DANIEL IRVING BERNSTEIN

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Research keywords: Nonlinear algebra, applied algebraic geometry, convex and discrete geometry, tropical geometry, matrix completion, rigidity theory, algebraic statistics, phylogenetics, graphical models

EDUCATION

2018	Ph.D.	North Carolina State University, Mathematics (advisor: Seth Sullivant)
2015	M.S.	North Carolina State University, Mathematics
2013	B.S.	Davidson College, Mathematics (departmental honors, *magna cum laude*)

EMPLOYMENT

2021 –	Assistant Professor, Tulane University
2021	Jerrold E. Mardsen Postdoc, Fields Institute
2020	Research Associate, Duke University
2018 – 2021	NSF Postdoc, Massachusetts Institute of Technology (Mentor: Caroline Uhler)
2018	ICERM Postdoc, Brown University

HONORS AND AWARDS

2021	Jerrold E. Marsden Postdoctoral Fellowship ($35,000 CAD)
2018	NSF Mathematical Sciences Postdoctoral Research Fellowship ($150,000 USD)
2018	NC State Math Department Winton-Rose Award ($1,000 USD)
2013	North Carolina State University Graduate Fellowship ($4,000 USD)
2013	Davidson College William G. McGavock Mathematics Award
2013	Patterson Prize for talk given at the MAA Southeastern Section Spring Meeting

PAPERS

Submitted papers and preprints

1. The pure condition for incidence geometries (with Signe Lundqvist). arxiv:2507.02661
2. Representability of Flag Matroids (with Nathaniel Vaduthala). arXiv:2504.00267
3. Maximum likelihood thresholds of generic linear concentration models (with Steven J. Gortler and Louis Theran). arXiv:2305.06280.

Journal Papers

4. Maximum likelihood thresholds of Gaussian graphical models and graphical lasso (with Hayden Outlaw). *Communications in Statistics – Simulation and Computation* (2025): 1-6. arXiv:2312.03145.
5. Matroid lifts and representability (with Zach Walsh). *Electronic journal of combinatorics*, Volume 31, Issue 1 (2024). arXiv:2306.12543.
6. Maximum likelihood thresholds via graph rigidity (with Sean Dewar, Steven J. Gortler, Anthony Nixon, Meera Sitharam, and Louis Theran). *Annals of applied probability* 2024, Vol. 34, No. 3, 3288-3319. arXiv:2108.02185.
7. Computing maximum likelihood thresholds using graph rigidity (with Sean Dewar, Steven J. Gortler, Anthony Nixon, Meera Sitharam, and Louis Theran). *Algebraic statistics* 14-2 (2023), 287–305. arXiv:2210.11081.

8. $K_{5,5}$ is fully reconstructible in $\mathbb{C}^3$ (with Steven J. Gortler). *Discrete Applied Mathematics*, 322 (2022): 442-447. arXiv:2110.10224.

9. Generic symmetry-forced infinitesimal rigidity: translations and rotations. *SIAM Journal on Applied Algebra and Geometry*, 6 (2), 190-215. arXiv:2003.10529.

10. Typical ranks in symmetric matrix completion (with Greg Blekherman and Kisun Lee). *Journal of Pure and Applied Algebra*, Volume 225, Issue 7, July 2021. arXiv:1909.06593

11. The algebraic matroid of the funtf variety (with Cameron Farnsworth and Jose Israel Rodriguez). *Journal of Pure and Applied Algebra*, Volume 224, Issue 8, August 2020. arXiv:1812.10353

12. L-infinity optimization to Bergman fans of matroids with an application to phylogenetics. *SIAM Journal on Discrete Mathematics*, 34, no. 1 (2020): 701-720. arXiv:1702.05141

13. The tropical Cayley-Menger variety (with Robert Krone). *SIAM Journal on Discrete Mathematics*, **33** (2019) no.3, pp. 1725–1742. arXiv:1812.09370

14. Typical and Generic Ranks in Matrix Completion (with Greg Blekherman and Rainer Sinn). *Linear algebra and its applications*, Volume 585, 15 January 2020, pp. 71-104. arXiv:1802.09513

15. Unimodular hierarchical models and their Graver bases (with Christopher O'Neill). *Journal of Algebraic Statistics*. **8** (2017) no. 2. arXiv:1704.09018

