ME Labs
Jack Button: Team Leader
Jack Dibachi: Technical Analyst
Scott Smith: Technical Writer

Experiment 2: Thermal Sensor Behavior and Lumped Capacitance Heat Transfer

Objectives

The purpose of this report is to describe the methods, results, and improvements upon using thermal measurements to gather heat transfer properties alongside the characteristics of 3 different temperature measuring sensors: a thermocouple, a thermistor, and a resistive thermal device (RTD).

Introduction

The ability to accurately and reliably measure the temperature of an environment or material is critical to any thermal analysis. There are many thermal measurement devices that can be appropriately used in a number of different scenarios, the most common of which are thermocouples, thermistors, and RTDs. All of these sensors behave differently, so it is important to understand the characteristics of each when selecting a device.

We are looking at zeroth, first, and second order sensors. Simply put, a zero order sensor acts in a purely linear relationship to an input, while a first order sensor takes time to approach a steady state reading, and a second order sensor responds dynamically by over and undershooting the response.

A thermocouple is a zero order sensor, it operates by measuring the voltage between two metals with different thermoelectric coefficients in series. Because their thermoelectric coefficients are different, the voltage across the thermocouple will change with temperature by the relationship:

(Equation 1) $$\Delta V = (\alpha_A - \alpha_B)(T - T_r)$$

Where α_A is the thermoelectric coefficient of metal A, α_B is the thermoelectric coefficient of metal B, T is the temperature being measured and T_r is the temperature of the base of the thermocouple. The fit of this sensor follows the relationship:

(Equation 2) $$T = a_1 + a_2V + a_3V^2 + ... + a_nV^{n-1}$$

Thermistors, a first order sensor, operate on the basis of a semiconductor's resistivity at temperature. The electrical resistance of a semiconductor decreases with rising temperature and this can be measured using a digital multimeter. Thermistors are highly nonlinear, making them more ideal measurement devices for low temperature (high sensitivity) and relatively unideal for higher temperatures (low sensitivity). Their fits follow:

(Equation 3) $$\frac{1}{T} = a_1 + a_2 ln(R) + a_3 ln(R)^2 + ... + a_n ln(R)^{n-1}$$

An RTD, a first order sensor, uses the change of a metal/conductor resistance with a change in temperature. The electrical resistance of a conductor increases with rising temperature and thus this resistance can be read using a digital multimeter. An RTD is more linear than a thermistor and follows the fit:

(Equation 4) $$T = a_1 + a_2 R + a_3 R^2 + ... + a_n R^{n-1}$$

These devices are often used in their respective limits to collect accurate material/thermal properties on a material of interest. However, they must be calibrated so that the coefficients of the curve fits can be found and a relationship between temperature and voltage or temperature and resistance can be found.

In this lab, a thermal measurement sensor must be used in order to know the temperature time response to thermal perturbation. This is a valuable data set for anyone attempting to understand the thermal performance of a material such as a fin or heat sink because it can be used to derive the thermal coefficient of that material at different temperatures or the thermal relaxation time of that material.

The heat transfer coefficient can be derived by finding the Nusselt number through:

(Equation 5-7) $$Nu = 2 + 0.6(Re^{1/2})(Pr^{1/3}),\ Re = \frac{UD}{\nu},\ Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Where Re Is the Reynolds number, Pr is the Prandtl number. The Reynolds number represents the inertial forces over the viscous forces of the fluid, while the Prandtl number is defined as the ratio of momentum diffusivity to thermal diffusivity. In these equations U is the fluid speed, D is the relevant dimensional length (in this experiment, the length is the diameter of a sphere), ν is the kinematic viscosity, μ is the dynamic viscosity, C_p is the specific heat at constant pressure, and k is the thermal conductivity of the material. These equations further describe the heat transfer coefficient through its relationship to the Nusselt number, which is simply a dimensionless presentation of the heat transfer coefficient:

(Equation 8) $$h = \frac{D}{kNu}$$

The time decay constant/thermal relaxation time τ with a dimension of time, can be found by using the data collected in response of a material to thermal perturbation alongside the resulting expected transient response of the material derived from an energy balance and assuming the lumped capacitance model to be applicable. The following equation describes this scenario:

(Equation 9) $\theta = \frac{(T(t) - T_\infty)}{(T_i - T_\infty)} = exp\left[-\frac{t}{\tau}\right]$

Where,

(Equation 10) $\tau = \frac{V\rho C_p}{hA}$

A test of energy balance is an excellent way to understand and quantify the losses inherent in the system set up in the lab. This can be done by allowing the sphere to cool in a known volume/known temperature of water and monitoring both temperatures as the system reaches equilibrium. Theoretically, the energy of this system should be conserved such that the total thermal energy of the system at the beginning of the lab and the total thermal energy at the end of the lab should be equal. This is inherently wrong though as an ideally insulating system is not going to be achieved. Some thermal energy is bound to be lost or gained from the surrounding environment, and an energy balance would be not applicable. It is, however, important to quantify these losses/gains to ensure that they are manageable and/or negligible when analyzing the results of this lab. The following equations will be used to show the total change in energy of the sphere and the surrounding water respectively.

(Equation 11) $\Delta Q = m_{sp} C_{p,sp} (T_{i,sp} - T_{f,sp})$

(Equation 12) $\Delta Q = m_w C_{p,w} (T_{i,w} - T_{f,w})$

Measurements have been taken on the water and the sphere in order to obtain volume values. With the known density of these materials, this data will be used to calculate the mass values of each that are present in equations 11-12.

Methods

Before performing any thermal measurements on the sphere, the thermocouple must be built and all 3 temperature measuring devices must be calibrated using an analog thermometer. The accuracy of the analog thermometer is easily determined by placing the thermometer in boiling water (held at 100C) and an ice bath (held at 0C). This is done first. It is determined that the thermometer can be accurately read within ± 0.25 C.

In order to build the thermocouple, two wires of two different materials (Chromium and Aluminum), are woven together and connected to a DAQ where the voltage data read from the thermocouple can be read using a LabVIEW VI. Once that is done, the three measurement devices can be calibrated.

