

1 **Title:** Empirical abundance distributions are more uneven than expected given their statistical baseline

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33 **Abstract**

34 The prevalence of the species abundance distribution's hollow-curve shape across many communities is
35 frequently assumed to reflect ecological processes structuring communities. However, this hollow curve
36 can also emerge as a statistical phenomenon of dividing a particular number of individuals into a given
37 number of species. While the hollow curve may be a statistical artefact, ecological processes may induce
38 subtle deviations between empirical species abundance distributions and their statistically most probable
39 forms. Examining ~22,000 communities, we found that empirical species abundance distributions are
40 more skewed and uneven than their statistical baselines. However, small communities – with few species
41 or individuals – exhibit poorly-resolved statistical baselines, thereby reducing our capacity to detect
42 deviations. The extraordinarily skewed and uneven nature of empirical species abundance distributions
43 provides new avenues for testing ecological theory, while the issues posed by small communities
44 illustrate the limitations of statistical baselines for studying ecological patterns in small samples.

45 Introduction

46 The distribution of how the total number of individuals in a community are divided among the species in
47 that community, or the species abundance distribution (SAD), is one of the few ecological patterns whose
48 shape is so consistent that it is often considered an ecological law (Lawton 1999, McGill et al. 2007).
49 Across varied ecosystems and taxa, the species abundance distribution is dominated by a few very
50 abundant species and a larger number of increasingly rare species, generating a distinctive hollow- or J-
51 shaped curve (Fisher et al. 1943). Community ecologists have used the SAD to test numerous theories
52 intended to determine which biological processes are most important for structuring assemblages of
53 species, by comparing theoretical predictions for the SAD to observed SADs (McGill et al. 2007). However,
54 this approach has proven inconclusive because many theories predict similar shapes for the SAD (McGill
55 et al. 2007), and even experimental manipulations generate little variation in the shape of the SAD (Supp
56 and Ernest 2014). After decades of attention to its consistent pattern and potential as a theoretical
57 benchmark, the utility of the SAD for assessing the processes structuring ecological communities remains
58 unclear.

59 Accumulating evidence suggests that statistical constraints may actually generate the most striking
60 feature of the species abundance distribution – the hollow curve (White et al. 2012, Locey and White
61 2013, Frank 2019). Indeed, power-law or log-series distributions (i.e. hollow curves) emerge from many
62 distinct generative processes for abundance distributions generally (Frank 2009, Frank 2019). In the
63 specific case of the SAD, approaches using both statistical mechanics (i.e. the Maximum Entropy Theory
64 of Ecology (METE); Harte et al. 2008, Harte 2011) and combinatorics (i.e. ‘the feasible set’; Locey and
65 White 2013) generate realistic hollow curves via the random division of the total number of individuals in
66 a community, N , into the total number of species present, S . Given that these stochastically generated
67 SADs are excellent empirical fits to those seen in nature (Harte 2011, White et al. 2012, Locey and White

2013), failing to account for these considerations may have led us to focus on a distinctive but relatively uninformative aspect of the SAD.

If SADs are statistically inclined to be hollow curves, it is no surprise that we have struggled to use the hollow curve to definitively identify the ecological processes that shape SADs. However, this does not necessarily mean that the SAD cannot be biologically informative. Biological factors may introduce subtle, but meaningful, deviations between the shapes of observed SADs and the shapes of the SADs expected due to the mathematical constraints imposed by S and N , which we hereafter refer to as the “statistical baseline” (Locey and White 2013, Harte and Newman 2014). Thus, it may be possible to use these *deviations* to detect strong ecological processes or evaluate theories (Harte and Newman 2014, Xiao et al. 2016). If the vast majority of mathematically achievable SADs for a community share a similar shape, an empirically observed SAD that deviates even slightly from this statistical baseline is unlikely to have emerged at random (Locey and White 2013). Such a deviation is therefore likely to be the signature of a non-random – i.e., biological – process operating on the relative abundances of species (Harte and Newman 2014). It may be possible to evaluate ecological processes and theories based on how well they predict these *deviations* between observed SADs and their statistical baselines.

