Week 7

Fourier transform

↳ F.T is defined for a continuous aperiodic signal

$$X(\omega) = F\{x(t)\}$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Inverse Fourier Transform.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$= F^{-1}(X(\omega))$$

$x(t),\ X(\omega) \rightarrow$ Fourier transform pair.

$$\boxed{x(t) \leftrightarrow X(\omega)}$$

Fourier Spectrum

$$X(\omega) = |X(\omega)|\, e^{j\phi(\omega)}$$

$$\phi(\omega) = \measuredangle X(\omega).$$

$|X(\omega)|$ = Magnitude spectrum of $x(t)$

→ versus ω.

$\measuredangle X(\omega) = \phi(\omega)$ = Phase spectrum of $x(t)$.

For real signals $x(t)$,

$$X(-\omega) = \int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, dt$$

$$(X(-\omega))^* = \left(\int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, dt\right)^*$$

$$= \int_{-\infty}^{\infty} \underbrace{x^*(t)}_{= x(t) \text{ for real signal}} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\underbrace{\qquad\qquad\qquad}_{X(\omega)}$$

$$\therefore \boxed{X^*(-\omega) = X(\omega)}$$

For real signal $x(t)$

$\Rightarrow |X(-\omega)| = |X(\omega)|$

$\Rightarrow$ Magnitude spectrum = even fxn of ω.

$\Rightarrow \phi(-\omega) = -\phi(\omega)$

$\Rightarrow$ Phase spectrum = odd fxn of ω

Conditions for convergence of $X(\omega)$:

Dirichlet conditions

Sufficient but not necessary

(1) $x(t)$ is absolutely integrable

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

finite qty

(2) $x(t)$ has a finite # of maxima & minima in any finite interval.

(3) $x(t)$ has a finite # of discontinuity in any finite interval and is finite at each of these discontinuity

Relation b/w Fourier and Laplace transform

$\rightarrow$ Fourier transform.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

Laplace transform

$s = \sigma + j\omega$ — σ: Real part, $j\omega$: Imaginary part

$$X(s) = X(\sigma + j\omega)$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma + j\omega)t}\, dt$$

$$= \int_{-\infty}^{\infty} \underbrace{x(t)\, e^{-\sigma t}\, e^{-j\omega t}\, dt}_{\text{Fourier transform of } x(t)\, e^{-\sigma t}}$$

$$\mathcal{L}\{x(t)\} = F\{x(t) e^{-\sigma t}\}$$

$s = \sigma + j\omega$

If $\sigma = 0$, $\Rightarrow s = j\omega$

$$X(j\omega) = F\{x(t)\}$$

Fourier transform at ω can be obtained by setting $s = j\omega$ in the Laplace transform.

<u>Note</u>:
This is only true if the signal $x(t)$ is <u>absolutely integrable</u>

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

Fourier transform of common <u>signals</u>

<u>UNIT IMPULSE FUNCTION</u>

$\delta(t)$

$$\mathcal{L}(\delta(t)) = 1$$

$$F(\delta(t)) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt$$

$$= e^{-j\omega t}\Big|_{t=0}$$

$\mathcal{F}(\delta(t)) = 1.$

This is similar to Laplace transform.

$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

absolutely integrable

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FOURIER TRANSFORM.

Exponential signal-

$x(t) = e^{-at} u(t).$

$a > 0$

absolutely integrable signal.

$$\mathcal{L}\{e^{-at} u(t)\} = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+s)t}\, dt$$

$$= \left. \frac{e^{-(a+s)}}{-(a+s)} \right|_{0}^{\infty}$$

$$= \frac{0 - 1}{-(s+a)}$$

$$= \frac{1}{(s+a)} \quad \text{If } Re\{s+a\} > 0$$

$\Rightarrow Re\{s\} > -a.$

ROC.

includes $Re\{s\} = 0$

$s = \sigma + j\omega$

$\sigma = 0$

$s = j\omega$ belongs to the ROC if $a > 0$.