All three devices are then placed in a bath of varying temperature. The temperature is increased over time to allow voltage data from the thermocouple and resistance data from the thermistor and RTD to be correlated to the known temperature read by the thermometer which is also in the bath alongside the sensors. This data is further used in the lab to produce a curve fit to the calibration data and later correlate other voltage output data to a temperature. It is important to note that the base of the thermocouple must be kept at a constant and known temperature because the device outputs a voltage dependent on not only the temperature of the metals at the end of the sensor, but also the temperature of the base/junction.

It's necessary to note that all sensors not only have an associated accuracy, but also an associated response time. If temperatures are changing faster than the sensors can equilibrate to, then temperature data will be considerably less accurate than reported. Also it is important to note this so that temperature for single data points are taken after, and only after, the temperature readout has reached equilibrium. The response time of these sensors can be gathered by subjecting them to a step upward in temperature and recording the transient response to this step upward. This data is collected in this lab and will be reported in further sections. It is collected by placing the ends of each sensor in an ice bath (0C) and allowing them to reach a steady-state readout and then recording the response of the sensors as they are placed in boiling water (100C) and allowed to again reach a steady state readout. The time response of these sensors is collected and further applied to equation 9-10 to find the time decay constant of each sensor. At this point the sensors are fully calibrated and understood. The sensors can be used to collect data.

In order to measure the transient response of a large aluminum sphere, embedded with an identical aluminum-chromium thermocouple, the sphere is allowed to reach boiling temperature in the boiling water previously used in this lab. Once the readout on the thermocouple/LabVIEW VI has reached steady-state, it is removed from the boiling water and placed inside an ice bath until it again reaches a steady-state voltage readout. The transient response is captured through the LabVIEW VI. The data collected from these measurements will be used to analyze the thermal properties of the sphere.

Further, an analysis of energy balance is performed by reheating the sphere to a known temperature and cooling it in a bath that is similar to the temperature of the room (to avoid heat loss to natural convection, which can't be measured using this lab setup), is above 0C, is contained within an insulated container, and is at a known temperature. Ensure that as little heat is lost to natural convection by covering the top of the insulated container. This analysis can be used to approximate the error of the lab setup over time.

Data

Table 1: Measured Values

Airstream Speed (m/s)	**Water Stirring Speed (rad/s)**	**Bucket Diameter (mm)**	**Stir Diameter (mm)**	**Water Stream Speed (m/s)**
13.7 ± 0.1	7.2 ± 0.5	177.80 ± 0.01	127.02 ± 0.01	0.46 ± 0.03

Table 2: Physical and Material Properties								
Water			**Aluminum**					
Volume (mL)	**Density (kg/m^3)**	**Heat Capacity (J/kgK)**	**Sphere Diameter (mm)**	**Surface Area (cm^2)**	**Volume (cm^3)**	**Density (kg/m^3)**	**Heat Capacity (J/kgK)**	**Thermal Conductivity (W/mK)**
850 ± 5	999.9	4187	50.78 ± 0.01	81.01 ± 0.03	68.56 ± 0.04	2699	900	210

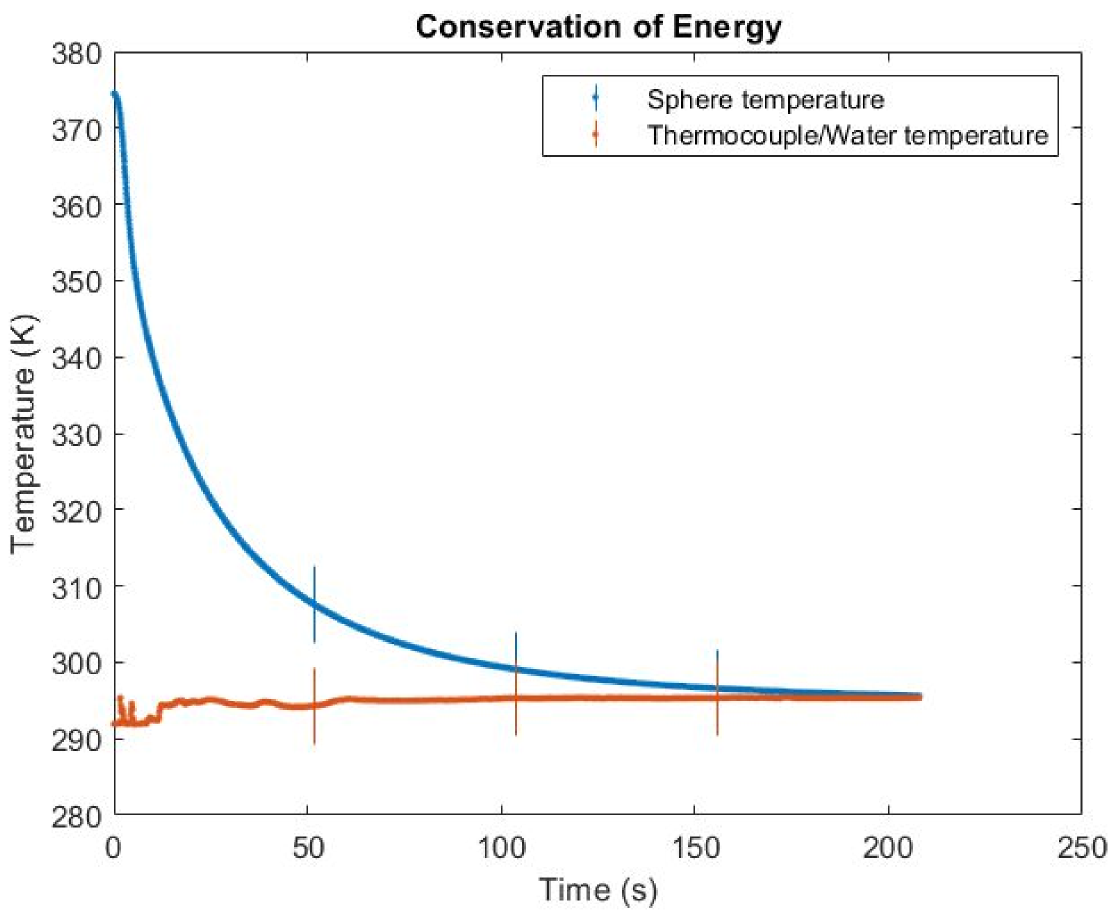


Figure 1: Water temperature and aluminum sphere temperature change over time, while inside an insulated container. The First Law of Thermodynamics will be applied to this data to characterize heat lost from the system.

