Assignment 7.

(1) anticausal.

$|z| < r_{min}$

Causal

$|z| > r_{max}$

(2) $\dfrac{2z^2 - 11z}{z^2 - z - 6} = \dfrac{N(z)}{D(N)}$

$\text{order}(N(z)) = \text{order}(D(z))$.

$$z^2 - z - 6 \overline{)\, 2z^2 - 11z \,}\,(\, 2$$
$$2z^2 - 2z - 12$$
$$- \quad + \quad +$$
$$-9z \quad + 12.$$

$$x(z) = 2 + \underbrace{\frac{(-9z + 12)}{z^2 - z - 6}}_{x_1(z)}.$$

$$x_1(z) = \frac{-9z + 12}{z^2 - z - 6}$$

$$= -\frac{9z + 12}{z^2 - 3z + 2z - 6}$$

$$= -\frac{3(3z - 4)}{z(z-3) + 2(z-3)}$$

$$= \frac{-3(3z-4)}{(z+2)(z-3)}$$

$$= \frac{c_1}{z+2} + \frac{c_2}{z-3}$$

$$c_1 = x_1(z)(z+2)\Big|_{z=-2}$$

$$= \frac{-9z + 12}{(z-3)}\Big|_{z=-2}$$

$$= \frac{18+12}{-5}$$

$$= -6$$

$$c_2 = X_1(z)(z-3)\Big|_{z=3}$$

$$= \frac{-9z+12}{z+2}\Big|_{z=3}$$

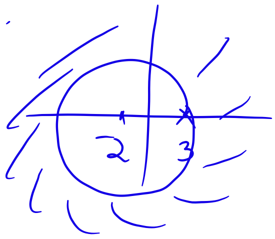
$$= \frac{-27+12}{5}$$

$$= -\frac{\cancel{15}^{\,3}}{\cancel{5}} = -3$$

$$X_1(z) = \frac{-6}{z+2} - \frac{3}{z-3}$$

$$\therefore X(z) = 2 - \frac{6}{z+2} - \frac{3}{z-3}$$

Causal $|z| > 3$.



$$X_1(z) = \underbrace{\frac{-6}{z+2}}_{X_2(z)} - \frac{3}{z-3}$$

$$X_2(z) = \frac{-6}{z+2}$$

$$x(n) = (-2)^n u(n) \longrightarrow \frac{z}{z+2}$$

$$(-2)^{n-1} u(n-1) \longrightarrow \frac{z^{-1} \cdot z}{z+2}$$

$$= \frac{1}{z+2}$$

$$\therefore x_2(n) = -6(-2)^{n-1} u(n-1)$$

III ry

$$\frac{-3}{z-3} \rightarrow -3(+3)^{n-1}u(n-1)$$

$$\therefore x(n) = 2\delta(n) - 6(-2)^{n-1}u(n-1) - 3(3)^{n-1}u(n-1)$$

Alternate approach.

$$\frac{2z^2-11z}{z^2-z-6} = \frac{Az}{z-3} + \frac{Bz}{z+2}$$

$$2z^2-11z = Az(z+2) + Bz(z-3)$$
$$= Az^2 + Bz^2 + 2Az - 3Bz$$
$$= (A+B)z^2 + (2A-3B)z$$

$A+B=2$, $2A-3B=-11$

$2(2-B)-3B=-11$.

$4-2B-3B=-11$

$-5B=-15$

$B=3$

$A=2-3=-1$

$$X(z) = \frac{-z}{z-3} + \frac{3z}{z+2}$$

$$x(n) = -(3)^n u(n) + 3(-2)^n u(n).$$

3 $\frac{z^3}{z-a}$

$$z^2\left(\frac{z}{z-a}\right)$$

$$a^n u(n) \longleftrightarrow \frac{z}{z-a}$$

$$x(n-n_0) \quad z^{-n_0}\frac{z}{z-a}$$

$n_0=-2$

$$x(n+2) \longrightarrow a^{n+2}u(n+2)$$

for causal ie $|z|>|a|$.

