Quorum Rules in a Multiple-Issue Referendum

Felipe Diaz Klaassen and Ricardo Pique*

November 2018

Abstract

Quorum rules are commonly used to increase turnout in referenda. The literature has argued that, a in single-issue referendum, these may actually increase the incentive to abstain among status-quo supporters. In this paper, we extend the ethical voter framework to analyze the effect of quorum rules when each of two political groups supports a different ballot measure. We show that bundling these two propositions in a single ballot can mitigate strategic abstention and help sustain no-quorum rules levels of turnout. This holds for both approval and participation quorum rules, which we show produce equivalent changes in voting behavior under both a single and a two-issue setting. Overall, our results show how the effects of referendum features such as ballot-measure bundling and quorum rules are interdependent.

Keywords: Ethical Voters, Quorum Rules, Referendum.

*Diaz Klaasen: University of Illinois at Urbana-Champaign, Champaign, IL 61820. Email: diazkla2@illinois.edu. Pique: Ryerson University, Toronto, Ontario, M5B 2K3, Canada. Email: rpique@ryerson.ca. We are grateful to Georgy Egorov, Timothy Feddersen and Jorge Lemus for helpful discussion and comments.

1 Introduction

In recent decades, referenda have become a popular mechanism to make policy decisions on a broad set of issues.[1] In many jurisdictions, a referendum leads to a change in the status quo only if a quorum rule is satisfied.[2] These rules are meant to increase turnout and provide legitimacy to the results (Qvortrup, 2002). However, low turnout is prevalent in areas that apply quorum rules.[3]

The theoretical literature has shown that quorum rules may not serve their purpose as these increase the incentive to abstain among status quo supporters (Aguiar-Conraria and Magalhães, 2010a; Herrera and Mattozzi, 2010; Hizen and Shinmyo, 2011). Previous models do not consider other important motives for voting such as ethical considerations,[4] which can be the main turnout driver for ballot measures related to abortion, gay marriage, and other significant social changes. Most importantly, previous results are based on single-issue referenda. In many cases, a referendum will involve multiple ballot measures or propositions.[5] It is common that each proposition will be backed by opposite political groups.[6] In this setting, the effect of quorum rules may differ from the single-issue case. Voters who are in favor of the status quo for one measure may turn out to give quorum to the measure they support.[7]

In this paper, we extend the ethical voter framework to analyze the effect of quorum rules on turnout when each of two political groups supports a different ballot measure. We assume

[1]For a comprehensive study of the history of referenda, check Qvortrup (2018).

[2]For a list of countries and regions with their respective quorum rules, check Aguiar-Conraria and Magalhães (2010b); Herrera and Mattozzi (2010)

[3]Aguiar-Conraria and Magalhães (2010b) finds a that participation quorum rules are negatively correlated with turnout. For specific examples of ballot measures which have failed due to lack of quorum, check Herrera and Mattozzi (2010); Maniquet and Morelli (2015).

[4]Herrera and Mattozzi (2010) present a group turnout mobilization model in which two parties motivate their voters to head to the polls through campaign expenditure. While ethical voter and mobilization models may produce similar results, they differ in their motivation, voter behavior assumption and solution concepts.

[5]For example, in the 2018 US midterm elections, there will be a total of 157 statewide ballot measures across 38 states. This gives an average of 4.1 measures per state.

[6]For instance, the California Democratic Party supports California's 2018 Proposition 10 on rent control and against Proposition 6 on repealing a gas tax, while the California Republican Party has opposite preferences.

[7]Italy is commonly used as an example of strategic abstention as numerous ballot measures have failed to pass due to lack of quorum, with few votes cast against these. The 1995 referendum, which featured multiple ballot measures supported by opposite political groups, recorded one of the largest turnout across Italian referenda. The outcome of the referendum was 5 measures approved and 7 rejected.

that a large electorate is divided into conservatives and liberals. The exact relative size of these groups is unknown. Conservatives support a conservative ballot measure and oppose a liberal one (i.e., they favor the status quo for that proposition), while liberals have opposite preferences. We analyze the effect of the two main types of quorum rules: participation rules, which require a given fraction of the electorate to participate for the referendum results to be valid, and approval rules, which require a given fraction of votes in favor of the ballot measure.

In our model, all voters are assume to be ethical or "rule-utilitarian" building on the framework proposed by Feddersen and Sandroni (2006a,b). These agents will turn out at a rate which maximizes the expected aggregate welfare of their political group, conditional on the behavior of voters from the opposite group. Hence, ethical voters may take a costly action (i.e., voting), even if this action has a negligible effect on the referendum results. In a referendum with no quorum rules, the voting rule that ethical voters follow trade-offs higher turnout costs for a higher probability of achieving a simple majority in both issues. However, when quorum rules are present, the trade-offs are different. In the case of a participation rule, ethical voters trade-off higher turnout costs and a higher probability of having quorum for the measure they oppose for a higher probability of achieving a simple majority in both issues and achieving quorum for the measure they support.

We first show how ethical considerations can also drive strategic abstention when only one measure is on the ballot. Similarly to Herrera and Mattozzi (2010) group mobilization approach, we find that when the quorum restriction is sufficiently strict, status quo supporters abstain from voting. That is, they rely on obtaining their preferred outcome due to lack of quorum. This behavior holds for both approval and participation rules. The result means that, in the single-issue setting, the introduction of quorum rules can only depress turnout and this negative effect can be observed for quorum requirements lower than 50%. However, if a quorum restriction is introduced, increasing the requirement may lead to higher turnout among those who favor the measure. These voters will be compelled to turn out at higher rates to partially off-set the increase in the probability that the status quo is upheld due to the higher quorum requirement. In addition, we show that, introducing a quorum rule may decrease the probability that the status quo is maintained.

We then analyze the effect of quorum rules in a two-issue setting in which group supports a different ballot measure. We show that bundling the two propositions in a single ballot can mitigate strategic abstention. We find that the no-quorum rules levels of turnout can be sustained for a larger set of quorum requirements. In particular, for ballot measures which are relatively important to voters, quorum rules with requirements lower than 50%, the upper bound requirement in most countries, will not change voter behavior. As in the single-issue case, we show that approval and participation rules produce equivalent results. However, under symmetric voting behavior, increasing the quorum requirement does not lead to higher turnout levels. Moreover, if the requirement is too strict, both groups will opt to abstain. This voting behavior implies that the probability that the status quo is maintained for either ballot measure is one half if the requirement is sufficiently low and 1 if it is sufficiently high. Hence, in the two-issue setting, quorum rules exhibit a weak bias towards the status quo.

