Assignment (W-9)

(1) Hilbert transform impulse response is $\frac{1}{\pi t}$.

(2) Parseval's relation for a cont time signal

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(w)|^2 dw..$$

(3) $$H(w) = \frac{(1w + jw)}{(1+jw)\,(10+jw)\,(1000+jw)^2}$$

$w >> 1000$

$1w + jw \approx jw.$

$1+jw \approx jw.$

$10+jw \approx jw$

$1000 + jw \approx jw.$

$$H(w) \approx \frac{\cancel{jw}}{\cancel{jw} \times jw \times (jw)^2}$$

$$= \frac{1}{(jw)^3}.$$

$$= +20\log_{10} \frac{1}{w^3}$$

$$= -60 \log_{10} 1/w.$$

$$= -60 \text{ db/dec.}$$

(4) 3dB freq

$w_0 = \frac{1}{RC}$ ✓

(5) $$H(w) = \frac{(1w + jw)^{\sim}}{(1+jw)(10+jw)(1000+jw)}$$

$1w \leq w \leq 1w0.$

$1w + jw \approx jw.$

$1+jw \approx jw$

$10+jw \approx jw.$

$w >> 1w.$

$w << 1000$

$1000 + jw \approx 1000$

$$= \frac{\cancel{(j\omega)^2}}{\cancel{(j\omega)} \times \cancel{(j\omega)} \times 1000}$$

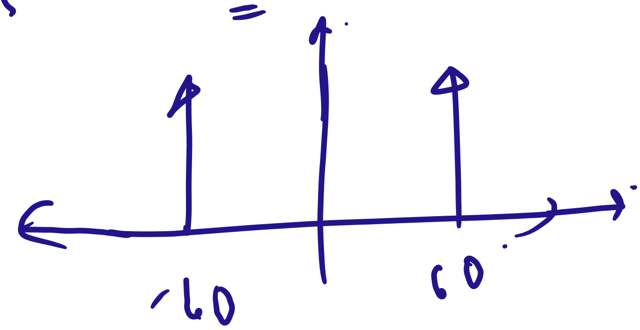
$$H(\omega) \simeq \frac{1}{1000}$$

$$|H(\omega)|_{dB} = 20 \log \frac{1}{1000}$$

$$= \rightarrow \underline{\text{constant}}$$

6. $f_m = 60\ Hz$ $\qquad \cos(2\pi f_m t)$

$f_s = 150\ \underline{Hz}$ $\qquad = \cos(2\pi 60 t)$



$f_s = 150\ Hz$

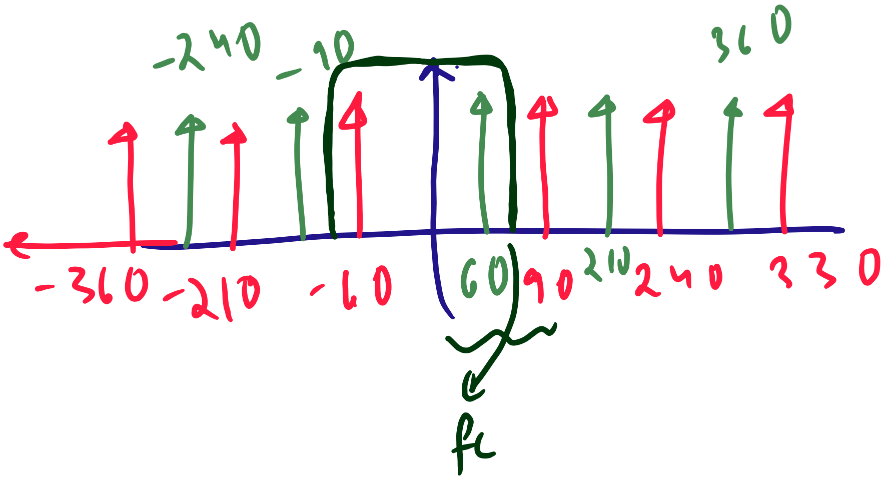
$t = nT_s$

$= \frac{n}{f_s}$

$= \cos\left(2\pi \frac{60n}{150}\right)$

$-60 + 150$

$= \underline{90}$ ✓



$$\underline{60 < f_c < 90}$$

7. $e^{-at}u(t) \longrightarrow \frac{1}{a + j\omega}$ $\qquad t\,x(t) \rightarrow j \frac{d}{d\omega} X(\omega)$

$; a > 0$

$$\underbrace{t\, e^{-at}\, u(t)}_{x(t)} \longrightarrow j \frac{d}{d\omega}\left(\frac{1}{a + j\omega}\right)$$

$$= j\,\frac{(a+j\omega)\cdot 0 - j}{(a+j\omega)^2}$$

$$= \frac{1}{(a+j\omega)^2}$$

8 Not done

9 $x(t) = e^{-a|t|} \longrightarrow \left.\frac{2a}{\omega^2+a^2}\right\}$ standard result

10 $a_k = 2\,\mathrm{Re}\{c_k\} = .$

8 official solution

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jk\omega_0 t}\, dt \quad = \frac{1}{T_0} \times T_0 \int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-jk\omega_0 t}\, dt$$

$$= 1$$