IV. Trajectory without drift

$restart : with(Physics) : with(Physics[Vectors]) : with(LinearAlgebra) : Setup(mathematicalnotation = true) :$

Making some assumptions

$Setup(realobjects = \{e, m, c\}) :$
$Setup(realobjects = \{\mathrm{t}, \mathrm{x}(\mathrm{t}), \Omega(t)\});$

$$[realobjects = \{\hat{i}, \hat{j}, \hat{k}, \hat{\phi}, \hat{r}, \hat{\rho}, \hat{\theta}, c, e, m, \phi, r, \rho, t, \theta, x, y, z, \Omega(t), x(t)\}] \quad \mathbf{(1)}$$

Cliford conjugation

$CliforConj := (X) \rightarrow \langle\langle X[2,2] | -X[1,2]\rangle, \langle -X[2,1] | X[1,1]\rangle\rangle :$

Define a contravariant vector v from a paravector P

$DefContrVect := (v, P) \rightarrow Define\left(v_{\sim \mathrm{mu}} = Simplify\left(Array\left(1..4, (l) \rightarrow \frac{1}{2} \cdot Trace(P.Psigma[l])\right)\right)\right) :$

Extract the electric field vector from the Faraday matrix

$GetElectricField := (F) \rightarrow Simplify\left(Vector\left(3, (l) \rightarrow \frac{1}{2} \operatorname{Re}(Trace(Psigma[l].F))\right).Vector([\_i, \_j, \_k])\right) :$

Extract the magnetic field vector from the Faraday matrix

$GetMagneticField := (F) \rightarrow Simplify\left(Vector\left(3, (l) \rightarrow \frac{1}{2 \cdot c} \operatorname{Im}(Trace(Psigma[l].F))\right).Vector([\_i, \_j, \_k])\right) :$

The radiated energy per unit of time is given by Larmor’s formula

$PowerEmitted := (U) \rightarrow \frac{e^2}{6 \cdot c \cdot \mathrm{Pi} \cdot \varepsilon_0} \cdot \frac{w}{2} \cdot Trace(diff(U, t).CliforConj(diff(U, t))) :$

The Lorentz factor

local γ :

$$\gamma := \left(1 - \left(\frac{diff(x(t), t)}{c}\right)^2\right)^{-\frac{1}{2}};$$

$$\gamma := \frac{1}{\sqrt{1 - \frac{\dot{x}(t)^2}{c^2}}} \tag{2}$$

Set the weight

$w := \gamma$:

Specifying the eigenspinor

$R := c \cdot t \cdot Matrix(Psigma[0]) + x(t) \cdot Matrix(Psigma[1]);$

$$R := \begin{bmatrix} c\,t & x(t) \\ x(t) & c\,t \end{bmatrix} \tag{3}$$

$U := \frac{\gamma}{c} \cdot diff(R, t)$:

$\Lambda := MatrixFunction(U, \mathrm{sqrt}(z), z).MatrixExponential\left(-\frac{I \cdot \Omega(t) \cdot Matrix(Psigma[2])}{2}\right)$:

The electromagnetic field (w is a parametric measure)

$\mathrm{F} := \mathrm{w} \cdot \frac{2 \cdot m \cdot c}{e} \cdot diff(\Lambda, t).CliffordConj(\Lambda)$:

Extract the electromagnetic fields from the matrix F

$E\_ := GetElectricField(F)$:

$'E\_[1]' = simplify(Component(E\_, 1))$ assuming $c > 0$ **and** $0 \leq \dot{x}(t) < c$;

$$\vec{E}_1 = -\frac{c^3\, m\, \ddot{x}(t)}{\sqrt{\dot{x}(t) + c}\, \left(\dot{x}(t)^2 - c^2\right) \sqrt{c - \dot{x}(t)}\; e} \tag{4}$$

$'E\_[1]' = simplify(Component(E\_, 1))$ assuming $c > 0$ **and** $-c < \dot{x}(t) \leq 0$;

$$\vec{E}_1 = -\frac{c^3\, m\, \ddot{x}(t)}{\sqrt{\dot{x}(t) + c}\, \left(\dot{x}(t)^2 - c^2\right) \sqrt{c - \dot{x}(t)}\, e} \tag{5}$$

'E_[2]'= *simplify*(*Component*(E_, 2));

$$\vec{E}_2 = 0 \tag{6}$$

'E_[3]'= *simplify*(*Component*(E_, 3));

$$\vec{E}_3 = -\frac{\dot{x}(t)\, \dot{\Omega}(t)\, m\, |c|^2}{\left(\dot{x}(t)^2 - c^2\right) e} \tag{7}$$

*B_* ≔ *GetMagneticField*(*F*) :

'B_[1]'= *simplify*(*Component*(B_, 1)) assuming $c > 0$ **and** $0 \leq \dot{x}(t) < c$;

$$\vec{B}_1 = 0 \tag{8}$$

'B_[1]'= *simplify*(*Component*(B_, 1)) assuming $c > 0$ **and** $-c < \dot{x}(t) \leq 0$;

$$\vec{B}_1 = 0 \tag{9}$$

'B_[2]'= *simplify*(*Component*(B_, 2));

$$\vec{B}_2 = \frac{\dot{\Omega}(t)\, c^2\, m}{\left(\dot{x}(t)^2 - c^2\right) e} \tag{10}$$

'B_[3]'= *simplify*(*Component*(B_, 3));

$$\vec{B}_3 = 0 \tag{11}$$