Successfully interpreting SADs in this fashion depends on our capacity to detect and quantify deviations between empirical observations and statistical baselines, which requires metrics and computational approaches that allow us to quantify and interpret whatever deviations may exist. Here, we build upon the combinatoric approach developed by Locey and White (2013) to define and explore the statistical baselines for SADs. For a given N (total number of individuals) and S (total number of species), there exists a finite (but potentially very large) set of possible distributions of individuals into species. Collectively, this set of possible SADs is referred to as the feasible set, with each possible SAD constituting a single element of the set. If an observed SAD is drawn at random from the set of mathematically possible SADs, it is likely to have a shape similar to the shapes most common in the feasible set. The feasible set can therefore be

used as a statistical baseline for assessing whether observed SADs deviate from what is likely to occur merely due to mathematical constraints, and to explore how the statistical baseline varies over across ranges of values of S and N (Locey and White 2013).

The specificity, or vagueness, of the expectations derived from the statistical baseline is critically important for disentangling the aspects of the SAD that are likely generated by statistical constraints from those generated by other processes. If the vast majority of mathematically possible SADs are similar in shape – generating a very specific, narrowly defined statistical baseline – then even small deviations between an observed SAD and this baseline can signal the operation of ecological processes. However, if many different shapes occur with more even frequency in the feasible set, the statistical baseline is less specific and less well defined, and our sensitivity for distinguishing biological signal from statistical constraints is greatly reduced. In general, a poorly defined statistical baseline is more likely to occur when the size of the community, in terms of S and N , is small, because in such cases there may be too few possible SADs in the feasible set for a particular shape to emerge as the most common, and therefore most likely, shape. When this occurs, we have reduced confidence that even an observation that deviates from the statistical baseline did not emerge at random from the relatively restricted pool of possible outcomes (Jaynes 1957). This general concern has been acknowledged in efforts to comparing ecological observations to statistical baselines (Harte 2011, White et al. 2012, Locey and White 2013) but there has not yet been a quantification of these effects for the SAD or an identification of the range of community sizes most strongly affected. Because ecologists study the SAD for communities varying in size from the very small – S and $N < 5$ – to the enormous – S and $N \gg 1000$ – identifying the community sizes for which we can and cannot confidently detect deviations from the statistical baseline is necessary to appropriately contextualize our interpretations.

Here we use the feasible set to define statistical baselines for empirical SADs for 22,000 communities of birds, mammals, trees, and miscellaneous other taxa. We then compare *observed* SADs to their

corresponding statistical baselines and evaluate 1) if the shapes of observed SADs consistently deviate from their statistical baseline, 2) how the specificity of the statistical baseline varies over ranges of S and N , and 3) whether this variation appears to be associated with variation in our capacity to detect deviations between observations and the corresponding baselines.

Methods

Datasets

We used a compilation of community abundance data for trees, birds, mammals, and miscellaneous additional taxa (White et al. 2012, data from Baldrige 2015 available at <https://doi.org/10.6084/m9.figshare.95843.v4>, Baldrige 2016, data from Baldrige 2016 available at <https://zenodo.org/record/166725>). This compilation consists of cleaned and summarized community abundance data for trees obtained from the Forest Inventory and Analysis (Woudenberg et al 2010) and Gentry transects (Phillips and Miller 2002), birds from the North American Breeding Bird Survey (Sauer et al. 2013), mammals from the Mammal Community Abundance Database (Thibault et al. 2011), and a variety of less commonly sampled taxa from the Miscellaneous Abundance Database (Baldrige 2015). Because characterizing the random expectation of the SAD is computationally intractable for very large communities, we filtered our datasets to remove communities with more than 40720 individuals, which was the largest community we successfully analyzed. This resulted in the removal of 4 communities from the Miscellaneous Abundance Database. We further filtered the FIA database. Of the 103,343 communities in FIA, 92,988 have fewer than 10 species. Rather than analyze all these small communities, we randomly selected 10,000 small communities to include in the analysis. We also included all FIA communities with more than 10 species, which added 10,355 FIA communities to the analysis and resulted in a total of 20,355 FIA communities. Finally, for sites that had repeated sampling over time, we followed White et al. (2012) and Baldrige (2016) and analyzed only a single, randomly selected, year of

data, because samples taken from a single community at different time points are likely to covary. It should be noted that our analyses include data from the Mammal Community Database and Miscellaneous Abundance Database that were collected over longer timescales and cannot be disaggregated into finer units of time. We also removed from our analyses any communities with only one species, or for which $N = S$ or $N = S + 1$, because these communities have only one mathematically possible SAD. Our final dataset consisted of ~22,000 communities with S and N ranging from 2 to 250 and 3 to 40714, respectively (see Figure S1 in Supporting Information). Details and code for the filtering process can be found in Appendix S2.

Generating the statistical baseline

We use the concept of the “feasible set” to establish a statistical baseline for the SAD (Locey and White 2013). For a given number of individuals N , there is a finite number of unique ways to partition those individuals into S species. The complete set of these unique partitions is the feasible set. Because, in this approach, neither species nor individuals are distinguishable from each other, partitions are unique if and only if they differ in the number of species that have a particular abundance (Locey and White, 2013). Operationally, this means that for $S = 3$ and $N = 9$, the species abundances $(1, 3, 5)$ and $(2, 2, 5)$ count as distinct partitions, but $(1, 3, 5)$ and $(3, 1, 5)$ are only one element of the feasible set because they each contain one species with an abundance 1, 3, and 5, respectively, and they differ only in the *order* of the numbers. In the absence of justification for additional assumptions regarding the distinguishability of species and/or individuals, we adopted this simple set of assumptions that has previously been shown to generate realistic statistical baselines (Locey and White 2013).

While it is possible to list all possible partitions in the feasible set for small S and N , the size of the feasible set increases rapidly with S and N . Therefore, characterizing the statistical properties of the feasible set for large S and N can be computationally intensive. This renders it necessary to draw samples from the

feasible set, rather than enumerating all of its elements. Unbiased sampling of large feasible sets is a nontrivial computational problem that has constrained previous efforts in this vein (Locey and White 2013). We developed an algorithm to efficiently and uniformly sample feasible sets even for large values of S and N . In brief, the algorithm takes a generative approach to sampling the feasible set. Individuals are allocated one species at a time, beginning with the least abundant species. At each step, the number of individuals to allocate for the current species is determined at random, with probability based on the number of feasible sets with that specific abundance for that species, and conditional on the individuals that have already been allocated. For example, if we have 3 species and 7 individuals, the least abundant species can have an abundance of 1 or 2. Allocating 1 individual to the least abundant species allows for the SADs (1, 1, 5), (1, 2, 4), and (1, 3, 3), and allocating 2 individuals to the least abundant species means the only possible SAD is (2, 2, 3). We therefore allocate 1 individual with probability 3/4, and 2 individuals with probability 1/4. If, at the first step, we allocated 1 individual to the least abundant species, the second species can have an abundance of 1, 2 or 3 with equal probability, because there is exactly 1 possible SAD with each of these possible abundances for the first two species. This process is continued until all individuals have been allocated. We implemented this algorithm in an R package, available on GitHub at www.github.com/diazrenata/feasiblesads; details of the sampling methodology are available in Appendix S3.

For every community in our database, we drew 4000 samples from the feasible set to characterize the distribution of statistically probable shapes for the SAD. We filtered the 4000 samples to unique elements. For small values of S and N , it can be impossible or highly improbable for the 4000 samples from the feasible set to all be unique, but for large communities, all 4000 are usually unique. We refer to this as the sampled feasible set.

Comparing observed SADs to their baselines

If all SADs in a feasible set are equally likely to occur, then an SAD with a particular S and N is likely to have a shape similar to the shape that is most common among the SADs in the feasible set for the same S and N ; in contrast, strong processes may cause observed SADs to have shapes that deviate from this statistical baseline (Locey and White 2013). We focus on two metrics to describe the shape of the SAD, skewness and Simpson's evenness. Skewness measures the asymmetry of a distribution around its mean, and Simpson's evenness is a commonly used metric in ecology for assessing how equitably abundance is distributed across species. By calculating these metrics for each of the samples in the community's sampled feasible set (see *Generating the statistical baseline*, above), we generated a distribution describing the general shape (i.e. evenness or skewness) expected from the randomly sampled SADs. Note that skewness, as implemented in the R package "e1071" (Meyer et al. 2019), always evaluates to 0 for distributions with only two species, and we therefore excluded those cases from analyses of skewness (but included those communities for analyses using Simpson's evenness).

