MA 242 Section 13 – Test 3 – Review

This review sheet contains questions that are similar to what I will ask on the test. The actual test will have fewer questions.

(1) 4.1.5: 13, 15, 17, 19, 21
(2) 4.2.5: 13, 19, 21, 27. I won't ask you to sketch anything on a test.
(3) 4.3.5: 19, 21, 25, 29
(4) 5.1.1: 1, 3, 14, 15, 19
(5) 5.2.1: 17, 19, 23, 25
(6) 5.3.1: 11, 13, 15, 17
(7) Questions on handwritten sheet

(1) Find the volume bounded above the disc $x^2+y^2 \le 4$ and under the cone $z=\sqrt{x^2+y^2}$

(2) Find the volume bounded above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$

(3) A swimming pool is circular with a 40 foot diameter. The depth is constant along east-west lines and increases linearly from 2 feet at the south end to 7 feet at the north end. Find the volume of water in the pool

(4) Evaluate $\iiint_E 6xy\,dV$ where E lies under the plane $z=1+x+y$ and above the region in the xy-plane bounded by the curves $y=\sqrt{x}$, $y=0$, and $x=1$.

(5) Evaluate $\iiint_E x^2\,dV$ where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$ and below the cone $z^2=4x^2+4y^2$

(6) Find the volume of the solid that lies within both the cylinder $x^2+y^2=1$ and the sphere $x^2+y^2+z^2=4$

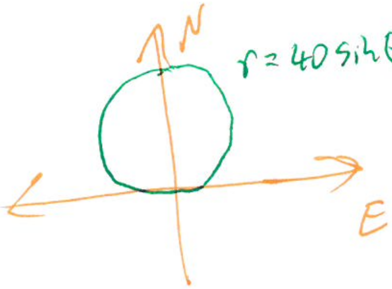
(7) Evaluate $\iiint_E z\,dV$ where E lies between spheres of radius 1 and 2, centered at $(0,0,0)$, in the first octant

(8) Find the volume of the smaller wedge cut from a sphere of radius 2 by two planes that intersect along a diameter at angle $\pi/6$

① $\int_0^{2\pi} \int_0^{2} \int_0^{r} r \, dz \, dr \, d\theta$

② $\int_0^{\pi/4} \int_0^{2\pi} \int_0^{1} \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

③ $\int_0^{\pi} \int_0^{40\sin\theta} \left(\frac{1}{8} r \sin\theta + 2\right) r \, dr \, d\theta$



$r = 40\sin\theta, \quad 0 \le \theta \le \pi$

$\text{depth} = \frac{5}{40} y + 2 = \frac{1}{8} r \sin\theta + 2$

4. $$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy \, dz \, dy \, dx = \frac{65}{28}$$

5. $$\int_0^{2\pi} \int_0^1 \int_0^{2r} r^2 \cos^2\theta \cdot r \, dz \, dr \, d\theta = \frac{2\pi}{5}$$

6. $$\int_0^{2\pi} \int_0^1 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta = \frac{4(8-3\sqrt{3})\pi}{3}$$

7. $$\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi = \frac{7\pi}{6}$$

8. $$\cancel{\int_0^{\pi/6} \int_{-\pi/2}^{\pi/2} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi = \frac{4(2-\sqrt{3})\pi}{3}}$$

Wrong answer

This gives volume cut out by the cone $\phi = \pi/6$, plane $y = 0$ and sphere $\rho = 2$.

$$\int_0^{\pi} \int_0^{\pi/6} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi = \frac{8\pi}{9}$$

Correct answer