Identification and Control of a Double-Cart System

Abstract

The identification and control of systems from experimental data is the core tenet of this course. This report discusses the representation of a driven two-cart dynamical system with a transfer function and the process of determining the transfer function without prior knowledge of the system parameters, such as cart mass, spring constant, etc. Attempts will be made to identify the transfer function using nonparametric and parametric identification with experimental data, then a controller will be designed to effectively control the position of the second cart.

1 Introduction

In this experiment, two carts on a track are attached to each other by a spring, and one of the two carts is driven by an electric motor. Unfortunately, the cart masses, spring stiffness, and motor constants remain unknown, and the only known information provided by the wheel diameter, encoder data on the two carts, and the voltage input to the motor. The goal of the series of experiments, then, is to obtain consistent estimates of the system transfer functions using experimental data, and to design a robust, efficient controller to move the cart from one end of the track to the other. The transfer function will be estimated using both nonparametric and parametric identification methods with numerous data sets, a controller will be designed and shaped using a standard lead-gain controller, and the system performance will be discussed later.

2 System Identification

2.1 Process to be controlled

As discussed in Section 1, the system represents carts 1 and 2 attached by a spring, with cart 1 driven by a DC motor. By Newton's laws, the equations of motion for the two carts[4][1] are

$$m\ddot{x}_1 = k(x_2 - x_1) + F \qquad m\ddot{x}_2 = k(x_1 - x_2), \tag{1}$$

where k is the spring stiffness in $[N/m]$, m_1 and m_2 are the cart masses in $[kg]$, x_1 and x_2 are the cart positions in $[m]$, and $\ddot{x}$ represents the second time derivative of x in $[m/s^2]$. F is the applied force from the DC motor in $[N]$, given by

$$F = \frac{K_m K_g}{R_m r}\left(V - \frac{K_m K_g}{r}\dot{x}_1\right), \tag{2}$$

where V is the input signal to the DC motor in $[V]$, K_m, K_g, R_m, and r are motor parameters with unspecified units. With the exception of the input signal V for a given experiment, none of the above parameter values are known. Although a transfer function relating the input voltage U and the output position of the second cart Y is given [4] by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2.97 \times 61.2}{s^4 + 13.24s^3 + 127.15s^2 + 810.37s}, \tag{3}$$

this is not the true transfer function for the two-cart system. To make a controller for the system, an estimate for the true transfer function must be obtained by experiment.

2.2 Identification

2.2.1 Nonparametric Identification

Nonparametric identification constructs a Bode plot of a system response without any prior information about the number of poles or zeros. In this identification experiment, a number of input sinusoids of various frequencies[5] are applied to the two-cart system, and the magnitude and phase for each test frequency are gathered by inspection and by the correlation method. The correlation method is used to remove the effects of noise that can dominate at higher

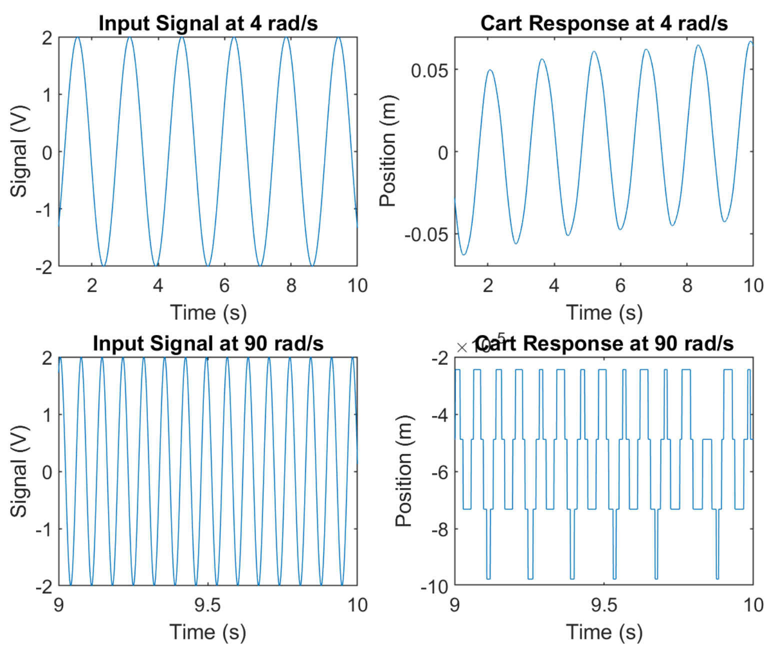


Figure 1: Cart responses for different frequencies of input signal. Notice the quantization error from the wheel encoder dominates the dynamics at higher input frequencies.

