SIGNALS AND SYSTEMS

Lec 01

properties of signals
properties of systems
&
principles

Interplay
b/w signals and systems

$x(t)$ → System → $y(t)$

signals interaction
b/w signals
and systems

Electrical Engineering
→ Power systems
→ Smart grid
→ Electronic
systems
TV, mobile phones
→ Communication
systems
(3G, 4G wireless systems
wifi 802.11)

Instrumentation & Control (eg- airport)

AIM → properties of Both continuous / Discrete
signals + systems

SIGNAL → Physical quantity
→ that conveys information
about phenomenon.
→ Typically exhibits variation
in space or time

image

eg EM wave
speech signal.

video → space + Time varying

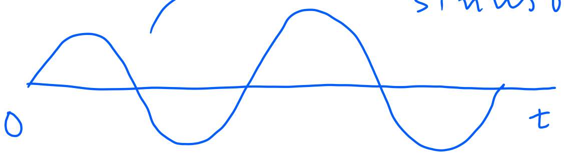
$x(t)$, $y(t)$ – Time varying
signals.

function of time

Classification of signals

↳ Continuous time signals → FM radio, TV, Broadcast, Telephony

$x(t) = \underbrace{\sin(2\pi t)}_{\text{sinusoid}}$

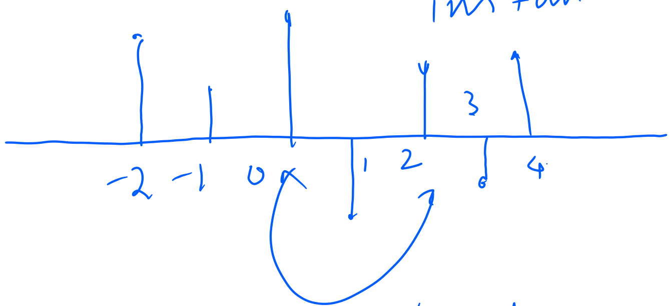


Defined for all time events in an interval $[t_1, t_2]$

Discrete time signals.

Modern 3G/4G/GSM system

$x(n)$ Defined at a discrete set of time instants



Defined only at discrete time instants

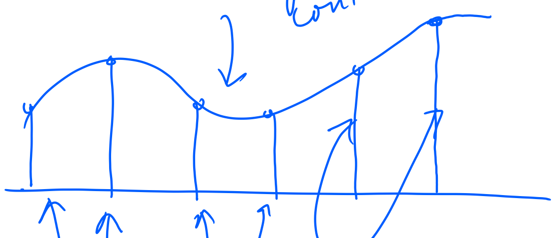
$\underbrace{x(-2), x(-1), x(0), x(1), x(2),}_{\text{Sequence or Series of Numbers}}$

Time series

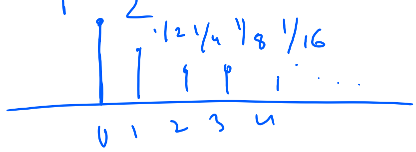
Discrete Time signal.

Discrete time signal

can be obtained by suitably sampling a continuous time signal.



$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



Discrete time signal

lec-02 Analog & Digital Signals

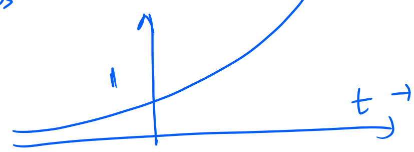
analog signal → continuous time signal

$x(t)$ can take values belonging to an interval $[a, b]$

$\sin(2\pi f t) \in [-1, 1]$

infinitely possible values

$e^t \in (0, \infty)$



takes all values in interval 0 to ∞

DIGITAL SIGNAL

Discrete time signal $x(n)$ that can take discrete set of values or finite values.

$x(n) \in \{-1, 1\}$

$x(n)$ either equals -1 or $+1$

can take only one of two values.

DIGITAL COMMUNICATION SYSTEM.

REAL AND COMPLEX SIGNALS

$x(t), x(n) \in \mathbb{R}$.

set of real number

signals that take values belonging to set of real numbers. — REAL SIGNALS.

eg: $\left.\begin{array}{l} \sin(2\pi f t) \\ e^{t} \end{array}\right\}$ Real signals.

$x(t), x(n) \in \mathbb{C}$ Complex numbers.

signals that take complex values. (belonging to set of complex numbers) $\longrightarrow$ COMPLEX SIGNALS.

