MA 242 Section 13 – Test 1 – Review

This review sheet contains questions that are similar to what I will ask on the test. The actual test will have fewer questions.

Vectors, lines and planes:

(1) 1.1.4: 4, 13, 15
(2) 1.2.7: 16, 19
(3) 1.3.4: 3, 5, 7, 9, 17, 19
(4) 1.4.4: 1, 3, 5, 9
(5) 1.5.4: 1, 2, 5, 6, 7, 11, 12, 13, 15, 16, 17
(6) For the lines given by the following equations, determine whether or not they intersect. If they do, find the point of intersection.
   - $\langle 1,3,1\rangle + t\langle 2,2,1\rangle$ and $\langle 3,-1,5\rangle + s\langle 4,-2,5\rangle$
   - $\langle 3,2,1\rangle + t\langle 5,-4,3\rangle$ and $\langle 4,4,3\rangle + s\langle -10,8,-6\rangle$.
   - $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$.

Curves:

(1) Section 2.2.5 problems 1, 3, 7, 15, 21, 23, 25, 27
(2) Section 2.3.4 problems 7, 9, 11, 13, 15
(3) Section 2.4.3 problems 7, 9, 21
(4) Let $\vec{r}(t) = \langle t^2, t\rangle$. I did the following calculations for you:

$$\begin{aligned} T(t) &= \langle \frac{2t}{\sqrt{1+4t^2}}, \frac{1}{\sqrt{1+4t^2}}\rangle \\ \frac{dT}{dt} &= \langle 2(4t^2+1)^{-3/2}, -4t(4t^2+1)^{-3/2}\rangle \\ N(t) &= \langle \frac{2}{\sqrt{(4+16t^2)}}, \frac{-4t}{\sqrt{(4+16t^2)}}\rangle \\ \kappa(t) &= \frac{2}{(4t^2+1)^{3/2}}. \end{aligned}$$

Now find an equation for the osculating circle to $\vec{r}(t)$ at the point $(1,1)$.

Fun stuff: We did not directly discuss how to do the following two problems but you might enjoy figuring them out (don't worry, you won't need to be able to do them on the test).

(1) 1.3.4: 12, 20
(2) Find the distance from the point $(1,2,3)$ to the plane with equation $x+y+z=2$.
(3) Find the distance from the point $(1,1,1)$ to the line with parametric equations $x = 3+t$, $y = 1-t$, $z = t$ (hint: one way to do this is as a minimization problem (which will involve a derivative)).
(4) Let $\vec{r}(t)$ be a vector valued function such that $\|\vec{r}(t)\| = 1$ for all t. Show that $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$ for all t.