Overall, our results show how the effects of referendum features such as ballot-measure bundling and quorum rules are interdependent. This conclusion is important for empirical studies on turnout in referenda and has implications for both political agents and policy makers regarding what measures to include and when to introduce quorum requirements.

This paper relates to two strands of the literature. First, our results complement previous findings on the effect of quorum rules on turnout. As aforementioned, these studies focus on single-issue referenda. Herrera and Mattozzi (2010) use a group turnout model to show that participation rules can only reduce turnout. Aguiar-Conraria and Magalhães (2010a); Hizen and Shinmyo (2011) reach a similar conclusion in a pivotal voter model. In particular, (Aguiar-Conraria and Magalhães, 2010a) point that approval rules also promote abstention among status-quo supporters and a decrease in turnout under most equilibria they compute. The empirical evidence on the topic is scarce and points to the effect of quorum rules on turnout being more ambiguous. (Aguiar-Conraria and Magalhães, 2010b) provide evidence of a negative effect of participation rules on turnout, though this effect is not statistically significant when the status-quo is the favored option. They also find no evidence of an effect of approval rules. (Aguiar-Conraria et al., 2016) show experimental evidence that both types of rules decrease turnout.

Our study complements this literature by using an alternative theoretical framework based on ethical voters extending the analysis to a two-issue referendum. Our results show how ethical considerations can help explain strategic abstention among status quo voters in the single-issue case and how this incentive disappears when ballot measures supported by opposing political groups are placed on the same ballot.

Our results also relate to the theoretical literature on the connection between "rule utilitarian" behavior and political outcomes (Harsanyi, 1977, 1992; Coate and Conlin, 2004; Feddersen and Sandroni, 2006a,b). In particular, we built on the ethical voter framework developed by Feddersen and Sandroni (2006a,b) to study turnout when quorum rules are imposed. This framework has also been used to study other political behaviors such as strategic voting (Bouton and Ogden, 2018; Li and Pique, 2018) and demand for political news (Piolatto and Schuett, 2015).

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we present our main results on voter behavior. We first extend our ethical voter framework to the single-issue case and then study the effect of quorum rules in a two-issue setting. Finally, Section 4 presents our concluding remarks.

2 The Model

We consider a model in which a large electorate has to vote on whether to accept or reject two propositions: one supported by a conservative group (Proposition C) and the other by a liberal group (Proposition L). Voting is voluntary. All ballot measures are printed on the same ballot and turnout is based on the number of ballots cast (i.e., turnout is not proposition-specific). Once a the polling booth, voters have to vote either in favor or against each proposition.

We approximate a large electorate by assuming a continuum of voters with measure 1. Voters belong to one of the two political groups: conservatives (c) and liberals (l). The measure of conservative voters is $0 \leq k \leq 1$. The exact value of k is unknown to voters but it is common knowledge that it is distributed uniformly on the interval [0,1]. The only difference between voters from the two political groups lies in their preference regarding the two propositions. A voter of group j receives utility w if proposition J is approved and 0 if it is rejected, while she receives an additional utility w if proposition $-J$ is rejected and 0 if it is approved. For simplicity, w is assumed to be constant across groups and issues. In addition, each voter faces a turnout cost which, for simplicity, is assumed to be constant and equal to $\theta > 0$.

A key feature of the model is that all voters are assumed to be ethical or "rule-utilitarian".[8] Similarly to (Feddersen and Sandroni, 2006a,b), these voters are assumed to follow a voting rule which maximizes the expected welfare of their political group, conditional on the voting behavior of the opposite group. In (Feddersen and Sandroni, 2006a,b), voters face different turnout costs and a voting rule requires voters with a cost lower than a cut-off to vote, and abstain otherwise. In our case, we assume turnout costs are constant. Therefore, so as not to deviate extensively from the original voting rule definition, we will assume that voters in each political group have a payoff-irrelevant type x, where x is distributed uniformly on the interval [0,1]. Given this, we define the rule for political group j as follows:

Definition 1. *A voting rule is a cut-off $\sigma_j \in [0,1]$ that specifies ethical voters with type $x \leq \sigma_j$ to vote in favor of proposition J and against proposition $-J$, and abstain otherwise.*

Alternatively, we can do without x and state that a voting rule specifies voters type j to turnout with probability σ_j.

Let P_J be the probability that Proposition J is approved. If voters act according to voting rule profile (σ_c, σ_l), then the expected aggregate welfare of conservative and liberal voters are:

$$G_c(\sigma_c, \sigma_l) = wP_C(\sigma_c, \sigma_l) + w(1 - P_L(\sigma_c, \sigma_l)) - \frac{\theta}{2}\sigma_c^2 \tag{1}$$

$$G_l(\sigma_c, \sigma_l) = w(1 - P_C(\sigma_c, \sigma_l)) + wP_L(\sigma_c, \sigma_l) - \frac{\theta}{2}\sigma_l^2 \tag{2}$$

Ethical voters are assumed to follow a voting rule which maximize the expected welfare of their group, subject to the behavior of other voters. Hence, as in (Feddersen and Sandroni, 2006b), the analysis will focus on ethical rules which satisfy the following consistency requirement:

[8]This assumption can be relaxed to include non-ethical voters who would always abstain without altering the qualitative nature of the results.

Definition 2. *A pair* (σ_c^*, σ_l^*) *is a consistent rule profile if* $G_c(\sigma_c^*, \sigma_l^*) \geq G_c(\sigma_c, \sigma_l^*)$ *for all* $\sigma_c \in [0,1]$, *and* $G_l(\sigma_c^*, \sigma_l^*) \geq G_l(\sigma_c^*, \sigma_l)$ *for all* $\sigma_l \in [0,1]$

In addition, since voters from both political groups face a symmetric turnout problem, we will restrict the analysis to symmetric voting rules. That is, we focus on rules where $\sigma_c^* = \sigma_l^*$.

3 Main Results

3.1 Single-Issue Referendum

We first check voting behavior in a single-issue setting. This will allow us to examine how turnout changes when measures are bundled together. The model detailed in Section 2 can be easily transformed to a single-issue context. Without loss of generality, we assume that only Proposition C is on the ballot and that voters receive utility $2w$ (instead of w) when the referendum outcome is their preferred outcome.

