INTEGRACION NUMÉRICA

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Curso

Métodos Numéricos

Universidad Surcolombiana

Neiva – Huila

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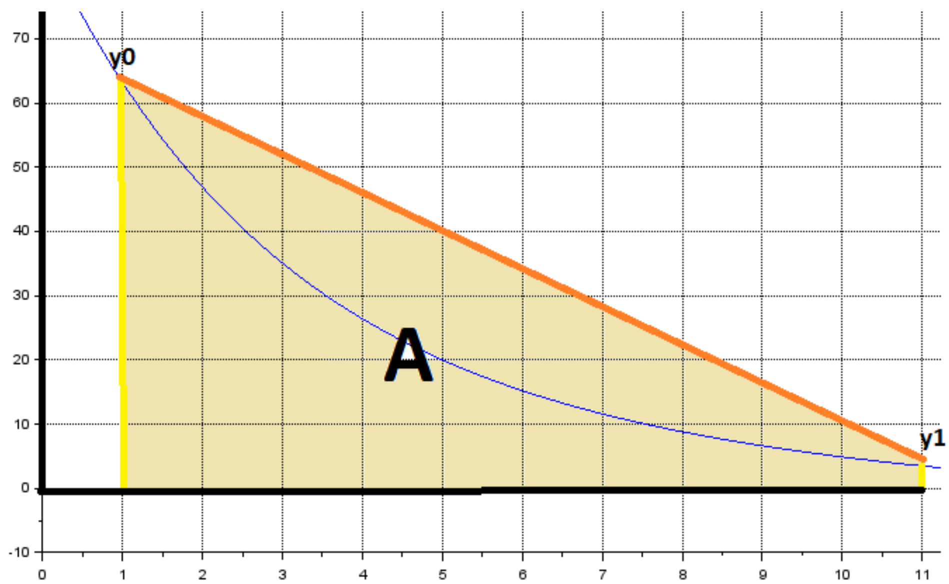
REGLA TRAPECIAL

$$\int_{1}^{11} 74.3e^{-0.34x} + (100 - 74.3)e^{-0.043x} - 14.3dx$$
$$\int 74.3e^{-0.34x} + (100 - 74.3)e^{-0.043x} - 14.3dx$$
$$\int \frac{743}{10} * \frac{1}{e^{0.34x}} + 25.7 * \frac{1}{e^{0.043x}} - \frac{143}{10} dx$$
$$\int \frac{743}{10e^{0.34x}} + \frac{257}{10} * \frac{1}{e^{0.043x}} - \frac{143}{10} dx$$
$$\int \frac{743}{10e^{0.34x}} + \frac{257}{10e^{0.043x}} - \frac{143}{10} dx$$
$$\int \frac{743}{10e^{0.34x}} dx + \int \frac{257}{10e^{0.043x}} dx - \int \frac{143}{10} dx$$
$$\left(-\frac{3715}{17e^{0.34x}} - \frac{25700}{43e^{0.043x}} - \frac{143}{10}\right)\Bigg|_{1}^{11}$$
$$-\frac{3715}{17e^{0.34*11}} - \frac{25700}{43e^{0.043*11}} - \frac{143}{10} * 11 - \left(-\frac{3715}{17e^{0.34*1}} - \frac{25700}{43e^{0.043*1}} - \frac{143}{10} * 1\right)$$
$$207.44$$