Results

Table 3: Time constants, R^2 fit, heat transfer coefficients, computed flow speed			
Natural Convection (Air)	**Natural Convection (Water)**	**Forced Convection (Air)**	**Forced Convection (Water)**
τ=1188 ± 8 seconds	**τ**=20 ± 2 seconds	**τ**=188.3 ± 0.4 seconds	**τ**=7.4 ± 0.1 seconds
R^2=0.9999	R^2=0.9998	R^2=0.9999	R^2=0.9991
h=17.3±0.1W/m^2/k	h=1032±103 W/m^2/k	h=109.4±0.3W/m^2/k	h=2793±38W/m^2/k
Fluid Velocity = 0	Fluid Velocity = 0	Fluid Velocity = 17.1 m/s	Fluid Velocity = 0.3 m/s

Table 4: Sensor Sensitivity			
Sensor Type	**Absolute Sensitivity**	**Full Scale**	**Percent Sensitivity**
Thermocouple	8.002 mV/K	273K-373K	1.00%
Thermistor	See Figure 2	273K-373K	See Figure 2
RTD	0.371 Ω/K	273K-373K	1.00%

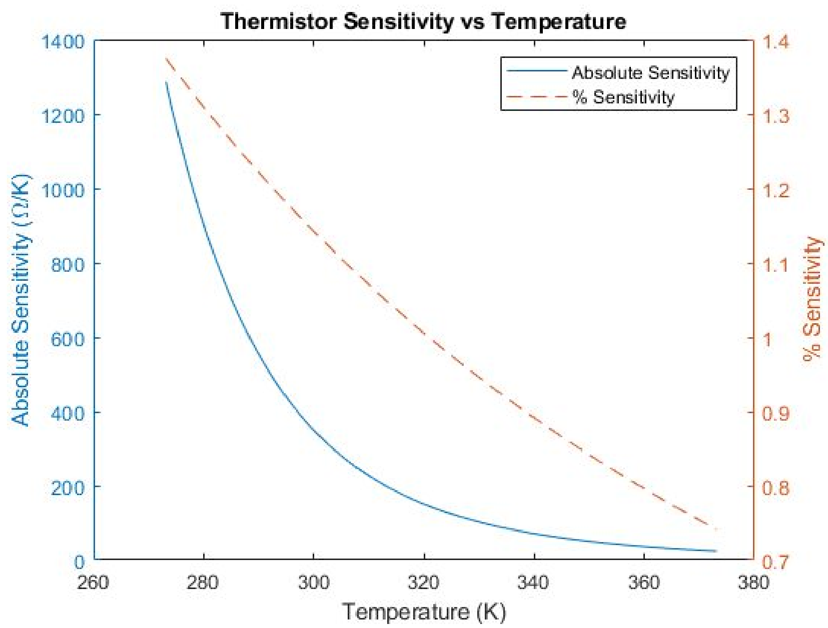


Figure 2: Thermistor sensitivity, expressed as an absolute and as a percentage, over the operating range of temperatures. The change in percent over the range is a result of the nonlinear sensor behavior.

Table 5: Sensor Response Times to a Step Change in Water Temperature

Sensor Type	One Time Constant τ (s)	Full Response Time 5τ (s)
Thermocouple	0.05	0.25
Thermistor	1.23	6.15
RTD	3.22	16.08

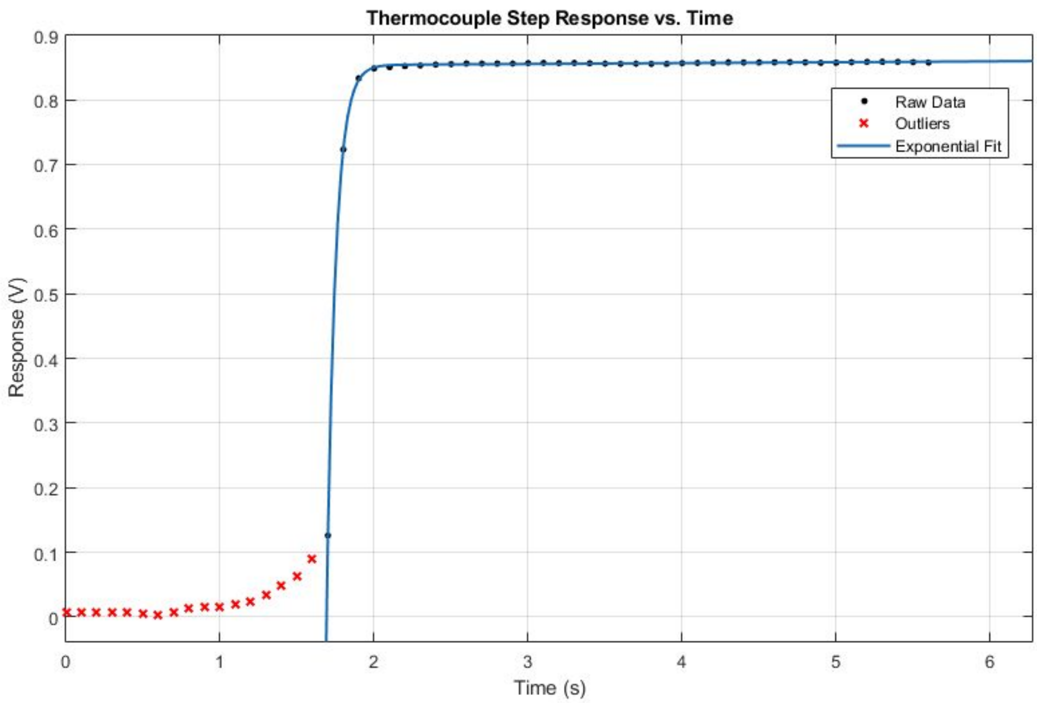


Figure 3: Thermocouple response to a step change in temperature from 0 °C to 100 °C water. The points marked with x represent the brief time the thermocouple was in air, and are omitted from the curve fit.

