general equilibrium
in a pure exchange economy

walrasian equilibrium
- definition
- aggregate excess demand
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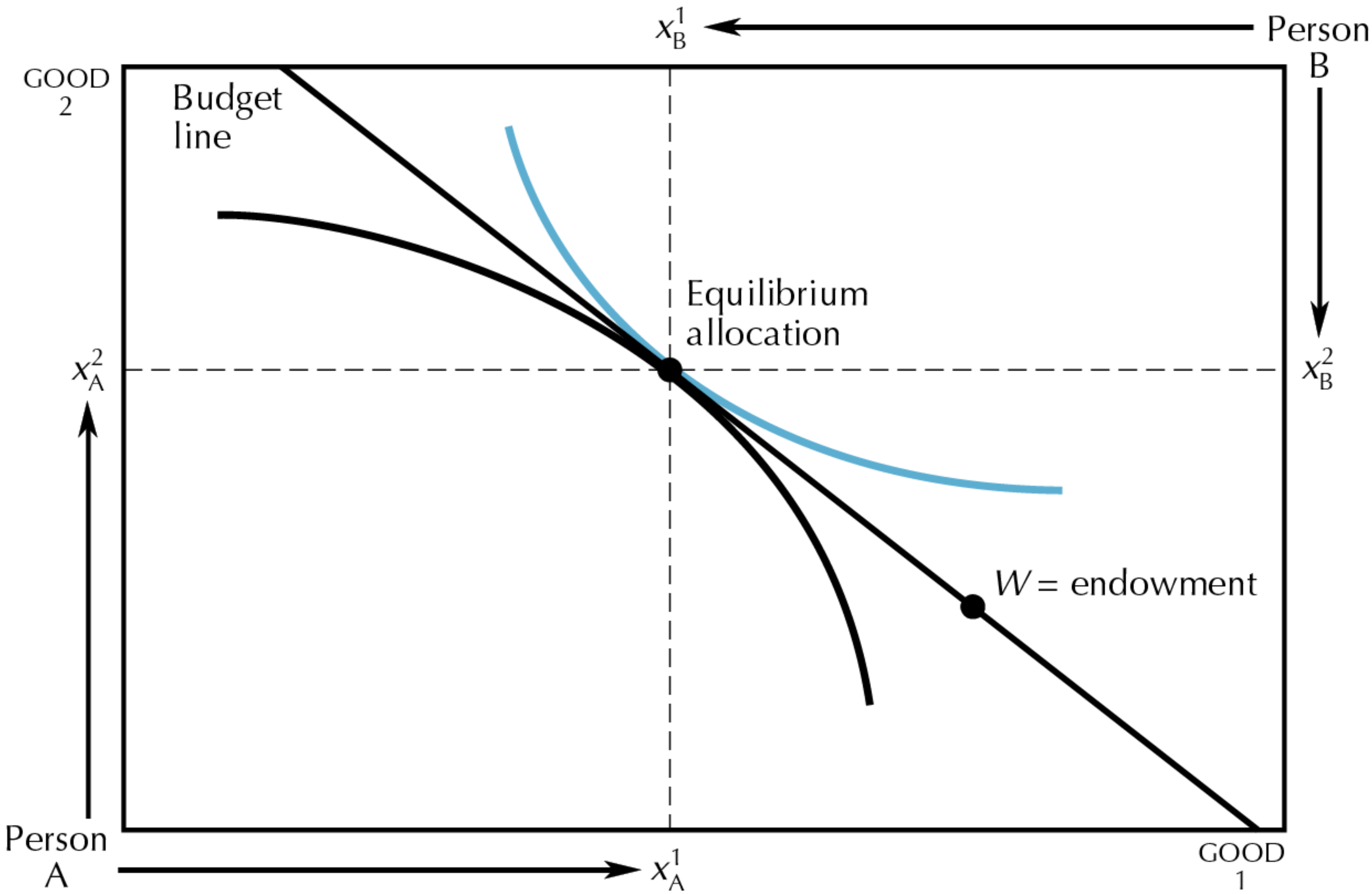
- Walras' law
- relative prices

