NPTEL MOOC,JAN-FEB 2015
Week 2, Module 5

DESIGN AND ANALYSIS OF ALGORITHMS

Merge Sort

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$O(n^2)$ sorting algorithms

- Selection sort and insertion sort are both $O(n^2)$
- $O(n^2)$ sorting is infeasible for n over 100000

A different strategy?

- Divide array in two equal parts
- Separately sort left and right half
- Combine the two sorted halves to get the full array sorted

Combining sorted lists

- Given two sorted lists A and B, combine into a sorted list C
  - Compare first element of A and B
  - Move it into C
  - Repeat until all elements in A and B are over
- Merging A and B

Merging two sorted lists

32 74 89

21 55 64

Merging two sorted lists

32 74 89

21 55 64

21

Merging two sorted lists

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Merging two sorted lists

21 32 55 64 74 89

Merge Sort

- Sort A[0] to A[n/2-1]
- Sort A[n/2] to A[n-1]
- Merge sorted halves into B[0..n-1]
- How do we sort the halves?
  - Recursively, using the same strategy!

Merge Sort

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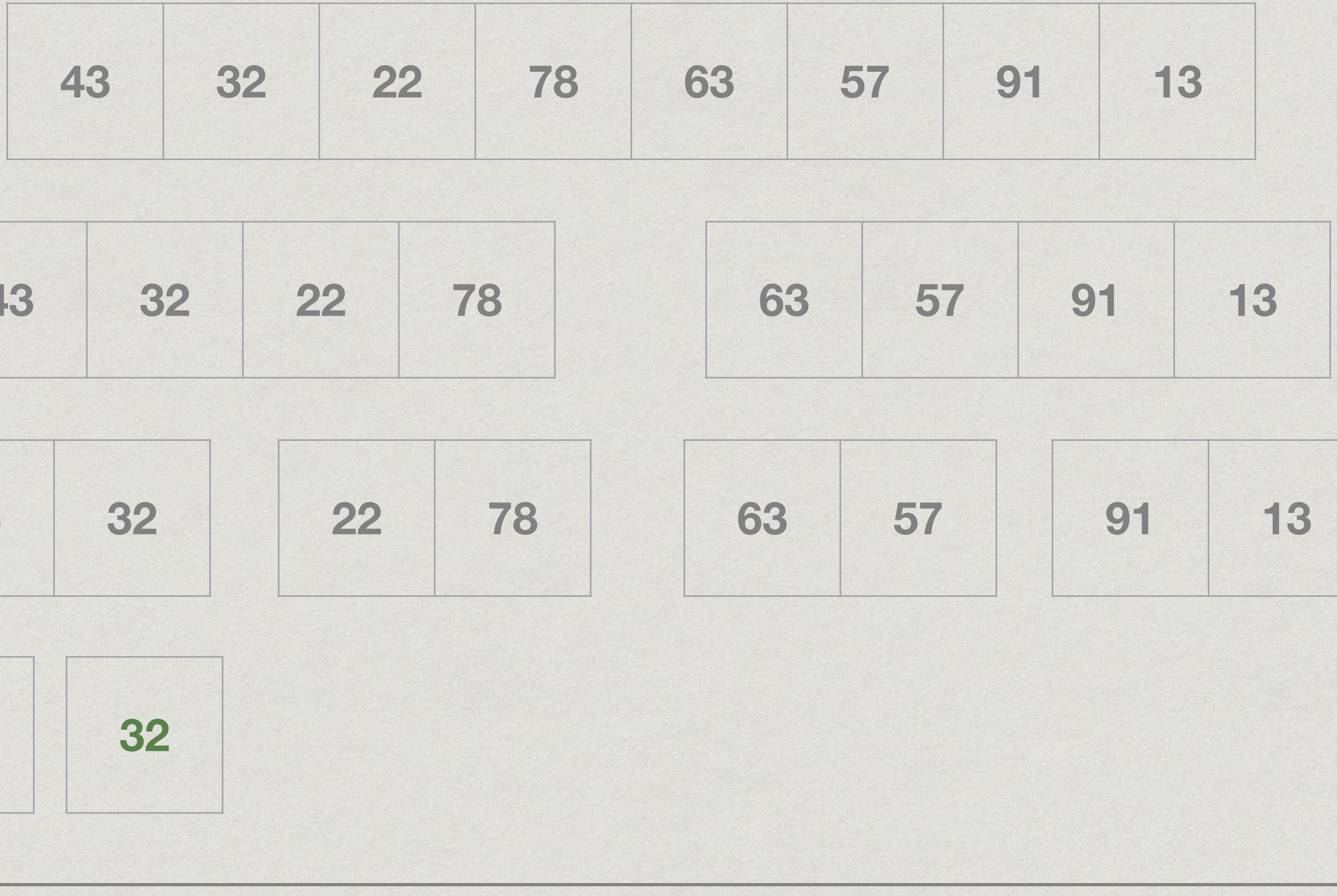
Merge Sort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Merge Sort


