Session

(TA)

Part I - Summary lectures of W I

(*) signals - a fxn that conveys information about some phenomenon.

eg $x(t): \mathbb{R} \to [-1, 1]$ given by
(time)

$x(t) = \sin(t) \; ; -\infty \le t \le \infty$

This signal for eg. can measure temperature of a city that varies sinusoidally with time

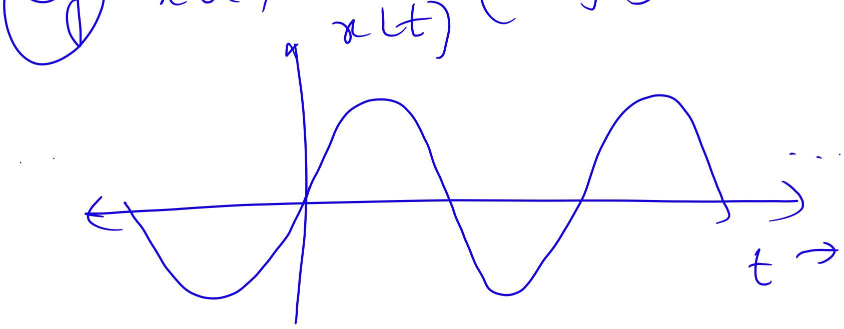
) Typically we encounter signals hat exhibit variation in time/space

Classification of signals

(1) Continuous time signals.

→ argument time 't' is continuous

eg $x(t) = \sin(2\pi f t)$

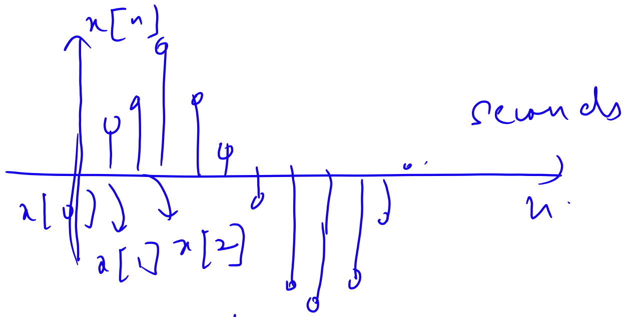


② Discrete time signals.

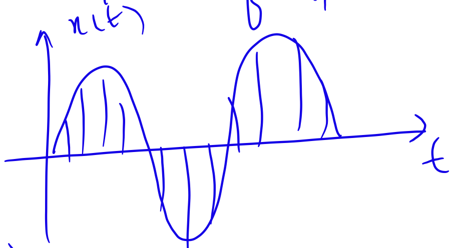
→ signal is measured only at discrete points in time.

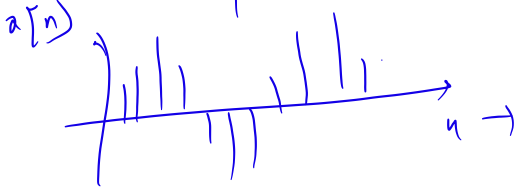
eg every 1 second, we measure signal amplitude.

eg $x[n] = \sin[w_0 n]$



→ It is a sequence of #s





→ can be obtained by sampling a continuous time signal

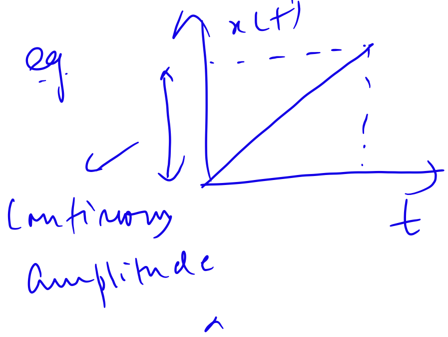
③ Analog signals

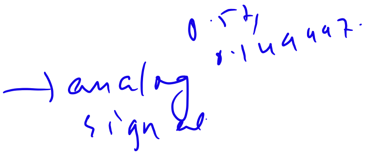
Signals that can take values in continuous amplitude domain.

