Assignment 12

(1) τ_g: group delay $= -\left.\frac{d\phi(w)}{dw}\right|_{w=w_0}$

τ_p: phase delay $= -\frac{\phi(w_0)}{w_0}$

(2) Not done

(3) Not done.

(4) $x_1(n) \cdot x_2(n) \rightarrow \frac{1}{N} X_1(k) \circledast X_2(k)$

(5) $x(n) = \cos\left(\frac{\pi n}{2}\right)$

$h(n) = \{2, -1, 3, -2\}$.

$x(n) = \{1, 0, -1, 0\}$

$h(n) = \{2, -1, 3, -2\}$. $N = 4$

$N-1$

$$y(n) = \sum_{m=0} x(m)\, h\left[(n-m)_N\right]$$

$$= \sum_{m=0}^{3} x(m)\, h\left((n-m) \bmod 4\right)$$

$$y(n) = x(0)\, h(n \bmod 4) + x(1)\, h((n-1) \bmod 4)$$

$$+ x(2)\, h((n-2) \bmod 4) + x(3)\, h((n-3) \bmod 4)$$

$y(0) = x(0)\,h(0 \bmod 4) + x(1)\,h(-1 \bmod 4)$
$+ x(2)\,h(-2 \bmod 4) + x(3)\,h(-3 \bmod 4)$.

$-1 + 4 = 3$

$= x(0)\,h(0) + x(1)\,h(3)$
$+ x(2)\,h(2) + x(3)\,h(1)$.

$x(n) = \{1, 0, -1, 0\}$
$h(n) = \{2, -1, 3, -2\}$.

$= 1 \cdot 2 + 0 + (-1) \cdot 3 + 0$

$= 2 - 3 = -1$

$y(1) = x(0)\,h(1 \bmod 4) + x(1)\,h(0 \bmod 4)$
$+ x(2)\,h(-1 \bmod 4) + x(3)\,h(-2 \bmod 4)$

$= x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(3)$
$+ x(3)\,h(2)$

$= -1 + 0 - 1 \times (-2) + 0$

$= -1 + 2 = 1$ ✓

$y(2) = x(0)\,h(2 \bmod 4) + x(1)\,h(1 \bmod 4)$
$+ x(2)\,h(0 \bmod 4) + x(3)\,h(3 \bmod 4)$

$= x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0)$
$+ x(3)\,h(3)$

$= 3 - 2$

$= 1$.

$\{-1, 1, 1, -1\}$.

(6) $x(n) \longrightarrow X(k)$

$X(n) \longrightarrow N\,x(-k \bmod N)$.

(7) $x(n) = \sin\left(\frac{\pi n}{2}\right), \quad 0 \le n \le 3$

$= \{0, 1, 0, -1\}$

$h(n) = \{4, 2, 1, 3\}$.

$y(n) = x(n) \circledast h(n)$

$= \sum_{m=0}^{N-1} x(m)\, h(((n-m))_N)$

$y(n) = x(0)\, h(n \bmod 4) + x(1)\, h((n-1) \bmod 4)$

$+ x(2)\, h((n-2) \bmod 4) + x(3)\, h((n-3) \bmod 4)$

$y(0) = x(0)\, h(0) + x(1)\, h(-1 \bmod 4)$

$+ x(2)\, h(-2 \bmod 4) + x(3)\, h(-3 \bmod 4)$

$= x(0)\, h(0) + x(1)\, h(3) + x(2)\, h(2)$

$+ x(3)\, h(1)$

$= 0 + 1 \times 3 + 0 + (-1) \times 2$

$= 3 - 2 = 1$.

(8) 2×2 IDFT

$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

(9) 8 pt DFT $\{4, 0, 0, 0, 4, 0, 0, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi/N\, nk}$$

$N = 8$

$$X(k) = \sum_{n=0}^{n=0} x(n)\, e^{-j\pi/4\, nk}$$

$$= x(0) + x(4)\, e^{-j\pi k}$$

$$X(0) = x(0) + x(4)$$

$$= 4 + 6 = 10$$

$$X(1) = x(0) + x(4)\, e^{-j\pi}$$

$$= 4 - 6 = -2.$$

$$X(2) = x(0) + x(4)\, e^{-j2\pi}$$

$$= 4 + 6$$

$$= 10$$

$\{10, -2, 10, \ldots\}$

(10) $x(n) = \{1, 0, -1, 0\}$

$h(n) = \{-1, 2, -3, 4\}$

$$y(n) = x(0)\, h(n \bmod 4) + x(1)\, h((n-1) \bmod 4)$$

$$+ x(2)\, h((n-2) \bmod 4) + x(3)\, h((n-3) \bmod 4)$$

$$y(0) = x(0)\, h(0) + x(1)\, h(3) + x(2)\, h(2) + x(3)\, h(1)$$

$$= -1 + 3 = 2.$$

$$y(1) = x(0)\, h(1) + x(1)\, h(0) + x(2)\, h(3) + x(3)\, h(2)$$

$$= 2 - 4 = -2.$$

$\{2, -2, 2, 2\}$

Assignment solutions. (official)

$$a^n u(n) \longrightarrow \frac{e^{-jw}}{1-ae^{-jw}}$$

$$a^{n-1} u(n-1) \longrightarrow \frac{e^{-jw}}{(1-a^{-jw})}$$

$$a^n u(n) * a^{n-1} u(n-1) \longleftrightarrow e^{-jw}\left(\frac{1}{1-ae^{-jw}}\right)^2$$

$$\downarrow w$$

$$= \sum_{m=-\infty} a^m u(m)\, a^{n-1-m} u(n-1-m)$$

$$= \left(\sum_{m=0}^{n-1} a^{n-1}\right) u(n-1) = n a^{n-1} u(n-1)$$

(3) DIT

$$X(k) = 2\sum_{n=0}^{3} e^{j2\pi\left(\frac{2nk}{8}\right)} + 3\sum_{n=0}^{3} e^{-j2\pi\frac{2nk}{8}} \times e^{j2\pi\frac{n}{8}}$$

$$= 2\sum_{n=0}^{3} e^{j\pi kn} + 3e^{j\pi k/4}\sum_{n=0}^{3} e^{j\pi k/2 n}$$

$$= \left(2 + 3e^{j\pi k/4}\right)\sum_{n=0}^{k} e^{j\frac{\pi k}{2}n}.$$

$$= \left(2 + 3e^{j\pi k/4}\right)\left(\frac{1-e^{ja\pi k}}{1-e^{j\pi k/2}}\right), \quad k = 0,4$$

$$0,\ w$$

$$X(0) = 5 \times 4 = 20$$

$$X(4) = (2-3) \times 4 = -4$$

$$X(k) = \{20,0,0,0,\ -4,0,0,0\}.$$