Edgeworth box

- 2-good, 2-agent case: walrasian equilibrium



walrasian equilibrium

$(p_1^*, p_2^*, x_1^{A*}, x_2^{A*}, x_1^{B*}, x_2^{B*})$

is a walrasian equilibrium if:

- all agents maximize utility subject to budget constraint
- all markets clear

walrasian equilibrium

$(p_1{}^*, p_2{}^*, x_1{}^{A*}, x_2{}^{A*}, x_1{}^{B*}, x_2{}^{B*})$

is a walrasian equilibrium if:

- for all i, $(x_1{}^{i*}, x_2{}^{i*})$ solve

$$\max u^i(x_1{}^i, x_2{}^i) \text{ s.t. } p_1x_1{}^i+p_2x_2{}^i \leq p_1\omega_1{}^i+p_2\omega_2{}^i$$

- all markets clear i.e.

$$x_1{}^A+x_1{}^B = \omega_1{}^A+\omega_1{}^B$$

$$x_2{}^A+x_2{}^B = \omega_2{}^A+\omega_2{}^B$$

aggregate excess demand

- gross demands

  $x_1^A(p_1, p_2)$, $x_2^A(p_1, p_2)$, $x_1^B(p_1, p_2)$, $x_2^B(p_1, p_2)$

- net or excess demands

  $e_1^A = x_1^A - \omega_1^A$; $e_2^A = x_2^A - \omega_2^A$