Eg $x(t) = e^{j2\pi f t}$

$= \cos(2\pi f t) + j \sin(2\pi f t)$

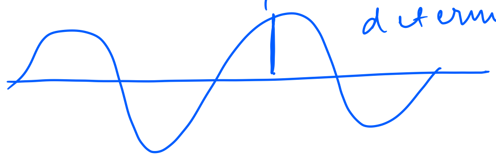
Complex sinusoid

where $j = \sqrt{-1}$

DETERMINISTIC AND RANDOM SIGNALS

$x(t)$ or $x(n)$ that is completely specified at any given instant of time

eg: $\sin 2\pi f_0 t$, e^t



t exactly determines the signal.

RANDOM SIGNAL

takes random values at different time instants.

Ex $\quad \underline{x[n]} = \begin{cases} +1, & \text{if outcome} = \text{heads} \\ -1, & \text{if outcome} = \text{tails} \end{cases}$

Represents a coin toss experiment

Noise

Random in nature

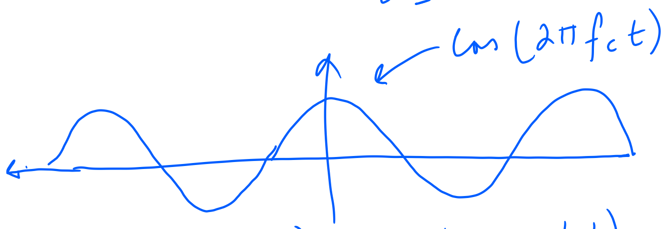
→ Fundamentally important to understand noise to characterise and analyse behaviour of systems.

→ Noise limits the performance of systems.

EVEN AND ODD SIGNALS

even signal : $x(t) = x(-t)$

$x[n] = x[-n]$

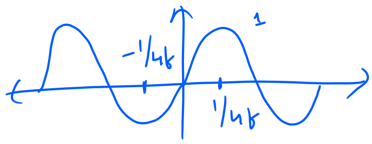


$$\cos(2\pi f t) = \cos(-2\pi f t) \longrightarrow \text{Even signal}$$

odd signal $\rightarrow$ $x(-t) = -x(t)$

$x[-n] = -x[n]$

$x(t) = \sin(2\pi f t)$



ODD SIGNAL

$$\sin(2\pi f t) = -\sin(-2\pi f t)$$

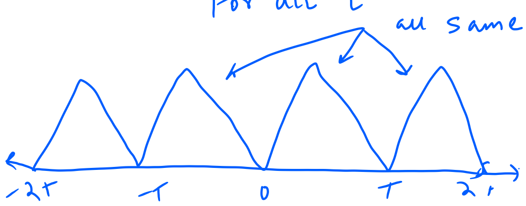
odd signal.

PERIODIC AND APERIODIC SIGNALS

very important classification

$x(t)$ is periodic if there exists a time period T such that $x(t+T) = x(t)$

For all t



eg $\sin(2\pi t)$

$= \sin(2\pi(t+1))$

$= \sin(2\pi t + 2\pi)$

$= \sin(2\pi t)$

$T = 1$ is period of $\sin(2\pi t)$

Fundamental Period

If T is a period of the periodic signal then mT is also a period for any integer m.

$$x(t+mT) = x(t)$$

$\underbrace{\hspace{4cm}}_{\text{For all } t}$

Fundamental Period T = smallest time period s.t.

$$x(t+T) = x(t)$$

$\underbrace{\hspace{4cm}}_{\text{For all } t}$

→ All other periods are Multiples of Fundamental period.

eg: $\sin(2\pi t)$

↳ $T = 1$ is a fundamental period

mT : $\underbrace{2T, 3T, -T, -2T}_{\text{periods}}$

For a discrete signal (periodic)

$$x(n+N) = x(n)$$

For all n.

→ Period of discrete time signal.

→ Smallest N for which this holds = Fundamental period N_0.