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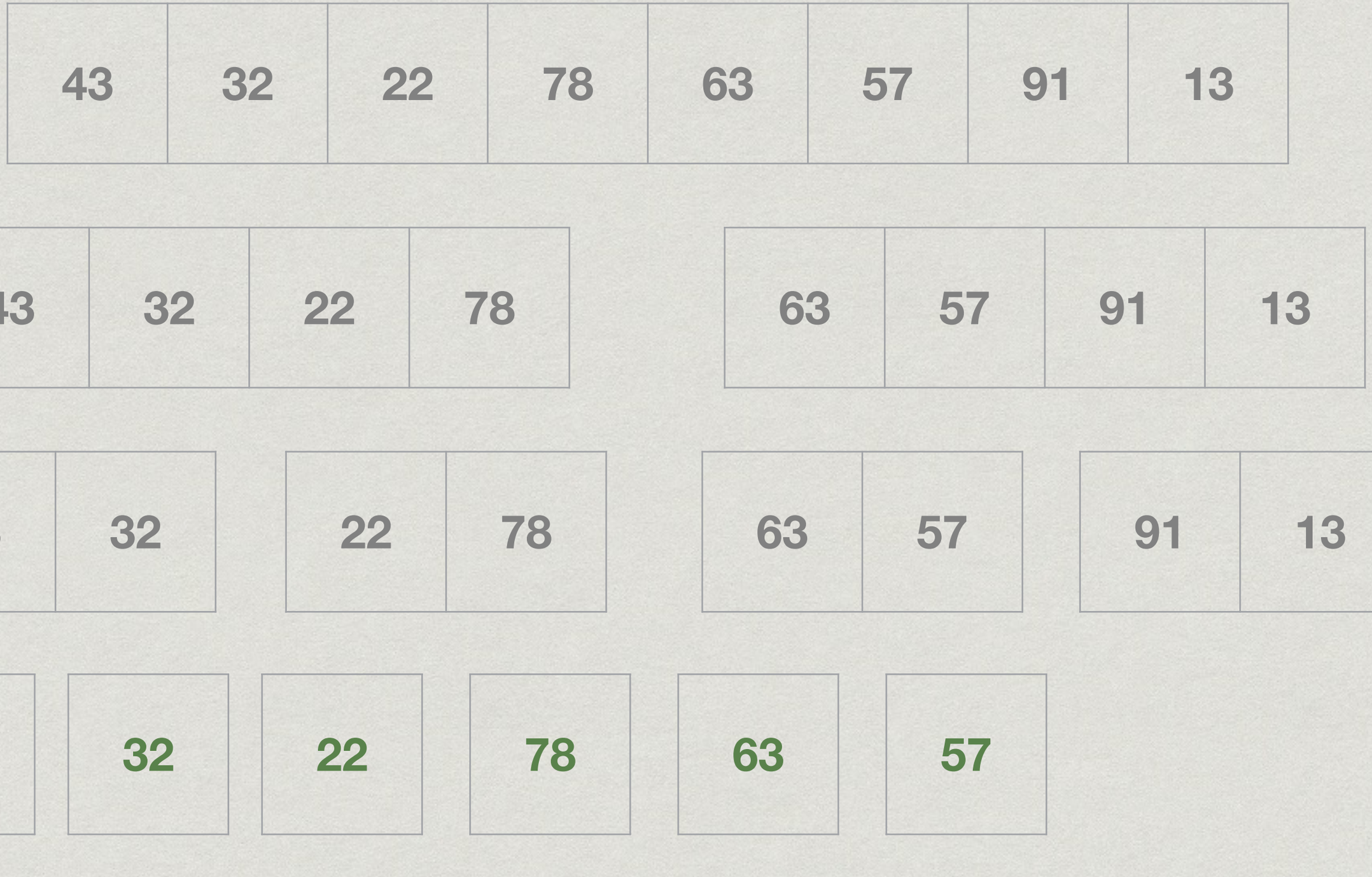
Merge