Denote the fraction of the electorate that votes in favor of proposition C as Y_C and the fraction that votes against it as N_C. Under no quorum rules, the expected aggregate welfare of conservative and liberal voters is given by:

$$G_c(\sigma_c, \sigma_l) = 2wP(Y_C > N_C) - \theta\sigma_c \tag{3}$$

$$G_l(\sigma_c, \sigma_l) = 2wP(N_C > Y_C) - \theta\sigma_l \tag{4}$$

In this setting, it can be shown that the unique consistent rule profile takes the following form:

Proposition 1. *In a single-issue referendum with no quorum rules, the unique consistent voting rule profile is* $(\sigma_{NQ}^{SIN}, \sigma_{NQ}^{SIN})$ *where*

$$\sigma_{NQ}^{SIN} = \min(1, \frac{1}{2\tilde{\theta}}) \tag{5}$$

where $\tilde{\theta} = \frac{\theta}{w} > 0$ *is the normalized cost of voting.*

The result is intuitive. When the cost of voting,*theta*, is sufficiently low or the importance of the referendum, $2w$ is sufficiently high, all voters turn out to the polls. For $\tilde{\theta} > \frac{1}{2}$, turnout is lower than 1 and is decreasing in the cost of voting and increasing in the importance of the referendum.

Now, assume that a participation rule, q_P, is introduced. In this case, for proposition C to be approved, $Y_C > N_C$ and $Y_C + N_C > q_P$. Hence, conservative voters must clear the quorum threshold in addition to obtaining a simple majority. In this case, the probability of the proposition being approved is given by:

$$P_C = \begin{cases} P(k > \max(\frac{\sigma_l}{\sigma_l+\sigma_c}, \frac{q-\sigma_l}{\sigma_c-\sigma_l})) & \text{if } \sigma_c > \sigma_l \\ P(\frac{\sigma_l}{\sigma_l+\sigma_c} < k < \frac{q-\sigma_l}{\sigma_c-\sigma_l}) & \text{otherwise} \end{cases} \tag{6}$$

Given this, the unique consistent voting rule profile will be:

Proposition 2. *In a single-issue referendum with a participation rule q_P, the unique voting rule profile is $(\sigma_{q_P}^{Y_{SIN}}, \sigma_{q_P}^{N_{SIN}})$ where*

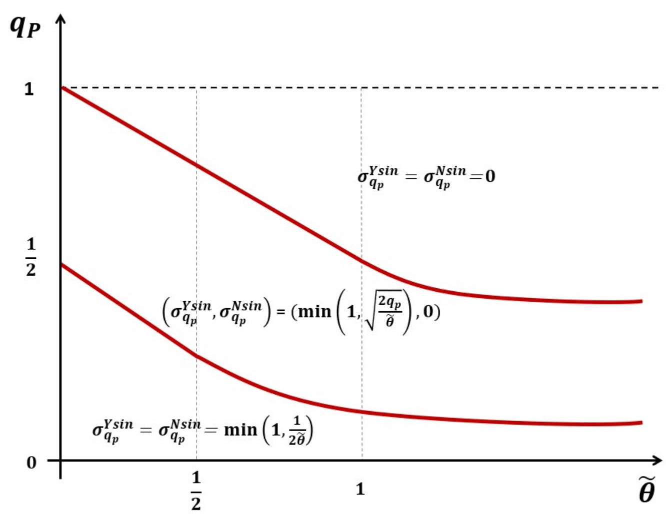
$$\sigma_{q_P}^{Y_{SIN}} = \begin{cases} 1 & \textit{if } \left(\tilde{\theta} \leq \frac{1}{2} \ \wedge \ q_P \leq 1 - \frac{\tilde{\theta}}{2}\right) \ \vee \ \left(\tilde{\theta} > \frac{1}{2} \ \wedge \ \tilde{\theta} \leq 1 \ \wedge \ q_P > \frac{\tilde{\theta}}{2} \ \wedge \ q_P \leq 1 - \frac{\tilde{\theta}}{2}\right) \\ \frac{1}{2\tilde{\theta}} & \textit{if } \tilde{\theta} > \frac{1}{2} \ \wedge \ q_P \leq \frac{1}{8\tilde{\theta}} \\ \sqrt{\frac{2q_P}{\tilde{\theta}}} & \textit{if } \tilde{\theta} > \frac{1}{2} \ \wedge \ q_P \leq \frac{\tilde{\theta}}{2} \ \wedge \ q_P \leq \frac{1}{2\tilde{\theta}} \ \wedge \ q_P > \frac{1}{8\tilde{\theta}} \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

and

$$\sigma_{q_P}^{N_{SIN}} = \begin{cases} 1 & \textit{if } \tilde{\theta} \leq \frac{1}{2} \ \wedge \ q_P \leq \frac{1}{2} - \frac{\tilde{\theta}}{2} \\ \frac{1}{2\tilde{\theta}} & \textit{if } \tilde{\theta} > \frac{1}{2} \ \wedge \ q_P < \frac{1}{8\tilde{\theta}} \\ 0 & \textit{otherwise} \end{cases} \tag{8}$$

The voting rule profile in Proposition 2 is illustrated in Figure 1. Note that if the quorum requirement, q_P is low enough (i.e., $\tilde{\theta} \leq \frac{1}{2} \ \wedge \ q_P \leq \frac{1}{2} - \frac{\tilde{\theta}}{2}$ or $\tilde{\theta} > \frac{1}{2} \ \wedge \ q_P < \frac{1}{8\tilde{\theta}}$) ethical voters follow the same behavior as if there was no quorum rule. For intermediate values of q_p (i.e, $\frac{1}{2} \ \wedge \ 1 - \frac{\tilde{\theta}}{2} \geq q_P \geq \frac{1}{2} - \frac{\tilde{\theta}}{2}$ or $\tilde{\theta} > \frac{1}{2} \ \wedge \ \frac{1}{2\tilde{\theta}} \geq q_P \geq \frac{1}{8\tilde{\theta}}$), only conservatives voters turn out to vote. Liberals prefer to stay at him as any gain for them in terms of increasing the probability that issue is rejecting by obtaining a simple majority of votes against is not enough to compensate the cost needed to do. In particular, if $q_p \geq \frac{1}{2}$, liberals maximize the probability that the status quo is kept by abstaining. Finally, when q_p is high enough (i.e., $\frac{1}{2} \wedge 1 - \frac{\tilde{\theta}}{2} \leq q_P$ or $\tilde{\theta} > \frac{1}{2} \wedge \frac{1}{2\tilde{\theta}} \leq q_P$, conservative voters also abstain. This is because the probability of approving the proposition under the optimal voting rule is not enough to compensate the voting cost.