  $e_1^B = x_1^B - \omega_1^B$; $e_2^B = x_2^B - \omega_2^B$

- aggregate excess demands

  $z_1 = e_1^A + e_1^B$

  $z_2 = e_2^A + e_2^B$

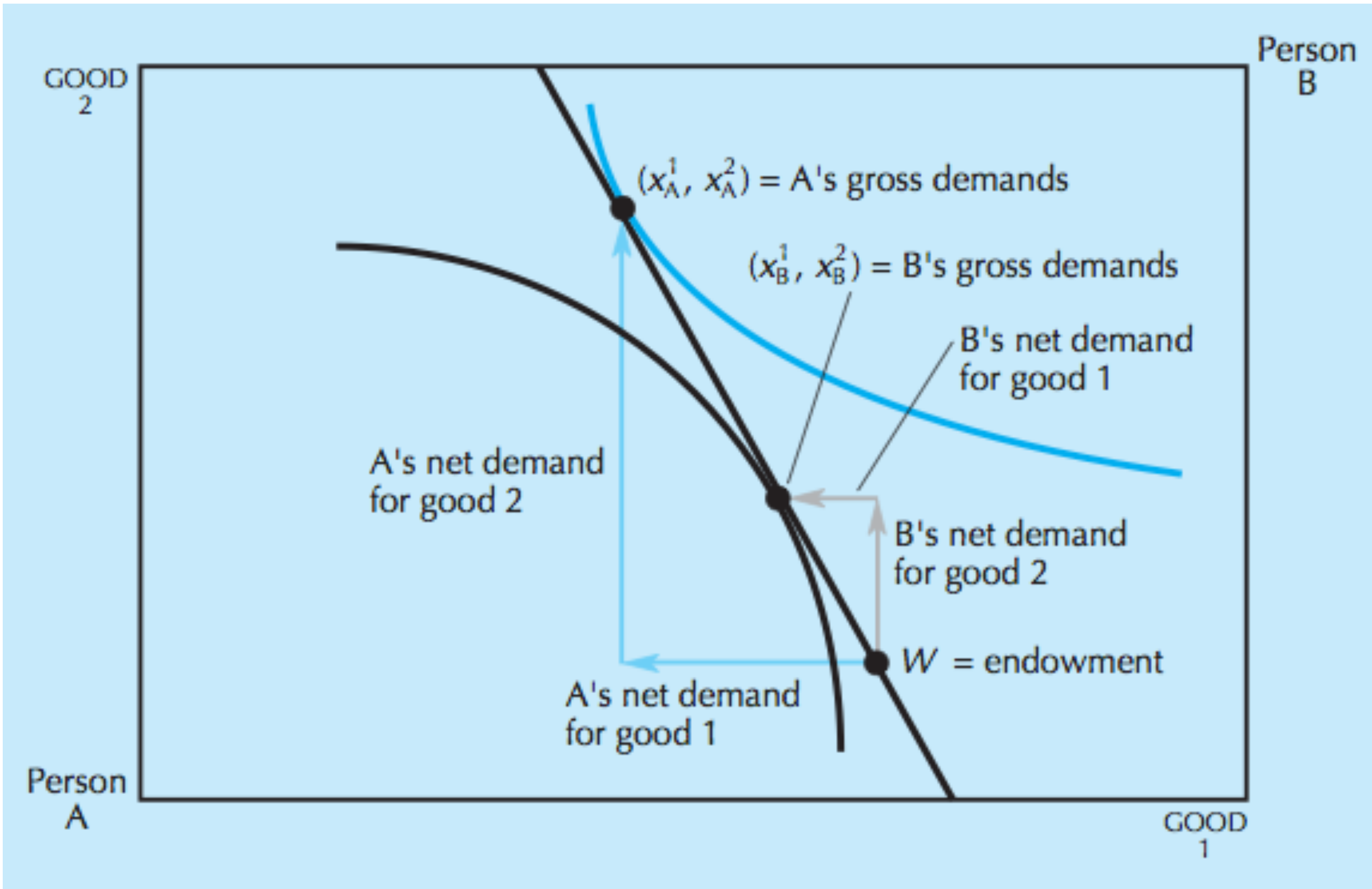
aggregate excess demand

$(p_1^*, p_2^*, x_1^{A*}, x_2^{A*}, x_1^{B*}, x_2^{B*})$

is a walrasian equilibrium if:

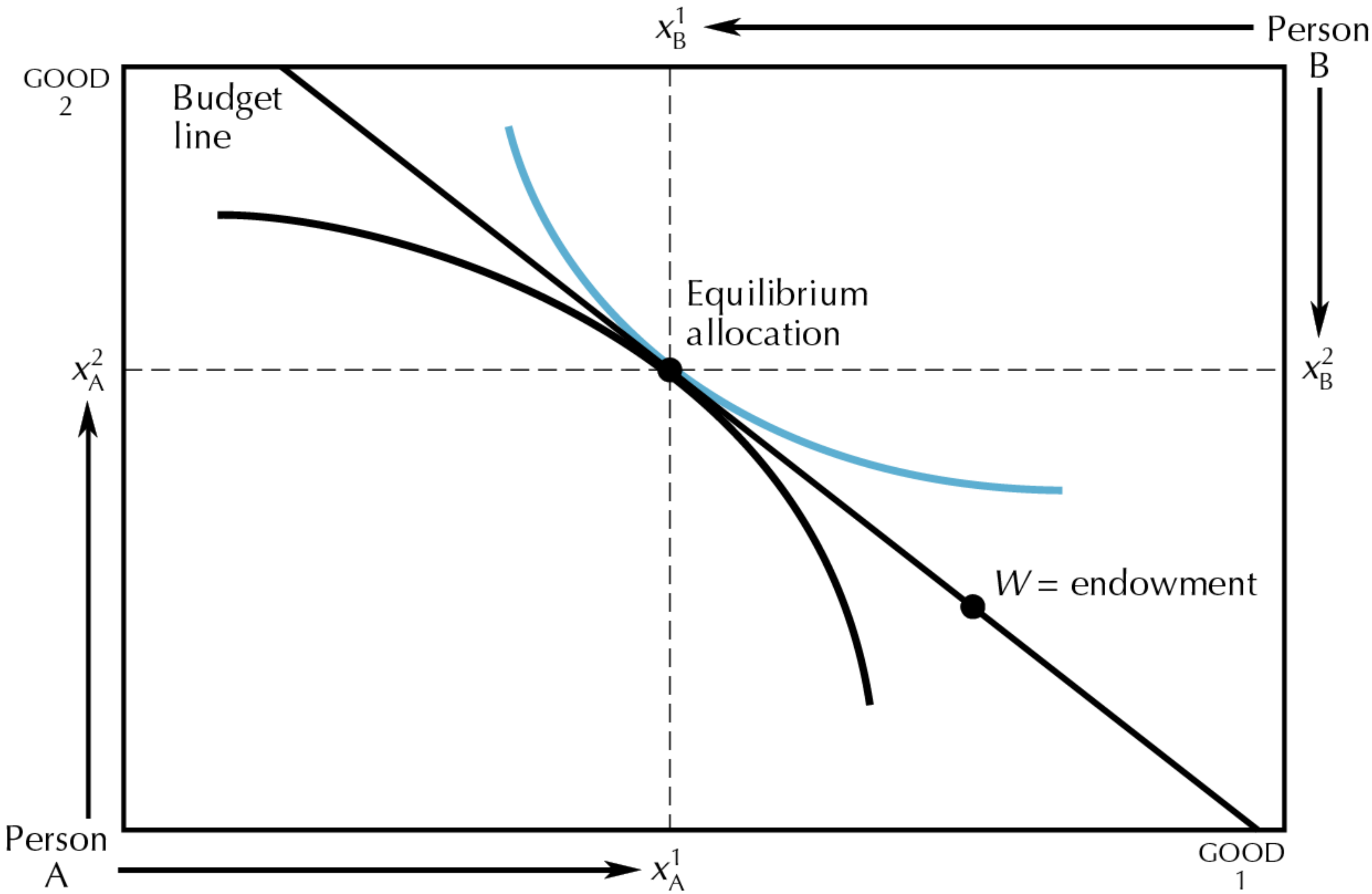
$$z_1(p_1^*, p_2^*) = 0$$
$$z_2(p_1^*, p_2^*) = 0$$

