MATH 7760 – GEOMETRIC COMBINATORICS

DANIEL IRVING BERNSTEIN

A polytope is the convex hull of a finite set of points in $\mathbb{R}^n$. They were studied by ancient cultures and are important in a wide array of fields today, from optimization to commutative algebra. This course will focus on the combinatorial aspects of polytopes. We will also learn about matroids, which are a combinatorial abstraction of linear independence that appear in rigidity theory, graph theory, tropical geometry, combinatorial optimization, and other areas of math. Highlights of the course include:

- techniques for visualizing polytopes in four and more dimensions
- a graph is 3-connected and planar if and only if it is the vertex-edge graph of a three-dimensional polytope (this result is often called "Steinitz's Theorem")
- proofs that certain sets of geometric constraints (e.g. certain subsets of points must lie on the same lower-dimensional subspace) are impossible to satisfy
- connections to optimization.

I hope that by the end of this class, you have:

- gained a lot of factual knowledge of polytopes and matroids and their uses
- sharpened your geometric and combinatorial intuition through many attempts (both successful and unsuccessful) to prove theorems, find counterexamples to false statements, and determine whether a given true-seeming statement is actually true
- improved your mathematical writing skills by submitting typed solutions to weekly problem sets
- engaged in frequent mathematical discussions with your classmates, e.g. while working together on problem sets.

Textbook

This course will draw from two textbooks that are both available (print and e-book) in the math library. I *highly* recommend that you read the textbook to reinforce what you learn in class for three reasons:

(1) the more different ways you take in the same information, the better you will learn,
(2) reading math is a skill that gets better with practice, and
(3) reading *well-written* math will improve your mathematical writing.

Resources and accommodations for student needs

I will make every reasonable effort to accommodate your needs including, but not limited to, religious observances, disabilities, and health (physical and mental). If you require accommodations of any kind, including deadline extensions, please let me know as soon as you are aware of a need. For disability accommodations, I may ask you to register with the Goldman Center for Student Accessibility (URL below).

You may find some of the following Tulane-wide resources useful:

- Goldman Center for Student Accessibility: `https://accessibility.tulane.edu`

- Center for Academic Equity: `https://academic-equity.tulane.edu/`
- Counseling Center: `https://campushealth.tulane.edu/counseling-center`
- Title IX Office: `https://allin.tulane.edu/titleix`

What to call me

In increasing order of formality, the names you can call me are: "Dan," "Daniel," "Dr. Bernstein," and "Professor Bernstein." I use he/him pronouns.

Logistics

When: MWF 1:00PM
Where: Gibson Hall 310
Office hours: Monday 2:00-4:00, also by appointment
Office: Gibson Hall 401A
Contact: dbernstein1@tulane.edu
Course website: `https://dibernstein.github.io/teaching/geometricCombinatorics.html`

Assessment

Problem sets. There will be roughly one problem set assigned each week and you will be responsible for turning in solutions to a subset of them. They will be posted on the course website. Solutions should be typed in LaTeX and written in a way that your classmates would be able to follow. Collaboration with your classmates is encouraged, but you must type up you own solutions in your own words. You must also clearly indicate who you collaborated with and on which problems. You are permitted, and moreover encouraged, to use computational algebra software such as Gap, Macaulay2, Maple, and Sage. You are prohibited from using resources beyond the textbook, computational algebra software, each other, and me, to find solutions to homework problems.

Grades. First, it probably goes without saying that the point of taking a class is to learn, not to get a particular grade. Moreover, almost nobody will care about the grades you get in graduate school. That said, the academic system as it currently functions requires me to assign grades. Your final numerical grade will be the average grade of your homework assignments. At the end of the semester, a conversion from numerical grades to letter grades will be established based on average class performance.