Fourier transform.

Fourier Transform

$$= \int_0^{\infty} e^{-at} e^{-j\omega t} \, dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^{\infty}$$

$$= \frac{0 - 1}{-(a+j\omega)}$$

$$X(\omega) = \frac{1}{(a+j\omega)}$$

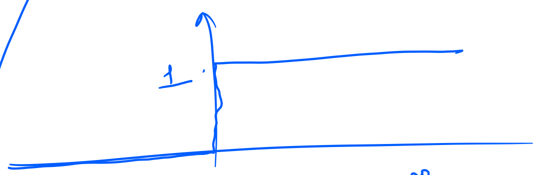
$$\lim_{t \to \infty} e^{-(a+j\omega)t} = 0$$

since $a > 0$

Fourier transform obtained by setting $s = j\omega$ in Laplace transform

$x(t) = u(t)$

unit step function



$$\int_{-\infty}^{\infty} |x(t)| \, dt = \int_0^{\infty} 1 \cdot dt$$

= Not finite

NOT absolutely

integrable.

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\mathcal{L}(u(t)) = 1/s.$$

ROC $= Re\{s\} > 0$

$\implies$ Does NOT include $s = j\omega$ in ROC

$s = j\omega \implies \sigma = 0$

$$F(u(t)) = \underbrace{\pi\delta(\omega) + \frac{1}{j\omega}}_{\text{NOT obtained}}$$

NOT obtained ~~from~~ Laplace transform by setting $s = j\omega$

PROPERTIES OF FOURIER TRANSFORM

① Linearity:

$x_1(t) \longleftrightarrow X_1(\omega)$

$x_2(t) \longleftrightarrow X_2(\omega)$

$$a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)$$

② TIME SHIFTING:

$x(t) \longleftrightarrow X(\omega).$

$x(t - t_0) \longleftrightarrow \tilde{X}(\omega)$

$x(t)$ delayed by t_0

$$\tilde{X}(\omega) = \int_{-\infty}^{\infty} x(t - t_0)\, e^{-j\omega t}\, dt$$

$t - t_0 = \tilde{t}$

$\Rightarrow dt = d\tilde{t}$

$$= \int_{-\infty}^{\infty} x(\tilde{t})\, e^{-j\omega(\tilde{t}+t_0)}\, d\tilde{t}$$

$$= \underbrace{\int_{-\infty}^{\infty} x(\tilde{t})\, e^{-j\omega\tilde{t}}\, d\tilde{t}}_{X(\omega)} \cdot e^{-j\omega t_0}$$

$$\tilde{X}(\omega) = \underbrace{X(\omega)\, e^{-j\omega t_0}}_{}$$

↳ complex exponential.

$$\boxed{x(t-t_0) \longleftrightarrow X(\omega)\, e^{-j\omega t_0}}$$

↳ shift in time $\longleftrightarrow$ Modulation in frequency.

③ FREQUENCY SHIFTING PROPERTY

$$\underbrace{e^{j\omega_0 t} x(t)}_{\text{Modulation in Time}} \longleftrightarrow \underbrace{X(\omega-\omega_0)}_{\text{Shift in Frequency Domain}}$$

④ Time scaling Property

$a > 0$

$$x(t) \longleftrightarrow X(\omega)$$

$$x(at) \longleftrightarrow$$

where $a > 0$ $\longleftrightarrow \frac{1}{a} X(\omega/a)$

↳ shrink / expand in Time

⇒ expand / shrink in Frequency.

$$\tilde{X}(\omega) = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

$at = \tilde{t}$

$dt = \frac{1}{a}\, d\tilde{t}$

$$= \int_{-\infty}^{\infty} x(\tilde{t})\, e^{j\omega \tilde{t}/a}\, d\tilde{t}/a$$

$$= 1/a \int_{-\infty}^{\infty} \underbrace{x(\tilde{t})\, e^{j\omega/a\, \tilde{t}}\, d\tilde{t}}_{X(\omega/a)}$$

$$\boxed{\tilde{X}(\omega) = \frac{1}{a} X(\omega/a)}$$

Fourier transform of $x(at)$

$a > 0$

(5) Time Reversal Property

$x(t) \longleftrightarrow X(\omega)$

$x(-t) \longleftrightarrow X(-\omega)$.

