$$= -\left[\frac{e^{j\Omega_0 n} - e^{-j\Omega_0 n}}{2j}\right].$$

$$= \frac{e^{-j\Omega_0 n}}{2j} - \frac{e^{j\Omega_0 n}}{2j}$$

$$= \frac{2\pi\delta(\Omega+\Omega_0)}{2j} - \frac{2\pi\delta(\Omega-\Omega_0)}{2j}$$

$$= \frac{\pi\delta(\Omega+\Omega_0)}{j} - \frac{\pi\delta(\Omega-\Omega_0)}{j}$$

$$= +\pi j\,\delta(\Omega-\Omega_0) - j\pi\delta(\Omega+\Omega_0).$$

(4) Not done.

(5) $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n]\, e^{j\frac{2\pi}{N}kn}.$

(6) infinite duration $\longrightarrow$ aperiodic.

(DTFT).

(7) Not done.

(8) $\cos(\Omega_0 n) = \frac{e^{j\Omega_0 n} + e^{-j\Omega_0 n}}{2}.$

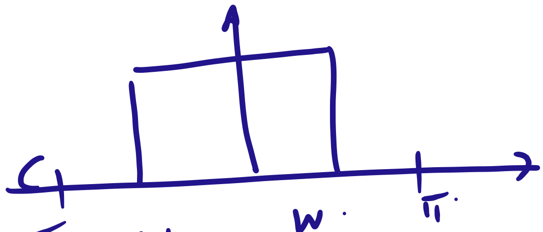
$$\xrightarrow{DTFT} \frac{2\pi\delta(\Omega-\Omega_0) + 2\pi\delta(\Omega+\Omega_0)}{2}$$

$1 \longrightarrow 2\pi\delta(\Omega)$

$e^{j\Omega_0 n} \longrightarrow 2\pi\delta(\Omega-\Omega_0)$

$$= \pi\delta(\Omega-\Omega_0) + \pi\delta(\Omega+\Omega_0).$$

(9)



IDTFT $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w) e^{jwn} dw$

$= \frac{1}{2\pi}\int_{-W}^{W} e^{jwn} dw = \frac{1}{2\pi} \frac{e^{jwn}}{jn}\Big|_{-W}^{W}$

$= \frac{2}{2\pi}\left[\frac{e^{jnW} - e^{-jWn}}{2jn}\right]$

$= \frac{1}{\pi n} \sin(nW)$

(10) DFT matrix $\begin{bmatrix} 1 & 1 \\ 1 & W_N^1 \end{bmatrix}$

$W_N^1 = e^{-j\frac{2\pi}{N}nk}$

$N=2$ $= e^{-j\frac{2\pi}{2}\cdot 1}$ $= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$= e^{-j\pi}$

$= -1$

Solutions (official).

(4) $x(n) = u\left(n + \frac{N-1}{2}\right) - u\left(n - \frac{N+1}{2}\right)$

$y(n) = u(n) - u(n-N)$.

$\Updownarrow$ DTFT

$e^{-jw\left(\frac{N-1}{2}\right)} \frac{\sin\left(\frac{wN}{2}\right)}{\sin(w/2)}$

$$x(n) = y\left(n + \frac{N-1}{2}\right) = u\left(n + \frac{N-1}{2}\right) - u\left(n - \frac{N+1}{2}\right)$$

$$e^{j\Omega \frac{N-1}{2}} e^{-j\Omega\left(\frac{N-1}{2}\right)} \frac{\sin(\Omega N/2)}{\sin(\Omega/2)} = \frac{\sin(\Omega N/2)}{\sin(\Omega/2)}.$$

(7) $$y[n] = \sum_{k=-\infty}^{n} x(k) = x(k) * u(k)$$

$$Y(\Omega) = X(\Omega)U(\Omega)$$

$$= X(\Omega)\left[\pi\delta(\Omega) + \frac{1}{1-e^{-j\Omega}}\right]$$

$$= \left[X(\Omega)\pi\delta(\Omega) + \frac{X(\Omega)}{1-e^{-j\Omega}}\right]$$

$$=$$