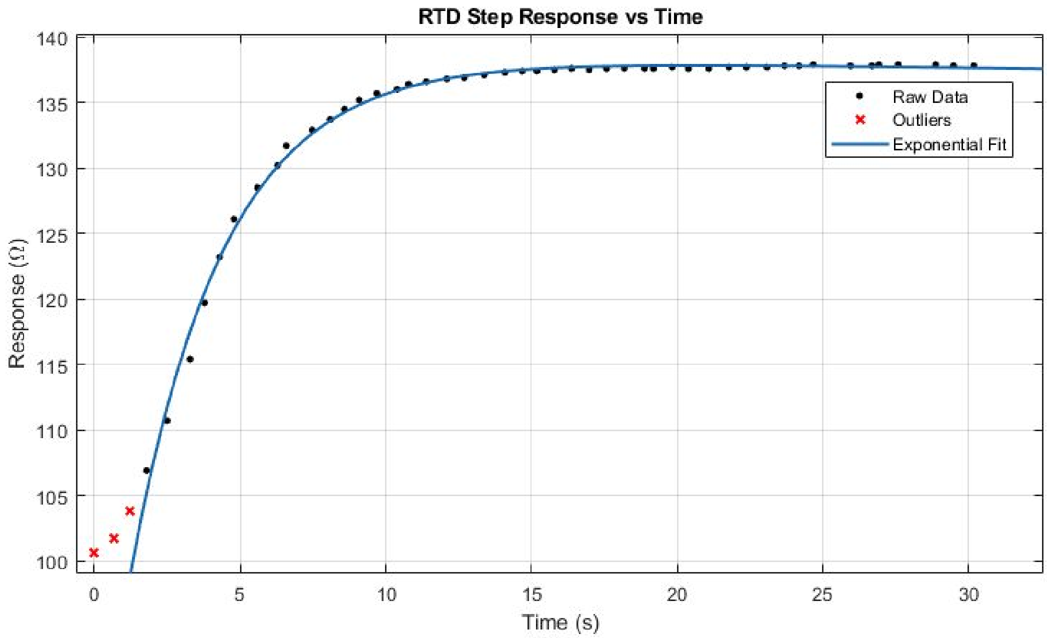


Figure 4: RTD response to a step change in temperature from 0 °C to 100 °C water.

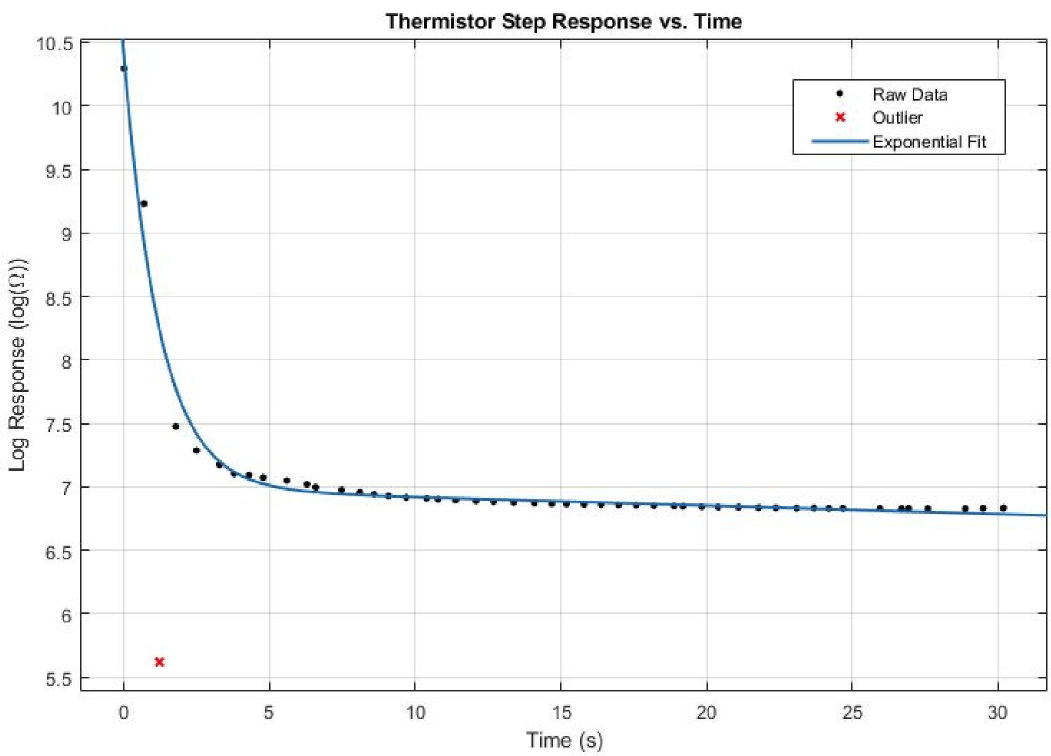


Figure 5: Thermistor response to a step change in temperature from 0 °C to 100 °C water. The response is plotted on a logarithmic scale to effectively demonstrate response time.