lec-3 ENERGY AND POWER SIGNALS

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

energy of signal

$$= \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If energy is finite

$\Rightarrow$ Energy signal

$0 < E < \infty$

eg $e^{-t} u(t)$

$e^{-n} u[n]$

Power $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

If power is finite $0 < P < \infty$

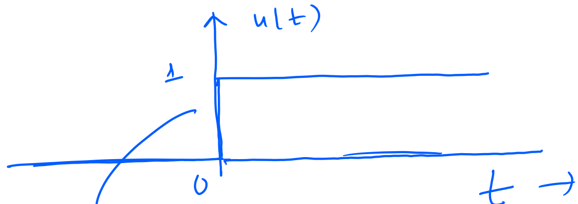
$\Rightarrow$ power signal

eg $\sin(2\pi f t)$

IMPORTANT CONTINUOUS TIME SIGNALS

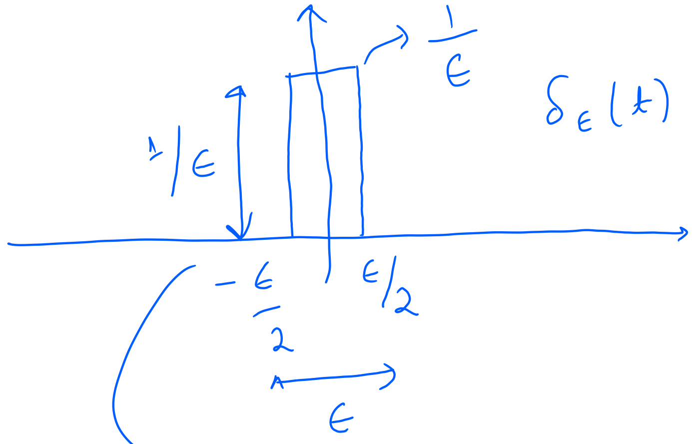
Unit step signal

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



unit step function

Unit Impulse function



Pulse : width $= \epsilon$

Height $= 1/\epsilon$

Area under pulse $= \epsilon \cdot 1/\epsilon$

$= 1.$

$$\int_{-\infty}^{\infty} \delta_\epsilon(t)\, dt = 1.$$

$$\delta(t) = \lim_{\epsilon \to 0} \delta_\epsilon(t)$$

impulse function

as $\epsilon \to 0$,

width $\epsilon \to 0$

Height $1/\epsilon \to \infty$

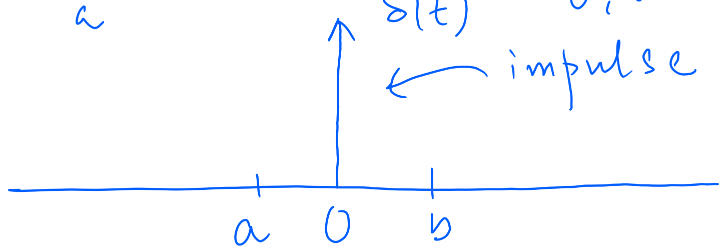
But, area $= 1$

$=$ constant

$$\int_a^b \delta(t)\, dt = 1 \text{ if } a < 0 < b$$

$\delta(t) = 0$, otherwise

impulse

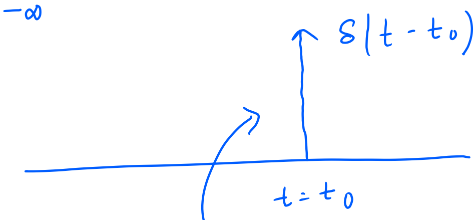


$a < 0 < b.$

Properties $\displaystyle\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$

Similarly

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$



impulse shifted to $t=t_0$

OTHER PROPERTIES OF IMPULSE FUNCTION

$$\delta(at) = \frac{1}{|a|}\,\delta(t)$$

$$\delta(-t) = \delta(t)$$

$$x(t)\,\delta(t) = x(0)\,\delta(t)$$

Further,

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t).$$

$$\tau - t = \tilde{t}$$

$$\tau = \tilde{t} + t$$

$$d\tau = d\tilde{t}$$

$$\int_{-\infty}^{\infty} x(\tilde{t}+t)\,\underbrace{\delta(-\tilde{t})}_{=\delta(\tilde{t})}\,d\tilde{t}$$

$$= \int_{-\infty}^{\infty} x(\tilde{t}+t)\,\delta(\tilde{t})\,d\tilde{t}$$

$$= x(\tilde{t}+t)\Big|_{\tilde{t}=0}$$

$$= x(t)$$

$$\boxed{\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)}$$

Sifting property of the impulse function

Complex exponential

$$x(t) = e^{j w_0 t} = \cos(w_0 t) + j \sin(w_0 t)$$

$$j = \sqrt{-1}$$

$$|x(t)| = \sqrt{\cos^2 w_0 t + \sin^2 w_0 t}$$

$$= 1$$

$$e^{j 2\pi f_0 t} = e^{j w_0 t}$$

$$\boxed{f_0 = w_0 / 2\pi}$$

Hz.

rad/s. (angular frequency)