Sort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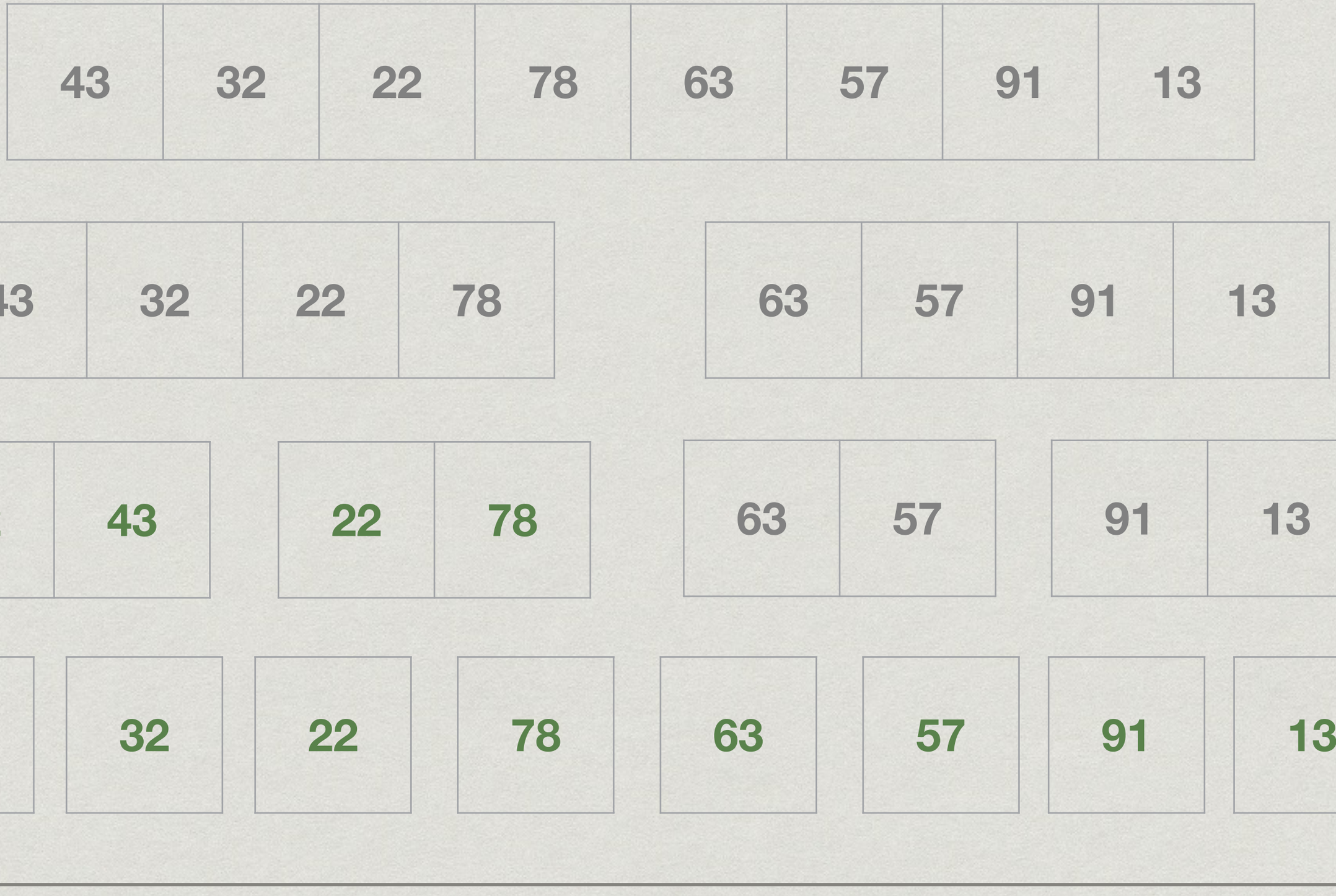
Merge Sort