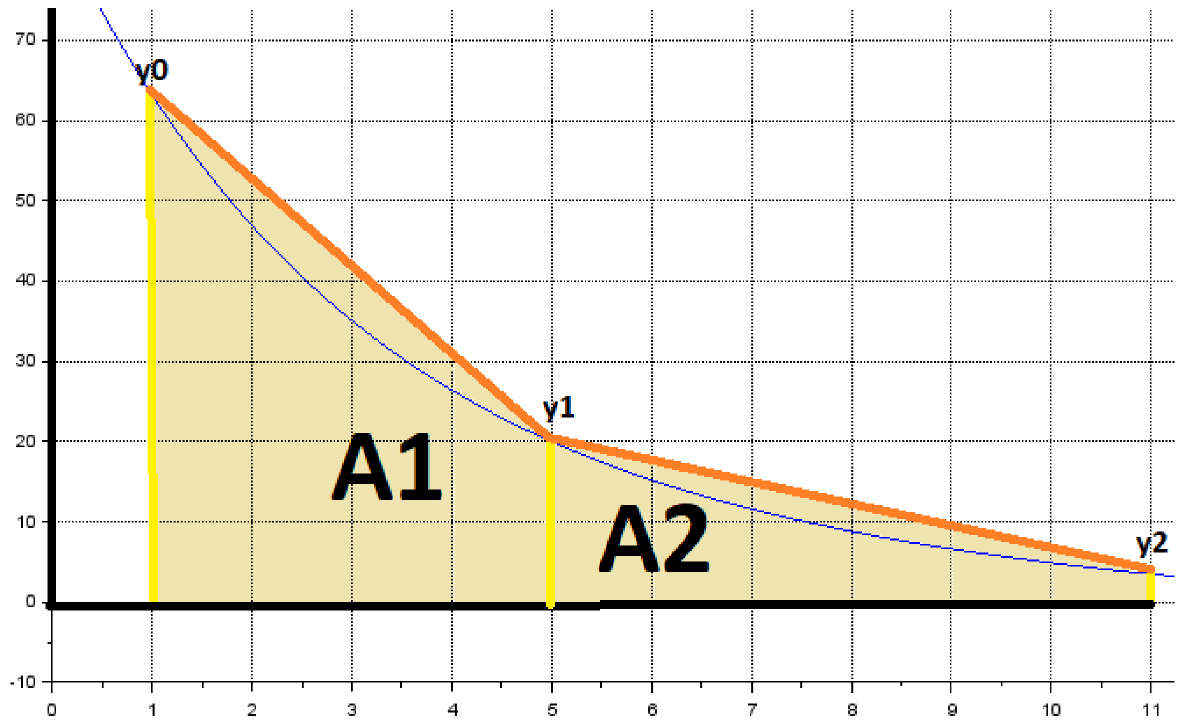
Trapecio una área.



$$A = \frac{y_0 + y_1}{2} * h$$
$$y_0 = f(1) = 74.3e^{-0.34(1)} + (100 - 74.3)e^{-0.043(1)} - 14.3$$
$$y_0 = 63.202$$
$$y_1 = f(11) = 74.3e^{-0.34(11)} + (100 - 74.3)e^{-0.043(11)} - 14.3$$
$$y_1 = 3.479$$
$$h = l_s - l_i = 11 - 1 = 10$$

$$A = \frac{63.202 + 3.479}{2} * 10$$
$$\boldsymbol{A = 333.405}$$
$$Area\ real = 207.44$$
$$\%\ Error = \frac{333.405 - 207.44}{207.44} * 100 = 60.723\%$$

Trapecio dos áreas



$$A = A1 + A2$$

$$A_1 = \frac{y_0 + y_1}{2} * h$$
$$y_0 = f(1) = 74.3e^{-0.34(1)} + (100 - 74.3)e^{-0.043(1)} - 14.3$$
$$y_0 = 63.202$$
$$y_1 = f(5) = 74.3e^{-0.34(5)} + (100 - 74.3)e^{-0.043(5)} - 14.3$$
$$y_1 = 20.001$$
$$h = l_s - l_i = 5 - 1 = 4$$
$$A_1 = \frac{63.202 + 20.001}{2} * 4$$
$$A_1 = 166.406$$

$$A_2 = \frac{y_1 + y_2}{2} * h$$
$$y_1 = f(5) = 74.3e^{-0.34(5)} + (100 - 74.3)e^{-0.043(5)} - 14.3$$
$$y_1 = 20.001$$
$$y_2 = f(11) = 74.3e^{-0.34(11)} + (100 - 74.3)e^{-0.043(11)} - 14.3$$
$$y_2 = 3.479$$

