Mini Project 2

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Contents

***Abstract*—This project is about IKP and FKP and Dynamics of Serial Robotic Manipulator. We have a simulink model of the robot so we can test our answers.**

***Index Terms*—**

I. Problem 1: DH parameters

We first need to draw Z and X axes on the robot joints.

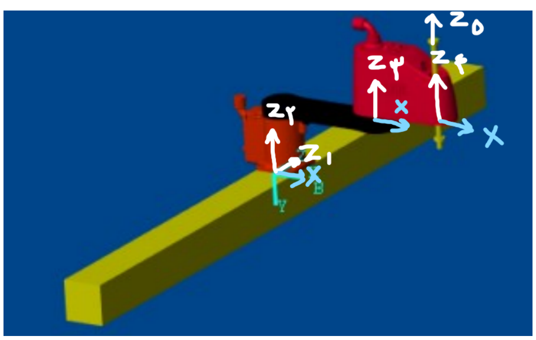

Fig. 1. Robot axes

Now we can find DH-parameters of out robot. We have the length of arms so we use them to find a_i and b_i.

i	a_i	b_i	α_i	θ_i
1	0	b1	$\frac{\pi}{2}$	0
2	400	191.6	0	θ_2
3	250	257.6-191.6=66	0	θ_3
4	0	b4	0	0

TABLE I
D-H parameters

II. Problem 2: FKP

We first have to find a_i which can be formed using this formula:

$$\vec{a_i} = \begin{bmatrix} a_i cos(\theta_i) \\ a_i sin(\theta_i) \\ b_i \end{bmatrix}$$

So we write down:

$$\vec{a_1} = \begin{bmatrix} 0 \\ 0 \\ b_1 \end{bmatrix}$$

$$\vec{a_2} = \begin{bmatrix} 400cos(\theta_2) \\ 400sin(\theta_2) \\ 191.6 \end{bmatrix}$$

$$\vec{a_3} = \begin{bmatrix} 250cos(\theta_3) \\ 250sin(\theta_3) \\ 66 \end{bmatrix}$$

$$\vec{a_4} = \begin{bmatrix} 0 \\ 0 \\ b_4 \end{bmatrix}$$

Now we write the Q transforms.

$$Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_3 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So now using the following equations we can solve the FKP and find P and Q.

$$\vec{P}_w = \vec{a}_1 + Q_1\vec{a}_2 + Q_1Q_2\vec{a}_3 + Q_1Q_2Q_3\vec{a}_4 \tag{1}$$

$$Q = Q_1 Q_2 Q_3 Q_4 \tag{2}$$

P=

$$\begin{pmatrix} 250\cos(\theta_2+\theta_3)+400\cos(\theta_2) \\ -b_4-\frac{1288}{5} \\ b_1+250\sin(\theta_2+\theta_3)+400\sin(\theta_2) \end{pmatrix}$$

Q=

$$\begin{pmatrix} \cos(\theta_2)\cos(\theta_3)-\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2)\sin(\theta_3)-\cos(\theta_3)\sin(\theta_2) & 0 \\ 0 & 0 & -1 \\ \cos(\theta_2)\sin(\theta_3)+\cos(\theta_3)\sin(\theta_2) & \cos(\theta_2)\cos(\theta_3)-\sin(\theta_2)\sin(\theta_3) & 0 \end{pmatrix}$$

The matlab code for this part can be found in Q2-3-4.mlx.

III. Problem 3: IKP

we have x and y and z.

$$x = 250\cos(\theta_2+\theta_3)+400\cos(\theta_2)$$

$$y = -b_4-\frac{1288}{5}$$

$$z = b_1+250\sin(\theta_2+\theta_3)+400\sin(\theta_2)$$

Now we can add the square of x and z and get rid of θ_2 so we can continue solving the problem using t:

$$\cos(\theta_1) = \frac{1-t^2}{1+t^2}$$

$$\sin(\theta_1) = \frac{2t}{1+t^2}$$

we solve this and get two answers for t which means two answers for θ_2, and for each θ_2 we get a θ_3.

$$\phi = \begin{pmatrix} \frac{500\,z-500\,b_1+\sigma_1}{{b_1}^2-2\,b_1\,z+x^2+500\,x+z^2+60900} \\ -\frac{500\,b_1-500\,z+\sigma_1}{{b_1}^2-2\,b_1\,z+x^2+500\,x+z^2+60900} \end{pmatrix}$$

where

$$\sigma_1 = \sqrt{-\left({b_1}^2-2\,b_1\,z+x^2+z^2-44100\right)\left({b_1}^2-2\,b_1\,z+x^2+z^2-84100\right)}$$

The matlab code for this part can be found in Q2-3-4.mlx.