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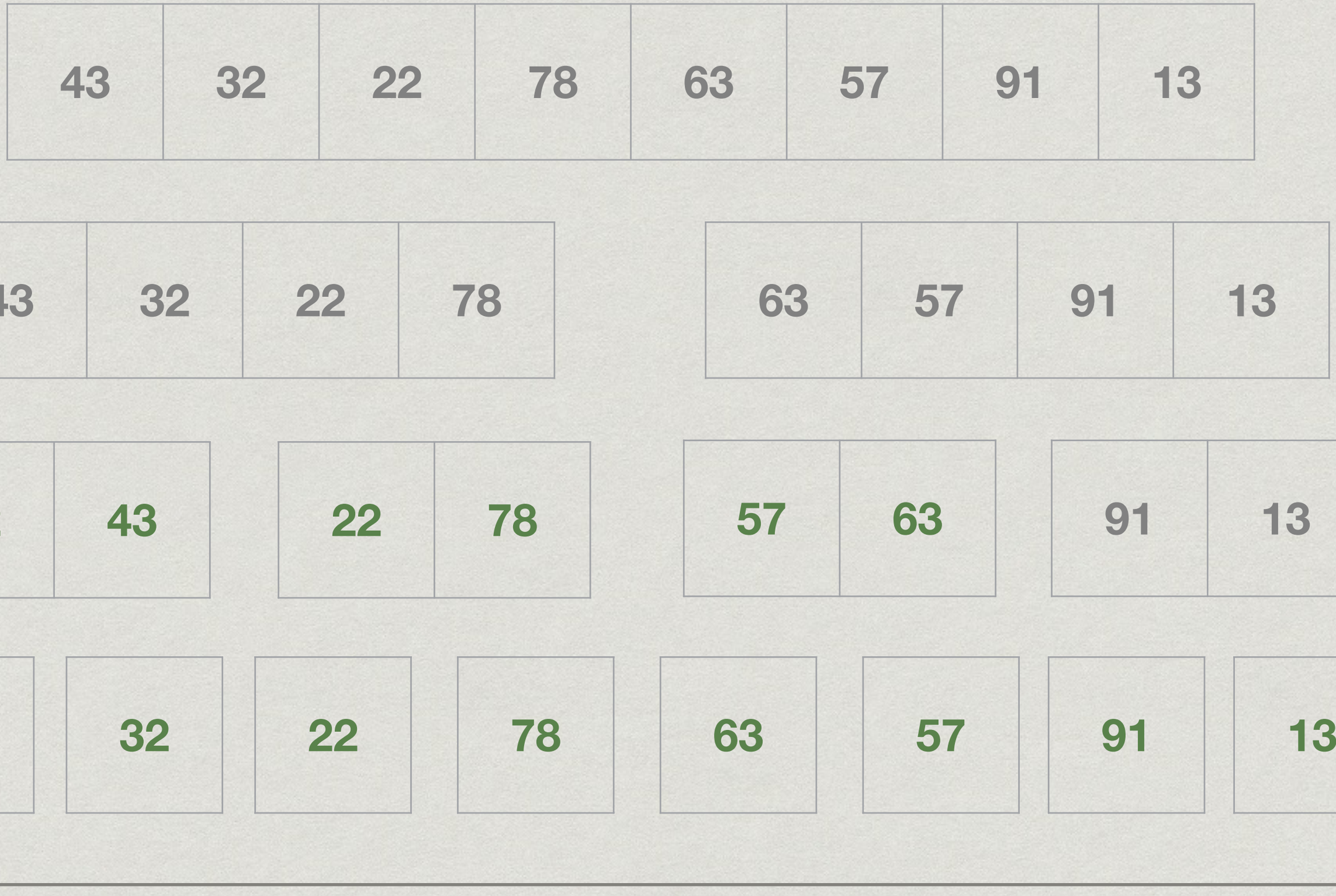
Merge Sort