Math 7120 – Homework 10 – Due: April 18, 2022

Practice problems:

Problem 1. Let K/F and K'/F' be field extensions and let $\phi : K \to K'$ be a field isomorphism such that $\phi(F) \subseteq \phi(F')$. Consider the map $\psi : \mathrm{Aut}(K/F) \to \mathrm{Aut}(K'/F')$ given by $\psi(\sigma) = \phi \circ \sigma \circ \phi^{-1}$. Prove that ψ is a group isomorphism.

Problem 2. Read the proof of Theorem 9 in 14.2.

Test prep:

Problem 3. Are $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ isomorphic?

Problem 4. Explicitly determine the automorphisms of the following field extensions:

(1) $\mathbb{C}/\mathbb{R}$
(2) $\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q}(\sqrt{2})$
(3) $\mathbb{Q}(\sqrt{2}, \sqrt{3})/\mathbb{Q}$

Type solutions to the following problems in LaTeX, and email the tex and PDF files to me at `dbernstein1@tulane.edu` by 10am on the indicated date. Please title them as [lastname].tex and [lastname].pdf. When preparing your solutions, you must follow the rules as laid out in the course syllabus.

Graded Problems: