$\boxed{\text{Week 4}}$

Laplace Transform

$\cos(w_0 t)\, u(t)$ → cosine signal.

$$\mathcal{L}\left(\cos(w_0 t)\, u(t)\right)$$

$$= \int_0^\infty \cos w_0 t \; e^{-st}\, dt$$

$$= \int_0^\infty \frac{1}{2}\left(e^{-jw_0 t} + e^{jw_0 t}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^\infty e^{-(s+jw_0)t}\, dt + \frac{1}{2}\int_0^\infty e^{-(s-jw_0)t}\, dt$$

$$= \frac{1}{2}\, \frac{e^{-(s+jw_0)t}}{-(s+jw_0)}\Bigg|_0^\infty + \frac{1}{2}\, \frac{e^{-(s-jw_0)t}}{-(s-jw_0)}\Bigg|_0^\infty$$

$$Re\{s\} > 0$$

$$= \frac{1}{2}\left[\frac{1}{s+jw_0} + \frac{1}{s-jw_0}\right]$$

$$= \frac{1}{2}\left[\frac{2s}{s^2+w_0^2}\right]$$

$$= s / s^2 + w_0^2$$

$$\mathcal{L}\left[\cos(w_0 t)\, u(t)\right] = \frac{s}{s^2+w_0^2}$$

Properties of Laplace Transform

$x_1(t) \longleftrightarrow X_1(s) \longrightarrow ROC = R_1$

$x_2(t) \longleftrightarrow X_2(s). \longrightarrow ROC = R_2$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 X_1(s) + a_2 X_2(s)$

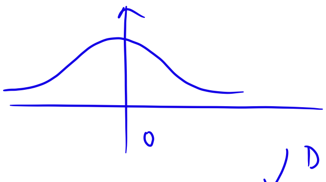
$\longrightarrow ROC = R_1 \cap R_2$

Time shifting Property

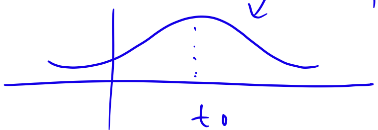
$x(t) \longleftrightarrow X(s)$

$x(t - t_0) \longleftrightarrow ?$

$x(t)$ delayed by t_0.



Delayed by t_0.



$$\int_{-\infty}^{\infty} x(t-t_0)\, e^{-st}\, dt$$

$$t - t_0 = \tilde{t}$$

$$dt = d\tilde{t}$$

$$= \int_{-\infty}^{\infty} x(\tilde{t})\, e^{-s(\tilde{t}+t_0)}\, d\tilde{t}$$

$$= e^{-st_0} \underbrace{\int_{-\infty}^{\infty} x(\tilde{t})\, e^{-s\tilde{t}}\, d\tilde{t}}_{\mathcal{L}(x(t)) = X(s)}$$

$$= e^{-st_0} X(s)$$

ROC $= R =$ ROC of $x(t)$.

$$\boxed{x(t-t_0) \longleftrightarrow e^{-st_0} X(s)}$$

Delay in time

Multiplication by e^{-st_0} in Transform Domain.

$$x(t) \longleftrightarrow X(s) \rightarrow \text{ROC} = R$$

$$e^{s_0 t} x(t) \longleftrightarrow ?$$

$$\int_{-\infty}^{\infty} e^{s_0 t}\, x(t)\, e^{-st}\, dt$$

$$= \underbrace{\int_{-\infty}^{\infty} x(t)\, e^{-(s-s_0)t}\, dt}_{X(s-s_0)}$$

Laplace transform evaluated at $s - s_0$.

$$s - Re\{s_0\} \in R$$

$$\underline{s \in R + Re\{s_0\}}.$$

ROC

<u>Differentiation in time</u>

$$x(t) \longleftrightarrow X(s)$$

$$\frac{dx(t)}{dt} \longleftrightarrow ?$$

$$\int_{-\infty}^{\infty} \frac{dx(t)}{dt}\, e^{-st}\, dt$$

$$= x(t)e^{-st}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} x(t)\, \frac{d e^{-st}}{dt}\, dt$$

$$= -\int_{-\infty}^{\infty} x(t)(-s)\, e^{-st}\, dt$$

$$= s\int_{-\infty}^{\infty} \underline{x(t) e^{-st} dt}$$

$$= s\,X(s) \qquad X(s)$$

ROC = R = ROC of $x(t)$

$$\boxed{\frac{dx(t)}{dt} \longleftrightarrow s\,X(s)}$$

Multiplication by s in Transform Domain.