$h = l_s - l_i = 11 - 5 = 6$

$A_2 = \frac{20.001 + 3.479}{2} * 6$

$A_2 = 70.44$

$A = A1 + A2$

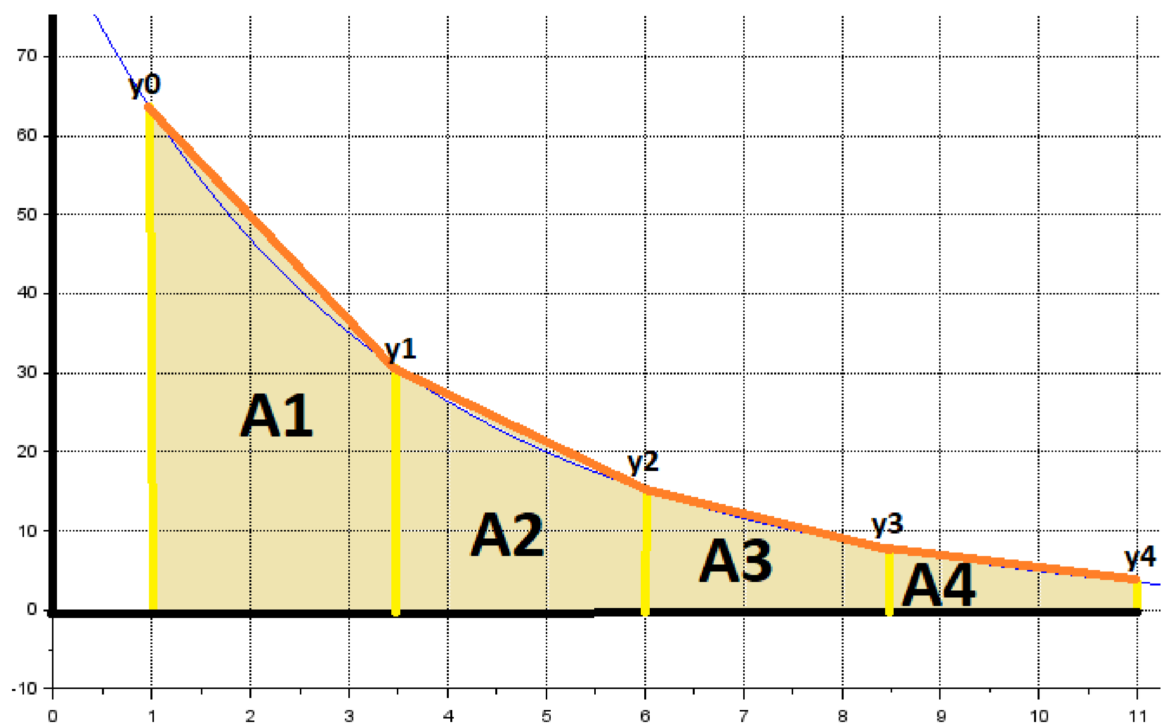
$A = 166.406 + 70.44$

$\boldsymbol{A = 236.846}$

$Area\ real = 207.44$

$\%\ Error = \frac{236.846 - 207.44}{207.44} * 100 = 14.175\%$

Trapecio cuatro áreas



$A = A1 + A2 + A3 + A4$

$A_1 = \frac{y_0 + y_1}{2} * h$

$y_0 = f(1) = 74.3e^{-0.34(1)} + (100 - 74.3)e^{-0.043(1)} - 14.3$

$y_0 = 63.202$

$y_1 = f(3.5) = 74.3e^{-0.34(3.5)} + (100 - 74.3)e^{-0.043(3.5)} - 14.3$

$y_1 = 30.412$

$h = l_s - l_i = 3.5 - 1 = 2.5$

$$A_1 = \frac{63.202 + 30.412}{2} * 2.5$$
$$A_1 = 117.01$$

$$A_2 = \frac{y_1 + y_2}{2} * h$$
$$y_1 = f(3.5) = 74.3e^{-0.34(3.5)} + (100 - 74.3)e^{-0.043(3.5)} - 14.3$$
$$y_1 = 30.412$$
$$y_2 = f(6) = 74.3e^{-0.34(6)} + (100 - 74.3)e^{-0.043(6)} - 14.3$$
$$y_2 = 15.216$$
$$h = l_s - l_i = 6 - 3.5 = 2.5$$
$$A_2 = \frac{30.412 + 15.216}{2} * 2.5$$
$$A_2 = 57.035$$

$$A_3 = \frac{y_2 + y_3}{2} * h$$
$$y_2 = f(6) = 74.3e^{-0.34(6)} + (100 - 74.3)e^{-0.043(6)} - 14.3$$
$$y_2 = 15.216$$
$$y_3 = f(8.5) = 74.3e^{-0.34(8.5)} + (100 - 74.3)e^{-0.043(8.5)} - 14.3$$
$$y_3 = 7.661$$
$$h = l_s - l_i = 8.5 - 6 = 2.5$$
$$A_3 = \frac{15.216 + 7.661}{2} * 2.5$$
$$A_3 = 28.596$$