example: disequilibrium



Edgeworth box

- 2-good, 2-agent case: walrasian equilibrium



Walras' law

Since for all i and for all (p_1, p_2),

$p_1 x_1^i(p_1, p_2) + p_2 x_2^i (p_1, p_2) = p_1 \omega_1^i + p_2 \omega_2^i$

then $p_1 z_1(p_1, p_2) + p_2 z_2 (p_1, p_2) = 0$

i.e. the value of aggregate excess demand is 0 for all prices

Implication: only 1 out of 2 equilibrium equations is linearly independent (and only k-1 out of k, in the general case with k goods)

relative prices

Multiplying all prices (p_1, p_2) by $t > 0$, gross demands remain the same since the budget constraint does not change.

But then only relative prices matter and we can fix one of the goods to become the numeraire.

Implication: only 1 out of 2 equilibrium prices needs to be determined (and only k-1 out of k, in the general case with k goods).

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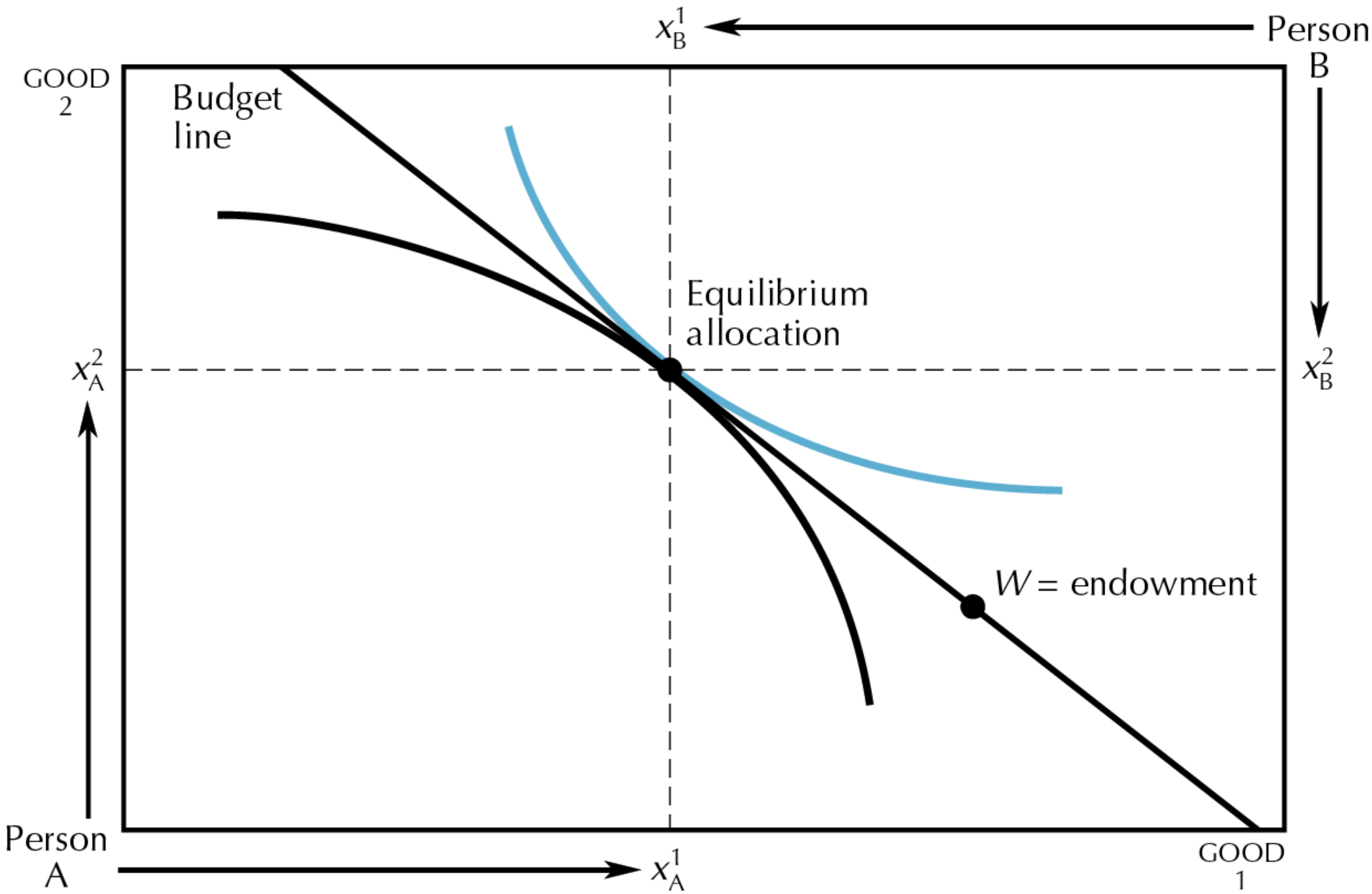
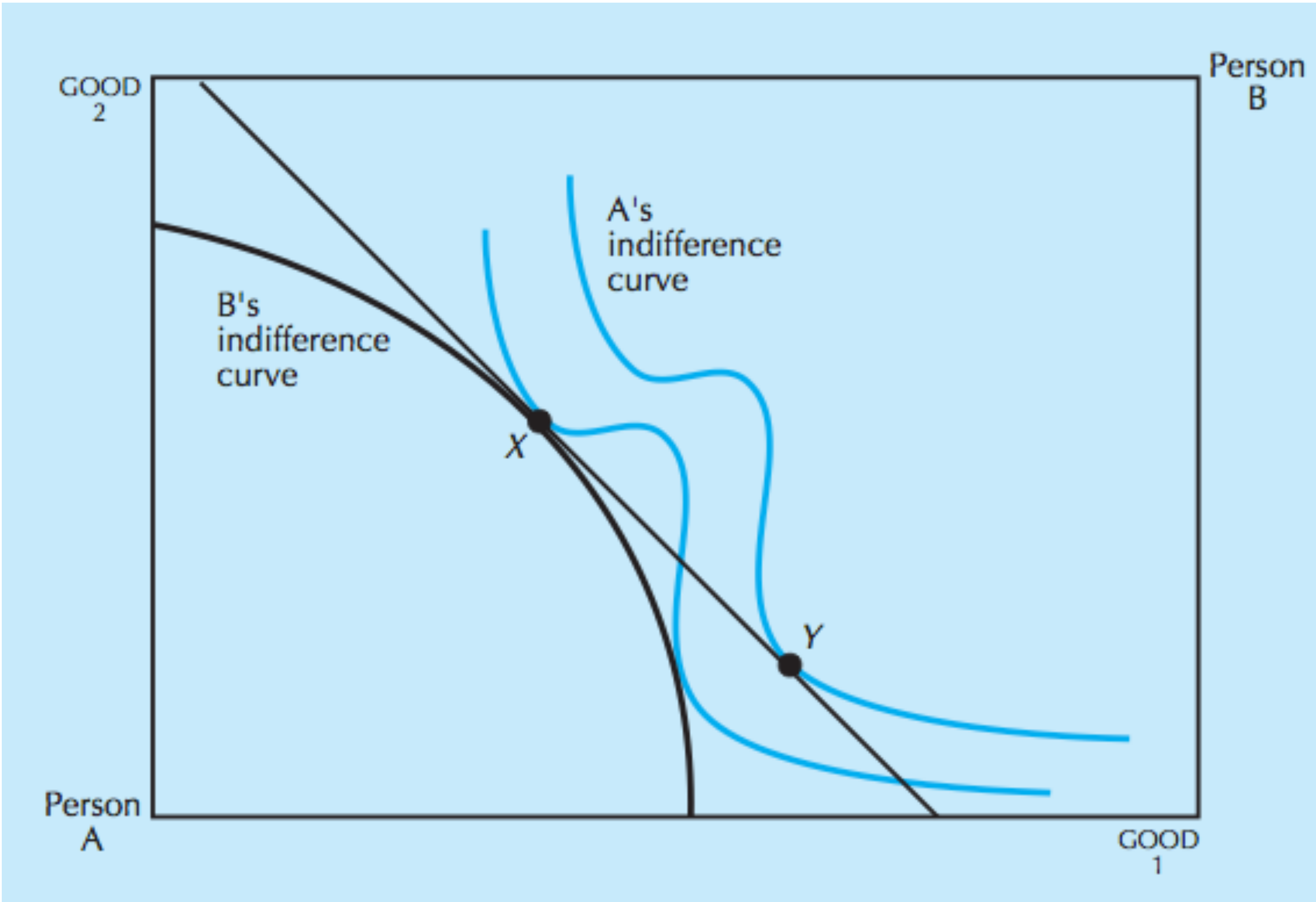
walrasian equilibrium
- existence
- first welfare theorem
- second welfare theorem

