Math 7760 – Homework 5 – Due: October 10, 2022

Practice Problems:

Problem 1. Let G be a graph. Describe the closure operator of $\mathcal{G}$ in graph-theoretic terms.

Problem 2. Oxley section 1.4 problem 6.

Problems to write up:

Problem 3. Prove that a graph with n vertices, c connected components, and at least $n-c+1$ edges has a cycle. Then let G be a graph with edge set E and show that the rank function of its matroid $\mathcal{M}(G)$ has the following rank function

$$\rho(S) = |V(S)| - c(S)$$

where $V(S)$ denotes the set of vertices of G that are incident to some edge in S and $c(S)$ denotes the number of connected components of the graph on vertex set $V(S)$ and edge set S.

Problem 4. Let E be a finite set and let $\mathcal{H} \subseteq 2^E$. Prove that $\mathcal{H}$ is the set of hyperplanes of a matroid if and only if

(1) $E \notin \mathcal{H}$,
(2) if $H_1, H_2 \in \mathcal{H}$ with $H_1 \subseteq H_2$, then $H_1 = H_2$, and
(3) if $H_1, H_2 \in \mathcal{H}$ and $e \notin H_1 \cup H_2$, then there exists $H \in \mathcal{H}$ such that $H \supseteq (H_1 \cap H_2) \cup e$.