16. Completion of tree metrics and rank 2 matrices. *Linear Algebra and its Applications*. **533** (2017), pp. 1-13. arXiv:1612.06797

17. L-infinity optimization to linear spaces and phylogenetic trees (with Colby Long). *SIAM Journal on Discrete Mathematics*. **31** (2017) no. 2, pp. 875-889. arXiv:1702.05127

18. Unimodular binary hierarchical models (with Seth Sullivant). *Journal of Combinatorial Theory, Series B*. **123** (2017), pp. 97-125. arXiv:1502.06131

19. Normal binary hierarchical models (with Seth Sullivant). *Experimental Mathematics*. **26** (2017) no. 2 pp. 153-164. arXiv:1508.05461

20. Bounds on the expected size of the maximum agreement subtree (with Lam Si Tung Ho, Colby Long, Mike Steel, Katherine St. John, and Seth Sullivant). *SIAM Journal on Discrete Mathematics*. **29** (2015) no. 4, pp. 2065-2074. arXiv:1411.7338

21. On three sets with nondecreasing diameter (with Carl Yerger and David J. Grynkiewicz). *Discrete Mathematics*. **338** (2015) no. 8, pp. 1328-1344. arXiv:1407.5122

Conference products

22. On Alignment in Deep Linear Neural Networks (with Adit Radhakrishnan, Eshaan Nichani, and Caroline Uhler). Companion paper to a poster presented at *ICML Workshop on "Over-Parameterization: Pitfalls and Opportunities" (ICML 2021)*. arXiv:2003.06340

23. Ordering-based causal structure learning in the presence of latent variables (with Basil Saeed, Chandler Squires, and Caroline Uhler). *The 23rd International Conference on Artificial Intelligence and Statistics (AISTATS 2020)*. arXiv:1910.09014

EXPOSITORY WRITING

1. Flag Matroids. Matroid union blog. February 2025. `http://matroidunion.org/?p=5644`.
2. Algebraic matroids. Matroid union blog. March 2024. `http://matroidunion.org/?p=5320`.
3. Matroid representability and non-rational polytopes. Matroid union blog. May 2023. `http://matroidunion.org/?p=4868`.
4. Matroids on graphs in applied algebraic geometry. Matroid union blog. January 2021. `http://matroidunion.org/?p=3675`.