Figure 1: Ethical Voter Behavior in a Single-Issue Referendum with a Participation Rule



The above results imply that, for the single-issue case, the ethical voter model produces similar predictions to those in the literature. In particular, it is easy to check that, for any normalized cost of voting, turnout under a participation rule cannot be higher than that under no quorum rules.[9] Interestingly, for intermediate values of q_p, turnout is weakly increasing in the strictness of the participation rule. For those values, the binding restriction for conservative voters is the quorum restriction. In this case, when q_p increases, so does the marginal benefit of voting for conservative voters (as long as $\sigma_c > q_p$). As the marginal cost is independent of q_p, conservative voters turnout at higher rates to try to achieve quorum.

While participation rules weakly decrease turnout, this negative effect does not hold for the probability that the status quo is maintained. Note that, in the absence of these rules,

[9]Note that, for the parameter space where the voting rule profile under a participation rule is different from that under no quorum rules, the maximum expected turnout under a participation rules is $\frac{1}{2}\min(1, \frac{1}{\tilde{\theta}}) = \min(\frac{1}{2}, \frac{1}{2\tilde{\theta}}) \leq \min(1, \frac{1}{2\tilde{\theta}})$.

the probability that the status quo is kept is always one half. When a participation rule is implemented, this probability is also equal to a half for sufficiently low values of q_P. For intermediate values of q_P, introducing a participation rule may increase the probability that the status quo is changed. In particular, the probability jumps from $\frac{1}{2}$ to $1-\tilde{\theta}$ at $q_P = \frac{1}{2} - \frac{\tilde{\theta}}{2}$ for $\tilde{\theta} \leq \frac{1}{2}$ and to $\frac{1}{4}$ at $q_P = \frac{1}{8\tilde{\theta}}$ for This means that if quorum rules are to be used to protect the status quo, then the rule has to be sufficiently strict. Otherwise, it is possible for the introduction of a participation rule to have a positive effect on the probability that a ballot measure is accepted by encouraging strategic abstention among status quo supporters.

Now, lets assume that, instead of a participation rule, an approval rule, q_A, is introduced. In this case, the quorum restriction implies that, for the proposition to be approved $Y_C \geq \max(N_C, q_A)$. Hence, the probability that the proposition is approved is given by:

$$P_C = P\left(k > \max\left(\frac{q_A}{\sigma_c}, \frac{\sigma_l}{\sigma_l + \sigma_c}\right)\right), \tag{9}$$

Given this, we have the following result:

Proposition 3. *In a single-issue referendum with an approval rule q_A, the unique voting rule profile is $(\sigma_{q_A}^{Y_{SIN}}, \sigma_{q_A}^{N_{SIN}})$ where*

$$\sigma_{q_A}^{Y_{SIN}} = \begin{cases} 1 & \textit{if} \left(\tilde{\theta} \leq \frac{1}{2} \wedge q_A \leq 1 - \frac{\tilde{\theta}}{2}\right) \vee \left(\tilde{\theta} > \frac{1}{2} \wedge \tilde{\theta} \leq 1 \wedge q_A > \frac{\tilde{\theta}}{2} \wedge q_A \leq 1 - \frac{\tilde{\theta}}{2}\right) \\ \frac{1}{2\tilde{\theta}} & \textit{if}\ \tilde{\theta} > \frac{1}{2} \wedge q_A \leq \frac{1}{8\tilde{\theta}} \\ \sqrt{\frac{2q_A}{\tilde{\theta}}} & \textit{if}\ \tilde{\theta} > \frac{1}{2} \wedge q_A \leq \frac{\tilde{\theta}}{2} \wedge q_A \leq \frac{1}{2\tilde{\theta}} \wedge q_A > \frac{1}{8\tilde{\theta}} \\ 0 & \textit{otherwise} \end{cases} \tag{10}$$

and

$$\sigma_{q_A}^{N_{SIN}} = \begin{cases} 1 & \textit{if}\ \tilde{\theta} \leq \frac{1}{2} \wedge q_A \leq \frac{1}{2} - \frac{\tilde{\theta}}{2} \\ \frac{1}{2\tilde{\theta}} & \textit{if}\ \tilde{\theta} > \frac{1}{2} \wedge q_P < \frac{1}{8\tilde{\theta}} \\ 0 & \textit{otherwise} \end{cases} \tag{11}$$

Proposition 3 implies that approval and participation rules alter voter behavior in a similar manner and have the same impact on voter turnout and on the probability that the status quo is maintained.

3.2 Two-Issue Referendum

We now turn back to the model in Section 2 and analyze how ethical voters would behave in a two-issue referendum with no quorum rules. To determine the consistent rule profiles, we focus on the consistency requirement for conservative voters as the case for liberal voters is analogous. Then, the expected aggregate welfare of conservative voters is given by:

Denote the fraction of the electorate that votes in favor of proposition $J \in \{C, L\}$ as Y_J and the fraction that votes against it as N_J. Hence, under no quorum rules, the expected aggregate welfare of conservative and liberal voters is given by:

$$G_c(\sigma_c, \sigma_l) = wP(Y_C > N_C) + wP(N_L > Y_L) - \frac{\theta}{2}\sigma_c^2 \tag{12}$$

where

$$P(Y_C > N_C) = P(N_L > Y_L) = P(k\sigma_c > (1-k)\sigma_l) = P(k > \frac{\sigma_l}{\sigma_l + \sigma_c}) = 1 - \frac{\sigma_l}{\sigma_l + \sigma_c} \tag{13}$$

Given this, we have the following result which characterizes ethical voter turnout under no quorum rules:

Proposition 4. *Under no quorum rules, the unique consistent voting rule profile is (σ_{NQ}, σ_{NQ}) where*

$$\sigma_{NQ} = \min(1, \frac{1}{2\tilde{\theta}}) \tag{14}$$

where $\tilde{\theta} = \frac{\theta}{w} > 0$.

Proposition 4 states that all ethical voters will turn out to vote as long as the cost of voting,*theta*, is sufficiently low or the value or importance of the propositions to voters,w, is sufficiently high. For $\tilde{\theta} > \frac{1}{2}$, turnout is lower than 1 and is decreasing in the cost of voting and increasing in the importance of the propositions. Alternatively, we can define $\frac{\theta}{w}$ as the normalized cost of voting and state that turnout is decreasing in these normalized cost.