Reversal in time leads to Reversal in Frequency

lec 38 FOURIER TRANSFORM PROPERTIES:

(6) Duality or symmetry:

$x(t) \longleftrightarrow X(\omega)$

$X(t) \longleftrightarrow ?$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

Inverse FT

interchange t, ω.

$$x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t)\, e^{j\omega t}\, dt$$

$$x(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t)\, e^{-j\omega t}\, dt$$

$$\Rightarrow 2\pi\, x(-\omega) = \underbrace{\int_{-\infty}^{\infty} X(t)\, e^{-j\omega t}\, dt}_{F\{X(t)\}}$$

$$\boxed{X(t) \longleftrightarrow 2\pi\, x(-\omega)}$$

Duality property
or
symmetry of F.T.

eg. $\delta(t) \longleftrightarrow 1$.

$$\underbrace{\delta(t-t_0)}_{x(t)} \longleftrightarrow \underbrace{e^{-j\omega t_0}}_{X(\omega)}$$

$$X(t) = e^{-jt t_0}$$

$$e^{-jt t_0} \longleftrightarrow 2\pi\, x(-\omega)$$
$$= 2\pi\, \delta(-\omega - t_0)$$
$$= 2\pi\, \delta(\omega + t_0)$$

$$e^{-jt t_0} \longleftrightarrow 2\pi\, \delta(\omega + t_0)$$

Replacing t_0 by $-\omega_0$

$$\Rightarrow \boxed{e^{(+)j\omega_0 t} \longleftrightarrow 2\pi\, \delta(\omega - \omega_0)}$$

impulse at ω_0 scaled by 2π

Differentiation in Time

$$x(t) \longleftrightarrow X(\omega)$$

$$\frac{d\,x(t)}{dt} \longleftrightarrow ?$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt} X(\omega)\, e^{j\omega t}\, d\omega.$$

$$= \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, j\omega\, e^{j\omega t}\, d\omega}_{\text{IFT of } j\omega X(\omega)}.$$

$$\boxed{\frac{d\,x(t)}{dt} \longleftrightarrow j\omega X(\omega)}$$

Fourier Transform of Derivative.

Differentiation in frequency

$$x(t) \longleftrightarrow X(\omega)$$

$$-jt\,x(t) \longleftrightarrow \frac{d}{d\omega} X(\omega)$$

Integration in time

$$x(t) \longleftrightarrow X(\omega)$$

$$\underbrace{\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \pi X(0)\,\delta(\omega) + \frac{1}{j\omega} X(\omega).}_{\text{Fourier transform of integral.}}$$

Convolution property of FT:

$$x_1(t) \longleftrightarrow X_1(\omega).$$

$$x_2(t) \longleftrightarrow X_2(\omega).$$

$$\tilde{x}(t) = x_1(t) * x_2(t)$$

convolution operator.

$$\tilde{x}(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\,x_2(t-\tau)\,d\tau.$$

$$\tilde{X}(\omega) = \int_{-\infty}^{\infty} \tilde{x}(t)\,e^{-j\omega t}\,dt$$

Substitute for $\tilde{x}(t)$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau \right) e^{-j\omega t}\, dt$$

Interchange order of integration.

$$= \int_{-\infty}^{\infty} x_1(\tau) \underbrace{\left(\int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-j\omega t}\, dt \right)}_{\text{Fourier Transform of delayed signal.}} d\tau$$

$=$

$$x_2(t-\tau) \xrightarrow{FT} = X_2(\omega)\, e^{-j\omega\tau}$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, X_2(\omega)\, e^{-j\omega\tau}\, d\tau$$

$$= X_2(\omega) \underbrace{\int_{-\infty}^{\infty} x_1(\tau)\, e^{-j\omega\tau}\, d\tau}_{X_1(\omega)}$$

$$\boxed{\tilde{X}(\omega) = X_1(\omega)\, X_2(\omega)}$$

$$x_1(t) * x_2(t) \longleftrightarrow X_1(\omega)\, X_2(\omega)$$

convolution in time

Multiplication in Freq.