$$A_4 = \frac{y_3 + y_4}{2} * h$$
$$y_3 = f(8.5) = 74.3e^{-0.34(8.5)} + (100 - 74.3)e^{-0.043(8.5)} - 14.3$$
$$y_3 = 7.661$$
$$y_4 = f(11) = 74.3e^{-0.34(11)} + (100 - 74.3)e^{-0.043(11)} - 14.3$$
$$y_4 = 3.419$$
$$h = l_s - l_i = 11 - 8.5 = 2.5$$
$$A_4 = \frac{7.661 + 3.419}{2} * 2.5$$
$$A_4 = 13.85$$

$$A = A1 + A2 + A3 + A4$$
$$A = 117.01 + 57.035 + 28..596 + 13.85$$
$$\boldsymbol{A = 216.491}$$
$$Area\ real = 207.44$$

$$\%\ Error = \frac{216.491 - 207.44}{207.44} * 100 = 4.363\%$$

ROMBERG

	Valor del trapecio	1 nivel	2 nivel
1 trapecio	333.405	204.669	210.04
2 trapecios	236.846	209.706	
4 trapecios	216.491		

$Area\ real = 207.44$

Primer Nivel

$$\frac{4^1 * 236.846 - 333.405}{4^1 - 1} = 204.659$$

$$\%\ Error = \frac{204.659 - 207.44}{207.44} * 100 = -1.34\%$$

$$\frac{4^1 * 216.491 - 236.846}{4^1 - 1} = 209.706$$

$$\%\ Error = \frac{209.706 - 207.44}{207.44} * 100 = 1.092\%$$

Segundo Nivel

$$\frac{4^2 * 209.706 - 204.669}{4^2 - 1} = 210.04$$