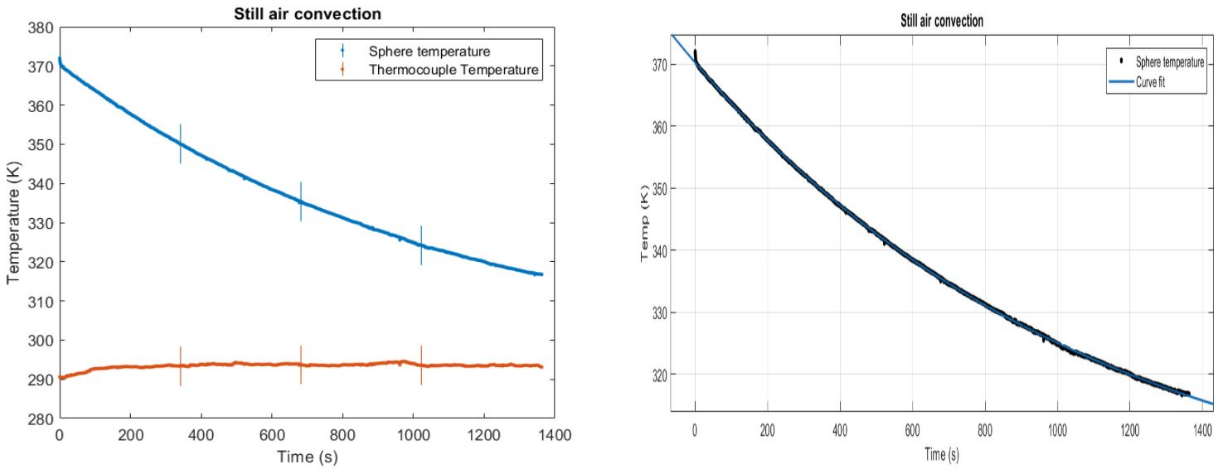


Figure 6: a) Thermal response of an aluminum sphere to a step change in temperature, from 100 °C water to still air. b) Curve fitted response used to calculate *h*

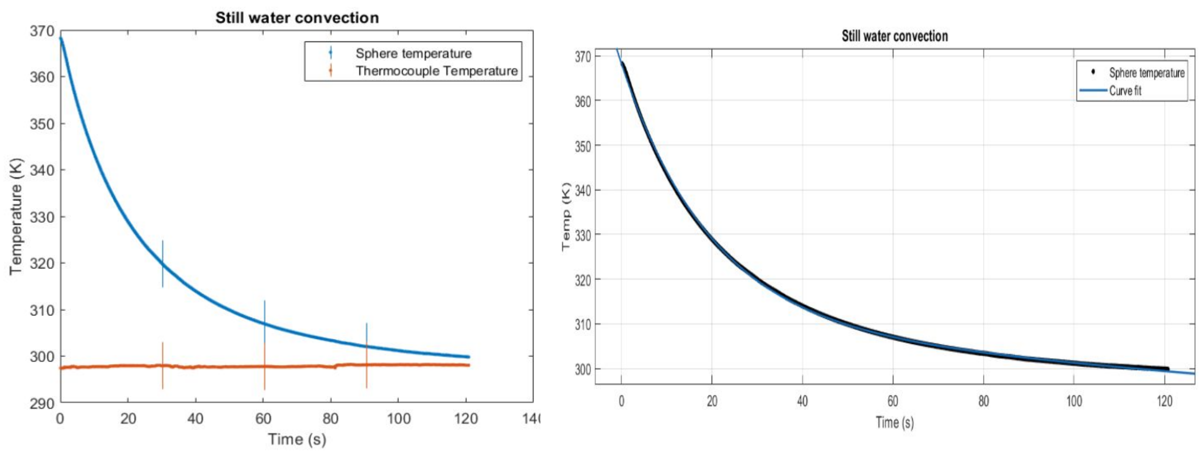


Figure 7: a) Thermal response of an aluminum sphere to a step change in temperature, from 100 °C water to still water. b) Curve fitted response used to calculate *h*

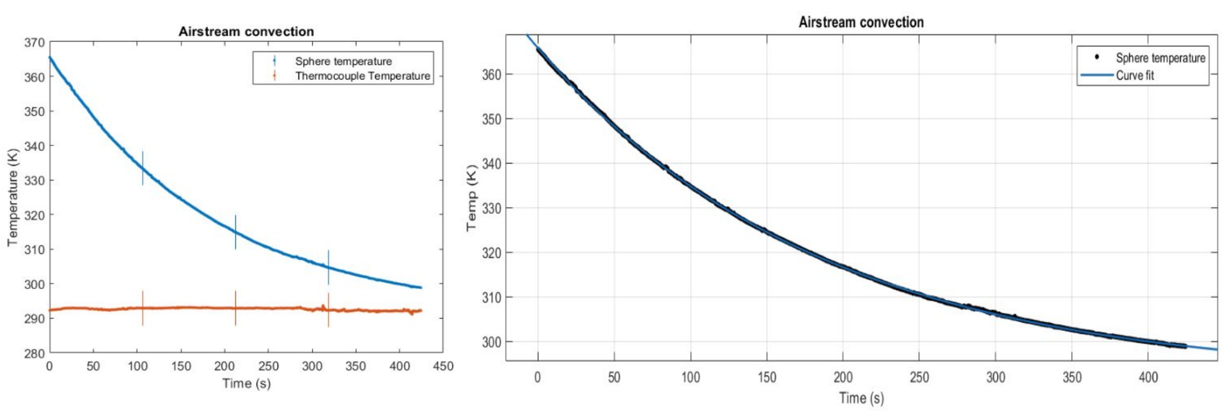


Figure 8: a) Thermal response of an aluminum sphere to a step change in temperature, from 100 °C water to moving air. b) Curve fitted response used to calculate *h*

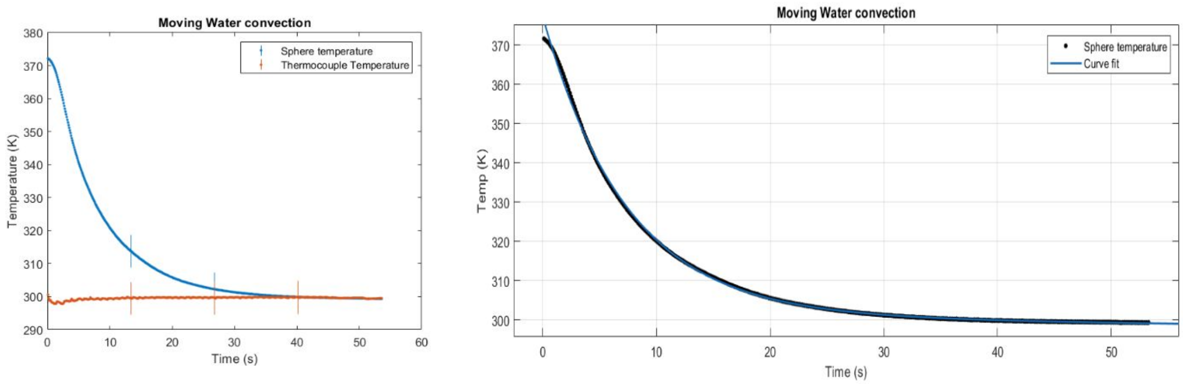


Figure 9: a) Thermal response of an aluminum sphere to a step change in temperature, from 100 °C water to moving water. b) Curve fitted response used to calculate *h*

