Particle in an electromagnetic plane wave

$restart : with(Physics) : with(Physics[Vectors]) : Setup(mathematicalnotation = true) :$

Making some assumptions

$Setup(realobjects = \{c, e, m\}) :$

$Setup\left(realobjects = \left\{\mathrm{eta}, \mathrm{xi}, \mathrm{tau}, G, v_x(\mathrm{xi}), v_y(\mathrm{xi})\right\}\right);$

$$\left[realobjects = \left\{G, \hat{i}, \hat{j}, \hat{k}, \hat{\phi}, \hat{r}, \hat{\rho}, \hat{\theta}, c, e, \eta, m, \phi, r, \rho, \tau, \theta, x, \xi, y, z, v_x(\xi), v_y(\xi)\right\}\right] \quad (1)$$

$PDEtools[declare]\left(\left(v_x, v_y\right)(\mathrm{xi})\right);$

$$v_x(\xi) \textit{ will now be displayed as } v_x$$

$$v_y(\xi) \textit{ will now be displayed as } v_y \quad (2)$$

Cliford conjugation

$CliffordConj := (X) \rightarrow \langle\langle X[2, 2] | -X[1, 2]\rangle, \langle -X[2, 1] | X[1, 1]\rangle\rangle;$

$$CliffordConj := X \mapsto \left\langle\left\langle X_{2, 2} \middle| -X_{1, 2}\right\rangle, \left\langle -X_{2, 1} \middle| X_{1, 1}\right\rangle\right\rangle \quad (3)$$

Extract the electric field vector from the Faraday matrix

$$GetElectricField := (F) \rightarrow Simplify\left(Vector\left(3, (l) \rightarrow \frac{1}{2}\,\mathrm{Re}(Trace(Psigma[l].F))\right).Vector([\_i, \_j, \_k])\right) :$$

Extract the magnetic field vector from the Faraday matrix

$$GetMagneticField := (F) \rightarrow Simplify\left(Vector\left(3, (l) \rightarrow \frac{1}{2 \cdot c}\,\mathrm{Im}(Trace(Psigma[l].F))\right).Vector([\_i, \_j, \_k])\right) :$$

Specifying the eigenspinor

$$\Lambda := Matrix\left(\left[\left[\exp\left(\frac{\mathrm{eta}}{2}\right), 0\right], \left[0, \exp\left(-\frac{\mathrm{eta}}{2}\right)\right]\right]\right).Matrix\left(\left[\left[1, v_x(\mathrm{xi}) - \mathrm{I} \cdot v_y(\mathrm{xi})\right], [0, 1]\right]\right);$$

$$\Lambda := \begin{bmatrix} e^{\frac{\eta}{2}} & e^{\frac{\eta}{2}}\left(v_x - I\, v_y\right) \\ 0 & e^{-\frac{\eta}{2}} \end{bmatrix} \tag{4}$$

The electromagnetic field (G is a parametric measure)

$$\mathrm{F} := \mathrm{G}\cdot\frac{2\cdot m\cdot c}{e}\cdot \mathit{diff}(\Lambda, \mathrm{xi}).\mathit{ClifordConj}(\Lambda);$$

$$F := \begin{bmatrix} 0 & \dfrac{2\, G\, m\, c \left(e^{\frac{\eta}{2}}\right)^2 \left(v_{x_\xi} - I\, v_{y_\xi}\right)}{e} \\ 0 & 0 \end{bmatrix} \tag{5}$$

(6)

Extract the elecromegnetic fields from the matrix F

$\mathit{GetElectricField}(F);$

$$\frac{\left(v_{x_\xi}\,\hat{i} + v_{y_\xi}\,\hat{j}\right) e^{\eta}\, G\, c\, m}{e} \tag{7}$$

$\mathit{GetMagneticField}(F);$

$$\frac{\left(v_{x_\xi}\,\hat{j} - v_{y_\xi}\,\hat{i}\right) e^{\eta}\, G\, m}{e} \tag{8}$$

Fix the parametric weight G

(9)

$\mathit{simplify}(\mathit{subs}(G = c\cdot\exp(-\mathrm{eta}), \mathit{GetElectricField}(F)));$

$$\frac{\left(v_{x_\xi}\,\hat{i} + v_{y_\xi}\,\hat{j}\right) c^2\, m}{e} \tag{10}$$

$\mathit{simplify}(\mathit{subs}(G = c\cdot\exp(-\mathrm{eta}), \mathit{GetMagneticField}(F)));$

$$\frac{\left(v_{x_\xi}\,\hat{j} - v_{y_\xi}\,\hat{i}\right) c\, m}{e} \tag{11}$$

$G := c\cdot\exp(-\mathrm{eta});$

$$G := c\, \mathrm{e}^{-\eta} \tag{12}$$

Extracting proper velocity

$u := simplify(\Lambda.Dagger(\Lambda));$

$$u := \begin{bmatrix} \mathrm{e}^{\eta}\left(v_x^{\,2} + v_y^{\,2} + 1\right) & v_x - \mathrm{I}\, v_y \\ v_x + \mathrm{I}\, v_y & \mathrm{e}^{-\eta} \end{bmatrix} \tag{13}$$

Extract trajectory

$Define\left(trajectory[\mathrm{mu}] = Int\left(\frac{c}{2 \cdot G} \cdot Trace(u.Psigma[\mathrm{mu}]), \mathrm{xi}\right)\right);$

Defined objects with tensor properties

$$\left\{\gamma_{\mu}, \sigma_{\mu}, \partial_{\mu}, g_{\mu,\nu}, \delta_{\mu,\nu}, \epsilon_{\alpha,\beta,\mu,\nu}, trajectory_{\mu}\right\} \tag{14}$$

the parameter ξ corresponds to the forward ligh-cone variable

$simplify(combine(trajectory[0] - trajectory[3]));$

$$\int 1\, \mathrm{d}\xi \tag{15}$$

Energy

$\frac{m \cdot c^2}{2} \cdot Trace(u.Psigma[0]);$

$$\frac{m\, c^2 \left(\mathrm{e}^{\eta}\left(v_x^{\,2} + v_y^{\,2} + 1\right) + \mathrm{e}^{-\eta}\right)}{2} \tag{16}$$