Sol^n By time shifting property

$$x(n) \longrightarrow X(\Omega)$$
$$x(n-n_0) \longrightarrow X(\Omega)\, e^{-j\Omega n_0}$$

In the Question
let $x(n)$ be same as prev. Q & $n_0 = 1$

$$\therefore \text{IDTFT} = x(n-1)$$
$$= n a^{n-1} u(n-1)$$

W-10 Assignment

① for gaussian pulse

$$e^{-at^2} \xleftrightarrow{FT} \sqrt{\pi/a}\; e^{-w^2/4a}.$$

$a=2$ $$\sqrt{2}\sqrt{\pi/2\times\cancel{2}}\; e^{-w^2/8} \xleftrightarrow{IFT} e^{-2t^2}$$

$$\Rightarrow \sqrt{\pi/4}\; e^{-w^2/8} \xleftrightarrow{IFT} \frac{e^{-2t^2}}{\sqrt{2}}.$$

② $x(t) = \cos(2\pi f_0 t)$

$$x(t) \longrightarrow \boxed{\text{shift by } \pi/2} \longrightarrow \hat{x}(t)$$

$$x(t-\pi/2) = \cos(2\pi f_0 t - \pi/2)$$
$$= \sin 2\pi f_0 t$$

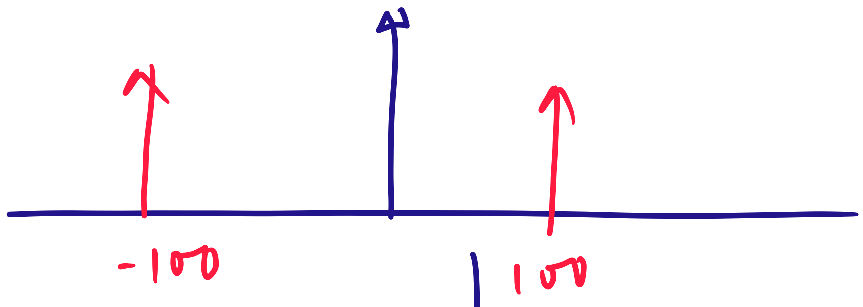
$$x(t)\cos(2\pi f_c t) - \hat{x}(t)\sin(2\pi f_c t)$$
$$= \cos(2\pi f_0 t)\cos(2\pi f_c t) - \sin(2\pi f_0 t)\sin(2\pi f_c t)$$

$\cos(A)\cos(B) - \sin(A)\sin(B)$
$= \cos(A+B)$
$= \cos[2\pi(f_0 + f_c)t]$.

③ $f_m = 100\ Hz$ (pure sinusoid)
$\cos(\underline{2\pi f_m t})$ ✓
$\updownarrow$ FT

OK

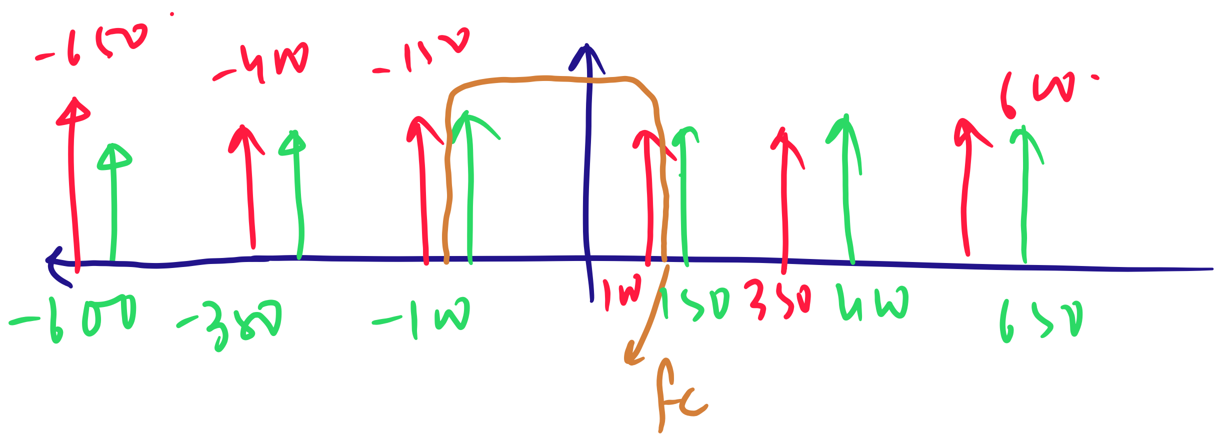
$-100 + 250 = 150$



sampling freq $f_s = 250\ Hz$

$$\begin{array}{r} 150 \\ +250 \\ \hline 400 \end{array}$$

$$\begin{array}{r} 350 \\ 250 \\ \hline \end{array}$$



$\underline{100 < f_c < 150}$ ✓

④ Not done

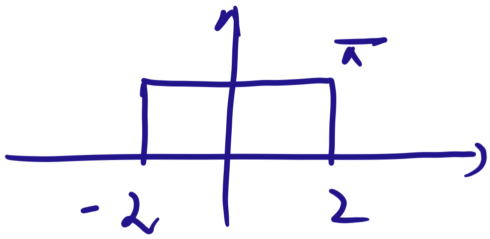
⑤ $x(t) = \sin 2\pi f_0(t) \longrightarrow \boxed{HT} \rightarrow \hat{x}(t) = \sin(2\pi f_0 t - \pi/2)$
$= -\cos 2\pi f_0 t$