MULTIPLICATION IN TIME

$$x_1(t) \longleftrightarrow X_1(\omega)$$

$$x_2(t) \longleftrightarrow X_2(\omega)$$

$$\boxed{x_1(t)\cdot x_2(t) \longleftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega)}$$

Multiplication in time.

Convolution in Frequency

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Parseval's relation: Fourier transform

$$x(t) \longleftrightarrow X(\omega)$$

$$x^*(-t) \longleftrightarrow ?$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega.$$

$$x^*(t) = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega\right)^*$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)e^{-j\omega t}\,d\omega.$$

$$\Rightarrow x^*(-t) = \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)e^{j\omega t}\,d\omega.}_{\text{Inverse F.T of } X^*(\omega).}$$

$$\boxed{x^*(-t) \longleftrightarrow X^*(\omega)}$$

Consider

$$\tilde{x}(t) = x(t) * x^*(-t)$$

$$\Rightarrow \tilde{X}(\omega) = F(\{x(t)\})\,F(\{x^*(-t)\})$$

$$\boxed{\tilde{x}(t) \longleftrightarrow |X(\omega)|^2}$$

$$\tilde{x}(t) = x(t) * x^*(-t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, x^*(-(t-\tau))\,d\tau$$

$$\tilde{x}(t) = \int_{-\infty}^{\infty} x(\tau)\, x^*(\tau-t)\,d\tau.$$

Setting $t=0$

$$\Rightarrow \tilde{x}(0) = \int_{-\infty}^{\infty} x(\tau)\, x^*(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} |x(\tau)|^2 d\tau.$$

$$\tilde{x}(0) = \int_{-\infty}^{\infty} |x(\tau)|^2 d\tau. \quad \text{--- (1)}$$

Energy of signal

$$\tilde{x}(t) = \mathcal{F}^{-1}\{|X(\omega)|^2\}$$

$$\tilde{x}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 e^{j\omega t} d\omega.$$

Set $t=0$

$$\Rightarrow \tilde{x}(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \quad \text{--- (2)}$$

From (1) and (2), we have

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$$

Parseval's Theorem.
Parseval's Relation.

Energy of $x(t)$

Energy of $X(\omega)$ scaling factor $= 1/2\pi$

$|X(\omega)|^2 \longrightarrow$ Energy spectral Density of $x(t)$

$\to$ Distribution of energy over spectrum.

$$\mathcal{E} = \frac{1}{2\pi}\underbrace{\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega}_{}$$

Integrating E.S.D over entire Frequency Band $-\infty$ to ∞

Also, $\tilde{x}(t) = x(t) * x^*(-t)$

$$\tilde{x}(t) = \int_{-\infty}^{\infty} x(\tau)\, x^*(\tau - t)\, d\tau \qquad \tilde{\tau} = \tau - t$$

$$= \underbrace{\int_{-\infty}^{\infty} x(\tilde{\tau} + t)\, x^*(\tilde{\tau})\, d\tilde{\tau}}_{}$$

Autocorrelation of signal $x(t)$

$R_{xx}(t)$.

Measure of self similarity.

