Assignment 6

(1) $x(n) \longleftrightarrow X(z)$

$$x(n+n_0) \longleftrightarrow z^{+n_0} X(z) = X_1(z)$$

$$x(-n+n_0) \longleftrightarrow X_1\left(\frac{1}{z}\right)$$

$$= \left(\frac{1}{z}\right)^{n_0} X(1/z)$$

$$= z^{-n_0} X(1/z).$$

(2) Impulse response is given by $H(z)$.

Now $H(z) = \dfrac{Y(z)}{X(z)}$ } Rational fxn.

(3) $5^{-n+1} u(n+1) - 4^n u(-n-1)$

LSS- $\dfrac{z}{z-4}$ $|z|<4$ ✓

$$x_1(n) = 5^{-n+1} u(n+1)$$

$$= 5 \cdot 5^{-n} u(n+1)$$

$$= 5 \cdot \left(\frac{1}{5}\right)^n u(n+1)$$

$$= 5 \cdot (1/5)^n \cdot \frac{(1/5)}{(1/5)} u(n+1)$$

$$= 25 \, (1/5)^{n+1} u(n+1).$$

$$\underbrace{(1/5)^n u(n)}_{x(n)} \longrightarrow \underbrace{\frac{z}{z-1/5}}_{X(z)}, \quad |z| > 1/5$$

$$x(n) \longleftrightarrow X(z)$$

$$x(n-n_0) \longleftrightarrow z^{-n_0} X(z)$$

$$x(n-(-1)) \longrightarrow z^{-(-1)} X(z)$$

$$\Rightarrow x(n+1) \longleftrightarrow z X(z)$$

i.e. $\left(\frac{1}{5}\right)^{n+1} u(n+1) \longleftrightarrow \dfrac{z^2}{z-1/5}$ $|z| > 1/5$

$\therefore$ ROC

$$1/5 < |z| < 4.$$

④ $\cos(\pi/3)\, n\, u(n)$.

From common z-transform pairs.

$$\cos(w_0 n)\, u(n) \longleftrightarrow \frac{1-\cos w_0 z^{-1}}{1-2\cos w_0 z^{-1} + z^{-2}}$$

$|z|>1$

$w_0 = \pi/3 \quad \cos \pi/3 = 1/2$.

$$X(z) = \frac{1 - z^{-1}/2}{1 - z^{-1} + z^{-2}}$$

$$= \left(z^2 - \frac{z}{2}\right) \Big/ \left(z^2 - z + 1\right)$$

⑤ $x(n) \longleftrightarrow X(z)$

$n\, x(n) \longleftrightarrow -z \underbrace{\frac{d X(z)}{dz}}_{X_1(z)}$ [property]

$n^2 x(n)$

$$\longleftrightarrow -z \frac{d X_1(z)}{dz}$$

$$= -z \left[\frac{d}{dz} \left(-z \frac{d}{dz} X(z) \right) \right]$$

$$= +z \left[+z \frac{d^2 X(z)}{dz^2} + \frac{d X(z)}{dz} \cdot 1 \right]$$

$$= z^2 \frac{d^2 X(z)}{dz^2} + z \frac{d X(z)}{dz}.$$

⑥ z, n = zero

z, n also

zero of X(z)

⑦ $x(n) \longleftrightarrow X(z)$

$$n\, x(n) \longleftrightarrow -z \frac{d X(z)}{dz}$$

⑧ $a^n u(n)$ ROC $|z|>|a|$

$$\longrightarrow \frac{z}{z-a}$$

$$n\, a^n u(n) \longrightarrow -z \frac{d}{dz} \left(\frac{z}{z-a} \right)$$

$$= -z\left[\frac{(z-a)\cdot 1 - z}{(z-a)^2}\right]$$

$$= -z\left[\frac{z - a - z}{(z-a)^2}\right]$$

$$= \frac{az}{(z-a)^2}$$

$$= \frac{az}{z^2(1-az^{-1})^2}$$

$$= \frac{az^{-1}}{(1-az^{-1})^2}$$

(9) $\left(\frac{1}{2}\right)^{-n+1} u(-n-4) + \left(\frac{1}{6}\right)^{n+2} u(n-2)$

$$x_1(n) = \left(\frac{1}{2}\right)^{-n+1} u(-n-4)$$

$$= \left(\frac{1}{2}\right)^{-n} \cdot \frac{1}{2}\, u(-n-4)$$

$$= \frac{2^n}{2}\, u(-n-4)$$

$$= \frac{1}{2}\, 2^n\, u(-n-4)$$

$2^n u(-n-1-3)$

$x(n) = 2^n u(-n-1)$.

ROC $|z| \leq 2$ ✓

$x_2(n) \left(\frac{1}{6}\right)^{n+2} u(n-2)$

$= \left(\frac{1}{6}\right)^n \cdot \frac{1}{36}\, u(n-2)$

$= \frac{1}{36}\left(\frac{1}{6}\right)^n u(n-2)$

ROC $|z| > \frac{1}{6}$

$\frac{1}{6} < |z| < 2$

(10) for neither causal nor anticausal

ROC must be of the form

$r_1 < |z| < r_2$.