$$\%\ Error = \frac{210.04 - 207.44}{207.44} * 100 = 1.254\%$$

MODELO SIMPSON 1/3

$$A = \frac{h}{3}\{y_0 + 4y_1 + y_2\}$$

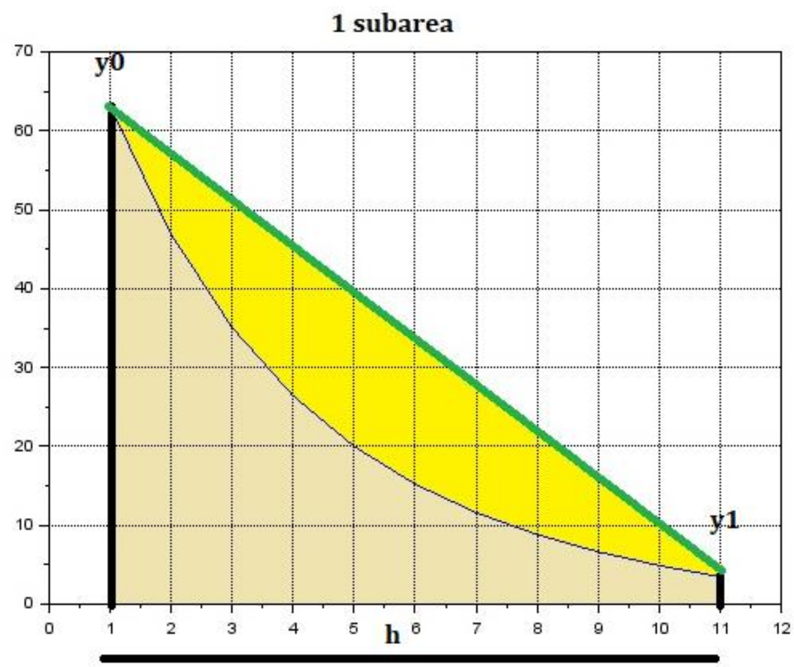
Función estudiantil

$$\int_1^{11} 74.3e^{-0.34x} + (100 - 74.3)e^{-0.043x} - 14.3dx$$

- **Una subárea**

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{1} = 10$$



$x_0 = 1$ $y_0 = 63.2$
$x_1 = 11$ $y_1 = 3.47$
$x_2 = 0$ $y_2 = 0$

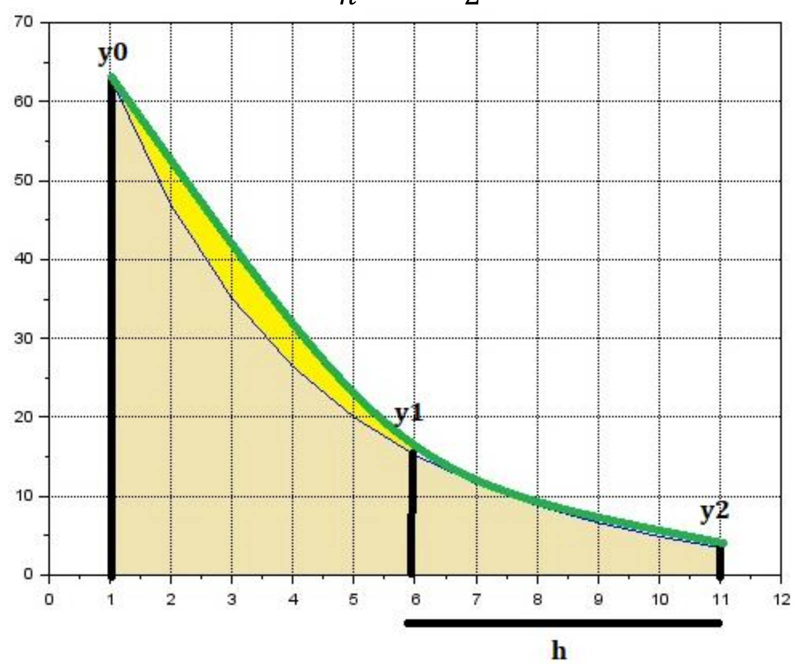
$$A = \frac{5}{3}\{63.2 + 4(3.47) + 0\}$$

$$A = 256.93$$

- **<u>Dos subáreas</u>**

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{2} = 5$$



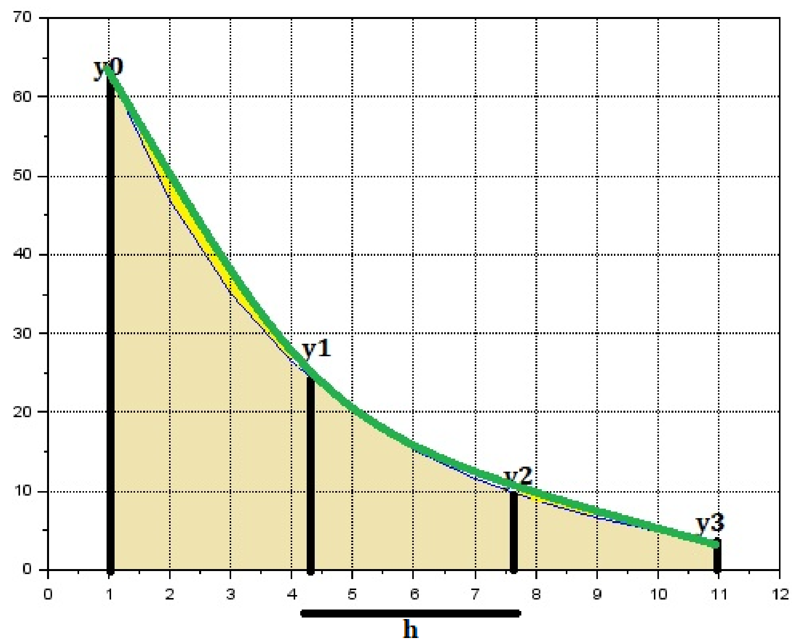
$x_0 = 1 \quad y_0 = 63.2$
$x_1 = 6 \quad y_1 = 15.21$
$x_2 = 11 \quad y_2 = 3.47$

$$A = \frac{5}{3}\{63.2 + 4(15.21) + 3.47\}$$
$$A = 212.51$$

- **Tres subáreas**

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{3} = 3.33$$



$x_0 = 1$ $y_0 = 63.2$
$x_1 = 4.33$ $y_1 = 24.07$
$x_2 = 7.66$ $y_2 = 9.68$
$x_3 = 11$ $y_3 = 3.47$

