Assignment 3.

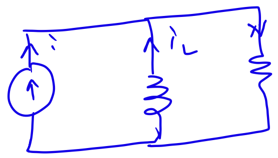
① $x(t) \rightarrow \boxed{T(\cdot)} \rightarrow y(t)$

$x(t-t_0) \rightarrow \boxed{T(\cdot)} \rightarrow y(t-t_0)$

Time invariance property

Time shift

③



$v = L\dfrac{di}{dt}$

$i = i_L + i_R$

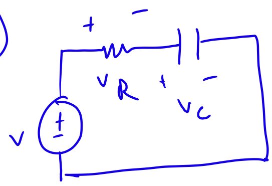
$\Rightarrow i(t) = i_L(t) + \dfrac{v}{R}$

$\Rightarrow i(t) = i_L(t) + \dfrac{L}{R}\dfrac{di_L(t)}{dt}$

$\Rightarrow i(t) = i_L(t) + 2\dfrac{di_L(t)}{dt}$ ✓

④ not done

⑤



$i = C\dfrac{dv}{dt}$

$v = v_R + v_C$

$v(t) = iR + v_C$

$\Rightarrow v(t) = C\dfrac{dv_c(t)}{dt}R + v_c(t)$

$v(t) = RC\dfrac{dv_c(t)}{dt} + v_c(t)$

⑥ $h(t) = e^{-t}u(t)$

$y(t) = x(t) * h(t)$

$= \displaystyle\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$

$= \displaystyle\int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, e^{1/2(t-\tau)}\, d\tau.$

$$x(\tau) = e^{\tau/2}$$

$$x(-\tau) = e^{-\tau/2}$$

$$x(t-\tau) = e^{\frac{(t-\tau)}{2}}$$

$$= \int_0^\infty e^{-\tau} e^{t/2} e^{-\tau/2} d\tau$$

$$= \int_0^\infty e^{-3\tau/2} e^{t/2} d\tau$$

$$= e^{t/2} \left. \frac{e^{-3\tau/2}}{-3/2} \right|_0^\infty$$

$$= e^{t/2} \left[\frac{-2}{3} [0 - 1] \right]$$

$$= 2/3 \, e^{t/2}$$

$\hookrightarrow$ eigenvalue

(7) $2y(n) = y(n-1) + 2x(n)$ Initial rest.

$2y(n) = y(n-1) + 2\delta(n)$ Impulse response.

$2y(0) = y(-1) + 2$

$\cancel{2}y(0) = \cancel{2}$

$y(0) = 1$

$2y(1) = y(0) + 2\cancel{x(1)}^{0}$

$2y(1) = 1$

$y(1) = 2^{-1}$

$2y(2) = y(1)$

$y(2) = 2^{-2}$

$\therefore$ $h(n) = 2^{-n} u(n)$

(8) Homogeneity principle

$x(t) \to \boxed{T(\cdot)} \to y(t)$

$\alpha x(t) \to \boxed{T(\cdot)} \to \alpha y(t)$.

(9) $\frac{dy(t)}{dt} + \alpha y(t) = x(t)$

$y(t) = y_p(t) + y_h(t)$

$y(0) = y_0, \quad x(t) = Ce^{-\beta t}$

let $y_p(t) = k e^{-\beta t}$

$$\frac{dy_p(t)}{dt} + \alpha y_p(t) = x(t)$$

$$\Rightarrow -\beta k \cancel{e^{-\beta t}} + \alpha k \cancel{e^{-\beta t}} = c \cancel{e^{-\beta t}}$$

$$\Rightarrow k(\alpha - \beta) = c$$

$$k = \left(\frac{c}{\alpha - \beta}\right)$$

$$y_p(t) = \frac{c}{\alpha - \beta} e^{-\beta t} \quad \left[\text{see class notes}\right]$$

(10) $\int_{-\infty}^{\infty} \sin u \, \delta(\alpha u - \beta\pi) \, du$

$$= \int_{-\infty}^{\infty} \sin u \, \delta\left(\alpha\left(u - \frac{\beta\pi}{\alpha}\right)\right) du$$

$$= \int_{-\infty}^{\infty} \sin u \, \frac{1}{\alpha} \delta\left(u - \frac{\beta\pi}{\alpha}\right) du$$

$$= 1/\alpha \, \sin u \Big|_{u = \beta\pi/\alpha}$$

$$= \frac{1}{\alpha} \sin\left(\frac{\beta\pi}{\alpha}\right)$$

(2) $2\frac{dy(t)}{dt} + 3y(t) = x(t)$

$$y(t) = y_p(t) + y_n(t)$$

let $y_p(t) = ke^{-t}$ (assume)

$$2k(-1)e^{-t} + 3ke^{-t} = e^{-t}$$

$$\Rightarrow -2k + 3k = 1$$

$$\underline{\underline{k = 1}}$$

$$y_p(t) = 1e^{-t}$$

$$= e^{-t}$$

(4) $h(n) = \alpha^n u(-n), \quad \alpha > 1$

$$x(n) = u(n) - u(n - N - 1)$$

$$h(n) = \alpha^n u(-n)$$

$$= \begin{cases} \alpha^n, & n \le 0 \\ 0, & n > 0 \end{cases}$$

$$x(n) = u(n) - u(n-N-1)$$

$$x(n) = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{else} \end{cases}$$

$$y(n) = x(n) * h(n)$$

$$= \sum_{m=0}^{N} h(n-m)$$

$$= \sum_{m=0}^{N} \alpha^{n-m} \underbrace{u(-(n-m))}_{=1 \text{ if}}$$

$-(n-m) \ge 0$

$n - m \le 0$

$m \ge n$

Case 1 $n > N$, then $m \ge n$ can't be satisfied

$\therefore y(n) = 0$, for $n > N$.

Case 2 $0 \le n \le N$.

$m = n, n+1, \ldots, N$.

$$y(n) = \sum_{m=n}^{N} \alpha^{n-m}$$

$$= \alpha^{n} \sum_{m=n}^{N} \alpha^{-m}$$

$$= \alpha^{n} \left[\frac{\alpha^{-n} \left(1 - \alpha^{-(N-n+1)}\right)}{1-\alpha^{-1}} \right]$$

$$= \frac{1 - \alpha^{-(N-n+1)}}{1-\alpha^{-1}}$$

$$= \frac{1 - \alpha^{n-N-1}}{1-\alpha^{-1}} \qquad 0 \le n \le N$$

Case 3 $n \le -1$, satisfied for $m \ge 0$ as $m \ge n$.

$$y(n) = \sum_{m=0}^{N} \alpha^{n-m}$$

$$= \alpha^{n} \sum_{m=0}^{N} \alpha^{-m}$$

$$= \alpha^{n} \left(\frac{1 - \alpha^{-(N+1)}}{1-\alpha^{-1}} \right)$$

$\forall\, n \le -1$