TALKS

Invited conference and colloquium talks

1. *Rigidity theory and Gaussian models.* April 18, 2025. Matroids, Rigidity, and Algebraic Statistics. ICERM, Brown University.
2. *Realizations of combinatorial incidence geometries.* January 11, 2025. Joint Math Meetings, AMS Special Session on Applications of Algebra and Geometry.
3. *Maximum likelihood thresholds of linear concentration models.* July 13th, 2023. Workshop on Geometric Constraints: Materials, Graphs and Matroids, Rigidity and Packings at the Fields Institute.
4. *Symmetry-forced rigidity.* January 4, 2023. March 18, 2023. AMS Southeastern Sectional in Atlanta. Special session on Combinatorics, Probability and Computation in Molecular Biology.
5. *Maximum likelihood thresholds of Gaussian models.* January 4, 2023. Joint Math Meetings in Boston. AMS Special Session on Applied Enumerative Geometry.
6. *Algebraic matroids and tropical geometry.* December 6, 2022. BIRS workshop 22w5022 on toric degenerations. Banff, Alberta, Canada.
7. *Maximum likelihood thresholds via graph rigidity.* July, 2022. British Combinatorial Conference, minisymposium on Matroids and Combinatorial Geometry, at Lancaster University, England, UK.
8. *Maximum likelihood thresholds via graph rigidity.* May 19, 2022. Algebraic Statistics at University of Hawai'i Manoa, Honolulu, HI.
9. *Rigidity theory for Gaussian graphical models: the maximum likelihood threshold of a graph.* June 10, 2021. Minisymposium on low-rank models and applications virtually "at" the Fields Institute, Toronto, Canada.
10. *Rigidity of plane frameworks with forced symmetry.* March 10, 2021. Pure mathematics colloquium, virtually "at" University of St. Andrews, Scotland.
11. *Generic symmetry-forced infinitesimal rigidity: translations and rotations.* February 24, 2021. Workshop on progress and open problems in rigidity theory, virtually "at" The Fields Institute, Toronto.
12. *Rigidity of symmetry-forced frameworks.* Would have been between June 22-26. Algebraic Statistics 2020 at University of Hawai'i at Manoa. Canceled due to the COVID-19 pandemic, became a contributed online talk.
13. Title not determined. Would have been between June 15-17. FOCM workshop in computational algebraic geometry at Simon Frasier University. Canceled due to the COVID-19 pandemic.
14. *The tropical Cayley-Menger variety.* April 4, 2020. Special Session on Rigidity Theory, Distance Geometry and Applications. AMS Spring Central Sectional Meeting at Purdue University. Canceled due to COVID-19 pandemic. Would have been between March 15-21.
15. *The algebraic matroid of a Hadamard product of linear spaces.* Would have been between March 15-21 at the Oberwolfach workshop, *Algebraic structures in statistical methodology.* Canceled due to COVID-19 pandemic.
16. *Tropical geometry in rigidity theory.* January 13, 2020. Discrete structures, a Heilbronn focused research group. Lancaster University, UK.
17. *Algebraic matroids in rigidity theory and matrix completion.* December 6, 2019. Matroids Day. University of Wisconsin-Madison.
18. *The tropical Cayley-Menger variety.* July 17, 2019. Algebra meets combinatorics in Neuchatel. University of Neuchatel, Switzerland.
19. **The tropical Cayley-Menger variety.* July 9, 2019. SIAM Conference on Applied Algebraic Geometry, minisymposium on algebraic geometry of low-rank matrix completion. Bern, Switzerland (*self-invite, I spoke at the session I organized).
20. *Nonlinear algebra and matrix completion.* November 16, 2018. Nonlinear algebra in applications workshop at Brown University, ICERM.
21. *Using tropical geometry to characterize the algebraic matroid for rank-2 matrix completion.* July 13, 2018. SIAM Annual Meeting, minisymposium on distance geometry. Portland, OR.
22. *Typical and generic ranks in low-rank matrix completion.* April 21, 2018. AMS Spring Eastern Sectional Meeting at Northeastern University. Special Session on Algebraic Statistics.

23. *Tropical linear spaces in phylogenetics.* September 23, 2017. AMS Fall Southeastern Sectional Meeting at University of Central Florida. Special Session on Mathematics of Biomolecules: Discrete, Algebraic, and Topological.

24. *Tropical Geometry for Rigidity Theory and Matrix Completion.* August 1, 2017. SIAM Conference on Applied Algebraic Geometry, minisymposium on algebraic methods in rigidity theory. Atlanta, Georgia

25. *Tropical linear spaces in phylogenetics.* May 27, 2017. Interactions between algebra and the sciences. Max Planck Institute for Mathematics in the Sciences. Leipzig, Germany

26. *Combinatorial properties of hierarchical models.* July 11, 2016. SIAM Annual Meeting, minisymposium on algebraic statistics

27. *Toric Varieties in Statistics.* April 9, 2016. Meeting on Algebraic Geometry for Applications. Clemson University. Clemson, SC.

28. *Hierarchical Models: Normality and Related Properties.* Oct 3, 2015. 2015 AMS fall central sectional meeting, special session on algebraic statistics and its interactions with combinatorics, computation, and network science. Loyola University. Chicago, IL.

Seminar talks and guest lectures

29. Algebra and combinatorics seminar at Tulane University (September 26, 2023)

30. Algebra and combinatorics seminar at Tulane University (October 20 and 27, 2021)

31. Algebra, geometry, and topology seminar at Tulane University (October 4, 2021)

32. Matroid union online seminar (April 19, 2021)

33. Guest lecture in Tony Nixon's graduate course on combinatorial and geometric rigidity virtually "at" the Fields Institute (February 5, 2021)