To assess whether the shape of an observed SAD was statistically unlikely, we calculated Simpson's evenness and skewness for the observed SAD and compared these observed values to the distributions of evenness and skewness obtained from that community's sampled feasible set. An observed SAD's deviation from its feasible set was determined by computing the percentile rank of its skewness and evenness relative to the sampled distributions for skewness and evenness, respectively. These percentile ranks are then comparable across different community sizes, allowing broad-scale assessment across wide ranges of S and N . After aggregating across communities, if observed SADs reflect random draws from their feasible sets, their percentile rank values should be uniformly distributed from 0 to 100. However, if observed SADs are consistently more skewed or even than their feasible sets, the percentile values will be disproportionately concentrated towards the extremes. Because an earlier survey in this space (Locey and White 2013) found that the tendency is for empirical SADs to be more skewed and less even than their feasible sets, we used one-tailed 95% confidence intervals and tested for unusually *high*

values for skewness and *low* values for evenness. This comparison is not meaningful if there are very few unique values in the distributions of skewness and evenness, which can occur for small feasible sets. We therefore excluded communities for which the distribution of skewness or evenness values from the sampled feasible set had fewer than 20 unique values (in these cases, it is impossible for an observation to fall above or below the 95th or 5th percentile, respectively). Our final aggregated analyses included 22,142 communities for evenness and 22,325 communities for skewness.

The narrowness of the expectation

We also used the distributions of skewness and evenness from the sampled feasible set to describe the relative specificity of the statistical baseline, in order to assess in what situations there could be challenges in determining whether observed communities differ from their statistical baselines. We quantified the narrowness of a distribution as the ratio of the range of values encompassed within a 95% density interval relative to the full range of values in the distribution (Figure 1). This breadth index for the statistical baseline ranges from 0 (a very narrow distribution and well-resolved baseline) to 1 (a very broad distribution), and is comparable across feasible sets for varying combinations of S and N . This metric corresponds qualitatively to more computationally-intensive approaches to measuring the self-similarity of the elements of feasible sets (see Appendix S4).

Results

Observed SADs compared to their feasible sets

For four of the five datasets we analyzed – BBS, Gentry, Mammal Communities, and Misc. Abund – empirical SADs are highly skewed and highly uneven relative to their feasible sets, much more frequently than would be expected by chance (Figure 2, Table 1). Combined across these four datasets, 16% of observed SADs are more skewed than 95% of their feasible sets, and 31% are less even than 95% of their feasible sets. For SADs randomly sampled from the feasible set, we would expect only 5% of observed

distributions to fall in these extremes. In contrast to the other datasets, the SADs from the FIA dataset exhibit percentile scores that are more uniformly distributed: 5% of observations are more skewed than 95% of their feasible sets, and 9% of observations are less even than 95% of their feasible sets.

The narrowness of the expectation

The ability to detect deviations from the statistical baseline depends on the distribution of SADs in the feasible set. Here, the statistical baseline for both skewness and evenness becomes more narrowly defined as the size of the feasible set increases (Figure 1; Figure S5), making even small deviations in skewness or evenness statistically meaningful and readily detectable. However, for communities with relatively small feasible sets – fewer than approximately 1000 elements for skewness, and approximately 200 elements for evenness – the breadth index approaches 1, meaning that a 95% density interval of the values in the distribution spans nearly the entire range of values (Figure S5). In particular, the FIA dataset is dominated by small communities for which the breadth index is very high, reflecting relatively broad and nonspecific statistical expectations for the shape of the SAD derived from the feasible set (Figure 3).