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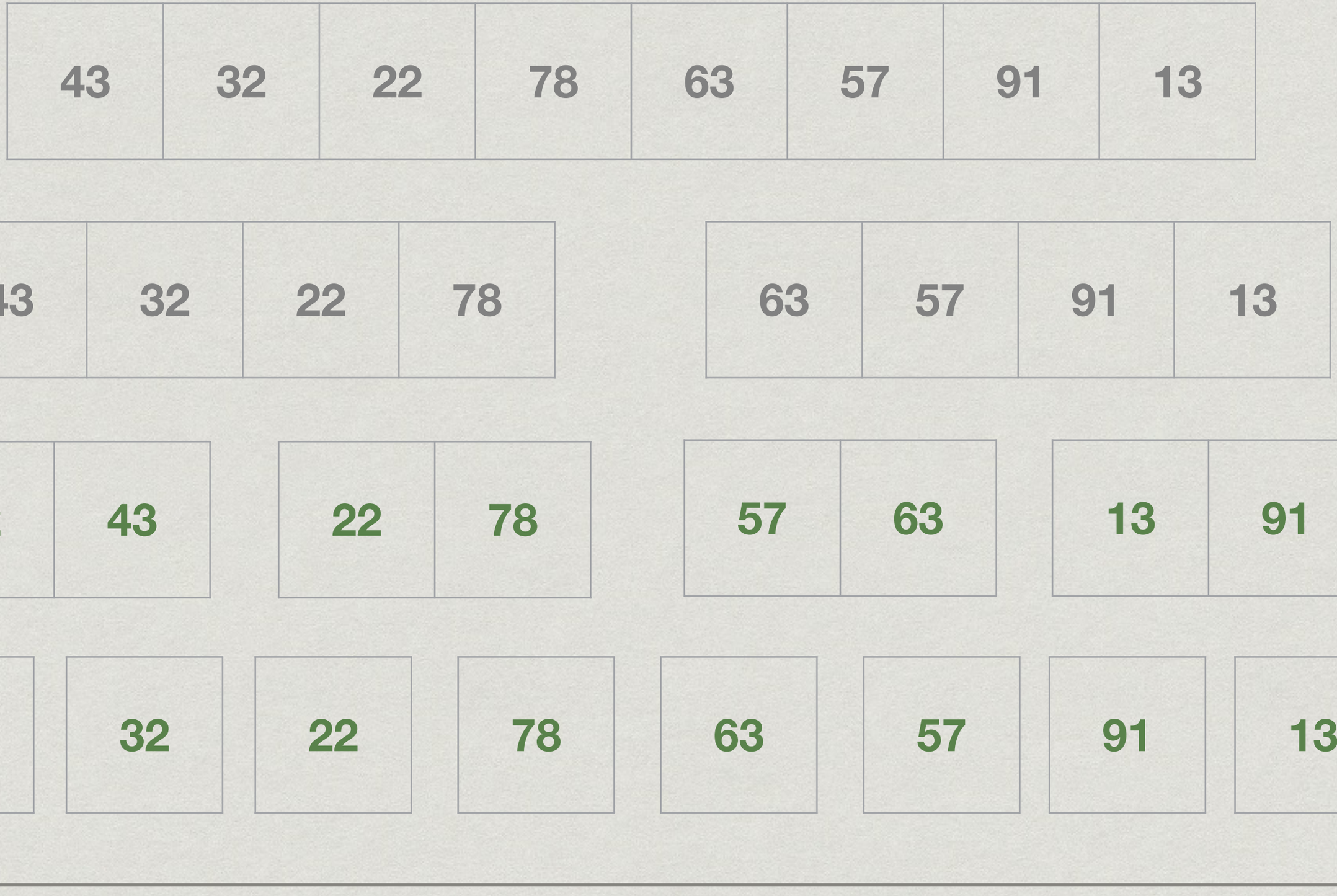
Merge Sort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