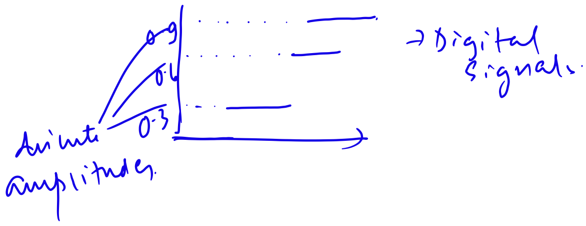
④ Digital signals

Digital signals signal take values in discrete amplitude domain.

eg







(5) Real Signals

$x(t) \in \mathbb{R}$ (i.e) amplitudes takes only real #s.

(eg) $x(t) = \sin(2\pi f t)$

(6) Complex signals.

Signals that take values belonging to set of complex #s.

eg $x(t) = e^{j 2\pi f_c t}$

$= \cos(2\pi f_c t) + j \sin(2\pi f_c t)$

(7) Deterministic (vs) Random signals.

$\sin(2\pi f t)$ $t = 0.1$
$t = 1 \omega$.

Deterministic → signal whose value can be specified at any time instant.

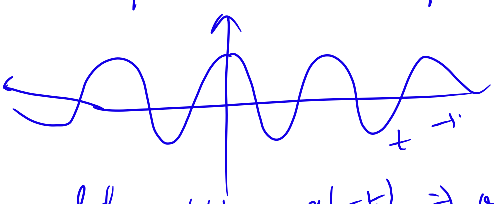
Random : at any time instant, any value can be assumed by the signal

(8) even vs odd signals

even : if $x(t) = x(-t)$ → even signal.

eg $x(t) = \cos(2\pi f_c t)$

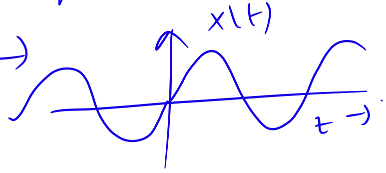
→ symmetric about y-axis.



odd $x(t) = -x(-t)$ ⇒ odd signal

eg $x(t) = \sin(2\pi f_c t)$

→ antisymmetrical about y-axis.

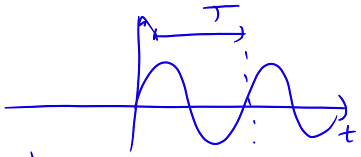


(9) Periodic signal

Periodic → if there exists a T

$\forall t \quad x(t+T) = x(t)$

eg. $\sin(2\pi f t)$, $T = 1/f$ Time period of this signal



$$x(t+T) = \sin(2\pi f(t+T))$$
$$= \sin(2\pi f t + 2\pi f T)$$
$$= \sin(2\pi f t)$$
$$= x(t).$$

Aperiodic → signals that are not periodic

(10) Energy of signal

for $x(t)$

$$\text{Energy } E_{x(t)} \overset{\Delta}{=} \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Defined as

|||ly for $x[n]$,

$$E_{x[n]} \overset{\Delta}{=} \sum_{n=-\infty}^{\infty} |x[n]|^2$$

(11) Power of signal

$$\text{Power} = \frac{\text{Energy}}{T}$$

For $x(t)$

$$\text{Power}: P_{x(t)} \overset{\Delta}{=} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

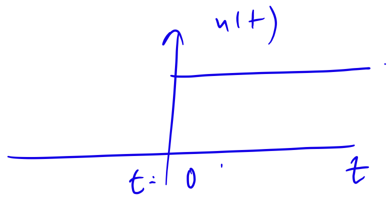
|||ly for $x[n]$

$$\overset{\Delta}{=} \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Some important signals.

(1) Unit step.