Conservation of Energy

A test of conservation of energy was performed in order to ensure that the sensor calibration suggests physically reasonable results. As shown by Figure 1 the thermocouple placed in the water read an initial temperature of 292 ± 0.25 K or 19 ± 0.25 C and read a final temperature of 296 ± 0.25K or 23 ± 0.25 C. The thermocouple embedded in the aluminum sphere read an initial temperature of 374 ± K or 101 ± C and a final temperature of 296 ± 0.25 K or 23 ± C.

Using Equations (11-12) and the values for sphere and water dimensions from Table 2, the resulting ΔQ_{sp} is shown to be -14.2 ± 0.0409 kJ and the resulting ΔQ_w is shown to be 13.0 ± 0.091 kJ.

Very clearly, energy was not perfectly conserved, but this is expected to be the case. This results in a total of 1.2 kJ to the environment which represents 8.45% of the total energy change of the sphere.

Discussion

Sensor Sensitivity and Response Time

The absolute sensitivity of any sensor is given by the change in sensor output per unit input. For the sensors used in this experiment, the absolute sensitivity is the slope of the response vs. temperature curves. For the thermocouple and RTD, the absolute sensitivities are given by the slope of the linear fit lines in Table 4, while the thermistor absolute sensitivity is given by the slope of the nonlinear response curve in Figure 2. The percent sensitivities are shown in these respective tables and figures to better characterize the relative change in each sensor response.

Since the thermocouple and RTD are operated over a range of 100 K, the percent sensitivity is precisely 1.00% for both sensors, since the percent sensitivity is the absolute sensitivity normalized to the full range of sensor outputs for a given input range. If the operating range spans a larger domain of temperatures, such as 1000 K, the percent sensitivity would be 0.10% for these two sensors. However, the thermistor has a nonlinear absolute sensitivity, so the percent sensitivity changes based on the range of operation and the measurement temperature. As a result, the thermistor has a higher percent sensitivity than the thermocouple and the RTD for temperatures lower than 320 K, as long as the range of operation is from 273 K to 373 K. If the operating range were shifted to be from 220 K to 320 K, the percent sensitivity of the thermistor would be significantly higher than that of the other two sensors, since the absolute sensitivity of the thermistor spikes in this temperature range.

The response times of each sensor are heavily influenced by the thermophysical properties of each sensor. The junction of a thermocouple is a very small, highly thermally conductive mass, so the temperature of the junction changes rapidly. The thermistor also has a small volume, but it operates using a semiconductor material rather than a metal, and is coated in an epoxy, so it is not as thermally conductive as the thermocouple. Therefore, the thermistor has a slower response time than the thermocouple, approaching steady state at 6.15 seconds while the thermocouple settles after 0.25 seconds. A RTD, the slowest responding sensor of the three tested, operates on the principle of changing metal resistivity, thus the wire inside the sensor is highly thermally conductive and small in volume. However, the sensing wire is wrapped around a ceramic core, which has low thermal conductivity, and is protected by a metal jacket on the outside of the sensor. In addition to the sensing wire, these additional components must also reach the sensing temperature before reaching a steady state output, resulting in a higher response time than either the thermocouple or the RTD.

Sphere Step Response

It is clear that each curve fit shown in Figure 6b, 7b, 8b, and 9b highly represents an exponential function with all R-squared values greater than 0.99. For each exponential fit we find the respective time constants with a 95% confidence interval given by MATLAB's curve fitting toolbox. From here we can use the lumped capacitance model to find the heat transfer coefficient (eqn 10). While the ambient temperature of the water convection experiments does change, the change in temperature is at most 2K. Since the error of our plots is ± 5K, logically we can neglect the change in ambient temperature and assume constant ambient temperature conditions.

The radiative heat transfer of the aluminum can be neglected due to the typically small emissivity of aluminum with an order of magnitude of 10^-2, else if the aluminum was a black body then radiation might be non negligible for air natural convection due to the low order of magnitude for the convective heat transfer coefficient range given in the prelab and verified in our results.

All of our heat transfer coefficients are in the standard range of values provided to us in the prelab, with the exception of water natural convection of 1032 ± 103 W/m^2/k, but the large uncertainty brings the possibilities of the actual heat transfer coefficient being in the accepted range of 10-1000 W/m^2/k.

Using the nusselt number correlation presented in equation 5, and known thermal properties of the aluminum and surrounding fluid we are able to approximately solve for the velocity of the forced convection experiments. We are unaware of how accurate this correlation for the dimensionless heat transfer coefficient is and thus it would be pointless to try and assign an uncertainty value to the computed velocity. The airstream velocity was measured to be 13.7 ± 0.1 m/s which is of the same order of magnitude for the computed value of 17.1 m/s. The water stream velocity was measured to be 0.46 ± 0.03m/s which again is of the same order of magnitude for the computed value of 0.27m/s. This indicates a fair amount of error accumulation in our measured values, the dimensionless heat transfer coefficient correlation, and along with the unquantifiable uncertainties discussed later on.

Conservation of Energy

For conservation of energy, though 8.45% of total change in thermal energy from the sphere might initially appear high, this value is significantly lower than with previous trials where water in the insulated container was in excess of 60 C. The high fluid temperature was the root of initial failures at this test, resulting in enormous amounts of heat loss to free convection.

