

# Homework 4

## Solutions

ECON 380  
UNC Chapel Hill

Name: \_\_\_\_\_

ONYEN: \_\_\_\_\_

This homework is due on **October 30** by **2:25PM**. You must turn in your work on a printed copy of this document in order for it to be graded. Your assignment must be stapled and in the correct order. Non-stapled assignments will automatically receive a 10 point deduction.

### Present Value

1. Suppose Mallory lives for two periods,  $t = 1, 2$ . In period 1, she could either directly enter the labor force and earn \$30,000, or she could enroll in college where her costs would be \$45,000. In period 2, Mallory will either earn \$30,000 if she chose to enter the workforce in period 1, or \$120,000 if she went to college in the first period. All transactions occur at the beginning of each period.

- (a) What is the net present value of Mallory's earnings if she decides to directly enter the labor force in period 1 and her discount rate is 5%? **[2 pts]**

**Solution:**

$$NPV_N = \$30,000 + \frac{\$30,000}{(1.05)} = \$58,571$$

Points: (1) NPV formula, (1) answer. If they missed up the NPV formula (i.e., discounted incorrectly), take off two points here but not on the following parts as long as they are consistent.

- (b) What is the net present value of Mallory's earnings if she decides to go to college in period 1 and her discount rate is 5%? **[2 pts]**

**Solution:**

$$NPV_C = -\$45,000 + \frac{\$120,000}{(1.05)} = \$69,286$$

- (c) At a discount rate of 5%, what path (college vs. non-college) maximizes Mallory's present value of lifetime earnings? **[2 pts]**

**Solution:** With  $r = 5\%$ ,  $NPV_C > NPV_N$ . Going to college maximizes her NPV of earnings.

- (d) At a discount rate of 25%, what path (college vs. non-college) maximizes Mallory's present value of lifetime earnings? [4 pts]

**Solution:**

$$NPV_N = \$30,000 + \frac{\$30,000}{(1.25)} = \$54,000$$

$$NPV_C = -\$45,000 + \frac{\$120,000}{(1.25)} = \$51,000$$

With  $r = 25\%$ , going straight into the labor force maximizes her NPV of earnings.

Points: (1)  $NPV_N$ , (1)  $NPV_C$ , (2) optimal choice

- (e) At what discount rate would Mallory be indifferent between directly entering the labor force and going to college? Call this rate  $r_{id}$ . [4 pts]

**Solution:** Mallory would be indifferent between the choices if  $NPV_N = NPV_C$ :

$$30,000 + \frac{30,000}{(1 + r_{id})} = -45,000 + \frac{120,000}{(1 + r_{id})} \Rightarrow$$

$$75,000 + \frac{30,000}{(1 + r_{id})} = \frac{120,000}{(1 + r_{id})} \Rightarrow$$

$$75,000(1 + r_{id}) + 30,000 = 120,000 \Rightarrow 75,000(1 + r_{id}) = 90,000 \Rightarrow$$

$$1 + r_{id} = 1.2 \Rightarrow r_{id} = .2 = 20\%$$

Points: (1)  $NPV_N = NPV_C$ , (2) work, (1)  $r_{id}$

- (f) How does Mallory's optimal decision change depending on the relationship between her discount rate  $r$  and  $r_{id}$ ? That is, what is Mallory's choice if  $r < r_{id}$ ? What if  $r > r_{id}$ ? [4 pts]

**Solution:** If  $r < r_{id}$ , Mallory's optimal choice would be to go to college since  $NPV_C > NPV_N$ . If  $r > r_{id}$ , Mallory's optimal choice would be to directly enter the labor force since  $NPV_C < NPV_N$ .

2. Kalani can either start to work the day she turns age 18 or go to college for 4 years before entering the labor force. Either way, she will retire the day she reaches age 65. If she starts work at 18, she will earn \$35,000 each year. If she goes to college, she will pay \$12,000 per year in tuition, but will earn \$55,000 per year once she starts to work. All payments (including tuition) are paid at the end of the year. Finally, Stella has a discount rate of 7%. Use the "NPV Template" Excel File on Sakai to answer the following questions.