Similarly:

$$t\,x(t) \longleftrightarrow -\frac{dX(s)}{ds}.$$

Integration in Time

$$x(t) \longleftrightarrow X(s)$$

$$\tilde{x}(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

o/p of integration circuit. Acts as a low pass filter.

$$\frac{d\tilde{x}(t)}{dt} = x(t)$$

$$s\tilde{X}(s) = X(s)$$

$$\boxed{\tilde{X}(s) \longleftrightarrow \frac{X(s)}{s}.}$$

ROC = R.

CONVOLUTION

$$x_1(t) \longleftrightarrow X_1(s) \qquad ROC = R_1$$

$$x_2(t) \longleftrightarrow X_2(s) \qquad ROC = R_2$$

$$x_1(t) * x_2(t) \longleftrightarrow ?$$

$$\underbrace{\int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau}_{\text{convolution}}\; e^{-st}\, dt}_{\text{Laplace Transform.}}$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, d\tau \int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, d\tau\, e^{-s\tau} \underbrace{\int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-s(t-\tau)}\, dt}$$

$t - \tau = \tilde{t}$

$dt = d\tilde{t}$

$$= \underbrace{\int_{-\infty}^{\infty} x_1(\tau) e^{-s\tau} d\tau}_{X_1(s)} \underbrace{\int_{-\infty}^{\infty} x_2(\tilde{t})\, e^{-s\tilde{t}} d\tilde{t}}_{X_2(s)}$$

$= X_1(s) X_2(s)$

$$\boxed{x_1(t) * x_2(t) \longleftrightarrow X_1(s) X_2(s)}$$

Convolution in Time

Product in L.T domain.

This property helps easily evaluate the convolution of two signals.

⇒ Convenient tool to analyse the output behaviour of LTI systems.

<u>POLES AND ZEROS:</u>

$X(s)$ = Rational function of s.

$$= \frac{a_0 s^m + a_1 s^{m-1} + \dots + a_m}{b_0 s^n + b_1 s^{n-1} + \dots + b_n}$$

Numerator polynomial

Denominator polynomial.

$$= \frac{a_0}{b_0} \frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

z_k, $1 \le k \le m$

= zeros of transfer function.

p_k = poles of transfer function

<u>Proper Rational Function</u>

if $m < n$ → Degree of Denominator polynomial

m → Degree of Num. polynomial

$m \geq n \Rightarrow$ Improper rational functions

Poles of $X(s)$ CANNOT LIE on ROC as $X(s) = \infty$ at poles.

$$X(s) = \frac{4s+3}{4s^2+6s+4}$$

$$= \frac{4(s+3/4)}{2(s+2)(s+1)}$$

$m=1$
$n=2$

$m < n$
$\Rightarrow$ Proper rational Function

$$= \frac{2(s+3/4)}{(s+2)(s+1)}$$

$z_1 = -3/4$: zero

$p_1 = -2$, $p_2 = -1$: poles

ROC cannot include poles $= -2, -1$

PROPERTIES OF ROC

(1) ROC does NOT contain poles

(2) For a finite duration signal

$x(t) = 0$ for $t < t_1$ or $t > t_2$

$\Rightarrow x(t) \neq 0$ only for $t_1 \leq t \leq t_2$

ROC : entire s-plane

(3) for right handed signal

$\Rightarrow x(t) = 0$ for $t < t_1$

ROC of form $Re\{s\} > \sigma_{max}$.

σ_{max} = max^m of real parts of poles of $X(s)$.

(4) for left handed signals

$\Rightarrow x(t) = 0$ for $t > t_1$

ROC of form $Re\{s\} < \sigma_{min}$

σ_{min} = minimum of Real parts of poles of $X(s)$.

(5) $x(t)$ = 2 sided signals

ROC of the form

$\sigma_1 < Re(s) < \sigma_2$

σ_1, σ_2 = real parts of poles of $X(s)$

PARTIAL FRACTION EXPANSION

$$X(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$m<n \Rightarrow$ proper rational function