34. Study Group on Graph Theory, Topology and Algorithms virtually "at" The University of Birmingham (January 25, 2021)

35. Connections between graphical models and rigidity theory "at" The Fields Institute (January 22, 2021)

36. Virtual seminar on algebraic matroids and rigidity theory (July 16, 2020)

37. Virtual seminar on algebraic matroids and rigidity theory (March 26, 2020)

38. Valley geometry seminar at UMass Amherst (October 25, 2019)

39. Discrete math seminar at Texas State University (May 3, 2019)

40. Guest lecture in Ngoc Tran's tropical geometry class at UT Austin (May 2, 2019)

41. Applied algebra and geometry seminar at Massachusetts Institute of Technology (Nov 21, 2017)

42. Algebra, geometry, and combinatorics seminar at San Francisco State University (Oct 19, 2016)

43. Combinatorics, algebra, convexity, algorithms and optimization seminar at UC Davis (Oct 17, 2016)

44. Graduate student algebra seminar at NC State, about once each semester while in grad school

COMMITTEES

1. Math department undergraduate studies committee. August 2023–

EDITORIAL POSITIONS

1. Matroid Union blog editor. January 2023–

EVENT ORGANIZING

1. SIAM Minisymposium on Combinatorial Algebraic Geometry and Rigidity Theory at 2023 TX-LA sectional meeting. Lafayette, LA. Nov 4, 2023.

2. Workshop on geometric constraints: materials, graphs and matroids, rigidity and packings. Fields Institute, Toronto CA. July 10-14 2023.

3. Workshop at Tulane for math graduate students on how to plan your graduate school career with an eye towards job market success. January 11, 2023.

4. Special session co-organizer (with Zvi Rosen) at the Southeastern International Conference on Combinatorics, Graph Theory, and Computing: *Matroids and rigidity theory*. March 7-11, 2022

5. Organizer of the Virtual Seminar on Algebraic Matroids and Rigidity Theory. Spring 2020-Fall 2020

6. SIAM minisymposium co-organizer (with Carlos Améndola): *Algebraic geometry of low rank matrix completion* at the SIAM Conference on Applied Algebraic Geometry. Bern, Switzerland, July 9, 2019.

7. Co-organizer (with Diego Cifuentes) of the MIT IDSS seminar in Algebra, Statistics and Optimization. Spring 2019-

8. Co-organizer (with Greg Blekherman and Rainer Sinn) of a working group in matrix completion at Brown University, ICERM (Fall 2018)

9. AMS special session co-organizer (with Nathaniel Bushek and Mateja Raic): *AMS Special Session on Algebraic Statistics (a Mathematics Research Communities Session)* at the Joint Mathematics Meetings. Atlanta GA, January 5, 2017.

10. Conference co-organizer (with Eva Brayfindley, Amanda Laubmeier, Molly Lynch, and Kristen Moody): *Second Triangle Area Math Graduate Conference*. Raleigh NC, October 24, 2015.

11. Conference co-organizer (with Eva Brayfindley, Amanda Laubmeier, Molly Lynch, Kristen Moody, Radmila Sazdanovic, and Seth Sullivant): *First Triangle Area Math Graduate Conference*. Raleigh NC, March 21, 2015.

TEACHING EXPERIENCE

Tulane University Department of Mathematics

1. *Topology 2* (graduate level): Spring 2024
2. *Topology I* (graduate level): Fall 2023
3. *Linear algebra*: Fall 2023, Fall 2024
4. *Calculus 3*: Spring 2023
5. *Geometric Combinatorics* (graduate topics course): Fall 2022
6. *Abstract Algebra 2* (graduate level): Spring 2022
7. *Abstract Algebra 1* (graduate level): Fall 2021

Duke University Department of Mathematics

1. *Real Analysis* (co-instructor with Mike Reed and Inmaculada C. Sorribes): Fall 2020

North Carolina State University Mathematics Department

1. *Calculus III*: Spring 2016, Fall 2016, Spring 2017, Fall 2017
2. *Calculus for Life and Management Sciences* (Recitation Leader): Spring 2014 and Fall 2014
3. *Applied Differential Equations* (Lecture Assistant): Fall 2013

STUDENTS

Graduate

1. Aalliyah Celestine, Tulane University. Fall 2024–
2. Nathaniel Vaduthala, Tulane University. Spring 2023–

Undergraduate

1. Julian Huddell, Tulane University. Spring 2024–
2. Mackenzie Bookamer, Tulane University. Spring 2023–Spring 2024.
3. Hayden Outlaw, Tulane University. Fall 2021–Spring 2024.
4. Ian Limarta, MIT. Summer 2019.