3.2.1 Participation Rules

Lets assume that a participation rule, q_P, is introduced. This means that, for proposition J to be approved, we require the number of votes in favor of it to be higher than the number of votes against it (i.e., $Y_J > N_J$), and the fraction of the electorate casting a ballot must be higher than q_P (i.e., $Y_J + N_J > q_P$). In this case, the probability that Proposition C is approved is:

$$P_C = \begin{cases} P(k > \max(\frac{\sigma_l}{\sigma_l+\sigma_c}, \frac{q-\sigma_l}{\sigma_c-\sigma_l})) & \text{if } \sigma_c > \sigma_l \\ P(\frac{\sigma_l}{\sigma_l+\sigma_c} < k < \frac{q-\sigma_l}{\sigma_c-\sigma_l}) & \text{otherwise} \end{cases} \tag{15}$$

and the probability that Proposition L is rejected is:

$$1 - P_L = \begin{cases} P(k < \frac{q-\sigma_c}{\sigma_c-\sigma_l} \cup k > \frac{\sigma_l}{\sigma_l+\sigma_c} & \text{if } \sigma_c > \sigma_l) \\ P(k > \min(\frac{\sigma_l}{\sigma_l+\sigma_c}, \frac{q-\sigma_l}{\sigma_c-\sigma_l})) & \text{otherwise} \end{cases} \tag{16}$$

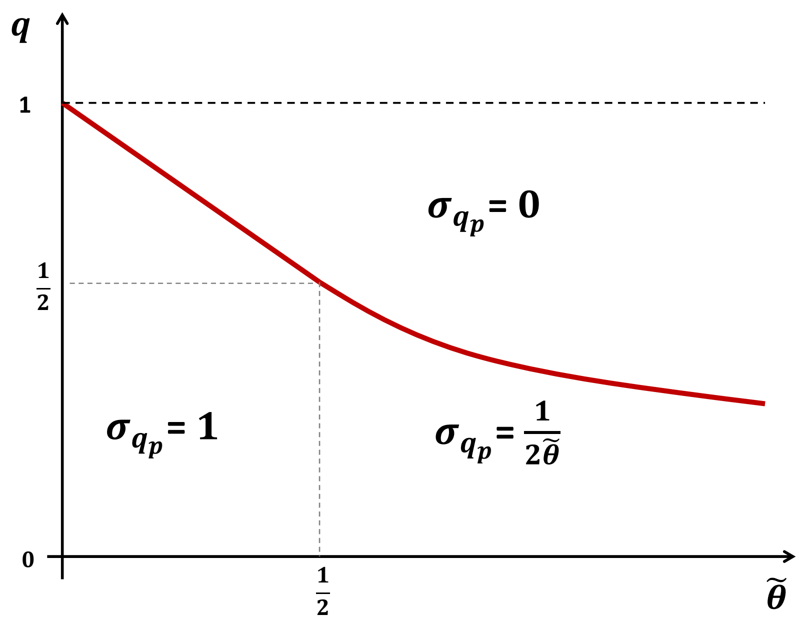
In this case, the expected aggregate welfare of conservative voters is given by equation 1, where P_C and $1 - P_L$ are defined as in equations 15 and 16, respectively. Ethical voter behavior can then be characterized as follows:

Proposition 5. *In a referendum with a participation rule q_p, the symmetric consistent voting rule profile is $(\sigma_{q_P}, \sigma_{q_P})$ where*

$$\sigma_{q_P} = \begin{cases} 1 & \textit{if } q_P < 1 - \tilde{\theta}, \tilde{\theta} \leq \frac{1}{2} \\ \frac{1}{2\tilde{\theta}} & \textit{if } q_P < \frac{1}{4\tilde{\theta}}, \tilde{\theta} > \frac{1}{2} \\ \{0, \min(1, \frac{1}{2\tilde{\theta}})\} & \textit{if } 1 - \tilde{\theta} = q_P, \tilde{\theta} \leq \frac{1}{2} \textit{ or } q_P = \frac{1}{4\tilde{\theta}}, \tilde{\theta} > \frac{1}{2} \\ 0 & \textit{otherwise} \end{cases} \tag{17}$$

In a single-ballot referendum, the consistent voting rule profile, illustrated in Figure 2, is similar to that under no quorum rules. The only difference is that when the value of the participation rule is high enough, ethical voters abstain from voting. Hence, the expected turnout is weakly decreasing in q_p and weakly lower than the turnout under no quorum. However, it should be noted that it is possible to sustain the same level of turnout as in referendum without quorum rules for relatively high values of q_p. In particular, for $\tilde{\theta}$ sufficiently low, impose a turnout requirement of more than half of the electorate may have no effect of turnout. This differs from single-issue and separate-ballot cases in which voters who oppose a proposition find it optimal to abstain when $q_p > 0.5$. In a multiple-issue, single-ballot referendum, voters from both political groups will turn out to vote at high rates as long as the policies on the ballot are sufficiently important or the cost of voting is sufficiently low.

Figure 2: Ethical Voter Behavior in a Referendum with Participation Rules



3.2.2 Approval Rules

Assume that an approval rule of value q_A is introduced. This implies that, in order for a proposition J to be approved, we require that not only the number of votes in favor of the proposition outnumber those against it, i.e., that $Y_J > N_J$, but also that the fraction of the electorate that votes in favor of the proposition be higher than q_A, that $Y_J > q_A$.

For the single ballot case the probability that proposition C is approved is:

$$P_C = P\left(k > \max\left(\frac{q_A}{\sigma_c}, \frac{\sigma_l}{\sigma_l + \sigma_c}\right)\right), \tag{18}$$

while the probability that proposition L is rejected is given by

$$1 - P_L = P\left(k > \min\left(\frac{\sigma_l - q_A}{\sigma_l}, \frac{\sigma_l}{\sigma_l + \sigma_c}\right)\right) \tag{19}$$

Now the expected aggregate welfare of conservative voters is given by equation (1), where P_C and $1-P_L$ are defined as in equations (15) and (16), respectively. Ethical voter behavior, then, can then be characterized as follows:

Proposition 6. *In a single-ballot referendum with an approval rule q_A, the unique symmetric consistent voting rule profile is $(\sigma_{q_A}, \sigma_{q_A})$ where*

$$\sigma_{q_A} = \begin{cases} 1 & \text{if } \tilde{\theta} \leq \frac{1}{2} \wedge q_A < 1 - \tilde{\theta} \\ \frac{1}{2\tilde{\theta}} & \text{if } \tilde{\theta} > \frac{1}{2} \wedge q_A < \frac{1}{4\tilde{\theta}} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

4 Conclusion

In this paper, we analyze the effect of quorum rules on turnout in an ethical voter model in which each of two political groups supports a different ballot measure. We examine voter behavior in both a single-issue and a two-issue referendum. We show how bundling the two measures in a single ballot can mitigate strategic abstention by encouraging each political group to turnout to support their proposition. In the two-issue case, even quorum requirements of more than 50% may not dampen turnout. In addition, we show that approval and participation rules produce equivalent results.

Our results highlight how various dimensions of referendum design can interact with each other. In particular, they show how the effect of quorum rules on turnout depends on ballot-measure bundling. To validate these results, more empirical evidence is needed. Causal estimates are scarce as the introduction of quorum rules is endogenous to the country or locality. New studies should consider that the effect of quorum rules may be heterogeneous and depend on the type of issues on the ballot.

This paper shows how the ethical voter framework can be extended to explain voting behavior under various circumstances. Our referendum model can be extended to account for behavior in other stages of the referendum process. For example, one could model which policies do political parties or interest groups put forward in a referendum and how this decision depends on quorum rules. This line of research should prove useful given the increase popularity of direct democracy mechanisms.

References

Aguiar-Conraria, L. and P. C. Magalhães (2010a). How Quorum Rules Distort Referendum Outcomes: Evidence from a Pivotal Voter Model. *European Journal of Political Economy 26*(4), 541–557.

Aguiar-Conraria, L. and P. C. Magalhães (2010b). Referendum Design, Quorum Rules and Turnout. *Public Choice 144*(1-2), 63–81.

Aguiar-Conraria, L., P. C. Magalhães, and C. A. Vanberg (2016). Experimental evidence that quorum rules discourage turnout and promote election boycotts. *Experimental Economics 19*(4), 886–909.

Bouton, L. and B. G. Ogden (2018). Ethical voting in multicandidate elections. Available at http://www.nber.org/papers/w23898.

Coate, S. and M. Conlin (2004). A Group Rule–Utilitarian Approach to Voter Turnout: Theory and Evidence. *The American Economic Review 94*(5), 1476–1504.

Feddersen, T. and A. Sandroni (2006a). A Theory of Participation in Elections. *The American Economic Review 96*(4), 1271–1282.

Feddersen, T. and A. Sandroni (2006b). Ethical Voters and Costly Information Acquisition. *Quarterly Journal of Political Science 1*(3), 287–311.

Harsanyi, J. C. (1977). Morality and the Theory of Rational Behavior. *Social Research*, 623–656.

Harsanyi, J. C. (1992). Game and Decision Theoretic Models in Ethics. *Handbook of game theory with economic applications 1*, 669–707.

Herrera, H. and A. Mattozzi (2010). Quorum and Turnout in Referenda. *Journal of the European Economic Association 8*(4), 838–871.

Hizen, Y. and M. Shinmyo (2011). Imposing a Turnout Threshold in Referendums. *Public Choice 148*(3-4), 491–503.

Li, C. and R. Pique (2018). Ethical voting in multicandidate elections. Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2318096.

Maniquet, F. and M. Morelli (2015). Approval Quorums Dominate Participation Quorums. *Social Choice and Welfare 45*(1), 1–27.

Piolatto, A. and F. Schuett (2015). Media Competition and Electoral Politics. *Journal of Public Economics 130*, 80–93.

Qvortrup, M. (2002). *Government by the People: A Comparative Study of Referendums*. Manchester University Press, Manchester.

Qvortrup, M. (2018). *Referendums Around the World*. Springer.

A Mathematical Appendix

Proposition 1

Proof. Notice that both G_j and G_{-j} are twice differentiable for all $\sigma_j,\ \sigma_{-j} \geq 0$ and that $\partial^2_{\sigma_j} G_j,\ \partial^2_{\sigma_{-j}} G_{-j} < 0$ for all $\sigma_j,\ \sigma_{-j} > 0$. Therefore, if they exist, both G_j and G_{-j} have unique nonzero global maximizers $\sigma^*_j(\sigma_{-j})$ and $\sigma^*_{-j}(\sigma_j)$. Solving for these leads to the unique (as it yields a strictly positive welfare) consistent voting rule profile $\sigma^*_j = \sigma^*_j = \min\left\{1, \frac{1}{2\tilde{\theta}}\right\}$, where $\tilde{\theta} = \frac{\theta}{w}$.

□

Proposition 2

Proof. Consider first the case where the simple majority restriction is binding. If $\sigma_j > q_P$ the situation is just as when there are no quorum rules, while $\partial_{\sigma_j} G_j > 0$ for any σ_{-j} if $\sigma_j \leq q_P$. Therefore, if the simple majority restriction binds the only consistent voting rule profile possible is as in Proposition 1. Now, if the quorum restriction is binding, group $-j$'s best response is to simply not turn up, as the status quo will be upheld regardless, and group j's best response, in this case, is either 0 as well or $\min\{1, \frac{2q_P}{\tilde{\theta}}\}$. We have, then, three candidates to consider as equilibria: the no quorum $(\sigma^{SIN}_{NQ}, \sigma^{SIN}_{NQ})$ profile, a quorum-busting $(\sigma^{SIN}_{QB} = \min\{1, \sqrt{\frac{2q_P}{\tilde{\theta}}}\}, 0)$ profile, and a no turnout profile.

Since $G_j(\sigma_j, \sigma^{SIN}_{NQ}) \leq 0$ and $\sigma^*_{-j}(\sigma_j) = 0$ for all σ_j such that the quorum restriction is binding, the no quorum profile will be consistent as long as neither group would rather cast zero ballots, that is as long as $G_j(\sigma^{SIN}_{NQ}, \sigma^{SIN}_{NQ}) \geq 0$ (which is always true) and $\Gamma(\tilde{\theta}, q_P) \coloneqq G_{-j}(\sigma^{SIN}_{NQ}, \sigma^{SIN}_{NQ}) - G_{-j}(\sigma^{SIN}_{NQ}, 0) \geq 0$; the quorum-busting profile will be consistent provided that group j prefers it to not vote and group $-j$ does not want to deviate outside of where participation binds, i.e., provided that $G_j(\sigma^{SIN}_{QB}, 0) \geq 0$ and $\Delta(\tilde{\theta}, q_P) \coloneqq G_{-j}(\sigma^{SIN}_{QB}, 0) - G_{-j}(\sigma^{SIN}_{QB}, \sigma^*_{-j}(\sigma^{SIN}_{QB})) \geq 0$; and the no turnout profile will be consistent as long as group j favors not voting over σ^{SIN}_{QB}, which is whenever $G_j(\sigma^{SIN}_{QB}, 0) \leq 0$.

