① $e^{i2\pi f_0 t}$ is eigen fn of LTI system.

② not done.

③ not done.

④ $T(x(t)) = x(t)\, e^{2\pi f_c t}$

Op 1 $\quad x(t) \to \boxed{\tau} \to x(t-\tau) \to \boxed{T(\cdot)} \to y_1(t)$

$x(t) \to \boxed{\tau} \to x(t-\tau) \to \boxed{T(\cdot)} \to x(t-\tau)\, e^{2\pi f_c t} = y_1(t)$

Op 2 $\quad x(t) \to \boxed{T(\cdot)} \to y(t) \to \boxed{\tau} \to y(t-\tau)$

$x(t) \to \boxed{T(\cdot)} \to x(t)\, e^{2\pi f_c t} \to \boxed{\tau} \to x(t-\tau)\, e^{2\pi f_c (t-\tau)} = y_2(t)$

$y_1(t) \neq y_2(t) \to$ TV ✓

Linearity

Op 1

$x_1(t) \xrightarrow{\times a_1}$, $x_2(t) \xrightarrow{\times a_2}$ $\to \Sigma \to a_1 x_1(t) + a_2 x_2(t) \to \boxed{T(\cdot)}$

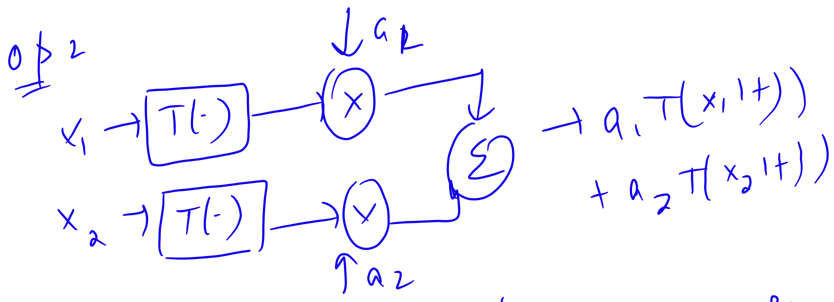
$\to T(\underbrace{a_1 x_1(t) + a_2 x_2(t)}_{x'(t)})$

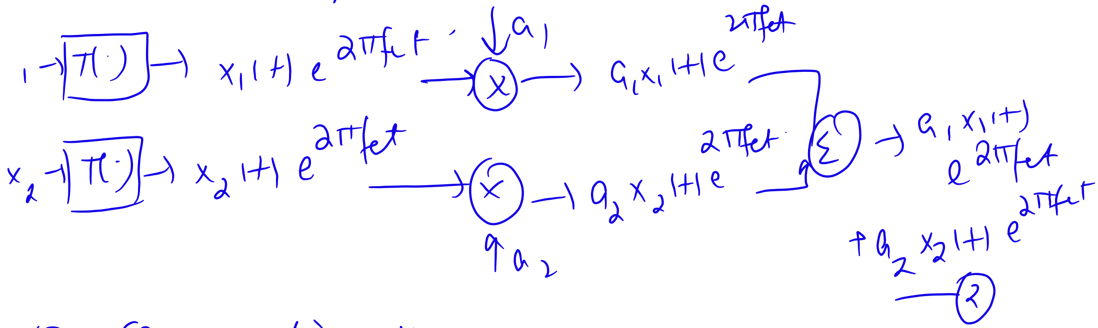
$a_1 x_1(t) + a_2 x_2(t) \to \boxed{T(\cdot)} \to (a_1 x_1(t) + a_2 x_2(t))\, e^{2\pi f_c t}$

$= a_1 x_1(t) + a_2 x_2 \cancel{(t)\, e}^{2\pi f_c t}$

$e^{2\pi f_c t}$

(1)





(1) = (2) ∴ linear

linear but time variant

(5) linear TIV systems has to satisfy Additivity, homogeneity and time invariance properties

(6) not done

(7) not done

(8) $x_i(t) \to \boxed{T(\cdot)} \to y_i(t)$

$\sum_{i}^{N} x_i(t) \to \boxed{T(\cdot)} \to \sum_{1}^{N} y_i(t)$

Additivity

(9) $e^{\sigma t}$

$x(t) = x_e(t) + x_o(t)$

$x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$= \dfrac{e^{\sigma t} + e^{-\sigma t}}{2}$

$= \cosh(\sigma t)$ ✓

(10) memoryless ✓ Causal

$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$

(2) $e^{i3\pi/7 t} + e^{i5\pi/9 t}$

$T_1 = \frac{2\pi}{3\pi/7} = 14/3$ $\qquad T_2 = \frac{2\pi}{5\pi/9} = \frac{18}{5}$

$T_1 = \frac{14}{3} \times 15 = 70$ $\qquad T_2 = 54$

$\text{Lcm}(70, 54) = \underline{1890}$

$\text{Lcm}(14/3, 18/5) = \frac{1890}{15}$

$= 126$ ✓

(3) $x(t) = \begin{cases} 1, & -1 \le t < 0 \\ -1, & 0 \le t < 1 \\ 0, & \text{else} \end{cases}$

$h(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{else} \end{cases}$

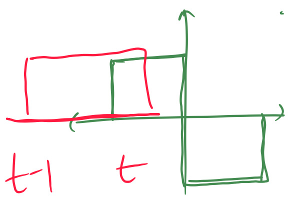
For LTI sys.

