IV. Trajectory without drift

$restart : with(Physics) : with(Physics[Vectors]) : with(LinearAlgebra) : Setup(mathematicalnotation = true) :$

Making some assumptions

$Setup\left(realobjects = \left\{e, m, c, \varepsilon_0\right\}\right) :$
$Setup(realobjects = \{t, x(t), \Omega(t)\});$

$$\left[realobjects = \left\{\hat{i}, \hat{j}, \hat{k}, \hat{\phi}, \hat{r}, \hat{\rho}, \hat{\theta}, c, e, m, \phi, r, \rho, t, \theta, x, y, z, \varepsilon_0, \Omega(t), x(t)\right\}\right] \quad (1)$$

Cliford conjugation

$CliffordConj := (X) \rightarrow \langle \langle X[2,2] | -X[1,2] \rangle, \langle -X[2,1] | X[1,1] \rangle \rangle :$

Define a contravariant vector v from a paravector P

$DefContrVect := (v, P) \rightarrow Define\left(v_{\sim mu} = Simplify\left(Array\left(1..4, (l) \rightarrow \frac{1}{2} \cdot Trace(P.Psigma[l])\right)\right)\right) :$

Extract the 3D vector from a paravector P

$ExtractVector := (P) \rightarrow \frac{1}{2} \cdot Simplify(Vector(3, (l) \rightarrow Trace(Psigma[l].P)).Vector([\_i, \_j, \_k])) :$

Extract the electric field vector from the Faraday matrix

$GetElectricField := (F) \rightarrow \frac{1}{2} \cdot Simplify(Vector(3, (l) \rightarrow \mathrm{Re}(Trace(Psigma[l].F))).Vector([\_i, \_j, \_k])) :$

Extract the magnetic field vector from the Faraday matrix

$GetMagneticField := (F) \rightarrow \frac{1}{2 \cdot c} \cdot Simplify(Vector(3, (l) \rightarrow \mathrm{Im}(Trace(Psigma[l].F))).Vector([\_i, \_j, \_k])) :$

Extracting the scalar and vector parts

$ScalarPart := (L) \rightarrow \frac{(L + CliffordConj(L))}{2} :$

$$VectorPart := (L) \rightarrow \frac{(L - CliffordConj(L))}{2} :$$

Extract the Poynting vector

$$PoyntingCliford := (F) \rightarrow \frac{c \cdot \varepsilon_0}{2} \cdot VectorPart(\ F.Dagger(F)\ ) :$$

$$PoyntingVector := (F) \rightarrow ExtractVector(PoyntingCliford(F)) :$$

The radiated energy per unit of time is given by Larmor's formula

$$PowerEmitted := (U) \rightarrow \frac{-e^2}{6 \cdot c \cdot \mathrm{Pi} \cdot \varepsilon_0} \cdot \frac{w^2}{2} \cdot Trace(diff(U, t).ClifordConj(diff(U, t))) :$$

The Lorentz factor

local γ :

$$\gamma := \left(1 - \left(\frac{diff(x(t), t)}{c}\right)^2\right)^{-\frac{1}{2}};$$

$$\gamma := \frac{1}{\sqrt{1 - \frac{\dot{x}(t)^2}{c^2}}} \qquad \mathbf{(2)}$$

Set the weight

$w := \gamma$:

Specifying the eigenspinor

$$R := c \cdot t \cdot Matrix(Psigma[0]) + x(t) \cdot Matrix(Psigma[1]);$$

$$R := \begin{bmatrix} c\,t & x(t) \\ x(t) & c\,t \end{bmatrix} \qquad \mathbf{(3)}$$

$$U := \frac{\gamma}{c} \cdot diff(R, t) :$$

$$\Lambda := MatrixFunction(U, \mathrm{sqrt}(z), z).MatrixExponential\left(-\frac{I \cdot \Omega(t) \cdot Matrix(Psigma[2])}{2}\right) :$$

The electromagnetic field (w is a parametric measure)

$$\mathrm{F} := \mathrm{w} \cdot \frac{2 \cdot m \cdot c}{e} \cdot diff(\Lambda, t).ClifordConj(\Lambda) :$$

Extract the electromagnetic fields from the matrix F

$E\_ := GetElectricField(F)$:

$'E\_[1]' = simplify(Component(E\_, 1))$ assuming $c > 0$ **and** $0 \le \dot{x}(t) < c$;

$$\vec{E}_1 = -\frac{c^3\, m\, \ddot{x}(t)}{\sqrt{\dot{x}(t) + c}\, \left(\dot{x}(t)^2 - c^2\right) \sqrt{c - \dot{x}(t)}\; e} \tag{4}$$

$'E\_[1]' = simplify(Component(E\_, 1))$ assuming $c > 0$ **and** $-c < \dot{x}(t) \le 0$;

$$\vec{E}_1 = -\frac{c^3\, m\, \ddot{x}(t)}{\sqrt{\dot{x}(t) + c}\, \left(\dot{x}(t)^2 - c^2\right) \sqrt{c - \dot{x}(t)}\; e} \tag{5}$$

$'E\_[2]' = simplify(Component(E\_, 2))$;

$$\vec{E}_2 = 0 \tag{6}$$

$'E\_[3]' = simplify(Component(E\_, 3))$;

$$\vec{E}_3 = -\frac{\dot{x}(t)\, \dot{\Omega}(t)\, m\, |c|^2}{\left(\dot{x}(t)^2 - c^2\right) e} \tag{7}$$

$B\_ := GetMagneticField(F)$:

$'B\_[1]' = simplify(Component(B\_, 1))$ assuming $c > 0$ **and** $0 \le \dot{x}(t) < c$;

$$\vec{B}_1 = 0 \tag{8}$$

$'B\_[1]' = simplify(Component(B\_, 1))$ assuming $c > 0$ **and** $-c < \dot{x}(t) \le 0$;

$$\vec{B}_1 = 0 \tag{9}$$

$'B\_[2]' = simplify(Component(B\_, 2))$;

$$\vec{B}_2 = \frac{\dot{\Omega}(t)\, c^2\, m}{\left(\dot{x}(t)^2 - c^2\right) e} \tag{10}$$

$'B\_[3]' = simplify(Component(B\_, 3))$;

$$\vec{B}_3 = 0 \tag{11}$$

$Simplify(PowerEmitted(U))$;

$$-\frac{\ddot{x}(t)^2\, c^3\, e^2}{6\, \varepsilon_0\, \pi \left(\dot{x}(t)^2 - c^2\right)^3} \tag{12}$$