$$A = \frac{3.33}{3}\{63.2 + 4(24.07) + 9.68\} = 187.76$$

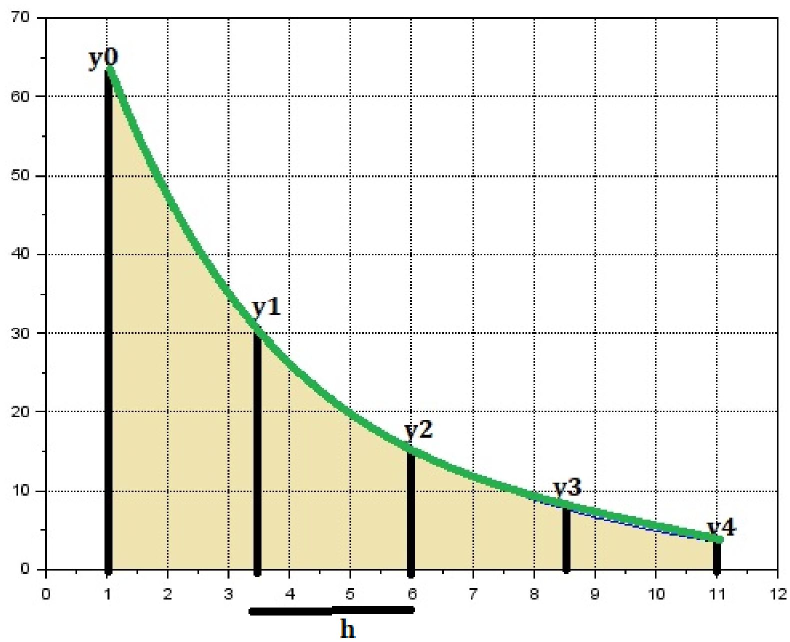
$$A = \frac{3.33}{3}\{9.68 + 4(3.47) + 0\} = 26.15$$

$$A_{total} = 187.76 + 26.15 = 213.91$$

- **<u>Cuatro subáreas</u>**

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{4} = 2.5$$



$x_0 = 1$ $\quad y_0 = 63.2$
$x_1 = 3.5$ $\quad y_1 = 30.41$
$x_2 = 6$ $\quad y_2 = 15.21$
$x_3 = 8.5$ $\quad y_3 = 7.66$
$x_4 = 11$ $\quad y_4 = 3.47$

$$A = \frac{2.5}{3}\{63.2 + 4(30.41) + 15.21\} = 166.70$$

$$A = \frac{2.5}{3}\{15.21 + 4(7.66) + 3.47\} = 41.09$$

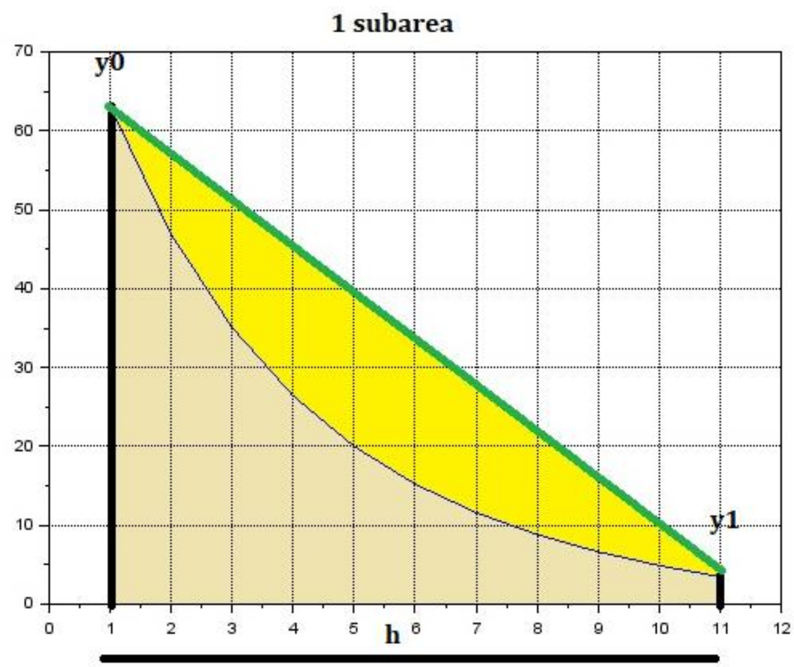
$$A_{total} = 166.70 + 41.09 = 207.79$$

MODELO SIMPSON 3/8

- ## Una subárea

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{1} = 10$$



$x_0 = 1 \qquad y_0 = 63.2$
$x_1 = 11 \qquad y_1 = 3.47$
$x_2 = 0 \qquad y_2 = 0$