$y(t) = x(t) * h(t)$

$h(t) = u(t) - u(t-1)$

$y(t) = \int_{t-1}^{t} x(\tau)\, d\tau$ ✓

$(t-1, t)$ overlap with $(-1, 1)$.



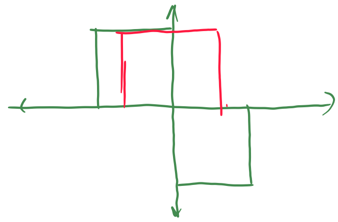
$-1 \le t \le 0$

$\int_{-1}^{t} x(\tau)\, dt$

$= \int_{-1}^{t} 1\, dt$

$= t+1$

$\frac{dy}{dt} = 1$. — (1)



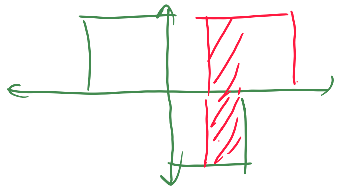
$\int_{t-1}^{0} 1\, dt + \int_{0}^{t} (-1)\, dt$

$= t\Big|_{t-1}^{0} + -t\Big|_{0}^{t}$

$= 0 - (t-1) + (-t)$

$= 1 - 2t$

$\frac{dy}{dt} = -2$ ✓



$$\int_{t-1}^{t+1} -1\, d\tau$$

$$= -|t|_{t-1}^{t+1} = -\left[t+1-(t-1)\right]$$

$$= -\left[-1-t+1\right]$$

$$= t-2$$

$$\frac{dy}{dt} = 1.$$

Max at $t=0$

$y(0) = 1.$

⑥ $x(n) = \alpha^n u(n)$

$h(n) = \beta^n u(-n)$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$x(k) \neq 0 \quad , k \geq 0$

$h(n-k) \neq 0 \quad n-k \leq 0$

$n \leq k \quad k \geq n.$

$K \geq \max(0, n)$

$$y(n) = \sum_{k=\max(0,n)}^{\infty} \alpha^k \beta^{n-k} = \beta^n \sum_{k=\max(0,n)} \left(\frac{\alpha}{\beta}\right)^k.$$

Geometric series

if $|\alpha/\beta| < 1.$

if

$n \geq 0.$

lower limit $k=n$

$$y(n) = \beta^n \sum_{k=n}^{\infty} (\alpha/\beta)^k.$$

$$= \beta^n \frac{(\alpha/\beta)^n}{1-(\alpha/\beta)}$$

$$= \frac{\alpha^n}{1-(\alpha/\beta)}$$

$n<0$

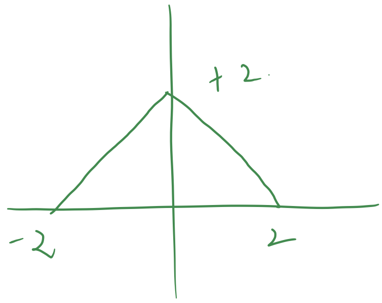
lower limit $k=0.$

$$y(n) = \beta^n \sum_{k=0}^{\infty} (\alpha/\beta)^k$$

$$= \frac{\beta^n}{1-(\alpha/\beta)}$$

⑦. $x(t) = 2-|t|.$ $\quad =$

$h(t) = u(t+3) - u(t-3)$



$$y(t) = x(t) * h(t)$$

$$= \int_{t-3}^{t+3} x(\tau)\, d\tau$$

$h(t)$ will cover $(-1, 1)$.

$$y(t) = \int_{-1}^{1} (2 - |\tau|)\, d\tau$$

for full overlap

$t - 3 \leq -1, \quad t + 3 \geq 1$

$t \leq 2, \quad t \geq -2$

$-2 \leq t \leq 2$.

Peak value.

$$\int_{-1}^{1} (2 - |\tau|)\, d\tau$$

$$= 2 \int_{0}^{1} (2 - \tau)\, d\tau$$

$$= 2 \; 2\tau\Big|_0^1 - \frac{\tau^3}{2}\Big|_0^1$$

$$= 2\left(2 - \tfrac{1}{2}\right)$$

$$= 3.$$

Peak value of 3 occurs at
$-2 \leq t \leq 2$.