

$$\begin{array}{c}
\frac{}{\Gamma, h \Rightarrow \Delta, h} \text{ Logical Axiom} \\
\frac{}{\Gamma, (\ulcorner h \urcorner = \ulcorner h \urcorner), h \Rightarrow \Delta, h} L_{wk} \\
\frac{}{\Gamma, (\ulcorner \psi \urcorner = \ulcorner h \urcorner), h \Rightarrow \Delta, \psi} h = \psi \\
\frac{}{\Gamma, (\ulcorner \psi \urcorner = \ulcorner h \urcorner) \wedge h \Rightarrow \Delta, \psi} R\wedge \\
\frac{}{\Gamma, ((\ulcorner \psi \urcorner = \ulcorner h \urcorner) \wedge h) \vee \tau(T)(\ulcorner \psi \urcorner) \Rightarrow \Delta, \psi} \\
\frac{}{\Gamma, (((\#0 = \ulcorner h \urcorner) \wedge h) \vee \tau(T))(\ulcorner \psi \urcorner) \Rightarrow \Delta, \psi} \text{ definition subst} \\
\frac{}{\Gamma, \tau([h, T])(\ulcorner \psi \urcorner) \Rightarrow \Delta, \psi} \text{ definition } \tau \\
\frac{}{\Gamma \Rightarrow \Delta, \tau([h, T])(\ulcorner \psi \urcorner) \supset \psi} R \supset \\
\hline
\Gamma \Rightarrow \Delta, \tau([h, T])(\ulcorner \psi \urcorner) \leftrightarrow \psi
\end{array}
\qquad
\begin{array}{c}
\frac{}{\Gamma, \perp \Rightarrow \Delta, h} L\perp \\
\frac{}{\Gamma, \tau(T)(\ulcorner h \urcorner) \Rightarrow \Delta, h} \neg h \text{ \underline{in} } T \\
\frac{}{\Gamma, \tau(T)(\ulcorner \psi \urcorner) \Rightarrow \Delta, \psi} h = \psi \\
\frac{}{\Gamma, ((\ulcorner \psi \urcorner = \ulcorner h \urcorner) \wedge h) \vee \tau(T)(\ulcorner \psi \urcorner) \Rightarrow \Delta, \psi} L\vee \\
\frac{}{\Gamma, \tau([h, T])(\ulcorner \psi \urcorner) \Rightarrow \Delta, \psi} \\
\frac{}{\Gamma \Rightarrow \Delta, \tau([h, T])(\ulcorner \psi \urcorner) \leftrightarrow \psi}
\end{array}
\qquad
\begin{array}{c}
\frac{}{\Gamma, h \Rightarrow \Delta, (\ulcorner h \urcorner = \ulcorner h \urcorner)} R = \frac{}{\Gamma, h \Rightarrow \Delta, h} \text{ Logical Axiom} \\
\frac{}{\Gamma, h \Rightarrow \Delta, (\ulcorner h \urcorner = \ulcorner h \urcorner) \wedge h} R\wedge \\
\frac{}{\Gamma, h \Rightarrow \Delta, (\ulcorner h \urcorner = \ulcorner h \urcorner) \wedge h, \tau(T)(\ulcorner h \urcorner)} R_{wk} \\
\frac{}{\Gamma, \psi \Rightarrow \Delta, (\ulcorner \psi \urcorner = \ulcorner h \urcorner) \wedge h, \tau(T)(\ulcorner \psi \urcorner)} h = \psi \\
\frac{}{\Gamma, \psi \Rightarrow \Delta, (\ulcorner \psi \urcorner = \ulcorner h \urcorner) \wedge h \vee \tau(T)(\ulcorner \psi \urcorner)} R\vee \\
\frac{}{\Gamma, \psi \Rightarrow \Delta, ((\#0 = \ulcorner h \urcorner) \wedge h) \vee \tau(T)(\ulcorner \psi \urcorner)} \text{ definition subst} \\
\frac{}{\Gamma, \psi \Rightarrow \Delta, \tau([h, T])(\ulcorner \psi \urcorner)} \text{ definition } \tau \\
\frac{}{\Gamma, \psi \Rightarrow \Delta, \tau([h, T])(\ulcorner \psi \urcorner)} R \supset \\
\frac{}{\Gamma \Rightarrow \Delta, \psi \supset \tau([h, T])(\ulcorner \psi \urcorner)} R \leftrightarrow
\end{array}$$