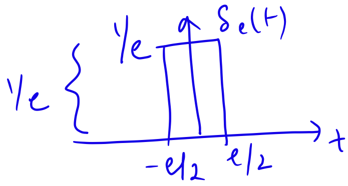
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{else.} \end{cases}$$



(2) Unit impulse :-

$$\delta(t) = \lim_{e \to 0} \delta_e(t) \quad \text{where}$$

$$\delta_e(t) = \begin{cases} 1/e, & -e/2 \le t \le e/2 \\ 0, & \text{else} \end{cases}$$



Area $= 1/e \cdot e = 1$

ie Area under $\delta_e(t) = 1$

(*) $\rightarrow \int_{-\infty}^{\infty} \delta_e(t)\, dt = 1$

(*) $\int_a^b \delta(t)\, dt = \begin{cases} 1, & \text{if } a < 0 < b \\ 0 & \text{else} \end{cases}$

|||ly $\int_{-\infty}^{\infty} x(t)\, \delta(t-t_0)\, dt$

$$= \int_{-\infty}^{\infty} x(t_0)\, \delta(t-t_0)\, dt$$

$$= x(t_0)$$

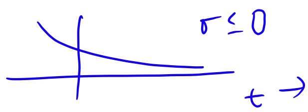
"Sifting Property"

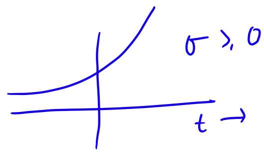
(3) Complex exponential

$$x(t) = A e^{j\omega_0 t}$$

(4) Real exponential

$$x(t) = e^{\sigma t} \geq 0$$





(5) Sinusoidal

$$x(t) = A\cos(\omega_0 t + \theta)$$

angular frequency

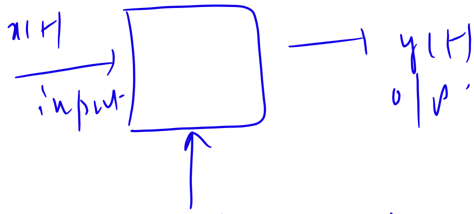
amplitude

phase

[similarly you can have discrete time versions of above signals]

Systems

Box (system)



audio player

$$y(t) = T[x(t)]$$

Classification of systems

(1) Memoryless systems.

The o/p depends only on current i/p.

i.e. $y(t) = f[x(t)]$ only

$y(t) \neq f(x(t), x(t-\epsilon), x(t+\epsilon))$

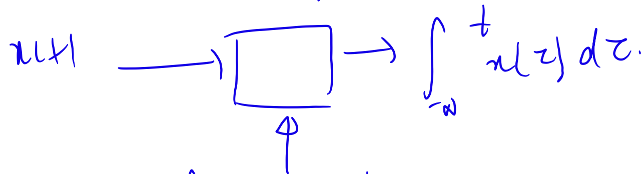
$\epsilon > 0$.

(2) Causal system

o/p depends only on present & past NOT on future values.

Eg (1) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

If we build a system such as



Causal s/s.

(2) $y(t) = \int_{-\infty}^{\infty} x(\tau)\, d\tau, \; t < \infty$

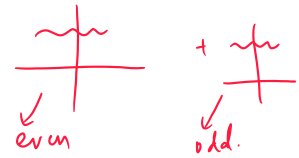
A system which builds this is NON-CAUSAL. They are not realizable in practice.

Part II Tutorials.

(1) Show that any signal $x(t)$ can be decomposed into an even signal and an odd signal.

$x(t) \neq x(-t)$
$x(t) \neq -x(-t)$

neither even nor odd.



Answer let $x(t)$ be a general signal.

$$\therefore x(t) = \frac{x(t) + x(t)}{2}$$

$$= \frac{x(t) + x(t)}{2} + \frac{x(-t) - x(-t)}{2}$$

$$= \underbrace{\frac{x(t) + x(-t)}{2}}_{\triangleq y(t)} + \underbrace{\frac{x(t) - x(-t)}{2}}_{\triangleq z(t)}$$

Claim 1 $y(t)$ is even signal.

$$y(-t) = \frac{x(-t) + x(-(-t))}{2}$$

$$= \frac{x(-t) + x(t)}{2}$$

$$= \frac{x(t) + x(-t)}{2}$$

$$= y(t)$$

$\Rightarrow y(t)$ is an even signal.

Claim 2 : $z(t)$ is an odd signal

$$z(-t) = \frac{x(-t) - x(-(-t))}{2}$$

$$= \frac{x(-t) - x(t)}{2}$$

$$= -\left[\frac{x(t) - x(-t)}{2}\right]$$

$$= -z(t)$$

$\therefore z(-t) = -z(t)$ (or) $z(t)$ is an odd signal.