- (a) What is the net present value of Kalani's earnings if she directly enters the labor force? [2 pts]

**Solution:** \$470,835.71

- (b) What is the net present value of Kalani's earnings if she goes to college before entering the labor force? [2 pts]

**Solution:** \$512,941.54

- (c) What is Kalani's optimal choice? [2 pts]

**Solution:** Go to college

- (d) State one discount rate that would change Kalani's optimal choice. [2 pts]

**Solution:** Any discount rate greater than or (almost) equal to 9% works.

### The Returns to Schooling

3. Consider the wage-schooling locus described in Table 1.

Table 1: Cyril and Pam's Wage-Schooling Locus

Years of Schooling	Earnings	MRR
11	\$20,000	—
12	\$25,000	25%
13	\$29,000	16%
14	\$32,500	12.1%
15	\$34,500	6.2%
16	\$36,000	4.3%
17	\$37,000	2.8%

- (a) Fill in the marginal rate of return to schooling for years 12 - 17. [3 pts]

1/2 point for each year's MRR

- (b) Suppose Cyril follows this wage-schooling locus. His discount rate is  $r_C = 9\%$ . What is his optimal level of schooling? [2 pts]

**Solution:** Cyril optimally chooses 14 years of schooling (his resulting wage is \$32,500).

- (c) Suppose Pam follows this wage-schooling locus. Her discount rate is  $r_P = 15\%$ . What is her optimal level of schooling? [2 pts]

**Solution:** Pam optimally chooses 13 years of schooling (her resulting wage is \$29,000).

Suppose Archer has the same discount rate as Cyril,  $r_A = 9\%$ . However, Archer has a higher ability level than Cyril,  $A^A > A^C$  and his wage-schooling locus is shown in Table 2.

Table 2: Archer's Wage-Schooling Locus

Years of Schooling	Earnings	MRR
11	\$22,000	—
12	\$32,000	45.4%
13	\$40,000	25%
14	\$47,000	17.5%
15	\$53,000	12.8%
16	\$58,000	9.4%
17	\$62,000	6.9%

- (d) Fill in Archer's marginal rate of return to schooling for years 12 - 17. [3 pts]

- (e) What is Archer's optimal schooling level? [2 pts]

**Solution:** Archer optimally chooses 16 years of schooling (his resulting wage is \$58,000).

- (f) Sketch the wage-schooling loci for Cyril and Archer on the same plot (years of school on the x-axis, earnings on the y-axis). Label the point on each respective locus that a researcher would observe. [4 pts]

**Solution:** Archer's wage schooling locus should be above Cyril's. We observe (16,\$58,000) on Archer's locus and (14,\$32,500) on Cyril's.

- (g) Suppose we don't account for the differing wage-schooling loci of these individuals and estimate the marginal rate of return by comparing Cyril's wage/schooling outcome to Archer's wage/schooling outcome. What is the estimated MRR? [4 pts]

**Solution:** We observe in the data that Archer chooses 16 years of schooling and earns \$58,000. Cyril chooses 14 years of schooling and earns \$32,500. If we naively estimate the MRR based on these results (i.e., we would be incorrectly assuming they are on the same wage-schooling locus), we would estimate the MRR per year to be

$$\widehat{MRR} = \frac{58,000 - 32,500}{2} \times \frac{1}{32,500} = 39.2\%$$

(2) set-up, (2) answer.

- (h) Compare your naive estimate of the marginal rate of return from part (f) to the true marginal rate of return to the 15<sup>th</sup> year of education for Cyril and Archer. Is our estimate higher than the true marginal rate of return, or lower than the true marginal rate of return for the 15<sup>th</sup> year of education for these individuals? [4 pts]

**Solution:** This estimated MRR is way higher than either of their true MRR's around the year they end schooling. Hence, failing to account for selection of schooling based on ability makes it appear as if the return to schooling is much higher than it actually is.

4. Read Heckman (2006). In 4-5 sentences, summarize the article. Type your response and attach it to the back of this packet. [4 pts]

**Solution:** The Heckman article focuses on the role of early age skills formation and how investments/interventions at this critical stage are important for future life outcomes. Gaps in skills present at the beginning of schooling do not significantly shrink or widen through schooling and these gaps in skills can largely be traced to differences in early family environments. Importantly, because later skills build on prior acquired skills, the rate of return to early interventions are high and thus these programs are more likely to be economically efficient than programs later in life.

## Signaling Model

5. Suppose there are two types of persons: high and low-ability. A particular diploma costs a high-ability person \$15,000 and costs a low-ability person \$25,000. Firms wish to use education as a screening device where they intend to pay \$40,000 to workers without a diploma and \$K to those with a diploma. In what range must K be to make this an effective screening device? [4 pts]

**Solution:** In order to be an effect screening device,  $K$  must be large enough to induce high-ability workers to obtain a diploma, but not so high that it also induces low-ability workers to obtain a diploma.