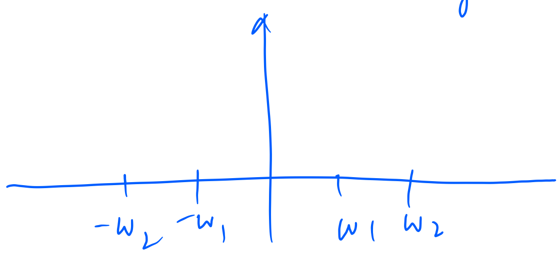
$R_{xx}(t) \longleftrightarrow |X(\omega)|^2.$

Auto correlation Function.

$= S_{xx}(\omega)$

Energy Spectral Density

ESD always ≥ 0.
Non-negative.



Band of interest

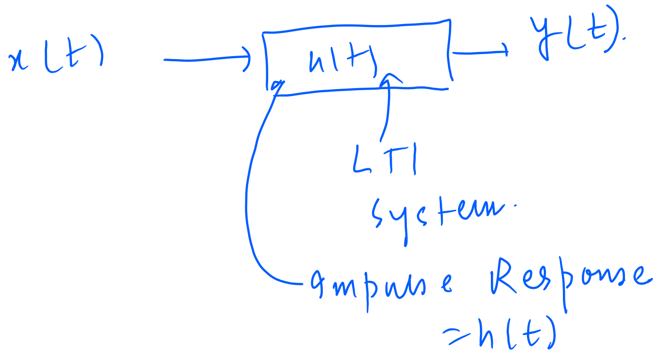
Energy in the band

$$= 2 \cdot \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} S_{xx}(\omega)\, d\omega.$$

$$= 2 \cdot \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} |X(\omega)|^2 d\omega.$$

So we are integrating ESD over band of interest.

FREQUENCY RESPONSE OF CONTINUOUS TIME LTI SYSEMS



$$y(t) = x(t) * h(t)$$

$$\Rightarrow Y(\omega) = X(\omega)\, H(\omega)$$

$$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Transfer function
Frequency Response of system.

$$H(\omega) = |H(\omega)|\, e^{j\theta_H}$$

$|H(\omega)|$ = Magnitude response of the system (LTI)

θ_H : Phase response of LTI system.

$$h(t) \longleftrightarrow H(\omega)$$

Frequency Response = FT of impulse Response

Consider $x(t) = e^{jw_0t}$

Complex Exponential

$X(w) = 2\pi\,\delta(w - w_0)$

$Y(w) = H(w)\,X(w)$.

$= 2\pi\,\delta(w - w_0)\,H(w)$

$= H(w_0)\,2\pi\,\delta(w - w_0)$

Taking IFT

$\Rightarrow y(t) = H(w_0)\,e^{jw_0t}$

input $= e^{jw_0t}$

$\Rightarrow$ output $= \underbrace{H(w_0)\,e^{jw_0t}}$

observed output

= scaled version of input

$\Rightarrow e^{jw_0t}$ = Eigen function of LTI system.

Eigen value of LTI system. for e^{jw_0t}

lec-40: Distortionless transmission:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$

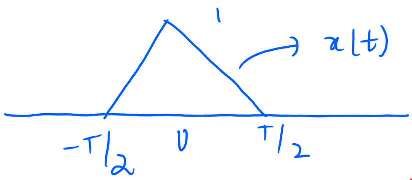
For distortionless transmission through LTI system

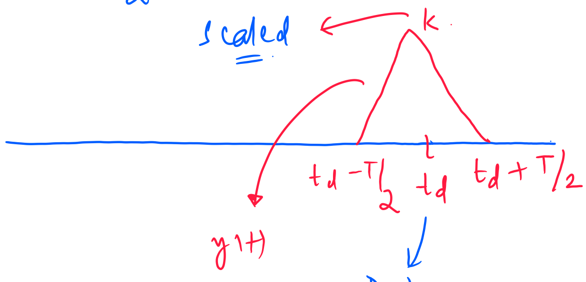
Termed Distortionless if $y(t)$ is a Delayed and Scaled version of $x(t)$.

$y(t) = k \cdot x(t - t_d)$

k → scaling factor.

t_d → Delay





Delay.