Simple poles

$\Rightarrow$ all poles are Distinct

$p_i \neq p_j$

$$X(s) = \frac{c_1}{s-p_1} + \frac{c_2}{s-p_2} + \cdots + \frac{c_n}{s-p_n}$$

partial Fraction Expansion

$$\boxed{c_k = (s-p_k)X(s)\Big|_{s=p_k}}$$

Coefficient in Partial Fraction expansion of $X(s)$

MULTIPLE POLE

$X(s)$ in denominator has a factor of form $(s-p_i)^r$

pole of p_i has multiplicity $= r$

$X(s)$ will have terms of form

$$\underbrace{\frac{\lambda_1}{s-p_i} + \frac{\lambda_2}{(s-p_i)^2} + \cdots + \frac{\lambda_r}{(s-p_i)^r}}_{\text{Terms for pole } p_i}$$

$$\boxed{\lambda_{r-k} = \frac{1}{k!}\frac{d^k}{ds^k}\left((s-p_i)^r X(s)\right)\Big|_{s=p_i}}$$

Coefficient in PF expansion of $X(s)$

LAPLACE TRANSFORM FOR LTI SYSTEMS.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

LTI system given by impulse response $h(t)$

$x(t)$ = input,
$y(t)$ = output

$$y(t) = \underbrace{x(t) * h(t)}_{\text{Convolution}}$$

$$Y(s) = X(s)\,H(s)$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

→ Transfer function of system.

$$\boxed{h(t) \longleftrightarrow H(s)}$$

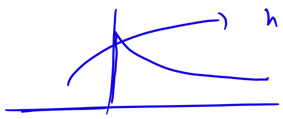
→ Laplace transform of impulse response of LTI system.

PROPERTIES OF LTI SYSTEMS.

CAUSAL SYSTEM

$h(t) = 0$ for $t < 0$

↳ Right-handed signal



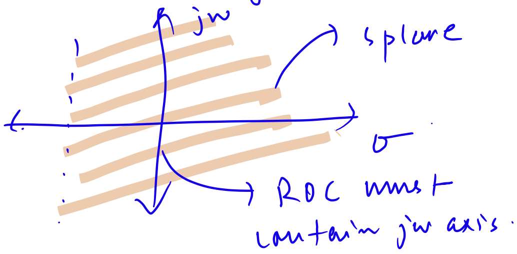
→ ROC of $H(s)$ is of form $\mathrm{Re}(s) > \sigma_{max}$.

STABILITY OF LTI SYSTEMS.

BIBO STABLE if

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

→ ROC contains $j\omega$ axis.



$$s = j\omega$$

$$|H(j\omega)| = \left| \int_{-\infty}^{\infty} h(t) e^{-j\omega t} \, dt \right|$$

$$\leq \int_{-\infty}^{\infty} |h(t)| \underbrace{|e^{-j\omega t}|}_{1} \, dt$$

$$= \underline{\int_{-\infty}^{\infty} |h(t)| \, dt < \infty}$$

finite

$$H(j\omega) = \text{finite} < \infty$$

$\Rightarrow j\omega \in ROC$

$\Rightarrow j\omega$ Belongs to ROC of $H(s)$

TRANSFER FUNCTION OF LTI SYSTEM DESCRIBED BY DE

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

$$y(t) \longleftrightarrow Y(s)$$

$$\frac{dy(t)}{dt} \longleftrightarrow sY(s)$$

$$\frac{d^k y(t)}{dt^k} \longleftrightarrow s^k Y(s)$$

From DE it follows

$$\sum_{k=0}^{N} a_k s^k Y(s) = \sum_{k=0}^{M} b_k s^k X(s)$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}}$$

T-F of the system

Rational function

EXAMPLE PROBLEMS

(1) $x(t) = e^{2t} u(t) + e^{-3t} u(-t)$

$\hookrightarrow$ Laplace Transform

$$e^{-at}u(t) = \frac{1}{s+a}$$

$$ROC = Re(s) > -a$$

$$-e^{-at}u(-t) = \frac{1}{s+a}$$

$$ROC = Re\{s\} < -a$$

$$e^{2t}u(t) = e^{-at}u(t)$$

$$a = -2$$

$$\longleftrightarrow \frac{1}{s-2}, \quad Re\{s\} > 2$$

$$e^{-3t}u(-t) \longleftrightarrow \frac{-1}{s+3}$$

$$Re\{s\} < -3.$$

NET ROC $= Re\{s\} > 2$

$\cap \, Re\{s\} < -3$

$= \phi \leftarrow$ empty set

$\Rightarrow$ ROC does not overlap

$\Rightarrow$ LT does not exist.