This was fixed by changing the water temperature to be that of the surrounding air before cooling the sphere. This minimized heat loss to natural convection by reducing the temperature difference between the water and the surrounding air. Furthermore, the container was provided with a lid with greater insulating capabilities.

Conclusion

In conclusion of this lab, it is found that the medium with which convection is occurring is hugely important to the thermal cooling time decay constant of a system. In this instance, the reduction of this constant from free convection in air to forced convection in water was $\boldsymbol{\tau}$=1188 ± 8 seconds to $\boldsymbol{\tau}$=7.4 ± 0.1 seconds. Utilizing this effect is advantageous in scenarios where the heat loss to the environment needs to be controlled (minimized or maximized). The transition from free convection in air to free convection in water was $\boldsymbol{\tau}$=1188 ± 8 to $\boldsymbol{\tau}$=20 ± 2 seconds respectively. While the transition from free convection in air to forced convection in air is $\boldsymbol{\tau}$=1188 ± 8 to $\boldsymbol{\tau}$=188.3 ± 0.4 seconds respectively. The medium, in this instance, had a significantly greater impact on the time decay constant than the transition toward forced convection.

Appendix I: Calibration

Table 6: Thermocouple calibration data.

Temperature (℃)	Sensor Output (V)
-1	0.046
13.5	0.12
18	0.145
22.5	0.19
33	0.288
43.5	0.383
55.5	0.464
61	0.522

Figure 10: Thermocouple calibration data with linear fit.

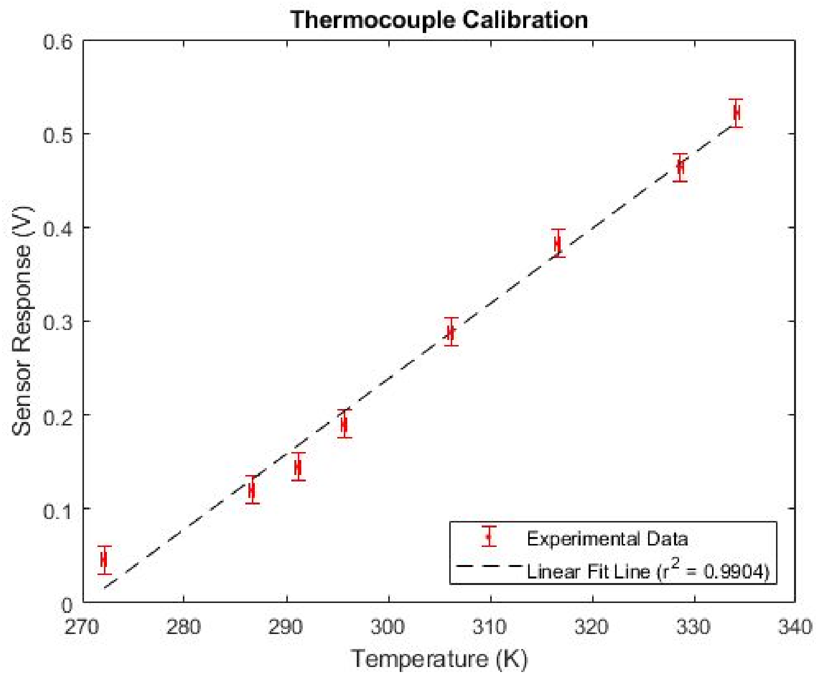


Table 7: Thermistor calibration data.	
Temperature (°C)	Sensor Output (kΩ)
-1	29.66
13.5	15.37
18	13.32
22.5	10.82
33	7.05
43.5	4.82
55.5	3.439
61	2.804

Figure 11: Thermistor calibration data with logarithmic linear fit.

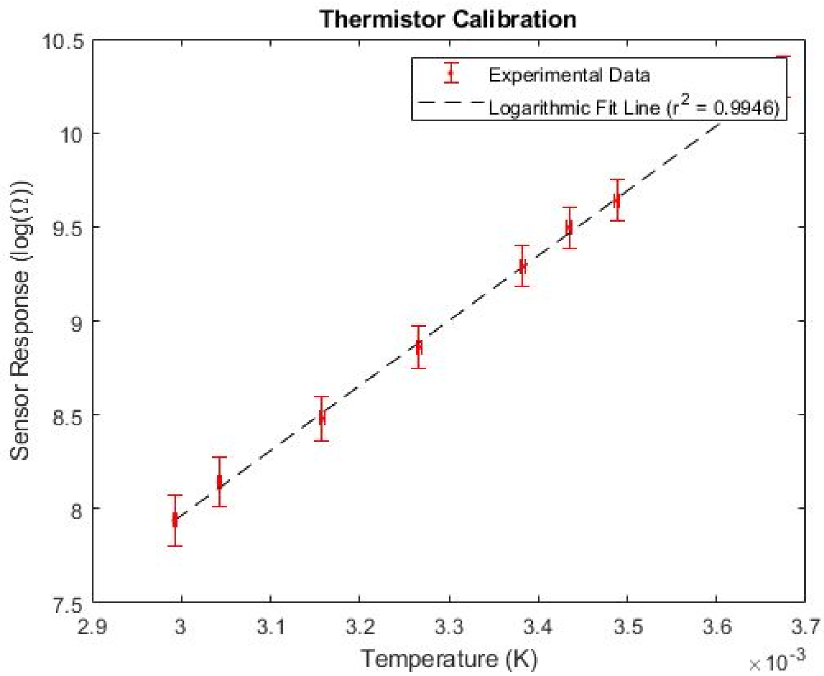
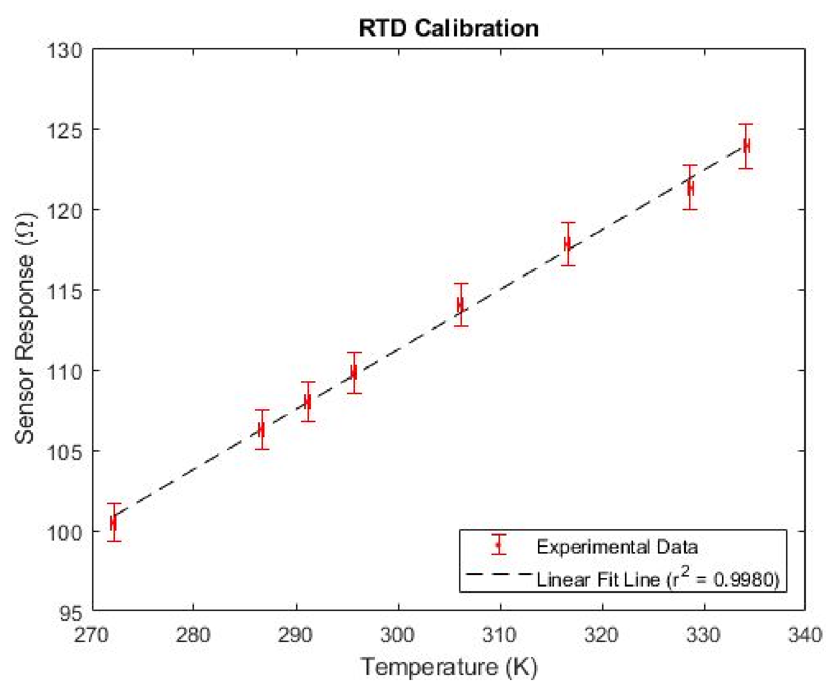