$y(t) = k\, x(t - t_d)$

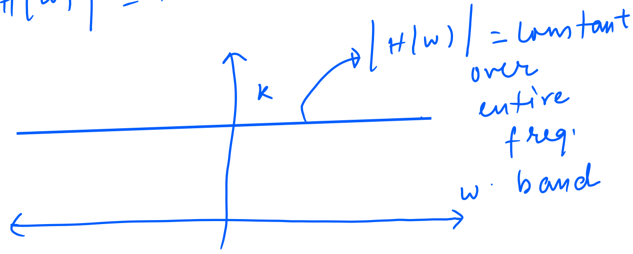
$\Rightarrow Y(w) = k\, X(w)\, e^{-jwt_d}.$

$Y(w) = \underbrace{k\, e^{-jwt_d}}_{H(w)}\, X(w)$

$H(w)$ → Frequency Response of Distortionless system.

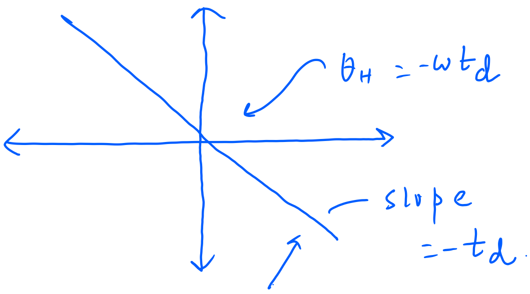
$H(w) = e^{-jwt_d} \cdot k.$

$|H(w)| = k$ → constant



$\theta_H = \measuredangle H(w) = -wt_d$

$-wt_d$ → Linear in w.



Phase = Linear in w.

<u>For distortionless transmission.</u>

LTI systems characterised by differential Equations.

constant coefficient differential Equation

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

constant coefficients

Taking Fourier Transform.

$$\sum_{k=0}^{N} a_k \cdot (jw)^k \cdot Y(w)$$

$$= \sum_{k=0}^{M} a_k (jw)^k X(w).$$

$$Y(w) \sum_{n=0}^{N} a_k (jw)^k = X(w) \sum_{k=0}^{M} b_k (jw)^k$$

$$\Rightarrow \frac{Y(w)}{X(w)} = \underbrace{\frac{\sum_{k=0}^{M} b_k (jw)^k}{\sum_{k=0}^{N} a_k (jw)^k}}_{H(w)}$$

Frequency response of LTI system.

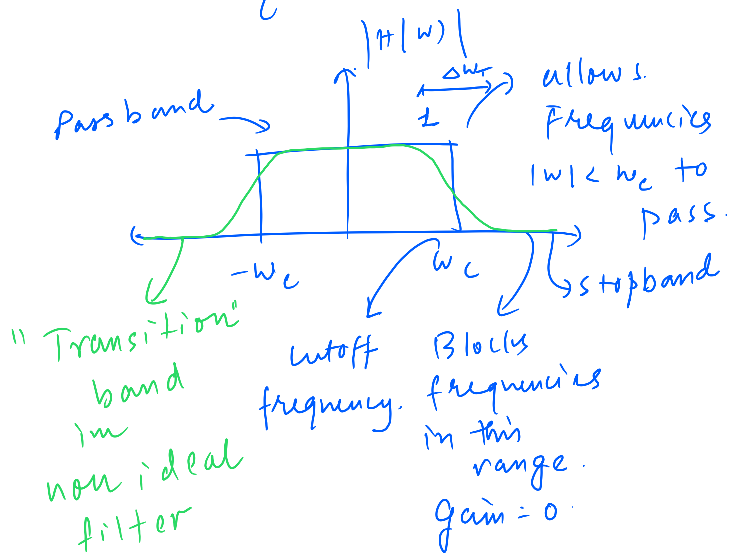
Ideal filters.

allow only specific set of frequencies to pass, blocks rest of frequencies.