$$x(t) = e^{-3t}u(t) + e^{2t}u(-t)$$

$a = 3$ $\qquad a = -2$

$$X(s) = \frac{1}{s+3} - \frac{1}{s-2}$$

ROC: $Re\{s\} > -3$ $\qquad Re\{s\} < 2$

ROC $= Re\{s\} > -3 \cap Re\{s\} < 2$

$= -3 < Re\{s\} < 2$

$$X(s) = \frac{-5}{(s+3)(s-2)}$$

ROC $= -3 < Re\{s\} < 2$

(2) $x(t) = e^{-2t}(u(t) - u(t-5))$

$= e^{-2t}u(t) - e^{-2t}u(t-5)$

$$e^{-2t}u(t) \longleftrightarrow \frac{1}{s+2}$$

$$Re\{s\} > -2$$

$e^{-2t} u(t-5)$

$= e^{-2(t-5)} e^{-10} u(t-5)$

$= e^{-10} \underbrace{\left[e^{-2(t-5)} u(t-5) \right]}_{x(t-t_0)}$

$\longleftrightarrow \quad e^{-10} e^{-5s} \times \dfrac{1}{(s+2)}$

$Re\{s\} > -2$

$X(s) = \dfrac{1}{s+2} - \dfrac{e^{-10} e^{-5s}}{s+2}$

$= \dfrac{1}{s+2}\left(1 - e^{-5(s+2)}\right)$

ROC : $Re\{s\} > -2$

(3) LT of $e^{-at} \cos(w_0 t)\, \underline{u(t)}$

$\cos(w_0 t)\, u(t) \longleftrightarrow \dfrac{s}{s^2 + w_0^2}$

ROC : $Re\{s\} > 0$

$x(t) \longleftrightarrow X(s)$

$e^{-at} x(t) \longleftrightarrow X(s+a)$

$e^{-at} \cos w_0 t\, u(t) \longleftrightarrow \dfrac{s+a}{(s+a)^2 + w_0^2}$

ROC : $Re\{s\} + a > 0$

$\Rightarrow \boxed{Re\{s\} > -a}$

LT $X(s)$.

(4) $X(s) = \dfrac{3s+5}{s^2+3s+2}$

ROC : $Re\{s\} < -2$

Inverse Laplace Transform

$x(t) = ?$

Rational function

$m = 1$ $\quad m < n$.

$n = 2$

$\Rightarrow$ Proper Rational function.

$$X(s) = \frac{(3s+5)}{(s+2)(s+1)}$$

$P_1 = -2,\ P_2 = -1 \rightarrow$ poles.

$z_1 = -5/3$.

Partial fraction Expansion.

$$X(s) = c_1/s+2 + \frac{c_2}{s+1}$$

$$c_1 = (s+2)\,X(s)\Big|_{s=-2}$$

$$= \frac{(3s+5)}{s+1}\Big|_{s=-2}$$

$$= -1/-1 = 1$$

$$\boxed{c_1 = 1}$$

$$c_2 = (s+1)\,X(s)\Big|_{s=-1}$$

$$= \frac{3s+5}{s+2}\Big|_{s=-1}$$

$$= 2/1 = 2$$

$c_2 = 2$

$$X(s) = \frac{1}{s+2} + \frac{2}{s+1} \quad \boxed{Re\{s\} < -2}$$

$-e^{-2t}u(-t)$

Left handed signal.

