$$\frac{1}{2} e^{-|t|} \xrightarrow{\text{FT}} \frac{1}{1+w^2}$$

$\underbrace{\quad}_{y(t)}$ $\underbrace{\quad}_{+(w)}$

Q: what is the FT of $y(t) = \frac{1}{1+t^2}$?

By Duality theorem

$$\mathcal{F}\left\{ \frac{1}{1+t^2} \right\} = 2\pi \, y(-w)$$

$$= 2\pi \cdot \frac{1}{2} e^{-|-w|}$$

$$= \pi e^{-|-w|}$$

$$= \pi e^{-|w|} \quad \blacksquare$$

W8 Assignment 8.

① $x(t)$ cannot be unbounded at a finite # of discontinuities in any interval.

② $u(t) \longrightarrow \pi\delta(w) + \frac{1}{jw}$ } (see theory by Prof. S+A)

③ Not done

④ $x(t) \longleftrightarrow x(w)$.

$x(t) \longleftrightarrow \dfrac{2\pi \, x(-w) \times 2\pi}{2\pi}$

$\boxed{\dfrac{X(t)}{4\pi^2}} \longleftrightarrow \dfrac{x(-w)}{2\pi} \checkmark$

$\searrow$ IFT

⑤ $x(t) \longrightarrow x(w)$

$\dfrac{dx(t)}{dt} \longleftrightarrow jw\, x(w)$.

⑥ $\cos^2\left(t + \pi/6\right)$

$\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos^2\theta = \dfrac{\cos 2\theta + 1}{2}$

$$= \dfrac{\cos\left(2\left(t + \pi/6\right)\right) + 1}{2}$$

$$= \dfrac{\cos\left(2t + \pi/3\right) + \frac{1}{2}}{2}$$

$\omega_0 = 2$.

$e^{j\pi/3}$

$= \cos\pi/3$
$\quad + j\sin\pi/3$

$= \frac{1}{2} + j\frac{\sqrt{3}}{2}$

$$= \dfrac{e^{j\left(2t + \pi/3\right)} + e^{-j\left(2t + \pi/3\right)}}{4} + \frac{1}{2}$$
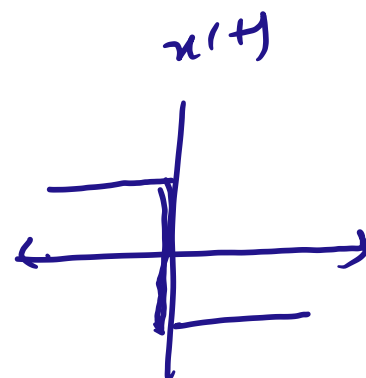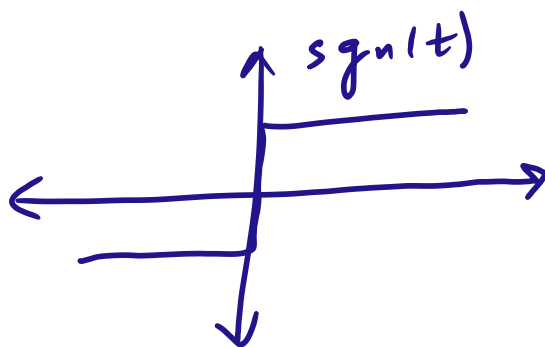
$$= \dfrac{e^{j2t} \cdot e^{j\pi/3}}{4} + \dfrac{e^{-j2t} \cdot e^{-j\pi/3}}{4} + \frac{1}{2}.$$

$C_1 = \dfrac{1}{4}\left(\dfrac{1}{2} + j\dfrac{\sqrt{3}}{2}\right)$

$C_{-1} = \frac{1}{4}\left(\frac{1}{2} - j\dfrac{\sqrt{3}}{2}\right)$,

$C_0 = \frac{1}{2}$

⑦ $x(t) = \begin{cases} -1, & t > 0 \\ 1, & t < 0 \\ 0, & t = 0 \end{cases}$

$sgn(t) \longleftrightarrow \dfrac{2}{j\omega}$

$sgn(-t) \longleftrightarrow \dfrac{-2}{j\omega}$

$-sgn(t)$

$x(t) = sgn(-t) = -sgn(t)$.

$x(\omega) = \dfrac{-2}{j\omega} = \dfrac{2j}{\omega}$.

⑧ $a_k = c_k + c_{-k}$

$\quad = 2\,\text{Re}\{c_k\}.$

⑨ Not done.

⑩ Not done

$3, \underset{=}{9}, 10$ ⌊ Solutions official

③ Fourier Series. Parseval's Theorem.

$$\sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} = \sum_{h=-\infty}^{\infty} |c_h|^2.$$

⑨ $e^{-t^2 a} \to \sqrt{\pi a}\; e^{-a\omega^2/4}.$

$\quad e^{-t^2 2a} \to \sqrt{\pi 2a}\; e^{-a\omega^2/2}$

$\qquad x(0) = 8$

$\qquad \Rightarrow \sqrt{2\pi}\, a = 8 \;\Rightarrow\; 2a\pi = 64$

$\qquad\qquad\qquad\qquad \Rightarrow a = 32/\pi.$

⑩ $x(\omega) * y^*(\omega) = \int_{-\infty}^{\infty} x(\omega)\, y^*(\tilde{\omega} - \omega)\, d\omega.$

$\qquad\qquad \uparrow$

$\quad 2\pi \quad x(t) \cdot y^*(-t).$

$\int_{-\infty}^{\infty} x(\omega)\, y^*(-\omega)\, d\omega\, \Big|_{\tilde{\omega}=0}.$

$\qquad f\left(2\pi\, x(t)\, y^*(-t)\right)\Big|_{\omega=0}$

$\qquad 2\pi \int_{-\infty}^{\infty} x(t)\, y^*(-t)\, e^{-j\omega t}\, dt\, \Big|_{\omega=0} = 2\pi \int_{-\infty}^{\infty} \frac{x(t)}{y^*(-t)}\, dt$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ See TA session.