frequencies, as shown in Figure 1, and the sine wave testing to inspect the magnitude and phase shift acts as a double check on the correlation method. For both methods, the data sets are trimmed to remove transient responses between $t = 1s$ and $t = 5s$ and to ensure a consistent number of periods of oscillation are observed. For each method at each input frequency, both the sine wave method and correlation method are carried out and compiled into a complex number representing the system transfer function at the input frequency.

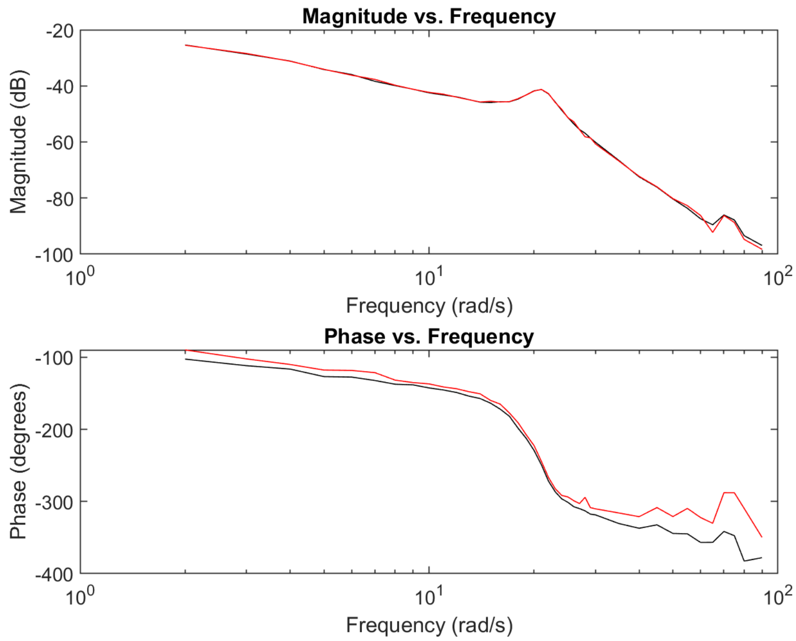


Figure 2: Nonparametric identification using sine wave inspection (red) and the correlation method (black).

Figure 2 demonstrates the two nonparametric methods producing nearly identical Bode plots. At frequencies above 30 rad/s, the two phase plots begin to deviate slightly from each other. The algorithm that automates the sine wave inspection identifies the local maxima of the cart response and compares the last of these maxima to the last maxima of the input signal to obtain the magnitude and phase information. At higher frequencies, the encoder quantization error dominates the shape of the output signal, which can flatten out the peaks of the cart response as shown in Figure 1. Thus, the detected peak may deviate from the true peak by a small number of sampling periods, which has a noticeable effect on the observed phase in Figure 2. The correlation method is more adept at detecting phase at these higher frequencies, so the data in black in Figure 2 will be used to estimate the transfer function.

To estimate the system transfer function, one simply observes the experimental Bode plot and uses known system properties to obtain the form of the transfer function. The numerical values are then tuned by simply guessing and checking. From physical intuition, the system has both an integrator and a pair of complex conjugate poles at the Bode plot inflection point at 21 rad/s. Furthermore, inspection of the phase plot in Figure 2 reveals there are four poles no zeros, as the phase approaches -360 degrees as $\omega \to \infty$. This information gives an estimated