Let's look at each of these conditions in more detail. $\Gamma(\tilde{\theta}, q_P) \geq 0$ if and only if

$$2w\frac{\sigma^{SIN}_{NQ}}{\sigma^{SIN}_{NQ} + \sigma^{SIN}_{NQ}} - \theta\sigma^{SIN}_{NQ} \geq 2w\frac{q_P}{\sigma^{SIN}_{NQ}},$$

which holds for $q_P \leq \frac{1}{2} - \frac{\tilde{\theta}}{2}$ if $\tilde{\theta} \leq \frac{1}{2}$, and for $q_P \leq \frac{1}{8\tilde{\theta}}$ otherwise. $\Delta(\tilde{\theta}, q_P) \geq 0$ is equivalent to

$$2w\frac{q_P}{\sigma_{QB}^{SIN}} \geq 2w\frac{\sigma_{-j}^*(\sigma_{QB}^{SIN})}{\sigma_{QB}^{SIN} + \sigma_{-j}^*(\sigma_{QB}^{SIN})} - \theta\sigma_{-j}^*(\sigma_{QB}^{SIN}),$$

which will be true as long as $q_P \geq \frac{1}{2} - \frac{\tilde{\theta}}{2}$ when $q_P > \frac{\tilde{\theta}}{2}$ and as long as $q_P \geq \frac{1}{8\tilde{\theta}}$ when it is not. And, lastly, $G_j(\sigma_{QB}^{SIN}, 0) \geq 0$, that is $2w\frac{\sigma_{QB}^{SIN} - q_P}{\sigma_{QB}^{SIN}} \geq \theta\sigma_{QB}^{SIN}$, will hold if and only if $q_P \leq 1 - \frac{\tilde{\theta}}{2}$ when $q_P > \frac{\tilde{\theta}}{2}$ and if $q_P \leq \frac{1}{2\tilde{\theta}}$ when $q_P \leq \frac{\tilde{\theta}}{2}$.

□

Proposition 3

Proof. Assume that political group j wants to change the status quo and group $-j$ wants to keep it. Note that, for any consistent voting rule profile, either the simple majority or the quorum restriction is binding (when the probability of measure j being approved is 0, the quorum restriction is binding). If the simple majority restriction is binding, then the voting rule profile is as in Proposition 1 (i.e., $\sigma_j = \sigma_{-j} = \sigma_{NQ}^{SIN} = \min\{1, \frac{1}{2\tilde{\theta}}\}$. If the quorum restriction is binding, then the best response for group $-j$ is 0. Given this, the best response of group j is either $min\{1, \frac{2q_A}{2\tilde{\theta}}\}$ or 0. Hence, the only consistent voting rule profiles are the no quorum profile $(\sigma_{NQ}^{SIN}, \sigma_{NQ}^{SIN}))$, the quorum-busting profile $(min\{1, \frac{2q_A}{2\tilde{\theta}}\}, 0)$ or a no voting profile $(0, 0)$.

First, lets check what happens when $q_A \geq \frac{1}{2}$. In this case, group $-j$ cannot increase the probability of maintaining the status quo by having a positive turnout rate. Hence, the best response for group $-j$ is 0 and the quorum restriction is the binding restriction. Group j will vote at rate $\min\{1, \frac{2q_A}{2\tilde{\theta}}$ as long as $G_j(\min\{1, \frac{2q_A}{2\tilde{\theta}}\}, 0) \geq G_j(0, 0)$. This holds if and only if $q_A \leq 1 - \frac{\tilde{\theta}}{2}$.

Now, assume that $q_A < \frac{1}{2}$. We first check where the no quorum profile is consistent. Say that $\min\{1, \frac{1}{2\tilde{\theta}}\} = 1$. Note that since $q_A < \frac{1}{2}$, profile $(1, 1)$ implies that the simple majority restriction is binding and that, for political group j, $\sigma_j = 1$ is as best response to $\sigma_j = 1$ for $\tilde{\theta} \leq \frac{1}{2}$

□

Proposition 4

Proof. Note that $G_j(\sigma_j, \sigma_{-j}) = 2w \max(0, 1 - \frac{\sigma_{-j}}{\sigma_{-j}+\sigma_j}) - \theta\sigma_j$. Hence, σ_j^* which maximizes the expected welfare of political group j conditional on the best response by group $-j$ is given by:

$$\frac{\partial G_j}{\partial \sigma_j} = 2w\frac{\sigma_{-j}^*}{(\sigma_{-j}^* + \sigma_j^*)^2} - \theta \geq 0 \tag{21}$$

Note that the above implies that $\sigma_j^* = \sigma_{-j}^*$. Solving for $\sigma_j^* = \sigma_{-j}^* = \sigma_{NQ}$ yields

$$\sigma_{NQ} = \min(1, \frac{1}{2\tilde{\theta}})$$

where $\tilde{\theta} = \frac{\theta}{w}$

□

Proposition 5

Proof. First, note that that if for the consistent rule $\sigma_c^* \geq q_p$ and $\sigma_l^* \geq q_p$, then $\sigma_c^* = \sigma_l^* = \sigma_{NQ}$ as the quorum restriction is not binding and equation 21 must hold for c and l. Also, note that if $\sigma_c^* < q_p$ and $\sigma_l^* < q_p$, then $\sigma_c^* = \sigma_l^* = 0$ will never be satisfied and voters will be better off by abstaining.

Say that $\sigma_c^* = \sigma_l^* = \min(1, \frac{1}{2\tilde{\theta}})$. For conservative voters, this will be a consistent voting rule as long as $G_c(\sigma_c^*, \sigma_l^*) \geq G_c(0, \sigma_l^*)$, where G_c is defined as in equation 1 and P_C and $1 - P_L$ are defined as in equations 15 and 16, respectively. Note that we do not need to check for other deviations as $\min(1, \frac{1}{2\tilde{\theta}}) = \arg\max_{\sigma_c \in [\bar{\sigma}_c, 1]} G_c(\sigma_c, \min(1, \frac{1}{2\tilde{\theta}}))$, where $\bar{\sigma}_c$ is such that $\frac{\sigma_l}{\sigma_l + \bar{\sigma}_c} = \frac{q - \sigma_l}{\bar{\sigma}_c - \sigma_l}$ (i.e., when the simple majority restriction is binding). Moreover, $G_c(0, \min(1, \frac{1}{2\tilde{\theta}})) > G_c(\sigma_c, \min(1, \min(1, \frac{1}{2\tilde{\theta}}))$ for $\sigma_c < \bar{\sigma}_c$ (i.e., when the only binding restriction is the quorum restriction).