For low-ability workers, their earnings without a degree must be higher than their net earnings would be with a degree in order for them to forgo the diploma:

$$\underbrace{40,000}_{\text{Earnings without a degree}} > \underbrace{K}_{\text{Earnings with a degree}} - \underbrace{25,000}_{\text{Cost of obtaining degree}}$$

$$\Rightarrow K < \$65,000$$

For high-ability workers, their earnings without a degree must be lower than their net earnings would be with a degree in order for them to get a diploma:

$$\underbrace{40,000}_{\text{Earnings without a degree}} < \underbrace{K}_{\text{Earnings with a degree}} - \underbrace{15,000}_{\text{Cost of obtaining degree}}$$

$$\Rightarrow K > \$55,000$$

Thus,  $\$55,000 < K < \$65,000$  in order for this screening device to be effective.

Points: (1) lower bound, (1) upper bound, (2) work

6. Suppose the market is populated by two types of workers. NeNes have an inherently low productivity level and make up 70% of the population. A NeNe produces \$1,000,000 for a firm over their lifetime. Cynthias, on the other hand, have an inherently high productivity level and make up 30% of the population and produce \$1,200,000 for a firm over their lifetime. Employers cannot observe a worker's type at the time of hiring. [4 pts]

Firms follow the rule of thumb that workers who obtain at least  $\bar{y}$  years of college are assumed to be Cynthias and are paid a lifetime salary of \$1,200,000. Workers with less than  $\bar{y}$  years of education are assumed to be NeNes and are paid \$1,000,000. Cynthias have a cost of \$30,000 for each year of college, while NeNes have a per year cost of \$50,000. What threshold for  $\bar{y}$  will create an equilibrium such that only Cynthias obtain the threshold education level?

**Solution:** Cynthias constraint:

$$1,200,000 - 30,000 \times \bar{y} > 1,000,000$$

$$\Rightarrow \bar{y} < 6.67$$

NeNes constraint:

$$1,200,000 - 50,000 \times \bar{y} < 1,000,000$$

$$\Rightarrow \bar{y} > 4$$

## Human Capital and Development

Directions: Type your answers to the following questions and attach them to the back of this packet.

7. Read Borjas 6.6 and the introduction to Duflo (2001). In 4-6 sentences, briefly summarize (i) the policy experiment analyzed in the paper and (ii) the results regarding how the policy affected schooling and labor market outcomes. What is estimated range of the rate of return to schooling? [4 pts]

### Solution:

- Policy: Large school construction program (INPRES) in Indonesia. Between 1973 and 1979, more than 61,000 primary schools were built, especially focused in areas with relatively low enrollment rates.
- Results: Years of education increased by .12 to .19 years (.11 using the DID approach in Borjas). Earnings for the first cohort affected by the program were also 1.5-2.7% higher (3% using the DID approach in Borjas)
- Estimated return to schooling: 6.8% to 10.6%

8. Read the introduction to Jensen (2010) and answer the following questions.

- (a) Jensen argues that the perceived rate of return to education is likely different than the actual rate of return to education, especially in developing countries. In 3-4 sentences, describe two of the factors he provides that might drive this difference and explain which rate of return (perceived versus actual) is likely smaller in low-income nations. [2 pts]

### Solution:

- Decision to drop out often made at a younger age
- Harder to obtain information (e.g., no guidance counselors or little information may available on labor market earnings)
- Perceived returns influenced by surroundings
  - Rural individuals may not know true potential returns in urban sector
  - Segregation by earnings income may depress perceived returns to schooling

The perceived return to education is likely smaller than the actual return to education in developing countries.

- (b) In 4-6 sentences, briefly describe (i) the intervention analyzed in the paper and (ii) the results of the intervention. Is the estimated effect of the intervention the same for students across income groups? [2 pts]

### Solution:

- Intervention: Students were asked about their perceived returns to schooling. In randomly selected schools, students were also provided information about the true returns to schooling.

- Results: Students in treatment schools reported much higher perceived returns to education upon reinterview. Additionally, treated individuals on average completed .20 more years of schooling over the next four years
- The effect varied across income groups. Low income individuals saw a large effect on schooling (.33 additional years of schooling), but there was no effect on the poorest students. Both groups' perceived *MRR* increased by the same amount.

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**FOR GRADING:**

Question:	1	2	3	4	5	6	7	8	Total
Points:	18	8	24	4	4	4	4	4	70
Score:									