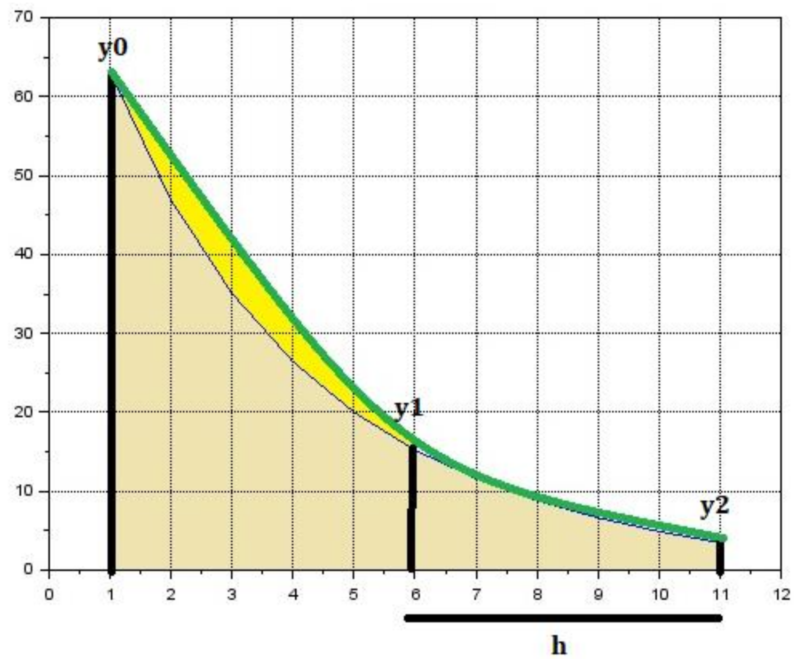
$$A = \frac{3 * 10}{8}\{63.2 + 3(3.47) + 3(0) + 0\}$$

$$A = 276.03$$

- **<u>Dos subáreas</u>**

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{2} = 5$$



$x_0 = 1 \quad y_0 = 63.2$
$x_1 = 6 \quad y_1 = 15.21$
$x_2 = 11 \quad y_2 = 3.47$