Moreover $x(t) = y(t) + z(t)$.

even odd. $\square$

Even component of $x(t)$: $\frac{x(t) + x(-t)}{2}$

odd component of $x(t)$: $\frac{x(t) - x(-t)}{2}$

TA

(2) Energy signal: signal for which energy is finite.

Power signal :- Signal for which power is finite.

a) Find the power of an energy signal.

b) Find the energy of a power signal.

Ans a) if $x(t)$ is an energy signal

$$\Rightarrow E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt = c$$

finite number.

$$P_{x(t)} = \lim_{T\to\infty} 1/T \int_{-T/2}^{T/2} |x(t)|^2 dt \leq \lim_{T\to\infty} 1/T \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} c = 0$$

$$\Rightarrow P_{x(t)} \leq 0 \to 0 \quad -(1)$$

But $P_{x(t)} \geq 0 \quad -(2)$

$\Big\} P_{x(t)} = 0$

Power of any energy signal $= 0$

b) let $y(t)$ be a power signal

$$\Rightarrow \lim_{T\to\infty} 1/T \int_{-T/2}^{T/2} |x(t)|^2 dt = d$$ → a finite #

$$\therefore \Rightarrow \frac{\lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt}{\lim_{T\to\infty} T} = d$$

$$\Rightarrow \underbrace{\lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt}_{\int_{-\infty}^{\infty} |x(t)|^2 dt} = d \lim_{T\to\infty} T$$

→ finite #

$$\Rightarrow E_x = d \lim_{T\to\infty} T = \infty$$

$\Rightarrow$ energy of any power signal is ∞

(Q3) Show that any periodic signal is a power signal.

HW

(4) Find the energy and power of the signal $x(t) = e^{-2|t|}$

Ans $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} (e^{-2|t|}) dt$

$= \int_{-\infty}^{\infty} e^{-4|t|} dt \qquad |t| = \begin{cases} t, \text{ if } t \geq 0 \\ -t, \text{ if } t < 0 \end{cases}$

$E_x = \int_{-\infty}^{0} e^{-4(-t)} dt + \int_{0}^{\infty} e^{-4(t)} dt$

$= \int_{-\infty}^{0} e^{4t} dt + \int_{0}^{\infty} e^{-4t} dt$

$= \frac{e^{4t}}{4}\Big|_{-\infty}^{0} + \frac{e^{-4t}}{-4}\Big|_{0}^{\infty}$

$= \frac{1 - e^{-\infty}}{4} - \frac{(e^{-\infty} - 1)}{4}$

$= \frac{(1-0) - (0-1)}{4} = 2/4 = \underline{\underline{1/2}}$

Energy signal

Power:

$P_{x(t)} = \lim_{T \to \infty} 1/T \int_{-T/2}^{T/2} |x(t)|^2 dt$

$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-4|t|} dt$

$= \lim_{T \to \infty} \frac{1}{T} \left[\int_{-T/2}^{0} e^{4t} dt + \int_{0}^{T/2} e^{-4t} dt \right]$

$= \lim_{T \to \infty} 1/T \left[e^{4t}/4 \Big|_{-T/2}^{0} + \frac{e^{-4t}}{-4} \Big|_{0}^{T/2} \right\}$

$= \lim_{T \to \infty} 1/T \left\{ \frac{1 - e^{-2T}}{4} - \frac{(e^{-2T} - 1)}{4} \right\}$

$= \lim_{T \to \infty} \frac{1}{T} \left\{ 1/2 - \frac{e^{-2T}}{2} \right\}$

$= \cancel{\lim_{T \to \infty} \frac{1}{2T}} - \lim_{T \to \infty} e^{-2T}/2T \quad \left(\frac{0}{\infty}\right)$

0

indeterminate form.

L-Hospital Rule

$\lim_{T \to \infty} \frac{e^{-2T}}{2T} = \lim_{T \to \infty} \frac{-2e^{-2T}}{2} = \underline{\underline{0}}$

$P_{x(t)} = 0 - 0 = \underline{\underline{0}}$

(5) let $x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

ie frequency $= 1/T = f$.

find the power of $x(t)$.