IV. Problem 4: Jacobian

For the Jacobian we first need r_i and e_i:

$$r2 = Q_1 * a_2 + Q_1 * Q_2 * a_3$$

$$r3 = Q_1 * Q_2 * a_3$$

$$r2 = \begin{pmatrix} 400\cos(\theta_2)+250\cos(\theta_2)\cos(\theta_3)-250\sin(\theta_2)\sin(\theta_3) \\ -\frac{1288}{5} \\ 400\sin(\theta_2)+250\cos(\theta_2)\sin(\theta_3)+250\cos(\theta_3)\sin(\theta_2) \end{pmatrix}$$

$$r3 = \begin{pmatrix} 250\cos(\theta_2)\cos(\theta_3)-250\sin(\theta_2)\sin(\theta_3) \\ -66 \\ 250\cos(\theta_2)\sin(\theta_3)+250\cos(\theta_3)\sin(\theta_2) \end{pmatrix}$$

$$e2 = Q_1 e1$$

$$e3 = Q_2 Q_1 e1$$

$$e4 = Q_3 Q_2 Q_1 e1$$

$$e1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$e_4 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

So we form the Jacobian matrix as follows:

$$J = \begin{bmatrix} 0 & e2 & e3 & 0 \\ e1 & e2*r2 & e3*r3 & e4 \end{bmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -400\sin(\theta_2)-\sigma_3-\sigma_2 & -\sigma_3-\sigma_2 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 400\cos(\theta_2)+\sigma_4-\sigma_1 & \sigma_4-\sigma_1 & 0 \end{pmatrix}$$

$$\sigma_1 = 250\sin(\theta_2)\sin(\theta_3)$$

$$\sigma_2 = 250\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_3 = 250\cos(\theta_2)\sin(\theta_3)$$

$$\sigma_4 = 250\cos(\theta_2)\cos(\theta_3)$$

The matlab code for this part can be found in Q2-3-4.mlx.

V. PROBLEM 5: DYNAMIC MODEL

The physical parameters of the links are given.
The DH transformations for each link:

$$T_0 = \begin{bmatrix} \text{round}(Q_x(-\pi/2)) & \frac{1}{1000}\begin{bmatrix}0 & 0 & 73.5\end{bmatrix}^\top \\ \begin{bmatrix}0 & 0 & 0\end{bmatrix} & 1 \end{bmatrix} \times \begin{bmatrix} I_3 & \begin{bmatrix}0 & 0 & 0.08\end{bmatrix}^\top \\ \begin{bmatrix}0 & 0 & 0\end{bmatrix} & 1 \end{bmatrix}$$

$$T = T_0 \times \begin{bmatrix} \text{round}(Q_x(\pi/2)) & \begin{bmatrix}-0.2 & 0 & 0\end{bmatrix}^\top \\ \begin{bmatrix}0 & 0 & 0\end{bmatrix} & 1 \end{bmatrix}$$

The inertia and COM matrices in the DH frame can be calculated. For example for link 1:

$$\text{com1\_dh} = T \times \begin{bmatrix} \text{com\_link1\_ros} \\ 1 \end{bmatrix}$$

$$I1\_dh = Q \times (I1\_ros + \text{mass\_link1} \times ((b^\top b) \times I_3 - b \times b^\top))/Q$$

Now we use lagrangian model to solve the problem.

$$L = T - V$$

$$T = \sum \left(\frac{1}{2} m_i \dot{c_i}^2 + \frac{1}{2} \omega_i^T I_i \omega_i \right)$$

$$V = \sum V_i = \sum m_i g h_i$$

Now we calculate the T matrix.