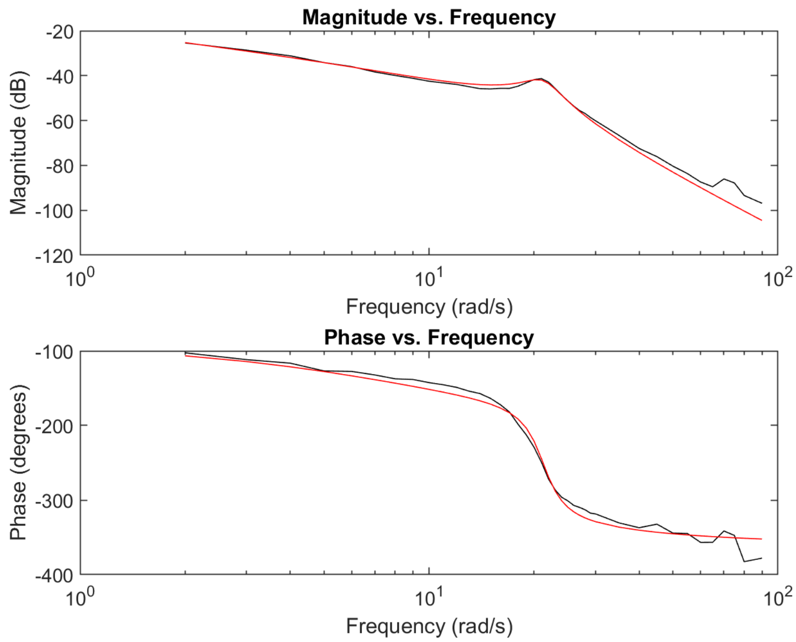


Figure 3: Transfer function estimate (red) shaped to correlation method experimental data (black).

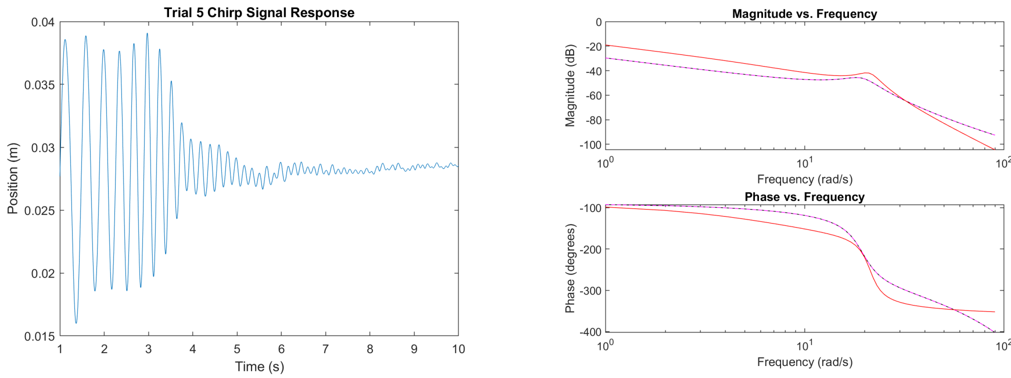
transfer function

$$G_{est}(s) = \frac{368}{s(s+7.24)(s^2+4.68s+451.9)}, \tag{4}$$

which closely corresponds with the experimental data as shown in Figure 3.

2.2.2 Parametric Identification

Parametric identification is the means of obtaining transfer function poles, zeros, and gain from experimental data if the form of the system is known beforehand. Since we have an analytical model of the system, we can perform a least-squares identification of the transfer function coefficients, and compare our findings to the nonparametric identification. The main distinction between the experimental setup for parametric and nonparametric identification is the type of signal applied and the number of trials. Where nonparametric identification uses several trials of single-frequency sinusoids to construct the Bode plot, parametric identification requires a single, frequency-rich input, such as a square wave or chirp signal, to obtain the transfer function coefficients. Eight different trials were conducted: six chirp signals of different ranges of frequencies, and two square pulses of one second duration. All signals have an amplitude of 2 volts. Most of these trials are either dominated by encoder quantization error or are not sufficiently rich, resulting in an estimated transfer function that does not properly represent the true system. Although the short duration of the chirp signals mute the true system dynamics at particularly sensitive frequencies, Trial 5, a chirp signal ranging from 1



(a) (b)

Figure 4: **a)** Experiment 5 cart response to a chirp signal ranging from 1 to 10 rad/s and **b)** parametric identification using data from Experiment 5. Matlab `arx`[2] identification (black) and `my_id`[3] identification (magenta) are overlapping, compared to nonparametric estimation (red).