$$A = \frac{3*5}{8}\{63.2 + 3(15.21) + 3(3.47) + 0\}$$
$$A = 223.57$$

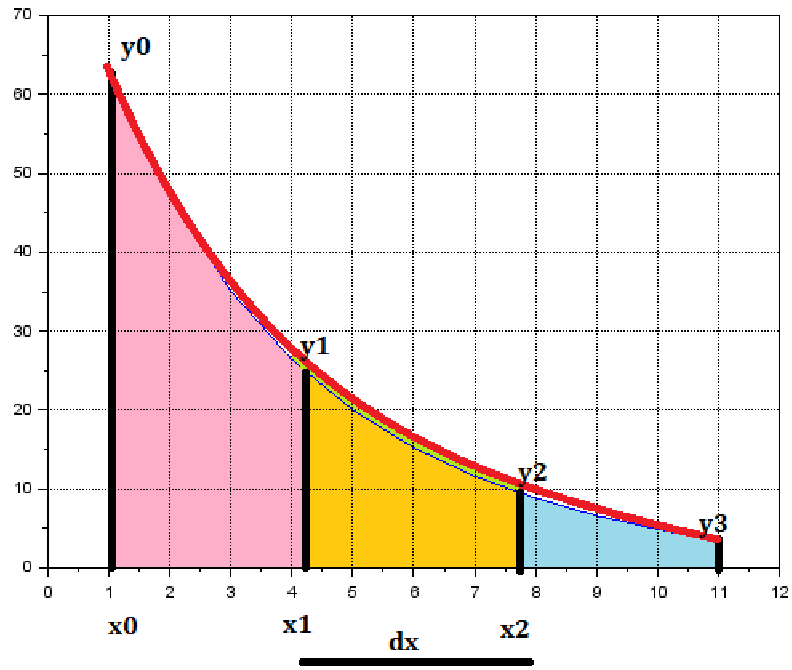
- **TRES SUBAREAS**

  A=11

  B=1

  N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{3} = 3.33$$



$x_0 = 1 \qquad y_0 = 63.2$

$x_1 = 4.33 \qquad y_1 = 24.07$

$x_2 = 7.66 \qquad y_2 = 9.68$

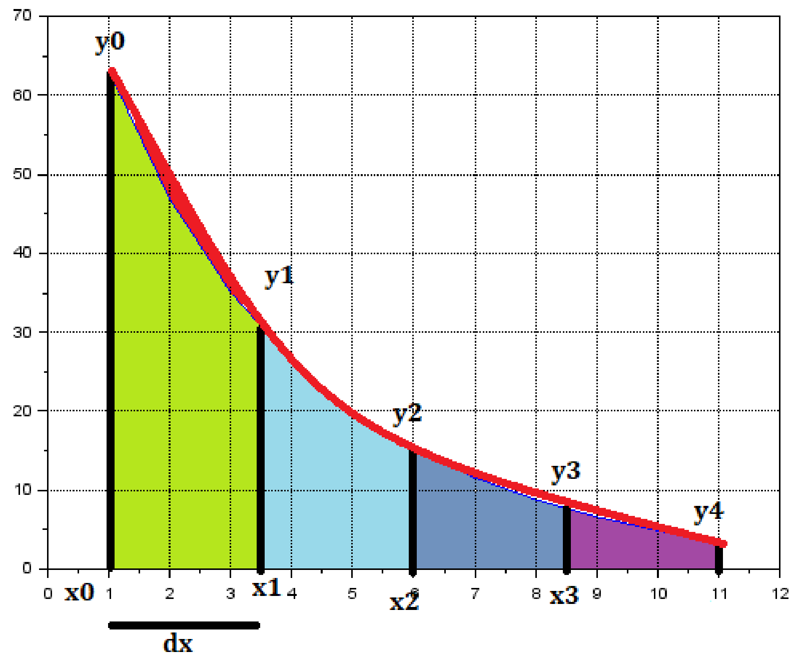
$x_3 = 11 \qquad y_3 = 3.47$

$$A = \frac{3*(3.33)}{8}\{63.2 + 3(24.07) + 3(9.68) + 3.47\} = 209.69$$

- **Cuatro subáreas**

A=11
B=1
N=número de particiones

$$h = \frac{b-a}{n} = \frac{1-11}{4} = 2.5$$



$x_0 = 1$ $y_0 = 63.2$
$x_1 = 3.5$ $y_1 = 30.41$
$x_2 = 6$ $y_2 = 15.21$
$x_3 = 8.5$ $y_3 = 7.66$
$x_4 = 11$ $y_4 = 3.47$

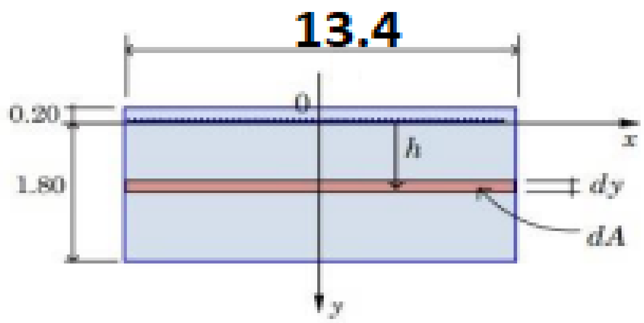
$$A = \frac{3 * 2.5}{8}\{63.2 + 3(30.41) + 3(15.21) + 7.66\} = 194.73$$

$$A = \frac{3 * 2.5}{8}\{7.66 + 3(3.47) + 3(0) + 0\} = 16.94$$

$$A_{total} = 194.73 + 16.94 = 211.67$$

Ejercicio de aplicación

La pared vertical de una piscina tiene forma rectangular de 13.4 metros de ancho en la parte superior y 2 metros de profundidad. Si el nivel del agua se encuentra 20 centímetros por debajo de la parte superior de la pared. Determine la fuerza debida a la presión del agua ejercida sobre la pared.



La figura muestra la compuerta situada en un sistema de coordenadas rectangulares, con el eje y positivo hacia abajo y el eje x en la parte superior de la compuerta.

La fuerza esta dada por

$$F = \int_a^b phdA$$

Como la superficie del agua coincide con el eje x se tiene que $h = y$

El diferencial de área tiene un ancho constante de 13.4 metros y una altura dy

$$dA = 13.4dy$$

Para establecer los limites de integración hay que observar que valor tiene y en donde se inicia la presión sobre la compuerta y que valor tiene y donde finaliza la presión sobre la misma . Para este problema la presión inicia en la parte superior del agua donde $y = 0$ y termina en el fondo de la piscina, donde $y = 1.8$.

El peso especifico del agua es una constante y se puede sustituir al finalizar los cálculos.

$$F = \int_a^b phdA = p\int_0^{1.8} y(13.4dy)$$

$$13.4p\int_0^{1.8} ydy = 13.4p\left(\frac{1}{2}y^2\right)\begin{matrix}1.8\\0\end{matrix}$$

$$13.4p\left(\frac{1}{2}1.8^2\right) = 21.70p$$

Como el peso especifico del agua es $1000\,\frac{kg}{m^3}$ o $9.8\,\frac{N}{m^3}$ la fuerza ejercida sobre la pared es.

$$F = (21.70) * (9.8) = 212.66\,N$$