# Assignment 5

(1) $X(s) = \frac{1}{(s-p_i)^r}$

Partial fraction

$$X(s) = \frac{k_1}{(s-p_i)} + \frac{k_2}{(s-p_i)^r}$$

$$k_2 = (s-p_i)^r X(s)\Big|_{s=p_i}$$

(2) $X(s) = \frac{-1}{(s+1)(s+2)^2}$

$$X(s) = \frac{4}{s+1} + \frac{\lambda_1}{\underline{(s+2)^1}} + \frac{\lambda_2}{(s+2)^r}$$

$r = 2 \quad k = 1$

$$\lambda(2-1) = \frac{1}{1!}\frac{d^1}{ds^1}\left((s+2)^2 X(s)\right)$$

$$= \frac{d}{ds}\left(\frac{-1}{(s+1)}\right)\Bigg|_{s=-1}$$

$$= -\left[\frac{(s+1)\cdot 0 - 1\cdot 1)}{(s+1)^2}\right)$$

$$= \frac{-1}{(s+1)^2}\Bigg|_{s=-2} = +1 \checkmark$$

(3) Not Done.

(4) $x(t) = e^{-5t}u(-t) + e^{3t}u(t)$.

$$= -\frac{1}{s+5} + \frac{1}{s-3}$$

$Re\{s\} < -5 \cap Re\{s\} > 3$

DNE

(5) $\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} = 3y(t) + x(t)$

$s^2Y(s) - 2sY(s) - 3Y(s) = X(s)$

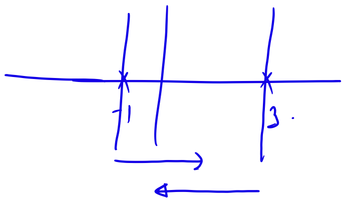
$Y(s)\left[s^2 - 2s - 3\right] = X(s)$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s^2-2s-3}$$

$$= \frac{1}{s^2-3s+s-3}$$

$$= \frac{1}{s(s-3)+1(s-3)}$$

$$= \frac{1}{(s+1)(s-3)} = \frac{A_1}{s+1} + \frac{A_2}{s-3}$$




$$A_1 = \frac{1}{s-3}\Big|_{s=-1} = -1/4$$

$$A_2 = \frac{1}{s+1}\Big|_{s=3} = 1/4$$

$$X(s) = \underbrace{\frac{-1}{4(s+1)}}_{RSS} + \underbrace{\frac{1}{4(s-3)}}_{LSS}$$

$$= -1/4\, e^{-t}u(t) - 1/4\, e^{3t}u(-t)$$

(6) $\frac{3}{(s+2)^2+9}$, $Re\{s\} > -2$

$$\frac{3}{s^2+9} \rightarrow \sin(3t)u(t)$$

$$e^{-2t}\sin(3t)u(t)$$

$$+ \quad \frac{3}{(s+2)^2+9}.$$

(7) $X(s) = \frac{s^2+9s+16}{(s+2)(s+3)^2}$.

$$A_{2\,1} = \frac{1}{1!}\frac{d}{ds}\left(\frac{s^2+9s+16}{s+2}\right)$$

$$= \frac{(s+2)(2s+9)-(s^2+9s+16)}{(s+2)^2}$$

$$= \frac{2s^2+\cancel{9s}+4s+18-s^2-\cancel{9s}-16}{(s+2)^2}$$

$$= \frac{s^2+4s+2}{(s+2)^2}\Big|_{s=-3} = \frac{9-12+2}{1} = -1$$

(8) for causal LTI system, for ROC of $H(s)$

$$Re\{s\} > \sigma_{max}$$

(9) ROC $Re\{s\} < 0 \rightarrow$ LSS.

$$\frac{s}{s^2+16} \rightarrow -\cos 4t \, u(-t) \ldots$$

(10) $\frac{1}{s+\alpha}$, $Re\{s\} > -\alpha$

$$e^{-\alpha t} u(t).$$

(3) $X(s) = \frac{(s+4)\, e^{-3s}}{(s^2+5s+6)}$ ROC $-3 < Re\{s\} < -2$

$$s^2+5s+6 = (s+2)(s+3)$$

$$X(s) = e^{-3s} \frac{(s+4)}{(s+2)(s+3)}$$

$X_1(s)$

$$X_1(s) = \frac{r_1}{s+2} + \frac{r_2}{s+3}$$

$$r_1 = \left.\frac{s+4}{s+3}\right|_{s=-2}$$

$$= \frac{2}{1} = 2$$

$$r_2 = \left.\frac{s+4}{s+2}\right|_{s=-3}$$

$$= -1$$

$$X(s) = e^{-3s}\left(\frac{2}{s+2} - \frac{1}{s+3}\right)$$

$$\frac{2}{s+2} \rightarrow -2e^{-2t} u(-t) \quad Re\{s\} < -2$$

$$e^{-3s}\frac{2}{s+2} \rightarrow -2e^{-2(t-3)} u(-(t-3))$$ LSS

$$= -2e^{-2t} \cdot e^{6} u(-t+3)$$

$\frac{-1}{s+3}$ $Re\{s\} > -3$ (RSS)

$$\hookrightarrow -e^{-3t} u(t)$$

$$-e^{-3s}\frac{1}{s+3} \rightarrow -e^{-3(t-3)} u(t-3)$$

$$= -e^{-3t} e^{9} u(t-3)$$

Final answer

$$= -e^{-3t} e^{9} u(t-3)$$
$$-2e^{6} e^{-2t} u(-t+3)$$