.)

Interference graph for rewritten code



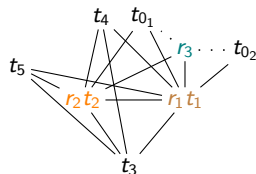
Nodes coalesced prior to the first potential spill may be kept

Register allocation — Take 2 (16)

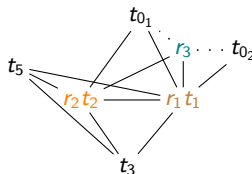
With coalescing and pre-coloured nodes

Example (18, cont.)

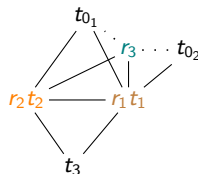
Colouring the new graph



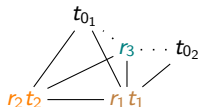
Initial graph



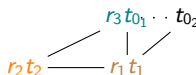
Simplify (t_4)



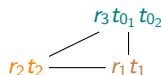
Simplify (t_5)



Simplify (t_3)



Coalesce r_3 and t_0



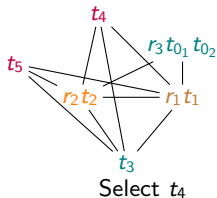
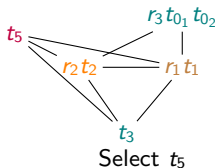
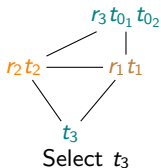
Coalesce $r_3 t_0$ and t_0

Register allocation — Take 2 (17)

With coalescing and pre-coloured nodes

Example (18, cont.)

Colouring the new graph (cont.)



It's done!

According to the resulting colouring, t_1 will use register r_1 , t_2 will use r_2 , t_3 , t_{01} and t_{02} will use r_3 , and both t_4 and t_5 will use r_4

The temporaries will now be replaced by the registers they will use

Register allocation — Take 2 (18)

With coalescing and pre-coloured nodes

Example (18, cont.)

```
 $r_3 \leftarrow i\_copy\ r_3$   
 $@s_1 \leftarrow i\_lstore\ r_3$   
 $r_1 \leftarrow i\_copy\ r_1$   
 $r_2 \leftarrow i\_copy\ r_2$   
 $r_3 \leftarrow i\_value\ 0$   
 $l_1 : r_3 \leftarrow i\_add\ r_3, r_2$   
 $r_4 \leftarrow i\_value\ 1$   
 $r_1 \leftarrow i\_sub\ r_1, r_4$   
 $r_4 \leftarrow i\_value\ 0$   
 $r_4 \leftarrow i\_lt\ r_4, r_1$   
 $cjump\ r_4, l_1, l_2$   
 $l_2 : r_1 \leftarrow i\_copy\ r_3$   
 $r_3 \leftarrow i\_lload\ @s_1$   
 $r_3 \leftarrow i\_copy\ r_3$   
 $i\_return\ r_1$ 
```

Removing useless copy
instructions

```
 $@s_1 \leftarrow i\_lstore\ r_3$   
 $r_3 \leftarrow i\_value\ 0$   
 $l_1 : r_3 \leftarrow i\_add\ r_3, r_2$   
 $r_4 \leftarrow i\_value\ 1$   
 $r_1 \leftarrow i\_sub\ r_1, r_4$   
 $r_4 \leftarrow i\_value\ 0$   
 $r_4 \leftarrow i\_lt\ r_4, r_1$   
 $cjump\ r_4, l_1, l_2$   
 $l_2 : r_1 \leftarrow i\_copy\ r_3$   
 $r_3 \leftarrow i\_lload\ @s_1$   
 $i\_return\ r_1$ 
```

Register allocation — Take 2 (19)

With coalescing and pre-coloured nodes

Example (19)

With one less register...

Using only 3 registers should result in something like the code on the right

```
@s1 ← i_lstore r3
@s2 ← i_lstore r2
r3 ← i_value 0
l1 : r2 ← i_lload @s2
     r3 ← i_add r3, r2
     r2 ← i_value 1
     r1 ← i_sub r1, r2
     r2 ← i_value 0
     r2 ← i_lt r2, r1
     cjump r2, l1, l2
l2 : r1 ← i_copy r3
     r3 ← i_lload @s1
     i_return r1
```