Discussion

We found widespread evidence that SADs for a range of real ecological communities are more skewed and less even than expected given the distribution of shapes within their feasible sets. These deviations may signal that ecological processes operate on top of statistical constraints, thereby driving the SAD away from common shapes that would be observed in the absence of a dominating non-statistical process. Our results suggest that the prevailing processes structuring these communities tend to be those that cause abundance distributions to be more uneven – rather than those that produce more even abundances across species. Ecological processes may lengthen the rare tail of the SAD, for example by promoting the persistence of rare species at very low abundances (e.g. Yenni et al. 2012). Or, they could drive abundant species to have larger populations that would be statistically expected, without driving

other species entirely to extinction (Chesson 2000). Although a disproportionate number of communities deviated statistically from their feasible sets, there were also many communities for which we did not detect deviations. In such cases, multiple ecological processes may operate simultaneously and with countervailing impacts on abundance distributions, resulting in no dominating net effect on the shape of the distribution beyond that imposed by fundamental constraints (Harte 2011; Harte and Newman 2014). Going forward, testing whether ecological theories or common functional approximations (e.g. the log-normal distribution) accurately predict this range of variation in deviations between observed SADs and their statistical baselines may be much more fruitful than focusing only on the general form of the SAD that emerges from statistical constraints (McGill et al. 2007; Locey and White 2013).

Unlike the other four datasets, communities in the FIA dataset showed weak or no evidence of deviations from their feasible sets. These results may reflect statistical phenomena related to community size. The FIA communities are by far the smallest across our datasets (Figure S1). Communities with small values of S and N have smaller feasible sets; when there are relatively few possible SADs, the distributions of evenness and skewness values from the feasible set are less narrowly peaked, meaning there is a weaker statistical distinction between “common” and “extreme” shapes for the SAD (Figure 1). In fact, across the range of community sizes present in our datasets, the feasible sets for small communities generally generated broader distributions of evenness, and especially skewness, than those for large communities (Figure S5). For such communities, the deviations – or lack thereof – that we perceive are less informative than for larger communities with more strongly defined statistical baselines (Jaynes 1957).

If the lack of discernable deviations from the feasible set for the FIA communities is indeed a byproduct of their generally small size, then we would expect similarly-sized communities from other datasets to have similar results. We identified 371 communities from other datasets with S and N matching communities from FIA. We found no difference in the distribution of percentile scores between communities from FIA and communities from other datasets (Figure 4; Table S6), confirmed via Kolmogorov-Smirnov tests (for

evenness, $D = 0.04$ and $p = 0.87$; for skewness, $D = 0.07$ and $p = 0.37$). Although 371 communities constitute a small sample relative to the 20,355 FIA communities we analyzed, these results point to community size, and not attributes specific to FIA, as a likely explanation for the weak evidence for deviations across the full FIA dataset.

If this is indeed the case, it means that small-community considerations may affect our capacity to meaningfully distinguish signal from randomness using this approach. FIA communities, with their broad distributions of shape metrics and overall lack of detectable signal, have on the order of 10 species and 50-100 individuals. While these values do not constitute hard thresholds, they may indicate a general range of values below which we have relatively diminished power to detect deviations from the statistical baseline represented by the feasible set. To meaningfully draw inferences from deviations in these small communities, we will likely need more sensitive metrics (than skewness and evenness), and/or theories that generate more specific predictions for the SAD. In the absence of such, we may stand to learn the most by focusing on SADs from relatively large communities.

It is also important to recognize that there are multiple plausible approaches to defining a statistical baseline for the SAD, of which we have taken only one (Haegeman and Loreau 2008, Locey and White 2013). Our approach follows Locey and White (2013) and reflects the random partitioning of individuals into species, with the resulting distributions considered unique if the species' abundance values are unique, regardless of the order in which the values occur. The philosophy behind the feasible set reflects a longstanding approach in the study of abundance distributions: to focus on the shape of the distribution without regard to species' identities (McGill 2007). To include differences in *order* in the statistical baseline would imply that identifying *which* species contain the most or least individuals is important for evaluating theory. Other formulations for the statistical baseline may be equally valid and generate different statistical expectations, including forms that approximate exponential, Poisson, or log-series

distributions (Harte et al. 2008, Favretti 2018). Comparing the results that emerge from different baselines will be an important next step towards reinvigorating the use of the SAD as a diagnostic tool.