Table 8: RTD calibration data.	
Temperature (°C)	Sensor Output (Ω)
-1	100.5
13.5	106.3
18	108
22.5	109.8
33	114
43.5	117.8
55.5	121.3
61	123.9

Figure 12: RTD calibration data with logarithmic linear fit.



Fitting parameters for sensor calibration:

Thermocouple:

Y intercept: -2.2 ± 0.3 V
Slope: 8.0e-3 ± 0.3e-3 V/K
R^2 : 0.9904

Thermistor:

Y intercept: -2.4 ± 0.4 log(Ω)
Slope: 3450 ± 40 log(Ω)/(1/K)
R^2 : 0.9992

RTD:

Y intercept: 0.0 ± 5.8 Ω
Slope: 0.372 ± 0.007 Ω/K
R^2 : 0.9980

Appendix II: Sources of Uncertainty

Sources of uncertainty for this experiment include the uncertainty associated with the thermometer readout, the uncertainty in the digital multimeter readout of resistance, and uncertainty in the voltage readout of the DAQ/LabVIEW VI.

Part of the uncertainty with the thermometer comes from the purity of water used which is currently unknown, i.e. milli-Q water, deionized water, or regular tap water. The concentrations of surfactants, ions, and other contaminants in the water can change the temperature at which it boils or freezes, indicating that we may not be fully measuring true 0C or 100C, respectfully.

As for the sensors it is assumed that there is zero error intrinsic to the physical sensors at steady-state. As long as the time decay constant of the sensors are significantly lower than the rate at which the temperature they are expected to measure is changing by, then all of the error associated with the temperature/voltage or resistance readings in the lab should be accumulated in the device reading these values (i.e. the digital multimeter and the DAQ).

Experimental uncertainty is generated by ignoring radiative effects when submerging the sphere in the water, as we do not know the emissivity of the containers used in this experiment.

Lastly, computational uncertainty arises from the thermal property values of the aluminum and its environment, as these values were pulled from tables that lack uncertainty estimates. Of course the material property values collected have a resolution orders of magnitude smaller than our physical measurements and thus can be neglected here.

Thermometer Readout Uncertainty

The resolution of the thermometer was determined to be 0.5C, resulting in the uncertainty of the thermometer as ± 0.25C.

DAQ Timing Uncertainty

The DAQ sampling frequency is 10 Hz, which affects the accuracy of time constants, curve fits, and other results derived from DAQ data.

Digital Multimeter Uncertainty

The resolution uncertainty of the digital multimeter is given by the following table:

Table 9: Tabulated resistance error for DMM readings.

Function	Range (Ohms)	Resolution (Ohms)	Accuracy
Resistance	400	0.1	±(0.8% Reading + 4 · Resolution)
	4000	1	±(0.8% Reading + 2 · Resolution)
	40000	10	±(1.0% Reading + 2 · Resolution)

This produces resistance readout uncertainties for the RTD and Thermistor. However, these devices are not used to make any measurements in this lab and thus this error is not propagated into any data sets. The table below shows the uncertainty and resolution in measurements for resistance data for the RTD and thermistor calibration.

Table 10: Tabulated resistance readings with associated error and resolution for thermistor and RTD.

Temp (C)	TC (V)	Thermistor (kΩ)	Resolution (kΩ)	Error ±(kΩ)	RTD (Ω)	Resolution (Ω)	Error ±(Ω)
-1	0.046	29.66	0.001	0.23928	100.5	0.1	1.204
13.5	0.12	15.37	0.001	0.12496	106.3	0.1	1.2504
18	0.145	13.32	0.001	0.10856	108	0.1	1.264
22.5	0.19	10.82	0.001	0.08856	109.8	0.1	1.2784
33	0.288	7.05	0.001	0.0584	114	0.1	1.312
43.5	0.383	4.82	0.001	0.04056	117.8	0.1	1.3424
55.5	0.464	3.439	0.0001	0.027912	121.3	0.1	1.3704
61	0.522	2.804	0.0001	0.022832	123.9	0.1	1.3912

Appendix III: Error Propagation

Propagated error was calculated using the standard form of the error propagation equation. The thermometer was found through observation to have a $\pm 0.25\ C$ uncertainty and the caliper was found to have a ± 0.001

$$\Delta y = \sqrt{\sum_{i=1}^{n} \left(\frac{\delta y}{\delta x_i} \Delta x_i\right)^2}$$

This leads to an equation for uncertainty in heat coefficient:

$$\frac{\Delta h}{h} = \sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2 + \left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta\tau}{\tau}\right)^2}$$

Where the variables on the right hand side from left to right represents density, radius, specific heat, and time constant of the aluminum sphere respectfully. The error in the time constant is given from the exponential curve fit values provided by matlab.

Works Cited:

Heat Library Property Tables
https://sites.me.ucsb.edu/~bennett/heatlib/index.html