$-2e^{-t}u(-t)$

$$\boxed{x(t) = -e^{-2t}u(-t) - 2e^{-t}u(-t)}$$

Inverse L.T

(5) Inverse Laplace Transform

$$X(s) = \frac{-s^2-2s+1}{(s+2)(s+3)^2}$$

ROC : $Re\{s\} > -2$

$P_1 = -2,\ P_2 = -3$

Multiplicity $= 2$

$r = 2$

$$X(s) = \frac{c_1}{s+2} + \frac{d_1}{s+3} + \frac{d_2}{(s+3)^2}$$

$$c_1 = (s+2)\,X(s)\Big|_{s=-2}$$

$$= -\frac{s^2-2s+1}{\cancel{(s+3)^2}}\Bigg|_{s=-2}$$

$$= \frac{-4+4+1}{1}\Bigg|$$

$$=1$$

$$c_1 = 1$$

$$d_{r-k} = \frac{1}{k!}\frac{d^k}{ds^k}(s-p_i)^r X(s)\Big|_{s=p_i}$$

$r=2$

$k=0$

$$d_2 = \frac{1}{0!}(s+3)^2 X(s)\Big|_{s=-3}$$

$$= -\frac{s^2-2s+1}{s+2}\Bigg|_{s=-3}$$

$$= \frac{-9+6+1}{-1} = 2$$

$$d_2 = 2$$

$$\underline{k=1}$$

$$d_1 = \frac{1}{1!}\frac{d}{ds}\left|(s+3)^2 X(s)\right|_{s=-3}$$

$$\frac{d}{ds}\left((s+3)^2 X(s)\right)$$

$$= d/ds\left[\frac{-s^2-2s+1}{s+2}\right]$$

$$= \frac{d}{ds}\left[\frac{-(s+2)s+1}{s+2}\right]$$

$$= \frac{d}{ds}\left[-s + \frac{1}{s+2}\right]$$

$$= -1 - \frac{1}{(s+2)^2}$$

$$\frac{d}{ds}\left((s+3)^2 X(s)\right)\Big|_{s=-3}$$

$$= -1 - 1/1 = -2$$

$$d_1 = -2$$

$$\boxed{X(s) = \frac{1}{s+2} \quad \frac{-2}{s+3} + \frac{2}{(s+3)^2}}$$

Two terms for $(s+3)$ since
$p_2=-3$ has multiplicity
$r=2$

ROC: $Re\{s\}>-2$
$\Downarrow$
Right handed signal.

$$\frac{1}{s+2} \longleftrightarrow e^{-2t}u(t)$$
$$\frac{1}{s+3} \longleftrightarrow e^{-3t}u(t)$$
$$\frac{1}{(s+3)^2} \longleftrightarrow ?$$

$$x(t) \longleftrightarrow X(s)$$
$$-tx(t) \longleftrightarrow \frac{dX(s)}{ds}$$
$$\frac{1}{(s+3)^2} = -\frac{d}{ds}\frac{1}{(s+3)}$$

$$X(s) = (s+3) \longleftrightarrow e^{-3t}u(t)$$

$$\frac{1}{(s+3)^2} = -\frac{d}{ds}\frac{1}{(s+3)}$$
$$\longleftrightarrow -t\left(-e^{-3t}u(t)\right)$$
$$= te^{-3t}u(t)$$

$$X(s) = \frac{1}{s+2} - \frac{2}{s+3} + \frac{2}{(s+3)^2}$$
$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$
$$e^{-2t}u(t) \quad -e^{-3t}u(t) \quad te^{-3t}u(t)$$

$$\boxed{x(t) = e^{2t}u(t) - 2e^{-3t}u(t) + 2te^{-3t}u(t)}$$

Inverse Laplace transform of $X(s)$.

⑥ Inverse Laplace Transform.

$$X(s) = \frac{3+4se^{-2s}}{s^2+6s+8}$$

ROC: $Re\{s\}>-2$
$\Rightarrow$ Right handed signal.

$$\frac{1}{s^2+6s+8} = \frac{1}{(s+2)(s+4)}$$
$$= \frac{1}{2}\frac{1}{s+2} - \frac{1}{2}\frac{1}{s+4}$$

$$\frac{3}{s^2+6s+8} \longleftrightarrow \frac{3}{2}e^{-2t}u(t) - \frac{3}{2}e^{-4t}u(t)$$

—— ①

$$\frac{s}{s^2+6s+8} = \frac{s}{(s+2)(s+4)}$$

$$= \frac{2}{s+4} - \frac{1}{s+2}$$

$$\longleftrightarrow 2e^{-4t}u(t) - e^{-2t}u(t)$$

$Y(s) \rightarrow y(t)$ Delayed signal

$$e^{-st_0}Y(s) \longleftrightarrow y(t-t_0)$$

$$e^{-2s}Y(s) \longleftrightarrow y(t-2)$$

$$4e^{-2s}Y(s) \longleftrightarrow 4y(t-2)$$

$$\frac{4e^{-2s}\cdot s}{s^2+6s+8} \longleftrightarrow 4\left\{2e^{-4(t-2)}u(t-2) - e^{-2(t-2)}u(t-2)\right\}$$

$$= 8e^{-4(t-2)}u(t-2) - 4e^{-2(t-2)}u(t-2) \quad \text{—— (2)}$$

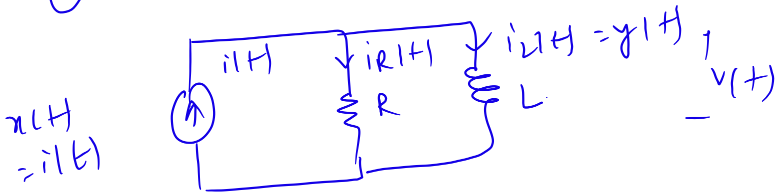
Net signal $x(t)$ given as

$$x(t) = (1) + (2)$$

$$= \tfrac{3}{2}e^{-2t}u(t) - \tfrac{3}{2}e^{-4t}u(t) + 8e^{-4(t-2)}u(t-2) - 4e^{-2(t-2)}u(t-2)$$

