

$$\begin{aligned}
 &= - \left[\frac{e^{jn_0 n} - e^{-jn_0 n}}{2j} \right] \\
 &= e \frac{e^{jn_0 n}}{2j} - e \frac{e^{-jn_0 n}}{2j} \\
 &= \frac{2\pi \delta(n+n_0)}{2j} - \frac{2\pi \delta(n-n_0)}{2j} \\
 &= \frac{\pi \delta(n+n_0)}{j} - \frac{\pi \delta(n-n_0)}{j} \\
 &= \pi j \delta(n-n_0) - j\pi \delta(n+n_0).
 \end{aligned}$$

(4) Not done.

(5) $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[k] e^{j \frac{2\pi}{N} kn}$.

(6) infinite duration \rightarrow Appendix.
(DTFT).

(7) NOT done.

(8) $\text{un}(n) = \frac{e^{jn_0 n} + e^{-jn_0 n}}{2}$

$\xrightarrow{\text{DTFT}}$

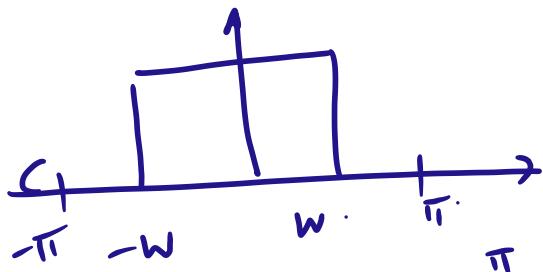
$1 \rightarrow 2\pi \delta(n)$

$e^{jn_0 n} \rightarrow 2\pi \delta(n-n_0)$

$\frac{2\pi \delta(n-n_0) + 2\pi \delta(n+n_0)}{2}$

$= \pi \delta(n-n_0) + \pi \delta(n+n_0)$.

(9)



$$\begin{aligned}
 \text{IDFT} \quad x(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} x(w) e^{jnw} dw \\
 &= \frac{1}{2\pi} \int_{-w}^w e^{jnw} dw = \frac{1}{2\pi} \left. \frac{e^{jnw}}{jn} \right|_{-w}^w \\
 &= \frac{2}{2\pi} \left[\frac{e^{jnw} - e^{-jnw}}{2jn} \right] \\
 &= \frac{1}{\pi n} \sin(nw)
 \end{aligned}$$

(10)

DCT matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & w_N^{-1} \end{bmatrix}$$

$$\begin{aligned}
 w_N^{-1} &= e^{-j\frac{2\pi}{N}nK} \\
 N=2 &= e^{-j\frac{2\pi}{2} \cdot 1} \\
 &= e^{-j\pi} \\
 &= -1
 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Solutions (official)

$$(5) \quad x(n) = u\left(n + \frac{N-1}{2}\right) - u\left(n - \frac{N+1}{2}\right)$$

$$y(n) = u(n) - u(n-N).$$

IDFT

$$e^{-jn\left(\frac{n-1}{2}\right)} \frac{\sin\left(\frac{\pi n}{2}\right)}{\sin(n/2)}$$

$$x(n) = y\left(n + \frac{N-1}{2}\right) = n\left(n + \frac{N-1}{2}\right) - n\left(n - \frac{N+1}{2}\right)$$

$$e^{jn\frac{N-1}{2}} e^{-jn\left(\frac{N-1}{2}\right)} \frac{\sin(\pi n/2)}{\sin(\pi/2)} = \frac{\sin(\pi n/2)}{\sin(\pi/2)}.$$

(7) $y[n] = \sum_{k=-\infty}^n x(k) = x(k) * u(k)$

$$\begin{aligned} y(n) &= x(n) * u(n) \\ &= x(n) \left[\pi \delta(n) + \frac{1}{1 - e^{-jn}} \right] \\ &= \left[x(n) \pi \delta(n) + \frac{x(n)}{1 - e^{-jn}} \right] \end{aligned}$$