

Math 7760 – Homework 1 – Due: August 31, 2022

Practice Problems:

Problem 1. Show that every compact convex set has an extreme point. Give an example of a non-compact convex set with an extreme point.

Problem 2. Prove that $x \in \text{Aff}(x_1, \dots, x_n)$ if and only if

$$\begin{pmatrix} 1 \\ x \end{pmatrix} \in \mathbb{R} \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{pmatrix}$$

and that $x \in \text{Conv}(x_1, \dots, x_n)$ if and only if

$$\begin{pmatrix} 1 \\ x \end{pmatrix} \in \mathbb{R}_{\geq 0} \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{pmatrix}$$

Problem 3. Let $X, Y \subseteq \mathbb{R}^d$ with $|X| = |Y| = d+1$. Assume X, Y are each affinely independent sets. Prove that $\text{Conv}(X)$ and $\text{Conv}(Y)$ are affinely isomorphic.

Type solutions to the following problems in L^AT_EX, and email the tex and PDF files to me at dbernstein1@tulane.edu on the due date. Please title them as [lastname].tex and [lastname].pdf. When preparing your solutions, you must follow the rules as laid out in the course syllabus.

Problems to write up:

Problem 4. Let $P, Q \subset \mathbb{R}^2$ be two-dimensional polytopes (i.e. polygons). For each of the following statements, either prove that they are true, or provide a counterexample.

- (1) If P and Q have the same number of edges, then they are affinely isomorphic.
- (2) If P and Q have the same number of edges, then they are combinatorially isomorphic.
- (3) If P and Q are both triangles, then they are affinely isomorphic.
- (4) If P is a square and Q is a parallelogram, then P and Q are affinely isomorphic. Begin by convincing yourself that it makes no difference if you assume that the vertices of P are $\{0, 1\}^2$ and that $(0, 0)$ is a vertex of Q .