walrasian equilibrium: existence

- 2-good, 2-agent case: walrasian equilibrium



example: inexistence



walrasian equilibrium: existence

In order to ensure existence, we need continuity of the aggregate excess demand – to ensure that there is a price that sets it equal to zero.

For that, we need

- either all individual demand curves to be continuous (for which convexity of preferences would be a sufficient condition)
- or, if some individual demand curves are not continuous, we need each consumer to be “small” relative to the market.

first welfare theorem

If $(p_1^*, p_2^*, x_1^{A*}, x_2^{A*}, x_1^{B*}, x_2^{B*})$ is a walrasian equilibrium, then $(x_1^{A*}, x_2^{A*}, x_1^{B*}, x_2^{B*})$ is Pareto efficient.

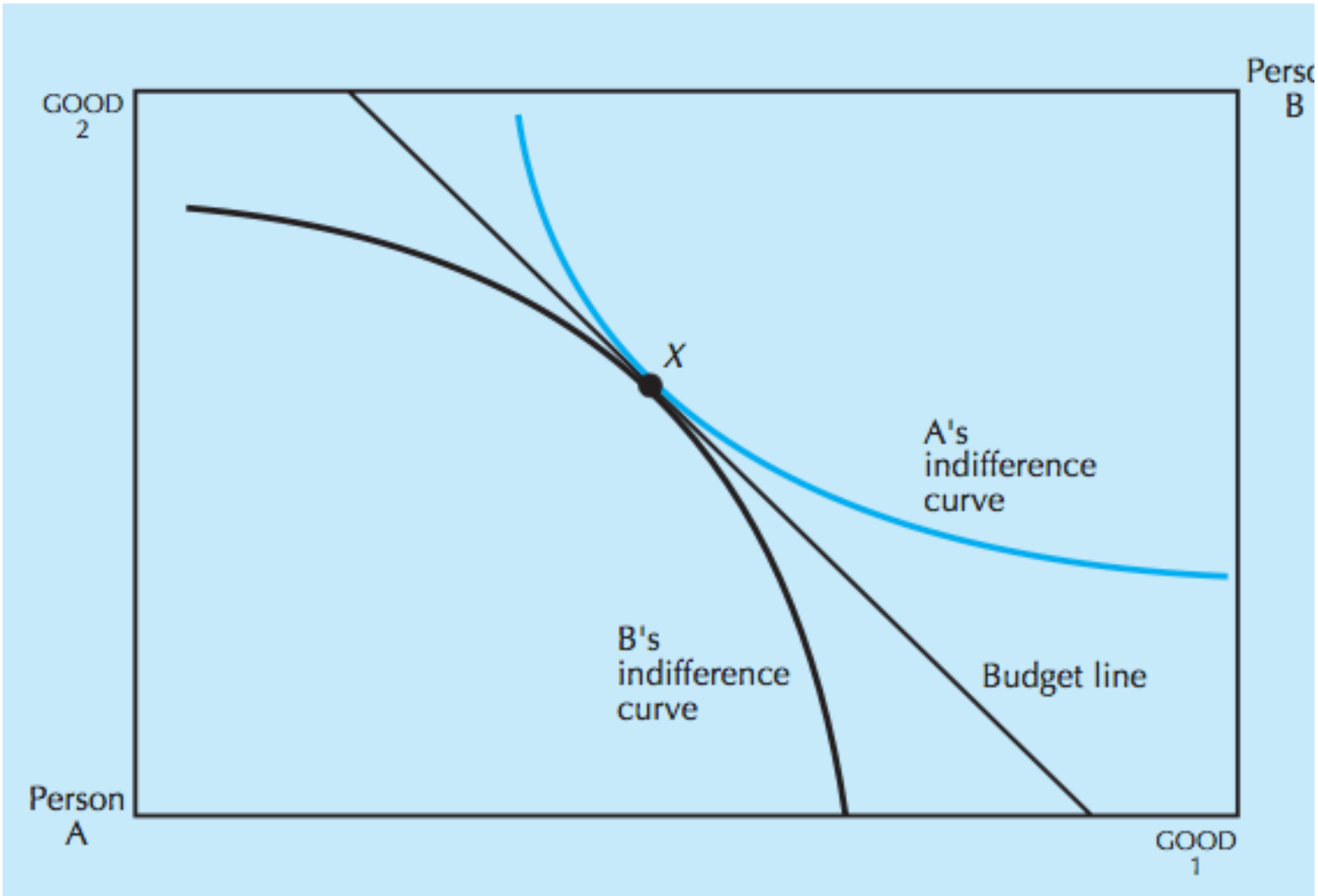
- If $MRS^i = p_1/p_2$ for all i, then $MRS^i = MRS^j$ for all i and j
- Consequences: info on prices is enough to make decisions; market ensures efficiency?
  - if equilibrium exists
  - if there are no externalities, public goods, market power, asymmetric information
  - and it tells us nothing on distribution