Ans $P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$

$T \to$ period of the signal

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A^2 \sin^2\left(\frac{2\pi t}{T}\right) dt$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\to \theta = \frac{2\pi t}{T}$

$= \lim_{T\to\infty} \frac{1}{T} A^2 \int_{-T/2}^{T/2} \left(\frac{1-\cos 2\theta}{2}\right)^2 dt$

$= \lim_{T\to\infty} \frac{1}{T}\cdot\frac{A^2}{2}\int_{-T/2}^{T/2} - \left[\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\cos\left(\frac{4\pi t}{T}\right) dt\right] \times A^2/2$

$\lim_{T\to\infty} \frac{1}{T}\frac{A^2}{2}\cdot \cancel{t} \to A^2/2$ $- \lim_{T\to\infty}\frac{A^2}{2T}\left.\frac{\sin\frac{4\pi t}{T}}{4\pi/T}\right|_{-T/2}^{T/2}$

$= A^2/2 - \lim_{T\to\infty}\frac{A^2}{2T}\frac{T}{4\pi}\left\{\sin\left(\frac{4\pi}{T}\cdot T/2\right) - \sin\left(\frac{4\pi}{T}(-T/2)\right)\right\}$

$= \frac{A^2}{2} - \lim_{T\to\infty} A^2/8\pi \left\{\sin(2\pi) + \sin(2\pi)\right\} \to 0$

$= A^2/2$.

Fact: The power of any sinusoidal (or) co-sinusoidal signal does not depend on frequency (or) phase. It is

It is always equal to

$P_x = \frac{(\text{amplitude})^2}{2}$

(Q): Find power of $x(t)$

$= 8\sin\left(\frac{2\pi t}{24T} + \pi/3\right)$.

Ans: $8^2/2 = 64/2 = 32$

(i.e. w/o integration to find power (by formula)).

$x(t)$
$x[n]$

⑥ Find the period of

$x[n] = \sin^2\left(\frac{\pi}{2}n\right)$; $n \in \mathbb{Z}$

integers.

Ans Recall that

$x[n]$ is periodic if

$x(n+N) = x(n)$; N - period!

$x(n+N) = \sin^2\left(\pi/_2 (n+N)\right)$

$= \sin^2\left(\pi/_2 n + \frac{\pi}{2}N\right)$

$= \frac{1-\cos 2\left(\pi/_2 n + \pi/_2 N\right)}{2}$

$= \frac{1-\cos(\pi n + \pi N)}{2} = x[n]$ — ①

∴ In order for $x[n]$ to be periodic, we need $\cos(\pi n + \pi N) = \cos(\pi n)$

Because, with $\cos(\pi n + \pi N) = \cos(\pi n)$

① becomes $\frac{1-\cos(\pi n)}{2}$

$= \sin^2(\pi/_2 n)$

$= x(n)$.

$\cos(\pi n + \pi N) = \cos(\pi n)$

$\Rightarrow \cancel{\pi}N = 2m\cancel{\pi}$, $m \in \mathbb{Z}$ → integer.

$\boxed{N = 2m}$; fundamental period

@ $m = 1$

$\boxed{N = 2}$ fundamental

∴ $N = 2$ is the fundamental period of $x[n]$.

$x(n) = x(n+2)$

$= x(n+4)$.

(7) Compute the value of

$$I = \int_{-\infty}^{\infty} e^{t}\, \delta\left(\tau - \frac{\alpha}{\beta} t\right) dt \quad , \alpha, \beta, \tau > 0$$

Ans

$$t' \overset{\Delta}{=} \frac{\alpha}{\beta} t \Rightarrow dt' = \alpha/_{\beta}\, dt$$

$$\Rightarrow dt = \beta/_{\alpha}\, dt'$$

$$I = \int_{-\infty \times \alpha/\beta}^{\infty \times \alpha/\beta} e^{(\beta/_{\alpha} t')} \delta(\tau - t') \frac{dt' \beta}{\alpha}$$

$$= \frac{\beta}{\alpha} \int_{-\infty}^{\infty} e^{\beta/_{\alpha} t'} \delta(\tau - t') dt'$$