But for anti causal

if $\underbrace{-a^n u(-n-1)}_{x(n)} \leftrightarrow \frac{z}{z-a}$, $|z|<|a|$

$x(n) = -a^n u(-n-1)$.

$x(n-n_0) = -a^{n-n_0} u(-(n-n_0)-1)$

$= -a^{n+2} u(-(n+2)-1)$

$= -a^{n+2} u(-n-3)$.

(4) Not done.

(5) for BIBO stability, ROC of z transform must include unit circle.

(6) $\frac{2z^2-5z}{z^2-5z+6} = \frac{Az}{z-3} + \frac{Bz}{z-2}$

$2z^2-5z = Az(z-2) + Bz(z-3)$

$= Az^2 - 2Az + Bz^2 - 3Bz$

$A+B=2, \quad -2A-3B=-5$

$2A+2B=4$

$-2A-3B=-5$

$-B=-1$

$\boxed{B=1}$

$A=1$

$= \frac{z}{z-3} + \frac{z}{z-2}$

LHS RSS

ROC required $2 < |a| 3$.

$= -(3)^n u(-n-1) + (2)^n u(n)$.

(7) Not Done

(8) $\frac{-z}{z+a}$. $|z|<|a|$

wkt

$-a^n u(-n-1) \rightarrow \frac{z}{z-a}, \; |z|<|a|$

$\boxed{a^n u(-n-1)} \rightarrow \frac{-z}{z-a} \quad |z|<|a|$

$(-a)^n u(-n-1) \rightarrow \frac{-z}{z+a} \quad |z|<|a|.$

(9) Not done

(10) for stable system, unit circle will be included.

$$\frac{3z^2-19z}{z^2-z-12} = \frac{3z^2-19z}{z^2-4z+3z-12}$$

$$= \frac{Az}{(z-4)} + \frac{Bz}{(z+3)}$$

$3z^2+19z \quad = Az^2+3Az + Bz^2-4Bz$

$A+B=3 \} \times 3$

$3A-4B=-19$

$\cancel{3A}+3B=9$

$\cancel{3A}-4B=-19$

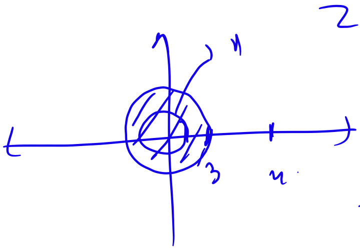
$- \quad + \quad +$

$7B=28$

$B=4.$

$A=3-4=-1.$

$$= \frac{z}{z-4} + \frac{4z}{z+3}$$



$= -(4)^n u(-n-1)$

$- 4\cdot(-3)^n u(-n-1)$

Solution 4,7,9 (official)

(9) $\lambda_{r-k} = \frac{1}{k!} \frac{d^k}{dz^k} (z-p_i)^r \frac{X(z)}{z} \Big|_{z=p_i}$

$k=r-1$

$$\lambda_1 = \frac{1}{(r-1)!} \frac{d^{r-1}}{dz^{r-1}} (z-p_i)^r \frac{x(z)}{z} \Big|_{z=p_i}$$

(7) $\sin(\omega_0 n)u(n) \rightarrow \dfrac{\sin(\omega_0) z}{z^2 - 2\cos\omega_0 z + 1}$

$$\frac{z}{z^2-\sqrt{2}z+1} = \sqrt{2} \frac{\sin \pi/4 \, z}{z^2 - 2\cos \pi/4 \, z + 1}, \quad |z|>1$$

inverse z tru

$$= \sqrt{2} \sin(\pi/4 \, n) \, u(n)$$

(4)

$e^{az^{-1}}$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

$$e^{az^{-1}} = 1 + az^{-1} + \frac{a^2 z^{-2}}{2!} + \cdots$$

$$= \frac{1}{n!} a^n u(n)$$