Econometrics Recitation Session 3

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Q3.7

Note that $PX = X(X'X)^{-1}X'X = X$ and $MX = (I - P)X = 0$ from the lectures.
But note that $PX \equiv P[X_1\ X_2] = [PX_1\ PX_2] = X \equiv [X_1\ X_2]$.
A similar argument with M helps us to prove that $PX_1 = X_1$ and $MX_1 = 0$□

Q3.8

First note that P is idempotent i.e. $P^2 = P$. This implies that $MM = (I - P)(I - P) = I - 2P + P^2 = I - 2P + P = I - P = M\square$

Q3.9

Some properties of the trace function.

- tr(A-B)=tr(A)-tr(B) **try to prove these**
- tr(AB)=tr(BA)
- tr(I_n)=n

Now note that

$$
\begin{aligned}
tr(M) &= tr(I_n - P) \\
&= tr(I_n) - tr(P) \\
&= n - tr(X(X'X)^{-1}X') \\
&= n - tr(X'X(X'X)^{-1}) \\
&= n - tr(I_k) \\
&= n - k
\end{aligned}
$$

$\square$

Q3.11

When X contains a constant then by the property that $X'e = 0$, we have $\frac{1}{n}\sum_{i=1}^{n} \hat{e}_i = 0$.
This implies that

$$\begin{aligned}
\bar{y} &= \frac{1}{n}\sum_{i=1}^{n} y_i \\
&= \frac{1}{n}\sum_{i=1}^{n}[\hat{y}_i + \hat{e}_i] \\
&= \frac{1}{n}\sum_{i=1}^{n}\hat{y}_i + \frac{1}{n}\sum_{i=1}^{n}\hat{e}_i \\
&= \frac{1}{n}\sum_{i=1}^{n}\hat{y}_i
\end{aligned}$$

$\square$