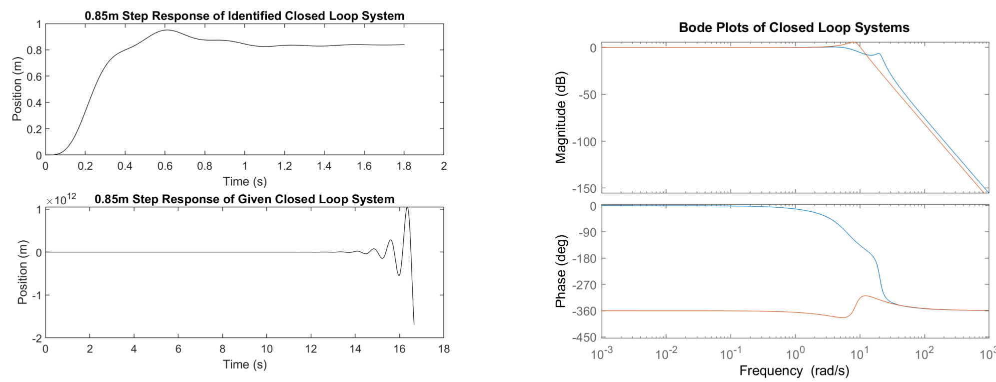
to 10 rad/s, allows for a system identification with similar behavior to the one identified in Section 2.2.1. The system transfer function is identified using Matlab's built-in `arx`[2] tool, as well as the `my_id`[3] function provided in class. Before the input-output data is sent to these functions, the data is down-sampled by a factor of 10 and scaled by a factor of 500, then the discrete-time integrator is removed. After the system is identified, the scaling is reversed and the discrete time integrator is restored, and the Bode plot for both identifications is compared to the estimated non-parametric system in Figure 4b).

2.3 Controller Design and Performance

While the parametric identification captures the conjugate poles at 20 rad/s, the magnitude slope is somewhat off from the nonparametric identification, and the phase plot does not have the proper shape. Since the nonparametric identification agrees both with correlation method data and direct inspection of the sine wave tests, the system given by Equation 4 will be used to represent the two-cart system and to create an effective controller. The closed loop controller used is a simple lead-gain controller[1] of the form

$$C(s) = k\frac{Ts+1}{\alpha Ts+1}, \tag{5}$$

where k = 4.5, T = 30, and $\alpha = 0.1$. To shape the controller, first α was gradually shifted from 1 to 0.1. This increased the amplitude of closed-loop oscillation, but did little to the rise time. At $\alpha = 0.1$, the value of T was shifted from T = 1 to 30. The system approaches the set point much more aggressively, but the settling time could be reduced further by implementing a positive gain. As k was varied from 1 to 4.5, the settling time decreased from 86 seconds and no overshoot to 0.96 seconds with a 13% overshoot, which meets the criterion set forth by the



(a) (b)

Figure 5: **a)** Closed-loop step response for the identified model in Equation 4 and the given model in Equation 3. **b)** Bode plots of the closed-loop identified system (blue) and the given system (orange).

RiseTime	0.2156 s
SettlingTime	0.9608 s
Overshoot	13.0441 %
Peak	0.9085 m
PeakTime	0.6084 s

Table 1: Closed-loop step response properties.

objective[4]. The relevant step response properties are in Table 1. Since the given system in Equation 3 is not the true system, or closely representative of it, the stability of the controller in a closed-loop with this system is not of concern. For reference, the Bode plots of the two closed loop systems are included in Figure 5b).

3 Conclusion and Future Work

Using voltage and encoder data obtained from the cart system, the system magnitude and phase plots were obtained and successfully cross-checked using sine-wave testing and the correlation method. The transfer function was then closely estimated with the help of physical knowledge of the system and Matlab's `tfest`[2] function. Although parametric identification failed to corroborate the transfer function obtained by nonparametric identification, this is merely a failing of data acquisition and not procedural accuracy. For the chirp signals to adequately capture the system dynamics for a broad range of frequencies, the rate of change of the frequency must be longer such that the system dynamics at resonance are not lost to noise or transient effects. Instead, a series of richer square wave inputs at higher frequencies would provide better parametric identification data, and thus a better estimated transfer function. In a normal lab setting where students could conduct the experiments themselves, there would be ample time to identify this problem and collect different parametric identification data.

Even so, a successful controller was developed and managed to move a cart to the end of a 1 meter track in less than a second. Again, in a normal lab setting, it would be nice to test this controller to ensure it behaves as intended, but the characteristics of the step response indicate the system behaves in a manner consistent with reality.

References

[1] Joao P. Hespanha. *Control Systems Design*. 2014.

[2] MathWorks. *Matlab Documentation*.

[3] Yasamin Mostofi. *System Identification Function*. 2016.

[4] Yasamin Mostofi. *Two-Cart with Spring*. 2020.

[5] Mert Torun. *Experimental Data*. 2020.