$\longrightarrow$ Solution

(7) Consider RL circuit.



$$v(t) = L\frac{di_L(t)}{dt}$$

$$i_R(t)\,R = v(t) = L\frac{di_L(t)}{dt}$$

$$\Rightarrow i_R(t) = \frac{L}{R}\frac{di_L(t)}{dt}$$

$$i(t) = i_R(t) + i_L(t)$$

$$i(t) = \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t)$$

$\uparrow$ $x(t)$

$$x(t) = \frac{L}{R}\frac{dy(t)}{dt} + y(t)$$

$$X(s) = \frac{L}{R} sY(s) + Y(s)$$

$$= Y(s)\left(1 + \frac{L}{R}s\right)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{1 + L/R\, s}$$

$$\boxed{H(s) = \frac{1}{1 + L/R\, s}}$$

$$= \frac{R/L}{s + R/L}$$

$p_1 = -R/L$

ROC : $s > -R/L$

$$\boxed{h(t) = R/L\, e^{-Rt/L} u(t)}$$

Impulse response of RL circuit.

Unilateral Laplace Transform.

one sided laplace transform

$$X_I(s) = \int_{0^-}^{\infty} x(t) e^{-st}\, dt$$

ROC : $Re\{s\} > \sigma_{max}$

ULT : use for causal system with causal input + non zero initial conditions.

Properties.

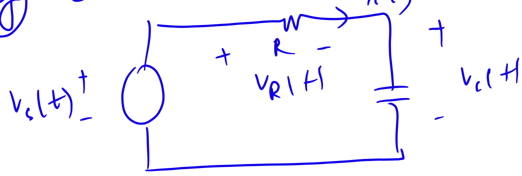
$$x(t) \longleftrightarrow X_I(s)$$

$$\frac{dx(t)}{dt} \longleftrightarrow sX_I(s) - x(0^-)$$

$$\int_{0^-}^{t} x(\tau)\, d\tau \longleftrightarrow \frac{X_I(s)}{s}$$

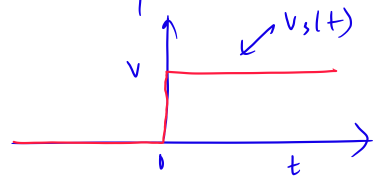
$$\int_{-\infty}^{t} x(\tau)\, d\tau \longleftrightarrow \frac{X_I(s)}{s} + 1/s \int_{-\infty}^{0^-} x(\tau)\, d\tau.$$

(eg) Consider RC circuit.



$v_c(0^-) = V_0$, $\quad v_s(t) = V u(t)$

initial voltage of capacitor



$v_c(t) = ?$ for $t \geq 0$

$v_c(t) + v_R(t) = v(t)$

$v_R(t) = i(t) R$

$i(t) = C \dfrac{d v_c(t)}{dt}$

$\Rightarrow v_R(t) = RC \dfrac{d v_c(t)}{dt}$

$\Rightarrow v_c(t) + RC \dfrac{d v_c(t)}{dt} = v_s(t)$

$= V u(t)$

Taking ULT,

$V_c(s) + RC\left(s V_c(s) - v_c(0^-)\right) = V/s$

$\Rightarrow V_c(s) + RC\left(s V_c(s) - V_0\right) = V/s$

$\Rightarrow V_c(s)(1 + RCs) = RCV_0 + V/s$

$$V_c(s) = \frac{RCV_0}{1+RCs} + \frac{V}{s(1+RCs)}$$

$$= \frac{V_0}{s + 1/RC} + V\left(\frac{1}{s} - \frac{RC}{1+RCs}\right)$$

Taking ILT

$v_c(t) = V_0 e^{-t/RC} u(t) + V\left(u(t) - e^{-t/RC} u(t)\right)$

$= V_0 e^{-t/RC} u(t) + V\left(1 - e^{-t/RC}\right) u(t)$

for $t \geq 0$

$$\boxed{v_c(t) = V_0 e^{-t/RC} + V\left(1 - e^{-t/RC}\right)}$$

o/p for $t \geq 0$.