Numerical Analysis Question Set

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1. A horizontal spring with a spring constant of 15 N/m is attached to a friction-less surface. A block of mass 2 Kg is attached to the end of the spring. A man spends 20 $Joule$ of energy to compress this spring. How far the block is from the equilibrium? Calculate the maximum kinetic energy and maximum potential energy of the spring when released? Draw the graph for the variation of kinetic energy and potential energy with distance and with time.

2. A Hydrogen ion is kept in an uniform electric field of 10 V/m. If the Hydrogen ion is released, draw the graph for the variation of ion velocity with time to the step of 0.1 *second*. (Total time given is 30 *second*). Show the variation of Kinetic Energy with time.

3. A ball is thrown from a building of height 10 m with velocity 14 m/s at an angle of 45°. Calculate the range, height and total time of flight. Draw the graph for the variation of potential energy and kinetic energy with height and distance. Show that the total energy is conserved.

4. A ball of mass 10 gm and charge 10 $Coulomb$ is let freely fall in an uniform electric field of 20 V/m (directed along y axis). If the acceleration due to gravity is 9.79 m/sec^2. Calculate the acceleration of the ball. Which direction will the ball move? Calculate the velocity.

5. A particle of mass 10 gm and charge 0.2 $Coulomb$ is moving under influence of $\vec{E} = 20\hat{z}\ V/m$ and gravitational field $\vec{g} = -9.8\hat{z}\ m/s^2$. At $t = 0$, $\vec{v} = -1.5\hat{z}\ m/s$ and vector $\vec{z} = 7.5\hat{z}\ m$ using **Runge-Kutta Forth order method** find position and velocity of the particle with suitable interval, until it gains it's velocity $1.5\hat{z}\ m/s$. Also calculate the error (in %) for both at each point. Plot the variation in position and velocity with time and as well as corresponding graph.

6. A hydrogen ion moves under the influence of $\vec{E} = 10\hat{x}\ V/m$. Initially the ion is at rest, at $x = 0\hat{x}$ then:

   (a) Over a duration of 15 sec, find the variation in the velocity, position and kinetic energy of the ion with an interval of 0.5 sec using **Simpson's 1/3 Rule.** Also plot the same.

   (b) Find the percentage error in position and kinetic energy at the end positions.

7. A particle of mass 2 Kg is attached with a string of force constant 2.5 N/m is making a horizontal oscillation. The equilibrium position of the particle is 1.5 m. The variation in velocity with time is given by $\vec{v} = 3.5\ sin\omega t\hat{z}$ where $\hat{z}$ and t are measured in m/s and sec. Consider minimum potential energy is 0. Using **Simpson's 1/3 Rule** find:

   (a) Its position with interval $\frac{1}{20}$ of its time period for 3 period.

   (b) Error in percentage at each point.

   (c) Its potential energy at the same point.

Also plot the variation of-

i. Position and potential energy with time.

ii. Potential and Kinetic energy with position from graph. Proof that the total energy is conserved.

8. A particle of mass $m = 15\ gm$ oscillating in a potential well. Its positions are given by $x = 6.5\ sin(15t)$; where x and t are measured in *cm* and *sec*. Using **Newton's Forward difference formula** with four points, find:

   (a) Its velocity with interval of $\frac{1}{20}$ of its period for four time period.

   (b) Error at each point.

   (c) Its kinetic energy at each point. Plot the variation of:

      i. Velocity and kinetic energy with time.

      ii. Velocity with position.

      iii. Error with velocity.

9. A particle of mass $m = 50\ gm$ is moving under the influence of force $F = 0.5\ N$ the initial velocity of the particle is $v_0 = 0.5\ m/s$ observed variation in the position of the particle with time, is given by,

t(s)	0	0.2	0.5	0.8	1.2	1.5	1.7	1.8	2.0
$x(m)$	1.5	1.7	2.8	5.3	8.8	14.1	15.1	17.8	21.5

   (a) Find the trajectory equation using above data using **Least Square Fitting** method.

   (b) Plot observed data and data from fitted equation, also plot theoretically observed data from theoretical equation.

(c) Compare initial position, initial velocity and acceleration with theoretical value.

(d) Find position at time 10 *sec* and compare with theoretical value.

10. Current in a semiconductor diode is given by the following equation:

$$I_D = I_S \left[exp \left(\frac{qV_D}{\eta K_B T} \right) - 1 \right]$$

where I_D is the diode current, $I_S = 1.5\ \mu m$ is the reverse biased saturation current, V_D is the voltage across diode, K_B is Boltzmann constant, q is electric charge and $n = 2$ is the ideality factor. At room temperature $T = 300K$ variation of diode current with applied diode voltage is given by the following table:

$V_D(V)$	0.12	0.15	0.2	0.3
$I_D(mA)$	0.137458	0.0257217	0.0700359	0.492515

$V_D(V)$	0.4	0.45	0.48	0.5
$I_D(mA)$	3.41009	8.9638	16.0062	23.5584

(a) Find the value of diode current using: **Lagrangian Interpo-Interpolation formula** at $V_D = 0.46\ V$, $V_D = 0.19\ V$ and $V_D = 0.1\ V$. Also calculate the percentage error in both case. Discuss the result.

11. Hydrogen ion is moving under the influence of magnetic field ($5\ KiloGauss\ \hat{z}$) in xy plane. It's position is $x = 1.4 - r_L\ sin\omega t$, $y = 2.5 + r_L\ cos\omega t$ where position and time are measured in cm and sec. ω is gyration frequency of the ion. The velocity of ion normal to the magnetic field is $1.5 \times 10^6\ m/s$.

    (a) Using (as a function) **Newton's Forward difference Formula**. Find it's momenta (P_x,P_y) with interval of 50^{th} of it's period over two period.

    (b) Draw -

        i. v_x and v_y in velocity space.

        ii. The trajectory of the ion in the phase space (x, P_x) and (y, P_y) using numerically calculated and theoretical value.

12. The variation of pressure of a gas in an enclosed vessel with respect to temperature is given in the following table :

$T(°C)$	35	40	45	50	55	60	65	70	75
$P(mbar)$	2.5	3.4	4.6	5.7	6.6	7.5	8.4	9.6	10.5

    (a) Find the value of pressure at T =$52.5°C$ and temperature when $P = 7.8$ milibar. Use at least two different method and compare the result.

    (b) Fit the pressure vs. temperature variation curve using suitable polynomial and obtain the interpolation value given in part-(a) of this question. Compare the result with interpolated value.