Recall: $\delta(t) = \delta(-t)$ — (A)

$$I = \frac{\beta}{\alpha} \int_{-\infty}^{\infty} \underbrace{e^{\beta/_{\alpha} t'} \delta(t' - \tau)\, dt'}_{\text{sifting property}}$$

[Due to eqn (A)]

$$= \frac{\beta}{\alpha} \times e^{\beta/_{\alpha} \tau}$$

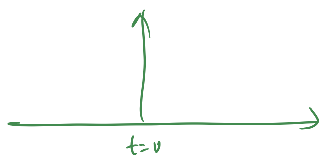
sifting property
$\int_{-\infty}^{\infty} x(t') \delta(t' - \tau) dt' = x(\tau)$

$e^{\beta/_{\alpha} t'}$

$$\boxed{I = \frac{\beta}{\alpha} e^{\beta/_{\alpha} \tau}}$$

(8) Plot $\delta(t)$

Ans



Plot $\delta(\cos(2\pi t))$ vs t

[composition of two signals]

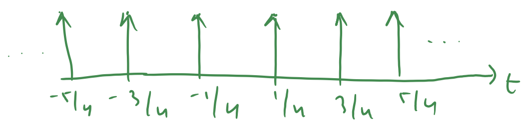
$f(t) = \delta(t)$
$g(t) = \cos(2\pi t)$
$x(t) = f(g(t))$.

(8) Check for all values of t s.t. $\cos(2\pi t) = 0$

because $\delta(0) \neq 0$
$\delta(\text{any other } \#) = 0$

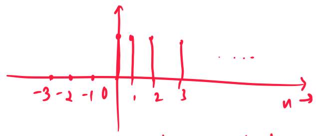
$$\Rightarrow 2\pi t = (2n+1)\pi/_2 \ , n \in \mathbb{Z}$$

$$t = (2n+1) \cdot 1/_4 \ ; n \in \mathbb{Z}$$



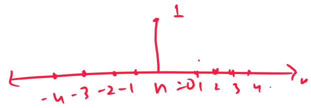
9. Discrete time unit step signal:

$$u[n] = \begin{cases} 1, & \text{if } n = 0, 1, 2, \dots \\ 0, & \text{if } n = -1, -2, \dots \end{cases}$$



Discrete time impulse signal.

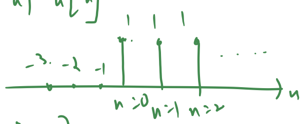
$$\delta[n] = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \neq 0 \end{cases}$$



a) express $\delta[n]$ in terms of $u[n]$

b) express $u[n]$ in terms of $\delta[n]$.

Ans a) $u[n]$

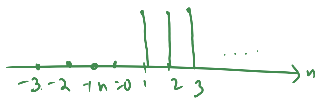


b) $u[n-1]$:

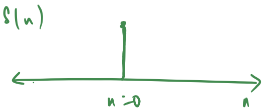
$u[n-1] = 1$ if $n - 1 \geq 0$

(or) $= 1$, if $n \geq 1$

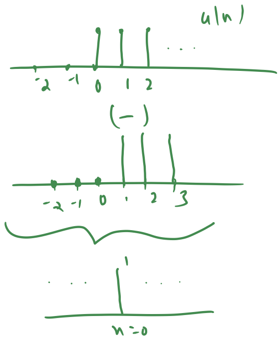
$u[n-1]$



Recall $\delta(n)$



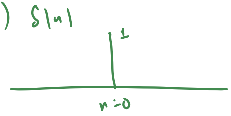
$u[n] - u[n-1] = \delta(n)$.



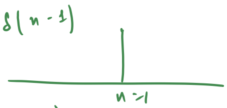
$= \delta(n)$

$\delta(n) = u[n] - u[n-1]$

(b) $\delta[n]$



$\delta(n-1)$



$\delta(n-2)$

$\delta(n-k)$

$$\therefore \boxed{u[n] = \sum_{k=0}^{\infty} \delta[n-k]}$$