IDEAL LOW PASS FILTER.

allows only lower frequencies to pass through.

$$|H(\omega)| = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \text{otherwise.} \end{cases}$$



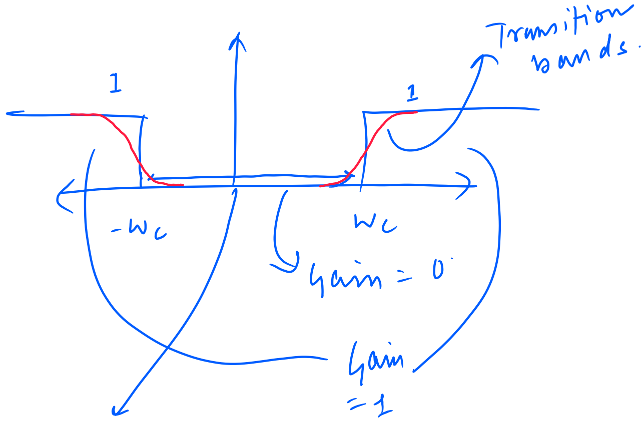
$\Delta\omega_T$ = Transition band.

(In practice, would like to have a very narrow transition band.

Ideal filters have very sharp edges

$\Rightarrow$ Difficult to Design.

IDEAL HIGH PASS FILTER.



Blocks all frequency components in $-\omega_c$ to ω_c.

$$|H(\omega)| = \begin{cases} 1, & |\omega| > \omega_c \\ 0, & \text{otherwise.} \end{cases}$$

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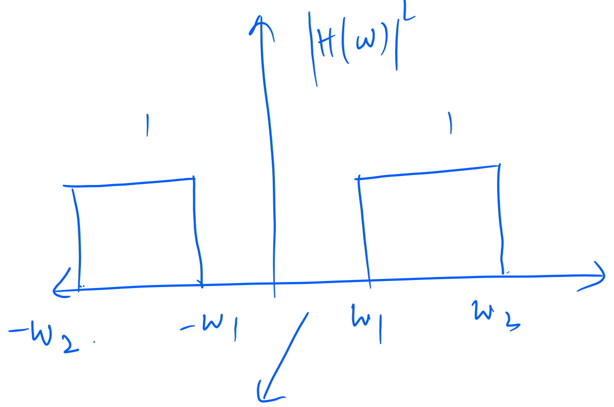
IDEAL FILTERS:

Ideal Bandpass filter:

$\omega_1 - \omega_2$ — Passband.

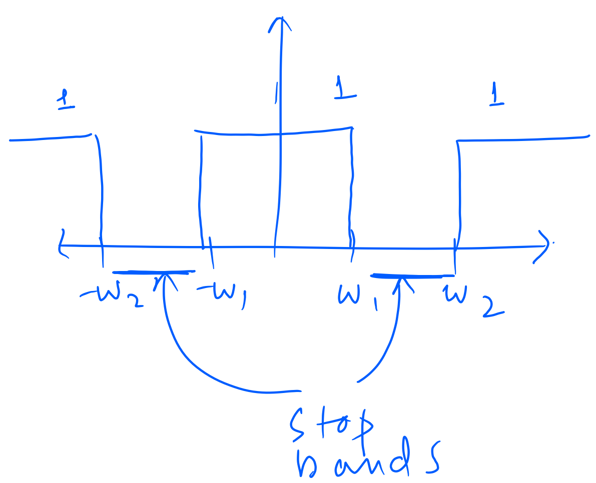
$$|H(\omega)| = \begin{cases} 1, & \omega_1 < |\omega| < \omega_2 \\ 0, & \text{otherwise.} \end{cases}$$



Allows only frequency components in band $\omega_1 - \omega_2$ to pass through.

$\omega_1 - \omega_2$ = Passband.

