Math 7120 – Homework 1 – Due: February 2, 2022

Practice problems:

Problem 1. Dummit and Foote, 10.1 problems 1 and 3.

Test prep:

Problem 2. Let $n \geq 2$ be an integer and let R be the ring of $n \times n$ matrices with entries in $\mathbb{C}$. Let M_l and M_r be the module structures on R defined by left and right multiplication (i.e. given $r, m \in R$, then multiplying r by m is rm when considering m as an element of M_l, and mr when considering m as an element of M_r). Let Z be the subset of R consisting of all matrices whose last $n-1$ columns are zero. Determine whether N is a submodule of M_l and M_r.

Problem 3. Determine which of the statements below are true. For those that are false, provide a counterexample:

(1) if R is a subring of S and M is an S-module, then M is an R-module
(2) if R is a subring of S and M is an R-module, then M is an S-module.

Type solutions to the following problems in LaTeX, and email the tex and PDF files to me at `dbernstein1@tulane.edu` by 10am on the indicated date. Please title them as [lastname].tex and [lastname].pdf. When preparing your solutions, you must follow the rules as laid out in the course syllabus.

Graded Problems:

Problem 4. The *annihilator* of an R-module M is defined to be

$$\{r \in R : rm = 0 \text{ for all } m \in M\}.$$

(1) Prove that the annihilator of an R-module is a two-sided ideal of R.
(2) Let M be a finitely generated abelian group, viewed as a $\mathbb{Z}$ module. What is the annihilator of M?

Problem 5. Let $F = \mathbb{R}$ and $V = \mathbb{R}^2$. Recall that each linear transformation $T : V \to V$ gives rise to an $F[t]$-module. For each of the following values of T, determine all $F[t]$-submodules of V:

(1) T is $90°$ clockwise rotation about the origin
(2) T is orthogonal projection onto the y-axis
(3) T is $180°$ clockwise rotation about the origin.