Assignment4

(1) For stability, TF of system must contain $j\omega$ axis in its ROC.

(2) $x(t) = e^{-3t}u(t-2) - e^{5t}u(4-t)$

1st term

$e^{-3(t-2)}u(t-2)\,e^{-6}$

$= e^{-6}\,e^{-3(t-2)}u(t-2)$.

$e^{-at}u(t) \rightarrow \frac{1}{s+a}$　　$e^{-3t}u(t) \rightarrow \frac{1}{s+3}$

$x(t) \rightarrow X(s)$

$x(t-t_0) \rightarrow e^{-st_0}X(s)$.

$e^{-3(t-2)}u(t-2)\,e^{-6} \rightarrow \frac{1}{(s+3)}e^{-2s} \times e^{-6}$

$= \frac{e^{-2s}e^{-6}}{(s+3)}$　　$Re\{s\} > -3$.

$-e^{5t}u(4-t)$

$= -e^{20}e^{5t}e^{-20}u(4-t)$

$= -e^{20}e^{5(t-4)}u(4-t)$

$= -e^{20}e^{5(t-4)}u(-(t-4))$

$= -e^{20}e^{-\alpha t}u(-t)$

$\hookrightarrow = e^{20}\frac{1}{s+\alpha}$　　$Re\{s\} < -\alpha$.

$-\alpha = 5$

$\alpha = -5$

$= e^{20}\frac{1}{s-\alpha}$　　$Re\{s\} < 5$

$= e^{20}\frac{1}{s-5}e^{-4s}$　　$Re\{s\} < 5$

$\therefore$ LT $= \frac{e^{-6}e^{-2s}}{s+3} + \frac{e^{20}e^{-4s}}{s-5}$　　ROC $-3 < Re\{s\} < 5$

③ $e^{-\alpha t} u(-t) \longrightarrow -\frac{1}{s+\alpha}$, $\operatorname{Re}\{s\} < -\alpha$.

$t\, x(t) \longrightarrow -\frac{d\, X(s)}{ds}$

$\underbrace{e^{-at} u(-t)}_{x(t)} \longrightarrow \underbrace{\frac{-1}{s+\alpha}}_{X(s)}$ $\operatorname{Re}\{s\} < -a$

$\underbrace{t\, x(t)}_{x_1(t)} \longrightarrow \underbrace{-\left[\frac{d}{ds} X(s)\right]}_{X_1(s)}$

$t\, x_1(t) \longrightarrow -\frac{d}{ds} X_1(s)$.

$\Rightarrow t\, t\, x(t) \longrightarrow -\frac{d}{ds}\left[-\frac{d}{ds} X(s)\right]$

$t^2 x(t) \quad = \frac{d^2}{ds^2} X(s)$

$= \frac{d^2}{ds^2}\left(\frac{-1}{s+a}\right)$

$= \frac{d}{ds}\left[-1\left(\frac{(s+a)\cdot 0 - 1\cdot 1}{(s+a)^2}\right)\right]$

$= \frac{d}{ds}\left[\frac{1}{(s+a)^2}\right]$

$= \frac{(s+a)^2 \cdot 0 - 2(s+a)}{(s+a)^4}$.

$= \frac{-2}{(s+a)^3}$. ✓

④ $\sin(at)\, u(t) \longrightarrow \frac{a}{s^2+a^2}$.

$\sin(-4t) \longrightarrow \frac{-4}{s^2+16}$

⑤ $x(t) \longrightarrow X(s)$

$x(-t) \longrightarrow X(-s)$

$t\,x(-t) \rightarrow \frac{-d}{ds} x(-s)$.

(6) $e^{-at}u(t) \rightarrow \frac{1}{s+a}$, $Re\{s\} > -a$.

$-e^{-at}u(-t) \rightarrow \frac{1}{s+a}$, $Re\{s\} < -a$

$e^{-at}u(-t) \rightarrow \frac{-1}{s+a}$, $Re\{s\} < -a$.

$e^{-2t}u(t) \longrightarrow \frac{1}{s+2}$, $Re\{s\} > -2$

$e^{+4t}u(-t) \rightarrow \frac{-1}{s-4}$ $Re\{s\} < 4$

$\frac{1}{s+2} \quad \frac{-1}{s-4}$

$= \frac{s-4-s-2}{(s+2)(s-4)}$

$= \frac{-6}{(s+2)(s-4)}$ $\quad -2 < Re\{s\} < 4$

(7) $\frac{Y(s)}{X(s)} = s^2 + 2s + 1$

$Y(s) = s^2X(s) + 2sX(s) + X(s)$

$y(t) = \frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} + x(t)$

A LCCDE involving $x(t)$, $y(t)$ & and higher order derivatives of $x(t)$ only.

(8) $x(t) \longrightarrow X(s)$

$x(at) \longrightarrow \frac{1}{|a|} X(s/a)$

$x(t/5) \rightarrow 5\,X(5s)$

$a = 4/5$

$x(-4/5\,t) \rightarrow \underline{5X(-5s)}$.

(9) $e^{-3t}u(t-4)$

$= e^{-3t}e^{+3\cdot 4} e^{-3\cdot 4}u(t-4)$

$= e^{-3(t-4)} e^{-12} u(t-4)$

$= e^{-12} e^{-3(t-4)} u(t-4)$

$\underbrace{e^{-at}u(t)}_{x(t)} \rightarrow \frac{1}{s+a}$

$x(t-4) \rightarrow \frac{e^{-4s}}{s+3} e^{-12}$

$Re\{s\} > -3$

$-e^{-at}u(-t) \rightarrow \frac{1}{s+a} \quad Re\{s\} < -a$

$-e^{-2t}u(2-t) = -e^{-2t} \cdot \frac{e^{2\cdot 2}}{e^{4}} u(2-t)$

$= -e^{-2(t-2)} e^{-4} u(2-t)$

$= -e^{-2(t-2)} u(-(t-2)) e^{-4}$

$-e^{-at}u(-t) \rightarrow \frac{1}{s+a} \quad Re\{s\} < -a$

$a = -2$

$-e^{+2t}u(-t) \rightarrow \frac{1}{s-a} \quad Re\{s\} < 2$

$= -e^{2(t-2)} u(-(t-2)) \rightarrow \frac{e^{-4s}}{s-2} e^{-4}$

LT

$= e^{-12}\frac{e^{-4s}}{s+3} + e^{4}\frac{e^{-2s}}{s-2}$

$-3 < Re\{s\} < 2$

(10) $\frac{1}{s-\alpha} \rightarrow Re\{s\} > \alpha$

$e^{\alpha t}u(t) \rightarrow \frac{1}{s-\alpha}, \; Re\{s\} > \alpha$