

Math 7110 – Homework 5 – Due: Oct 15, 2021

Practice Problems:

Problem 1. All the problems you missed on the test.

Definition. Let R be a ring and let I, J be ideals of R . Define IJ to be the ideal generated by $\{fg : f \in I, g \in J\}$ and $I + J$ to be the ideal in R generated by $\{f + g : f \in I, g \in J\}$.

Problem 2. Let R be a ring and let $I, J \subseteq R$ be ideals.

- (1) Is $I + J$ always equal to $\{f + g : f \in I, g \in J\}$? Is IJ always equal to $\{fg : f \in I, g \in J\}$?
- (2) Prove that $I \cap J$ is an ideal containing IJ .
- (3) Let $I = (2, x) \subseteq \mathbb{Z}[x]$ and use this to show that the containment in the previous part may not be strict.
- (4) Prove that if R is commutative and $I + J = R$ then $IJ = I \cap J$.

Test practice:

Problem 3. Determine whether or not $2\mathbb{Z}$ and $3\mathbb{Z}$ are isomorphic as rings.

Problem 4. Dummit and Foote 7.3 question 10.

Type solutions to the following problems in L^AT_EX, and email the tex and PDF files to me at dbernstein1@tulane.edu by 10am on the due date. Please title them as [lastname].tex and [lastname].pdf. When preparing your solutions, you must follow the rules as laid out in the course syllabus.

Graded Problems:

Problem 5. Dummit and Foote 7.3 question 17.

Problem 6. Dummit and Foote 7.3 questions 26 and 28.