(Radians/sec)

$$\boxed{\text{Period} \quad T = 1/f_0 = 2\pi / w_0}$$

$$f_0 = 5\,\text{Hz}.$$

$$\Rightarrow \quad T = 1/5 = 0.2\,s.$$

General complex exponential

$$s = \sigma + j w.$$

$$x(t) = e^{st}$$

$$= e^{(\sigma + j w) t}$$

$$= e^{\sigma t} \left(\cos(w t) + j \sin(w t) \right)$$

General complex exponential

lec-04

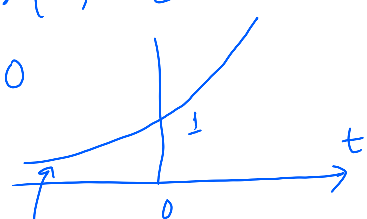
* REAL exponential signals

"exponential" signals

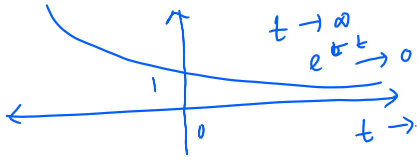
always +ve signal

$$x(t) = e^{\sigma t} \geq 0$$

$\sigma \geq 0$



$t \to -\infty, \; e^{\sigma t} \to 0, \; \sigma > 0$



$e^{\sigma t}$ For $\sigma < 0$

Decreasing function

SINUSOIDAL SIGNAL

$$x(t) = A\cos(\omega_0 t + \theta)$$

amplitude

angular freq. (rad/s)

phase offset

$$f_0 = \frac{\omega_0}{2\pi}$$

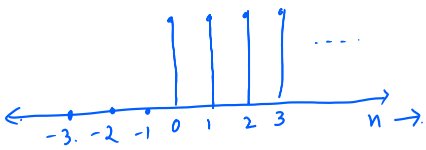
fundamental frequency.

Periodic signal.

BASIC DISCRETE TIME SIGNALS

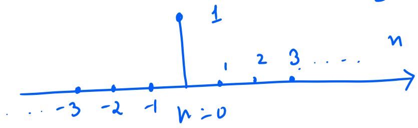
$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

unit step function.



UNIT IMPULSE FUNCTION

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



$$\sum_{n=-\infty}^{\infty} \delta(n) = 1$$

$$\sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) = x(n)$$

Sifting property for Discrete time signals.

$$\delta(n) = u(n) - u(n-1)$$

$$u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

COMPLEX Exponential

DISCRETE TIME

$x(n) = e^{j\Omega_0 n}$

$= \cos(\Omega_0 n) + j\sin(\Omega_0 n)$

Periodic?

$f_0 = \frac{\Omega_0}{2\pi}$

$e^{j2\pi f_0 n} = e^{j2\pi f_0 (n+N)}$

$= e^{j2\pi f_0 n} \underbrace{e^{j2\pi f_0 N}}_{1}$

$e^{j2\pi f_0 N} = 1$

Periodic if

$f_0 N = k$

$\Rightarrow \boxed{f_0 = k/N}$

Rational Number.

Therefore, $e^{j\Omega_0 n}$ is periodic only if $\Omega_0/2\pi = m/N$.

Rational Number.

Lec-05 Memory | Memory-less and Causal | Non Causal systems

* CLASSIFICATION OF SYSTEMS

System Representation

"System"

Mathematical Model for a physical process / Device that describes the o/p signal for any given input signal

$$\boxed{y(t) = T(x(t))}$$

o/p signal

Captures the system Transformation. Mapping.

input signal

i/p

$$x(t) \longrightarrow \boxed{T(\,)} \longrightarrow y(t)$$

o/p.

Mapping

$$\boxed{y[n] = T(x[n])}$$

Discrete time.

$$x[n] \longrightarrow \boxed{T(\,)} \longrightarrow y[n]$$

CLASSIFICATION OF SYSTEMS

SYSTEMS WITH MEMORY AND MEMORYLESS SYSTEMS

Memoryless system

⇒ output depends only on current input ~ i/p at current time instant

eg $y(t) = k x(t)$.

$v(t) = R\, i(t)$

v → voltage, R → Resistance, i → current

$y[n] = K\, x[n]$

→ Memoryless systems.

Systems with Memory

↳ o/p $y(t_0)$ at time t_0 depends NOT only on $x(t_0)$. but also $x(t)$ for $t < t_0$.

↙ o/p $y(t_0)$ depends also on past values of signal $x(t)$.

Ex: $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau$

voltage

current

Capacitance.

Capacitor is a system with Memory

Causal AND Non Causal SYSTEMS

Causal: output depends only on $x(t)$, $t \leq t_0$

"Causality Principle"

→ o/p depends only on PAST values of $x(t)$ and NOT Future values of $x(t)$.

Non Causal ⇒ system is NOT Causal.

→ o/p signal depends also on future values of input signal.

$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

depends on $x(\tau)$ $\tau \leq t$

Causal system.

$$y[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} x(n+k)$$

$y[n]$ depends on

$x[n-N]$, $x[n-N+1]$

... $x[n+N]$

depends also on future values of $x[n]$,

Hence, NON-CAUSAL.