Band stop Filter:

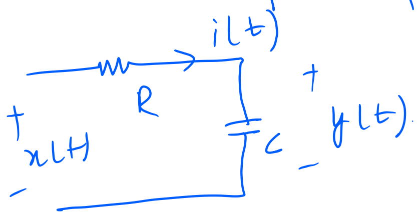


$$H(\omega) = \begin{cases} 0, & \omega_1 < |\omega| < \omega_2. \\ 1; & \text{otherwise.} \end{cases}$$

Blocks all frequencies in w_1 to w_2, $-w_2$ to $-w_1$.

To avoid distortions, all filters should have a linear phase characteristic

<u>Non ideal low pass filter.</u>



RC circuit

$$x(t) = i(t)R + y(t)$$

$$i(t) = C\frac{dy(t)}{dt}$$

$$\Rightarrow x(t) = RC\frac{dy(t)}{dt} + y(t)$$

constant coefficient differential equation for RC circuit.

Take FT

$$X(w) = RC\,jw\,Y(w) + Y(w)$$

$$= Y(w)(1 + jwRC)$$

$$\Rightarrow \frac{Y(w)}{X(w)} = \frac{1}{1 + jwRC}$$

$$H(w) = \frac{1}{1 + j\left(\frac{w}{w_0}\right)}.$$

where.

$$w_0 = 1/RC$$

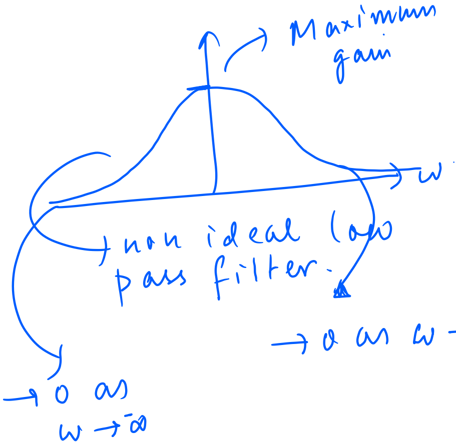
$$|H(w)| = \frac{1}{\sqrt{1 + \left(\frac{w}{w_0}\right)^2}}$$

2

$$|H(w)| = \frac{1}{1+(w/w_0)^2}$$

at $w = 0$ $|H(w)|^2 = 1$

$w \to \infty$, $|H(w)|^2 \to 0$



3 dB bandwidth.

w_{3dB} : value of w s.t power decreases by a factor of 1/2 compared to peak.

⇒ amplitude decreases by factor of $1/\sqrt{2}$

$|H(0)| = 1$

$$\Rightarrow |H(w_{3dB})| = \frac{1}{\sqrt{2}} \times 1$$

$$\Rightarrow |H(w_{3dB})|^2 = 1/2 \times 1 = 1/2$$

$$|H(w_{3dB})|^2 = \frac{1}{1+\frac{w_{3dB}^2}{w_0^2}} = \frac{1}{2}$$

$$1 + \frac{w_{3dB}^2}{w_0^2} = 2$$

$$\Rightarrow \frac{w_{3dB}^2}{w_0^2} = 1.$$

$$\Rightarrow \boxed{w_{3dB} = w_0} = \frac{1}{RC}$$

$$|H(w_{3dB})|^2 = 1/2$$

$$10 \log_{10} |H(w_{3dB})|^2$$

$$= 20 \log_{10} |H(w_{3dB})|$$

$$= 10 \log 1/2$$

$$\boxed{\simeq -3\,dB}$$

3 dB Reduction in power.

SIGNAL BANDWIDTH :

$$x(t) \longleftrightarrow X(w)$$

$w_{3dB} = w$, such that

$$|X(w_{3dB})| = \frac{1}{\sqrt{2}} |X(0)|$$

$$|X(w_{3dB})|^2 = \frac{|X(0)|^2}{2}$$

Band limited signal

$x(t)$ is Band limited signal

if $x(t) \longleftrightarrow X(w)$

$|X(w)| = 0$ for $w > w_m$

Maximum frequency.