$x(t)\cos(2\pi f_c t) - \hat{x}(t)\sin(2\pi f_c t)$
$= \sin 2\pi f_0 t \cos 2\pi f_c t + \cos 2\pi f_0 t \sin 2\pi f_c t$
$= \sin(2\pi(f_0 + f_c)t)$

⑥ $$\frac{\sin(w_0 t)}{t} \rightarrow \pi\left[u(w+w_0) - u(w-w_0)\right]$$

$$w_0 = 2$$

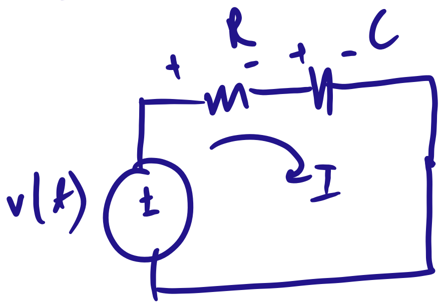
$$\frac{\sin(2t)}{t} \rightarrow \pi\, u(w+2) - u(w-2)$$



$$\downarrow$$

$$= \begin{cases} \pi & \text{if } |w| \le 2, \\ 0, & o/w \end{cases}$$

⑦



$$i = C\frac{dv}{dt}$$

$$v(t) = v_R(t) + v_C(t)$$
$$= iR + v_C(t)$$

$$v(t) = C\frac{dv_C(t)}{dt}R + v_C(t)$$

$$\Rightarrow \frac{dv_C(t)}{dt}RC + v_C(t) = v(t)$$

$$\Rightarrow (sRC + 1)V_C(s) = V(s)$$

$$\Rightarrow V_C(s) = \frac{V(s)}{1+sRC} = \frac{V(s)}{RC(s + 1/RC)}$$

$$= \frac{1/s}{RC(s + 1/RC}$$

$$= \frac{1}{RC} \quad \frac{1}{s(s+1/RC)}$$

$$\frac{1}{s}\frac{1}{(s+1/RC)} = A/s + \frac{B}{s+1/RC}$$

$$A = \left.\frac{1}{s+1/RC}\right|_{s=0} = RC$$

$$B = \left.\frac{1}{s}\right|_{s=-1/RC} = -RC$$

$$= 1/RC\left[\frac{RC}{s} - \frac{RC}{s+1/RC}\right)$$

$$v_c(s) = \frac{1}{s} - \frac{1}{s+1/RC}$$

$$v_c(t) = \left(1 - e^{-t/RC}\right)u(t)$$

$RC = 10260 \times 10^{6} \times 10^{-12}$

$= 10260 \times 10^{-6}$.

as $t \to \infty$ $\quad v_c(t) = 1 \to$ final value

$\underline{10\%}$ of final value $= 0.1$

$$0.1 = 1 - e^{-t/RC}$$

$$= 1 - e^{-t \times 10^{6}/10260}$$

$$0.1 = 1 - e^{-97.46t}$$

$$\Rightarrow e^{-97.46t} = 0.9$$

$$\Rightarrow -97.46t = -0.1053$$

$$= 0.00108\,s \quad \text{or } 1.08\,ms$$

$0.9 = 1 - e^{-97.46t}$

$\Rightarrow e^{-97.46t} = 0.1$

$-97.46t = -2.302$

$t = 23.6\text{ ms}.$

$23.6 - 1.08 \simeq \underline{22.5\text{ms}}.$

$x(t) \to X(w)$

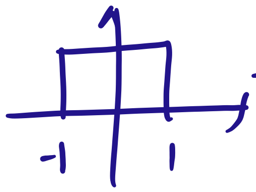
$X(t) \to 2\pi x(-w)$

(8) $X(w) = \dfrac{2\sin 2w \cos w}{w}.$

$\cos w_0 t \longrightarrow \pi[\delta(w - w_0) + \delta(w + w_0)]$

$\pi[\delta(t - w_0) + \delta(t + w_0)] \longrightarrow 2\pi\cos(-ww_0)$
$= 2\pi\cos(ww_0)$

