Appendix S3: Sampling the Space of Feasible Sets

# Species Abundance Distribution (SAD) Definition

For , we define a SAD as:

where

In effect, a SAD is a division of individuals among classes (i.e. taxonomic categories), and we are interested only in the sizes of those classes, disregarding their specific identity (i.e. the classes are interchangeable).

Note that we deviate slightly from ecological convention in that we order the sizes of the classes in ascending, as opposed to descending order.

# Feasible Sets

A **Feasible Set**, , is the set of all possible SADs for a particular and , and is the cardinality of (i.e. the size of the Feasible Set). The values of are identical to the number of partitions of into exactly parts[[1]](#footnote-1).

## Alternative Representation

First, we describe an alternative representation for SADs, where we describe the relative differences in sizes of consecutive classes instead of the sizes of the classes directly:

where

Thus,

and

## Generating SADs

We describe a generative approach for creating a SAD.

1. Set , the index of the class whose size will be determined next. Set , the number of individuals remaining to be allocated. Set , the number of classes reamining to be filled.
2. Choose a value for . The possible values are .
3. Compute , the remaining un-allocated individuals, as . Compute , the remaining un-allocated classes, as . Increment .
4. Choose a value for . The possible values are .
5. Repeat steps 3 and 4, until the SAD is fully generated.

## Counting Feasible Sets

Mirroring the generative approach, we can define recursively.

1. Define to be the number of ways of allocating individuals among classes, and allowing the sizes of the classes to include 0.

where (there is only way to allocate individuals among 1 class).

## Sampling Feasible Sets

We can also sample uniformly, using a similarly recursive approach.

1. For all , compute .
2. The sum of these values of determine the relative probabilities for the corresponding value of .
3. Choose based on the corresponding weights,
4. Choose the remaining according to the corresponding values of as used to count the feasible set.

## References

Dickson, Leonard E. 1919. *History of the Theory of Numbers*. Vol. 2: Diophantine Analysis. Carnegie Institution of Washington. <https://archive.org/details/historyoftheoryo02dickuoft/page/100/mode/2up>.

Euler, Leonhard. 1862. *Sex Litterae Ad Nicolaum Bernoullium Ii, Basileensem J. U. D. Datae 1742 Ad 1745*. 820. Euler Archive - All Works. <https://scholarlycommons.pacific.edu/euler-works/820>.

1. , the number of partitions of into exactly parts is more commonly expressed via the related quantity, , the number of partitions of into at most parts; the two are related by . Some of the earliest calculations of were performed by Euler in a 10 November 1742 letter to Bernoulli, in which values were computed via recurrence relation (1862); for a more extensive history of the topic, see Dickson (1919). [↑](#footnote-ref-1)