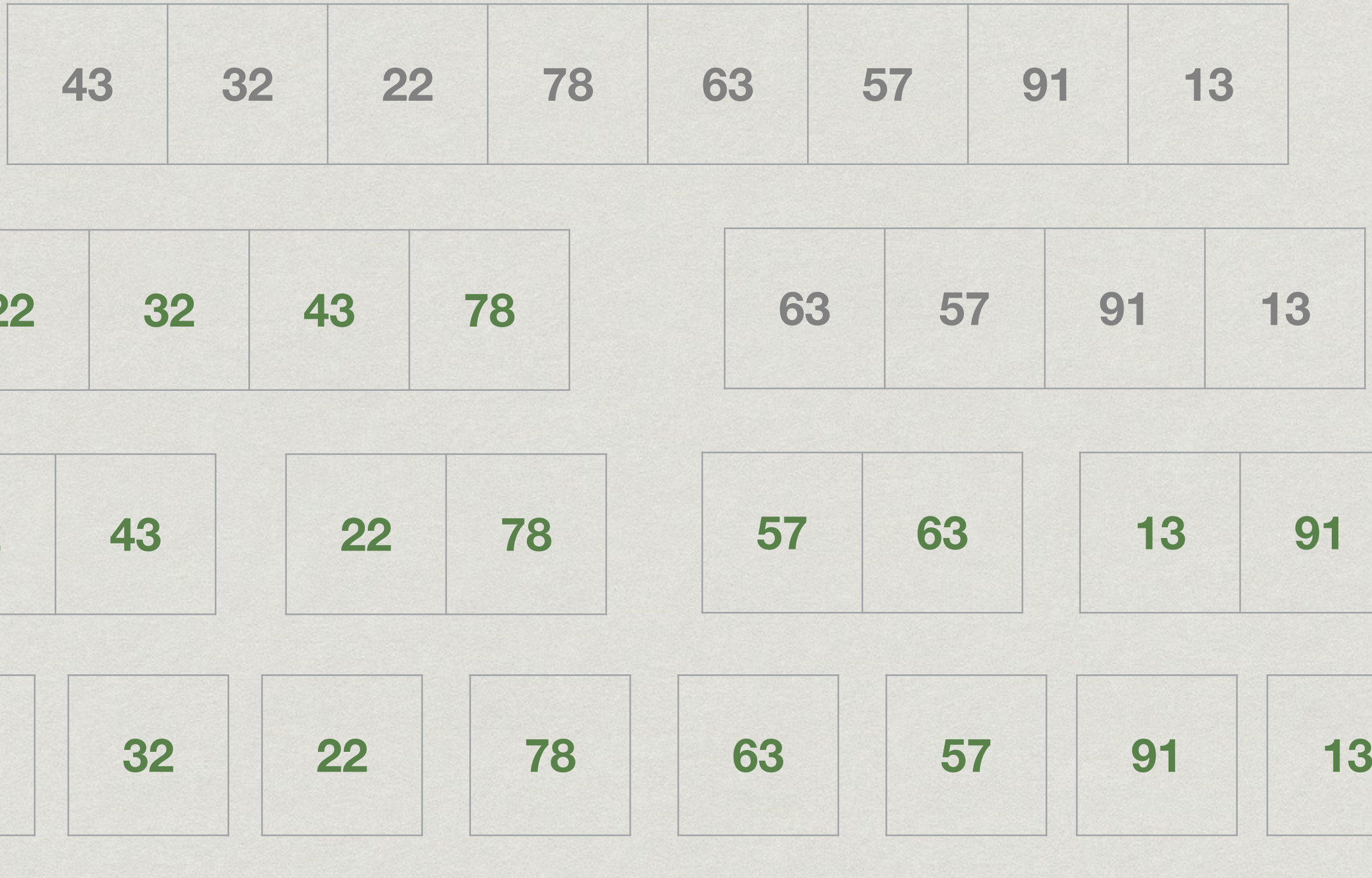
Merge Sort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