$$T = T1 + T2 + T3 + T4$$

where:

$$T1 = \text{mass\_link1} \times N1^\top \times N1 + W1^\top \times I1\_dh \times W1$$

$$T2 = \text{mass\_link2} \times N2^\top \times N2 + W2^\top \times Q_z(\theta_2 - \theta_{2,0}) \times I2\_dh \times Q_z(\theta_2 - \theta_{2,0})^\top \times W2$$

$$T3 = \text{mass\_link3} \times N3^\top \times N3 + W3^\top \times Q_z(\theta_3 - \theta_{3,0}) \times I3\_dh \times Q_z(\theta_3 - \theta_{3,0})^\top \times W3$$

$$T4 = \text{mass\_link4} \times N4^\top \times N4 + W4^\top \times I4\_dh \times W4$$

To have the T as in the formula we have to :

$$Tnew = 0.5 \times \theta_{\text{dot}}^\top \times T \times \theta_{\text{dot}}$$

The equations are given:

$$\tau = T \times \theta_{\text{ddot}} + n$$

where

$$n = T_{\text{dot}} \times \theta_{\text{dot}} - \text{jacobian}(Tnew, \theta)^\top + \text{jacobian}(V, \theta)^\top$$

So here τ is calculated. And the fuction is created automatically.

VI. PROBLEM 6: SIMSCAPE MODEL

Here we check our results with the simscape model: First, we run the motions-for-simulink.m.Then we run the Simulink model(after adding the STL files). Using torques-using-model.m and adding 4-5-6-7 interpolation to it, we calculate the force and torque with the function we exported earlier in section 5. Using the plotting-results.m file, we plot the torques both from the dynamic model and the simulation.

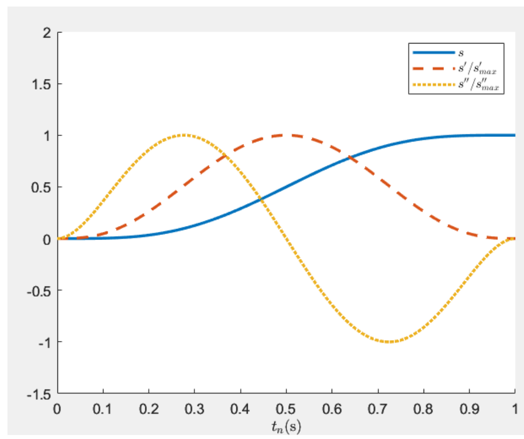

Fig. 2. 4-5-6-7 Interpolation

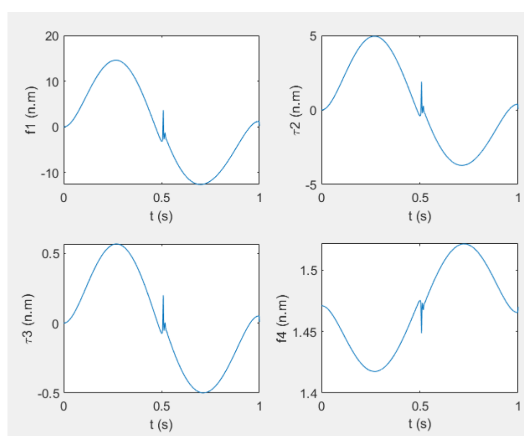

Fig. 3. Robot simulink results

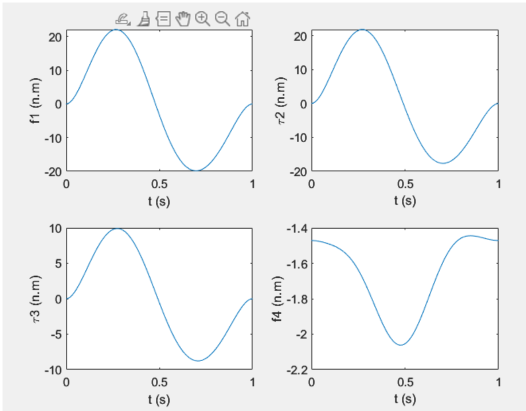

Fig. 4. Robot my codes results

We see that they are quite similar.