Our study demonstrates the utility, and the potential challenges, of applying tools from the study of complex systems and statistical mechanics to the study of ecological communities (Haegeman and Loreau 2008, Harte 2011, White et al. 2012, Harte and Newman 2014). While concepts such as maximum entropy and the feasible set are promising new horizons for macroecology, the small size of some ecological communities may present difficulties that do not occur as often in the domains for which these tools were originally developed (Jaynes 1957, Haegeman and Loreau 2008). When the observed numbers of species and individuals are too small to generate highly resolved statistical baselines, these approaches will be less informative than we might hope – as appears to be the case for the smallest communities in our analysis. In larger communities, where mathematical constraints have stronger effects on the general form of the SAD, our results show that these constraints alone do not fully account for the extremely uneven SADs we often observe in nature – leaving an important role for ecological processes. This ability to detect and diagnose the specific ways in which empirical SADs deviate from randomness can generate new avenues for understanding how and when biological drivers affect the SAD. There are, of course, still many elements to be improved in our ability to distinguish biological signal from randomness, including assessing alternative statistical baselines and calibrating our expected power to detect deviations, especially for small communities. Indeed, more sensitive metrics could also enable identification of processes that operate through time (note that, in this analysis, we sampled only one time point for each community). Continuing to explore and account for the interplay between statistical constraint and biological process constitutes a promising and profound new approach to our understanding of this familiar, yet surprisingly mysterious, ecological pattern.

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Figure legends

Figure 1. Large feasible sets may allow better detection of deviations from the statistical baseline by generating more specific, narrowly-defined baselines. We illustrate this phenomenon using 3 hypothetical communities: a small community ($S = 4$, $N = 34$; top row), an intermediate community ($S = 7$, $N = 71$; middle row), and a large community ($S = 44$, $N = 13360$; bottom row). The large community has approximately 6.59×10^{70} possible SADs in its feasible set, while the intermediate community has 60,289 and the small community has only 297. For every SAD sampled from the feasible set (left column), we calculate the skewness (color scale) and evenness (not shown). The distributions of these values (right column) constitute the statistical baseline. We define a “breadth index” as the ratio of the range encompassed in the one-tailed 95% density interval (distance between red lines, right), compared to the full range of values for the statistic (distance between the maximum and minimum values). As S and N increase, the size of the feasible set increases, resulting in a narrower statistical baseline (smaller breadth index) – thus enabling easier detection of deviations that may be the result of ecological processes affecting the SAD.

Figure 2. Many ecological communities are more skewed (left) or uneven (right) than their statistical baselines. Percentile ranks are calculated by comparing each community to its sampled feasible set, with very high or very low percentile ranks reflecting extreme values relative to statistical baselines. The vertical red line marks the 95th percentile for skewness and the 5th percentile for evenness. Species abundance distributions that are sampled at random from the feasible set will produce percentile ranks that are uniformly distributed from 0 to 100, with approximately 5% of values above or below the 95th and 5th percentiles, respectively. In contrast, most datasets have more communities that are highly skewed or uneven than would be expected by chance.

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409 Figure 3. Breadth indices of skewness (left) and evenness (right) indicate varying ability to detect the
410 deviations between observations and the statistical baseline. The breadth index (see Figure 1) quantifies
411 how narrowly-defined the statistical baseline is; high values indicate broad, poorly-defined statistical
412 baselines that may impede our ability to confidently detect deviations between observations and what is
413 expected given the baseline. Most datasets contain a mixture of communities with broad and narrow
414 statistical baselines, but some – particularly the skewness baseline for the Forest Inventory and Analysis –
415 have consistently high breadth indices across all of their communities, suggesting that skewness is not an
416 effective metric for distinguishing empirical observations from the feasible set. In general, the breadth
417 index for evenness (right panels) indicates more narrow statistical baselines than those for skewness.

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419 Figure 4. Small communities exhibit consistently broad statistical baselines (top), and consistently weak
420 evidence of deviations for observed SADs (bottom), regardless of the originating dataset. For a subset of
421 371 communities from the Forest Inventory and Analysis with communities in other datasets with
422 matching S and N , we generate distributions of breadth indices for skewness and evenness (top) and
423 compute corresponding percentile ranks for the observed SADs (bottom). Visually, there is no difference
424 between FIA (left panels) and other datasets (right panels), when they are matched in S and N . This is
425 confirmed by Kolmogorov-Smirnov tests for the breadth indices (for evenness, $D = 0.04$ and $p = 0.91$; for
426 skewness, $D = 0.03$ and $p > 0.99$) and percentile ranks (for evenness, $D = 0.04$ and $p = 0.87$; for skewness,
427 $D = 0.07$ and $p = 0.37$).

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