If $\min(1, \frac{1}{2\tilde{\theta}}) = 1$, the restriction on G_c holds if and only if $q_P \leq 1 - \theta$. If $\min(1, \frac{1}{2\tilde{\theta}}) = \frac{1}{2\tilde{\theta}} < 1$, the restriction holds if and only if $q_P \leq \frac{1}{4}\tilde{\theta}$.

Say that $\sigma_c^* = \sigma_l^* = 0$. In this case, we have to check whether $\sigma_c = \min(1, \sqrt{\frac{q_P}{\tilde{\theta}}}) = \arg\max_{\sigma_c} G_c(\sigma_c, 0) = w(1 - \frac{q_P}{\sigma_c}) + w - \theta\sigma_c$ provides a higher welfare than $\sigma_c = 0$. Note that $\min(1, \sqrt{\frac{q_P}{\tilde{\theta}}}) > q$ as otherwise voters are better off by abstaining. If $\min(1, \sqrt{\frac{q_P}{\tilde{\theta}}}) = 1$, then $G_c(0, 0) \geq G_c(1, 0)$ if and only if $q_P \geq 1 - \theta$. If $\min(1, \sqrt{\frac{q_P}{\tilde{\theta}}}) = \frac{q_P}{\tilde{\theta}}$, then $G_c(0, 0) \geq G_c(\frac{q_P}{\tilde{\theta}}, 0)$ if and only if $q_P \geq \frac{1}{4}\tilde{\theta}$.

$\square$

Proposition 6

Proof. Consider that the expected aggregate welfare of political group j for the single-ballot case is given by

$$w\left[P\left(k>\max\left(\frac{q_A}{\sigma_j},\frac{\sigma_{-j}}{\sigma_{-j}+\sigma_j}\right)\right)+P\left(k>\min\left(\frac{\sigma_{-j}-q_A}{\sigma_{-j}},\frac{\sigma_{-j}}{\sigma_{-j}+\sigma_j}\right)\right)\right]-\theta\sigma_j$$

Notice also that $\frac{q_A}{\sigma_j}\geq\frac{\sigma_{-j}}{\sigma_{-j}+\sigma_j}$ iff $\frac{\sigma_{-j}-q_A}{\sigma_{-j}}\leq\frac{\sigma_{-j}}{\sigma_{-j}+\sigma_j}\Leftrightarrow\sigma_{-j}\leq\frac{q_A\sigma_j}{\sigma_j-q_A}$. If it is the case that $\sigma_{-j}\geq\frac{q_A\sigma_j}{\sigma_j-q_A}$ conservative voters will be maximizing $w\left(\frac{\sigma_j}{\sigma_{-j}+\sigma_j}+\frac{\sigma_j}{\sigma_{-j}+\sigma_j}\right)-\theta\sigma_j$, while if $\sigma_{-j}\leq\frac{q_A\sigma_j}{\sigma_j-q_A}$ they will be trying to maximize $w\left(\frac{\sigma_j-q_A}{\sigma_j}+\frac{q_A}{\sigma_{-j}}\right)-\theta\sigma_j$. Their best-response function is, then,

$$\sigma_j^*\left(\sigma_{-j}\right)=\begin{cases}\min\{1,\sqrt{\frac{q_A}{\tilde{\theta}}}\} & \text{if } \sigma_{-j}\leq\frac{q_A\sigma_j^*(\sigma_{-j})}{\sigma_j^*(\sigma_{-j})-q_A}\\ \min\{1,\sqrt{\frac{2}{\tilde{\theta}}}\sqrt{\sigma_{-j}}-\sigma_{-j}\} & \text{if } \sigma_{-j}\geq\frac{q_A\sigma_j^*(\sigma_{-j})}{\sigma_j^*(\sigma_{-j})-q_A}\end{cases}$$

We can see that the only possible asymmetric equilibria could only take place on a subset of the parameter space of measure zero, as they would have to satisfy $\sigma_j^*=\frac{q_A\sigma_{-j}^*}{\sigma_{-j}^*-q_A}$, since otherwise we would have that both $\sigma_j^*<\frac{q_A\sigma_{-j}^*}{\sigma_{-j}^*-q_A}$ and $\sigma_j^*>\frac{q_A\sigma_{-j}^*}{\sigma_{-j}^*-q_A}$. On the other hand, we have three candidates for symmetric equilibria: $\sigma^*=1$, $\sigma^*=\frac{1}{2\tilde{\theta}}$, and $\sigma^*=\sqrt{\frac{q_A}{\tilde{\theta}}}$, which will hold if $\left(\tilde{\theta}\leq q_A\wedge q_A\geq\frac{1}{2}\right)\vee\left(\tilde{\theta}\leq\frac{1}{2}\wedge q_A\leq\frac{1}{2}\right)$, if $\tilde{\theta}>\frac{1}{2}\wedge q_A\leq\frac{1}{4\tilde{\theta}}$, and if $\tilde{\theta}>q_A\wedge q_A\geq\frac{1}{4\tilde{\theta}}$, respectively. We need to check now if voters would rather deviate and not vote altogether, i.e., whether $G_j\left(\sigma^*,\sigma^*\right)\geq G_j\left(0,\sigma^*\right)$. For $\sigma^*=\frac{1}{2\tilde{\theta}}$ this is equivalent to $w-\theta\frac{1}{2\tilde{\theta}}\geq 0\Leftrightarrow\frac{1}{2}\geq 0$, thus voters will not deviate. In the case of $\sigma^*=1$ voters will favor casting zero ballots if $w-\theta<wq_A\Leftrightarrow q_A>1-\tilde{\theta}$, while for $\sigma^*=\sqrt{\frac{q_A}{\tilde{\theta}}}$, since $w-\theta\sqrt{\frac{q_A}{\tilde{\theta}}}<w\frac{q_A}{\sqrt{\frac{q_A}{\tilde{\theta}}}}\Leftrightarrow q_A>\frac{1}{4\tilde{\theta}}$, voters will always want to deviate (except for $q_A=\frac{1}{4\tilde{\theta}}$). $\square$