# III. Particle in an electromagnetic plane wave

$restart : with(Physics) : with(Physics[Vectors]) : with(LinearAlgebra) : Setup(mathematicalnotation = true) :$

Making some assumptions

$Setup(realobjects = \{c, e, m\}) :$

$Setup\left(realobjects = \left\{\text{xi, tau}, \Omega(\xi), \eta(\xi), w(\xi), v_x(\xi), v_y(\xi)\right\}\right);$

$$\left[realobjects = \left\{\hat{i}, \hat{j}, \hat{k}, \hat{\phi}, \hat{r}, \hat{\rho}, \hat{\theta}, c, e, m, \phi, r, \rho, \tau, \theta, x, \xi, y, z, \Omega(\xi), \eta(\xi), v_x(\xi), v_y(\xi), w(\xi)\right\}\right] \quad \mathbf{(1)}$$

Cliford conjugation

$CliffordConj := (X) \rightarrow \langle \langle X[2,2] | -X[1,2] \rangle, \langle -X[2,1] | X[1,1] \rangle \rangle :$

Define a contravariant vector $v$ from a paravector $P$

$DefContrVect := (v, P) \rightarrow Define\left(v_{\sim mu} = Simplify\left(Array\left(1..4, (l) \rightarrow \frac{1}{2} \cdot Trace(P.Psigma[l])\right)\right)\right) :$

Extract the electric field vector from the Faraday matrix

$GetElectricField := (F) \rightarrow Simplify\left(Vector\left(3, (l) \rightarrow \frac{1}{2} \operatorname{Re}(Trace(Psigma[l].F))\right).Vector([\_i, \_j, \_k])\right) :$

Extract the magnetic field vector from the Faraday matrix

$GetMagneticField := (F) \rightarrow Simplify\left(Vector\left(3, (l) \rightarrow \frac{1}{2 \cdot c} \operatorname{Im}(Trace(Psigma[l].F))\right).Vector([\_i, \_j, \_k])\right) :$

**(2)**

Specifying the eigenspinor

$\Lambda := MatrixExponential\left(\frac{\eta(\xi)}{2}\cdot Matrix(Psigma[3])\right).Matrix\left(\left[\left[1, \frac{v_x(\xi)}{c} - \frac{I\cdot v_y(\xi)}{c}\right], [0, 1]\right]\right)$
$.MatrixExponential\left(-I\cdot\frac{\Omega(\xi)}{2}\cdot Matrix(Psigma[3])\right);$

$$\Lambda := \left[\left[e^{\frac{\eta(\xi)}{2}}\left(\cos\left(\frac{\Omega(\xi)}{2}\right) - I\sin\left(\frac{\Omega(\xi)}{2}\right)\right), e^{\frac{\eta(\xi)}{2}}\left(\frac{v_x(\xi)}{c} - \frac{I\,v_y(\xi)}{c}\right)\left(\cos\left(\frac{\Omega(\xi)}{2}\right) + I\sin\left(\frac{\Omega(\xi)}{2}\right)\right)\right], \left[0, e^{-\frac{\eta(\xi)}{2}}\left(\cos\left(\frac{\Omega(\xi)}{2}\right) + I\sin\left(\frac{\Omega(\xi)}{2}\right)\right)\right]\right] \quad (3)$$

Extracting proper velocity

$U := simplify(\Lambda.Dagger(\Lambda));$

$$U := \begin{bmatrix} \frac{e^{\eta(\xi)}\left(v_y(\xi)^2 + v_x(\xi)^2 + c^2\right)}{c^2} & \frac{-I\,v_y(\xi) + v_x(\xi)}{c} \\ \frac{I\,v_y(\xi) + v_x(\xi)}{c} & e^{-\eta(\xi)} \end{bmatrix} \quad (4)$$

Define contravariant vector of the velocity

$DefContrVect(u, U);$

*Defined objects with tensor properties*

$$\left\{\gamma_{\mu}, \sigma_{\mu}, \partial_{\mu}, g_{\mu,\nu}, u^{\mu}, \delta_{\mu,\nu}, \epsilon_{\alpha,\beta,\mu,\nu}\right\} \quad (5)$$

$combine(expand(u[\sim 3]));$

$$\frac{e^{\eta(\xi)}\,v_y(\xi)^2}{2\,c^2} + \frac{e^{\eta(\xi)}\,v_x(\xi)^2}{2\,c^2} + \frac{e^{\eta(\xi)}}{2} - \frac{e^{-\eta(\xi)}}{2} \quad (6)$$