$\frac{\pi}{j}[\delta(t - w_0) - \delta(t + w_0)] \to 2\pi\sin(-w_0 w)$

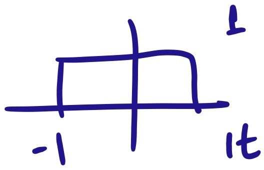


$\longrightarrow 2\dfrac{\sin w}{w}.$

$1 = T_1 = T/2.$ $\quad \text{rect}(t/2) \to 2\sin$

$\Rightarrow T = 2.$ $\quad \cancel{A}\,\text{rect}(t/\tau) \to \dfrac{2\cancel{A}}{w}\sin(w\tau/2).$

$\tau = 2$



$\longleftarrow \text{rect}(t/2) \to 2\dfrac{\sin w}{w}.$ ✓

$[\delta(t - w_0) + \delta(t + w_0)] \longrightarrow 2\cos(-ww_0)$
$= 2\pi\cos(ww_0)$

$w_0 = 2$ $\quad \frac{1}{2}(\delta(t-2) + \delta(t+2)) \longrightarrow \cos(2w)$

$\text{rect}(t/2) \rightarrow \dfrac{2\sin w}{w}$

$\frac{1}{2}\left(\delta(t-2) + \delta(t+2)\right) \rightarrow \cos 2w.$

$\text{rect}(t/2) * \frac{1}{2}\delta(t-2)$

$+ \text{rect}(t/2) * \frac{1}{2}\delta(t+2).$

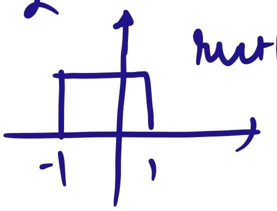
$\cos 2w \;, \; \dfrac{2\sin w}{w}$

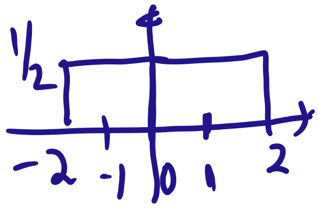
$x(t) * \delta(t-t_0) = x(t-t_0)$

$= \frac{1}{2}\text{rect}\left(\frac{t-2}{2}\right) + \frac{1}{2}\text{rect}\left(\frac{t+2}{2}\right)$

$\text{rect}(t/2 - 1)$

$\text{rect}(t/2)$





$x(-2) = 1/2.$

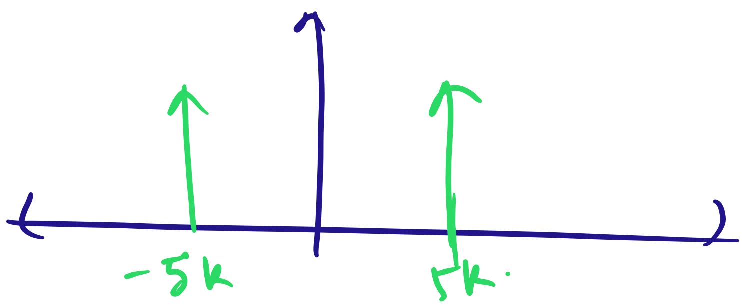
(9) $\dfrac{\sin 8\pi t \cos 8\pi t \; 2}{2\,(2\pi t)} = \dfrac{\sin(16\pi t)}{4\pi t}$

$2\cancel{\pi}f = 16\cancel{\pi}$

$f = 8$

$f_s = 2f_m = 16 \; \underline{Hz}$

(10)



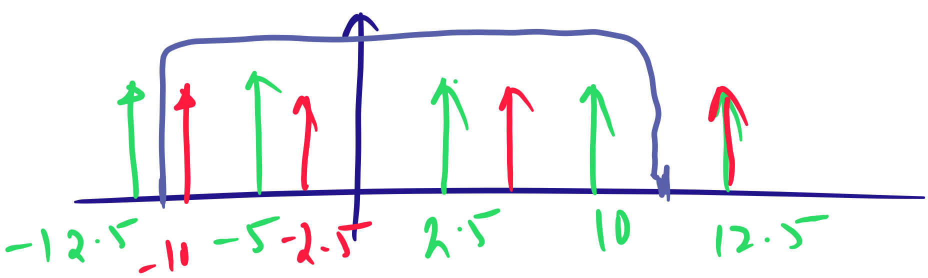
$f_m = 5k.$

$f_s = 7.5k$

-2.5
-7.5

7.5
-5

-5
-7.5
-12.5



$f_c = 11 \text{ KHz}$

freq at o/p $= 2.5, 5, 10 \text{ KHz}$

(4) $T_s = 2\pi / w_s$. (official solution).

$$\frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(w - n w_s) = w_s / 2\pi \sum_{k=-\infty}^{\infty} X(w - k w_s).$$

↳ when sampled by

$$\sum_{k=-\infty}^{\infty} \delta(t - k\, 2\pi / w_s).$$

$$\frac{w_s}{2\pi} \sum_{k=-\infty}^{\infty} \delta(t - k\, 2\pi / w_s) \longrightarrow \frac{w_s^2}{2\pi} \sum_{k=-\infty}^{\infty} X(w - k w_s).$$