Angel and Shreiner: Interactive Computer Graphics, Seventh Edition

Chapter 6 Solutions

6.1 Point sources produce a very harsh lighting. Such images are characterized by abrupt transitions between light and dark. The ambient light in a real scene is dependent on both the lights on the scene and the reflectivity properties of the objects in the scene, something that cannot be computed correctly with OpenGL. The Phong reflection term is not physically correct; the reflection term in the modified Phong model is even further from being physically correct.

6.2 With smooth shading, the colors at the vertices are interpolated across the polygon. If the polygon is large, the viewer can be close to points in the center of the polygon and far from the vertices. Consequently, the interpolative value is not close to what the Phong model would place at the center. The solution is to divide the large polygon into smaller polygons, thus placing more vertices near the viewer.

6.3 If we were to take into account a light source being obscured by an object, we would have to have all polygons available so as to test for this condition. Such a global calculation is incompatible with the pipeline model that assumes we can shade each polygon independently of all other polygons as it flows through the pipeline.

6.4 A viewer sees a point on an illuminated surface as a source of light. Hence, the amount of light received by the viewer from each point on the source is inversely proportional to the square of the distance between the viewer and the surface.

6.5 Materials absorb light from sources. Thus, a surface that appears red under white light appears so because the surface absorbs all wavelengths of light except in the red rangea subtractive process. To be compatible with such a model, we should use surface absorption constants that define the materials for cyan, magenta and yellow, rather than red, green and blue.

6.6 See problem 1.2. We also can argue from symmetry as follows. Suppose that we place one of the vertices at $(0, 1, 0)$. We can place the other three in the plane $y = ?d$. If the center of mass of the resulting tetrahedron is to be at the origin, we must have $d = 1/3$. We can place one of the three vertices in this plane at $(0, ?1/3, z)$. If this point is a unit distance from the origin, then $z = 2p2/3$. The final two points are at $(x, ?d, z)$ and $(?x, ?d, z)$. Using the values of z and d, and the requirements that these points be a unit distance from the origin, we find $x = p6/3$.

6.7 Let ψ be the angle between the normal and the halfway vector, ϕ be the angle between the viewer and the reflection angle, and θ be the angle between the normal and the light source. If all the vectors lie in the same plane, the angle between the light source and the viewer can be computer either as $\phi + 2\theta$ or as $2(\theta + \psi)$. Setting the two equal, we find $\phi = 2\psi$. If the vectors are not coplanar then $\phi < 2\psi$.

6.8 If the surface is curved, we have to compute the normal at each point; if it is flat the normal is constant. If both the viewer and the light source are far, we can use the halfway vector, computing it only once. If the source is far and the surface is flat, the diffuse terms are constant.

6.13 Without loss of generality, we can consider the problem in two dimensions. Suppose that the first material has a velocity of light of v_1 and the second material has a light velocity of v_2. Furthermore, assume that the axis $y = 0$ separates the two materials.

Place a point light source at $(0, h)$ where $h > 0$ and a viewer at (x, y) where $y < 0$. Light will travel in a straight line from the source to a point $(t, 0)$ where it will leave the first material and enter the second. It will then travel from this point in a straight line to (x, y). We must find the t that minimizes the time travelled.
Using some simple trigonometry, we find the line from the source to $(t, 0)$ has length $l_1 = \sqrt{h^2 + t^2}$ and the line from there to the viewer has length $l_2 = \sqrt{y^2 + (x - t)^2}$. The total time light travels is thus $\frac{l_1}{v_1} + \frac{l_2}{v_2}$.
Minimizing over t gives desired result when we note the two desired sines are $\sin\theta_1 = \frac{h}{\sqrt{h^2+t^2}}$ and $\sin\theta_2 = \frac{-y}{\sqrt{(y^2+(x-t)^2}}$.

6.14 If we orient the surface so it points in the direction of the halfway vector, then we have equal angles between the halfway vector and the viewer and between the halfway vector and the light source. This configuration is exactly that of a mirror which reflects all the light towards the viewer.

6.19 Shading requires that when we transform normals and points, we maintain the angle between them or equivalently have the dot product $\mathbf{p} \cdot \mathbf{v} = \mathbf{p'} \cdot \mathbf{v'}$ when $\mathbf{p'} = \mathbf{Mp}$ and $\mathbf{n'} = \mathbf{Mp}$. If $\mathbf{M}^T\mathbf{M}$ is an identity matrix angles are preserved. Such a matrix ($\mathbf{M}^{-1} = \mathbf{M}^T$) is called orthogonal. Rotations and translations are orthogonal but scaling and shear are not.

6.21 Probably the easiest approach to this problem is to rotate the given plane to plane $z = 0$ and rotate the light source and objects in the same way. Now we have the same problem we have solved and can rotate everything back at the end.

6.23 A global rendering approach would generate all shadows correctly. In a global renderer, as each point is shaded, a calculation is done to see which light sources shine on it. The projection approach assumes that we can project each polygon onto all other polygons. If the shadow of a given polygon projects onto multiple polygons, we could not compute these shadow polygons very easily. In addition, we have not accounted for the different shades we might see if there were intersecting shadows from multiple light sources.

6.26 The normal at each point on the sphere points from the origin to that point. Given this normal n and the location of the viewer p, we can compute the reflection vector $r = 2(p \cdot n)n - p$ as in Section 5.5. The viewer would see the color of the first opaque object along this reflected vector from the sphere.