The electromagnetic field (w is a parametric measure)

$F := w(\xi)\cdot\frac{2\cdot m\cdot c}{e}\cdot diff(\Lambda, \xi).ClifordConj(\Lambda) :$

Extract the electromagnetic fields  from the matrix F

$E\_ := GetElectricField(F);$

$$\vec{E} := \frac{1}{e}\left(\left(\left(v_y(\xi)\, e^{\frac{\eta(\xi)}{2}} \left(\frac{d}{d\xi}\,\Omega(\xi)\right)\hat{i} - v_x(\xi)\, e^{\frac{\eta(\xi)}{2}} \left(\frac{d}{d\xi}\,\Omega(\xi)\right)\hat{j} + c\, e^{-\frac{\eta(\xi)}{2}} \left(\frac{d}{d\xi}\,\eta(\xi)\right)\hat{k} + \left(\frac{d}{d\xi}\,v_y(\xi)\right) e^{\frac{\eta(\xi)}{2}}\,\hat{j} + \left(\frac{d}{d\xi}\,v_x(\xi)\right) e^{\frac{\eta(\xi)}{2}}\,\hat{i}\right) e^{\frac{\eta(\xi)}{2}}\, w(\xi)\, m\right)\right)$$

$Simplify(Component(\vec{E}, 1));$

$$\frac{\left(v_y(\xi)\left(\frac{d}{d\xi}\,\Omega(\xi)\right) + \frac{d}{d\xi}\,v_x(\xi)\right) e^{\eta(\xi)}\, w(\xi)\, m}{e} \quad \textbf{(8)}$$

$Simplify(Component(\vec{E}, 2));$

$$-\frac{\left(v_x(\xi)\left(\frac{d}{d\xi}\,\Omega(\xi)\right) - \left(\frac{d}{d\xi}\,v_y(\xi)\right)\right) e^{\eta(\xi)}\, w(\xi)\, m}{e} \quad \textbf{(9)}$$

$combine(Component(\vec{E}, 3));$

$$\frac{w(\xi)\, m\, c\left(\frac{d}{d\xi}\,\eta(\xi)\right)}{e} \quad \textbf{(10)}$$

$B\_ := GetMagneticField(F);$

$$\vec{B} := \frac{1}{e\,c}\left(\left(\left(v_y(\xi)\, e^{\frac{\eta(\xi)}{2}} \left(\frac{d}{d\xi}\,\Omega(\xi)\right)\hat{j} + v_x(\xi)\, e^{\frac{\eta(\xi)}{2}} \left(\frac{d}{d\xi}\,\Omega(\xi)\right)\hat{i} - c\, e^{-\frac{\eta(\xi)}{2}} \left(\frac{d}{d\xi}\,\Omega(\xi)\right)\hat{k} - \left(\frac{d}{d\xi}\,v_y(\xi)\right) e^{\frac{\eta(\xi)}{2}}\,\hat{i} + \left(\frac{d}{d\xi}\,v_x(\xi)\right) e^{\frac{\eta(\xi)}{2}}\,\hat{j}\right) e^{\frac{\eta(\xi)}{2}}\, w(\xi)\, m\right)\right) \quad \textbf{(11)}$$

$Simplify(Component(\vec{B}, 1));$

$$\frac{\left(v_x(\xi)\left(\frac{d}{d\xi}\,\Omega(\xi)\right) - \left(\frac{d}{d\xi}\,v_y(\xi)\right)\right) e^{\eta(\xi)}\, w(\xi)\, m}{e\,c} \quad \textbf{(12)}$$

$Simplify(Component(\vec{B}, 2));$

$$\frac{\left(v_y(\xi)\left(\frac{d}{d\xi}\Omega(\xi)\right)+\frac{d}{d\xi}v_x(\xi)\right)e^{\eta(\xi)}\,w(\xi)\,m}{e\,c} \tag{13}$$

$combine\left(Component\left(\vec{B}, 3\right)\right);$

$$-\frac{w(\xi)\,m\left(\frac{d}{d\xi}\Omega(\xi)\right)}{e} \tag{14}$$

Energy

$Simplify\left(\frac{m\cdot c^2}{2}\cdot Trace(U.Psigma[0])\right);$

$$\frac{m\left(e^{\eta(\xi)}\left(v_y(\xi)^2+v_x(\xi)^2+c^2\right)+e^{-\eta(\xi)}c^2\right)}{2} \tag{15}$$