second welfare theorem

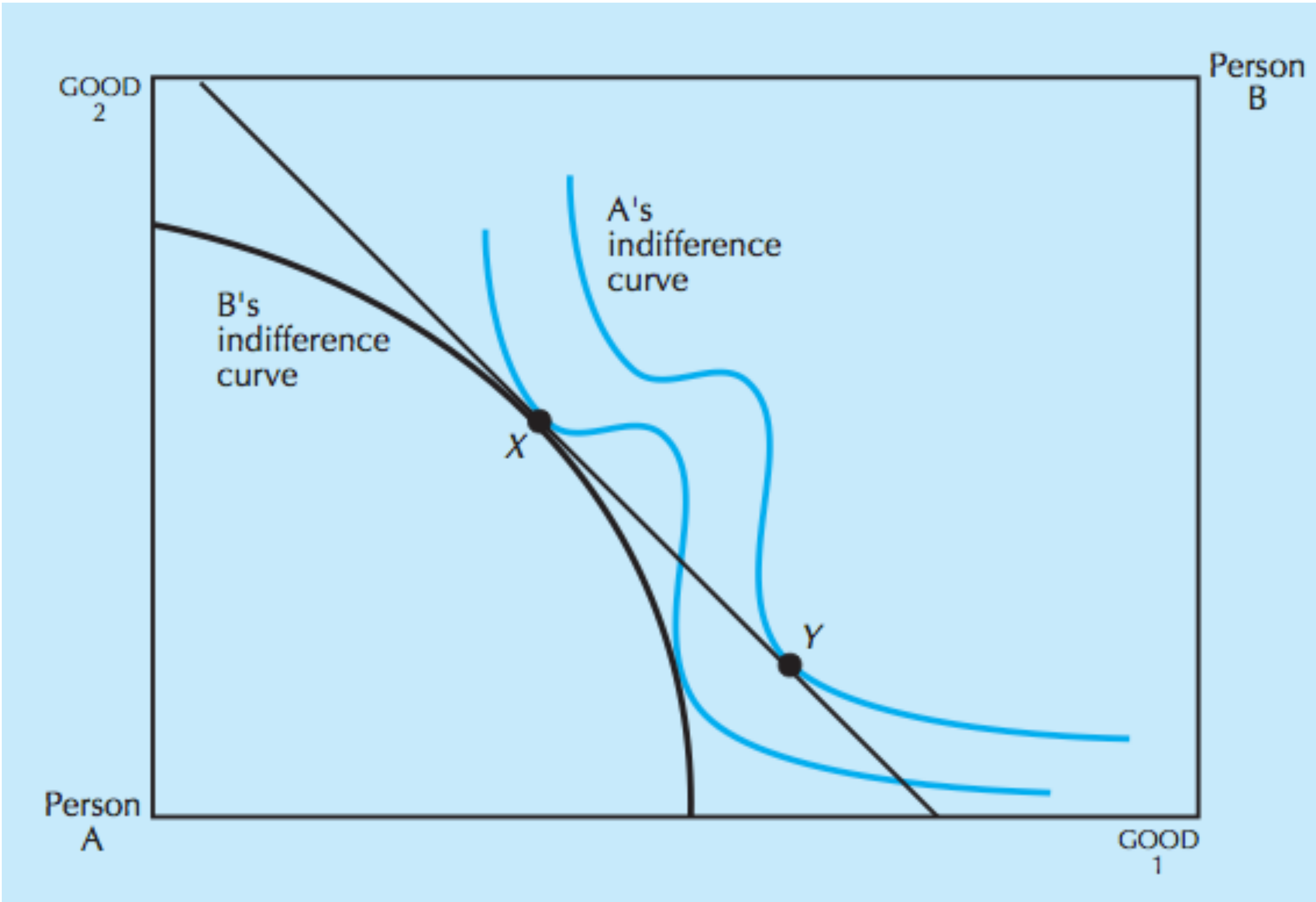
If $(x_1^{A*}, x_2^{A*}, x_1^{B*}, x_2^{B*})$ is Pareto efficient, then there is a price vector (p_1^*, p_2^*) and a redistribution of the endowment such that $(p_1^*, p_2^*, x_1^{A*}, x_2^{A*}, x_1^{B*}, x_2^{B*})$ is a walrasian equilibrium.

- If $MRS^i = MRS^j$ for all i and j, we can set $p_1/p_2 = MRS^i$
- for the allocation to become an equilibrium, we need to make sure it is on the budget constraint for all agents, which may require reallocation of the endowment

second welfare theorem



second welfare theorem: convexity



second welfare theorem

- prices have allocative and distributive roles: separate the two, letting prices focus on reflecting scarcity
- separate efficiency from distribution
- but lump-sum reallocation of endowments and not changes involving marginal decisions – labor tax already involves distortion...