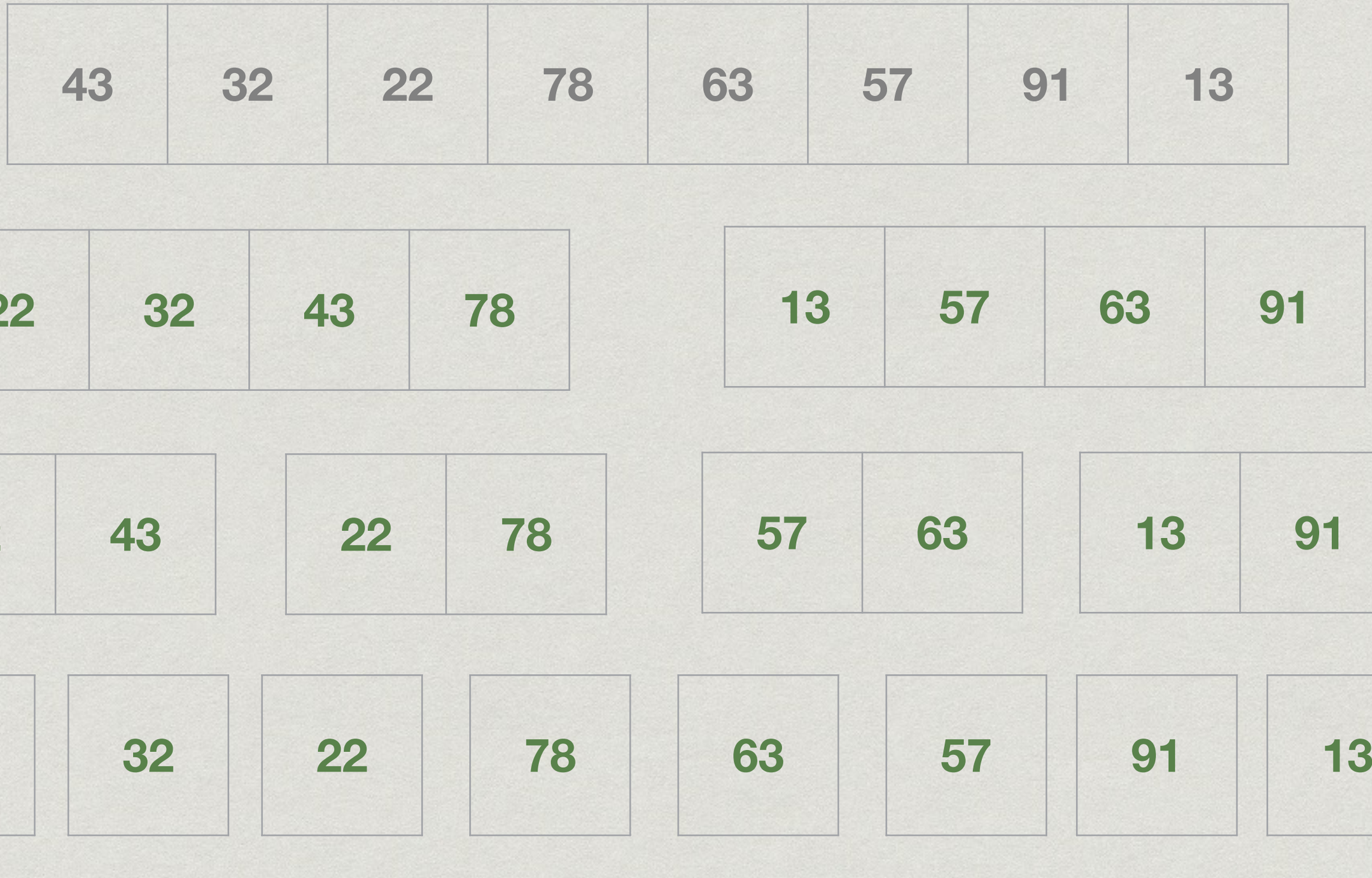
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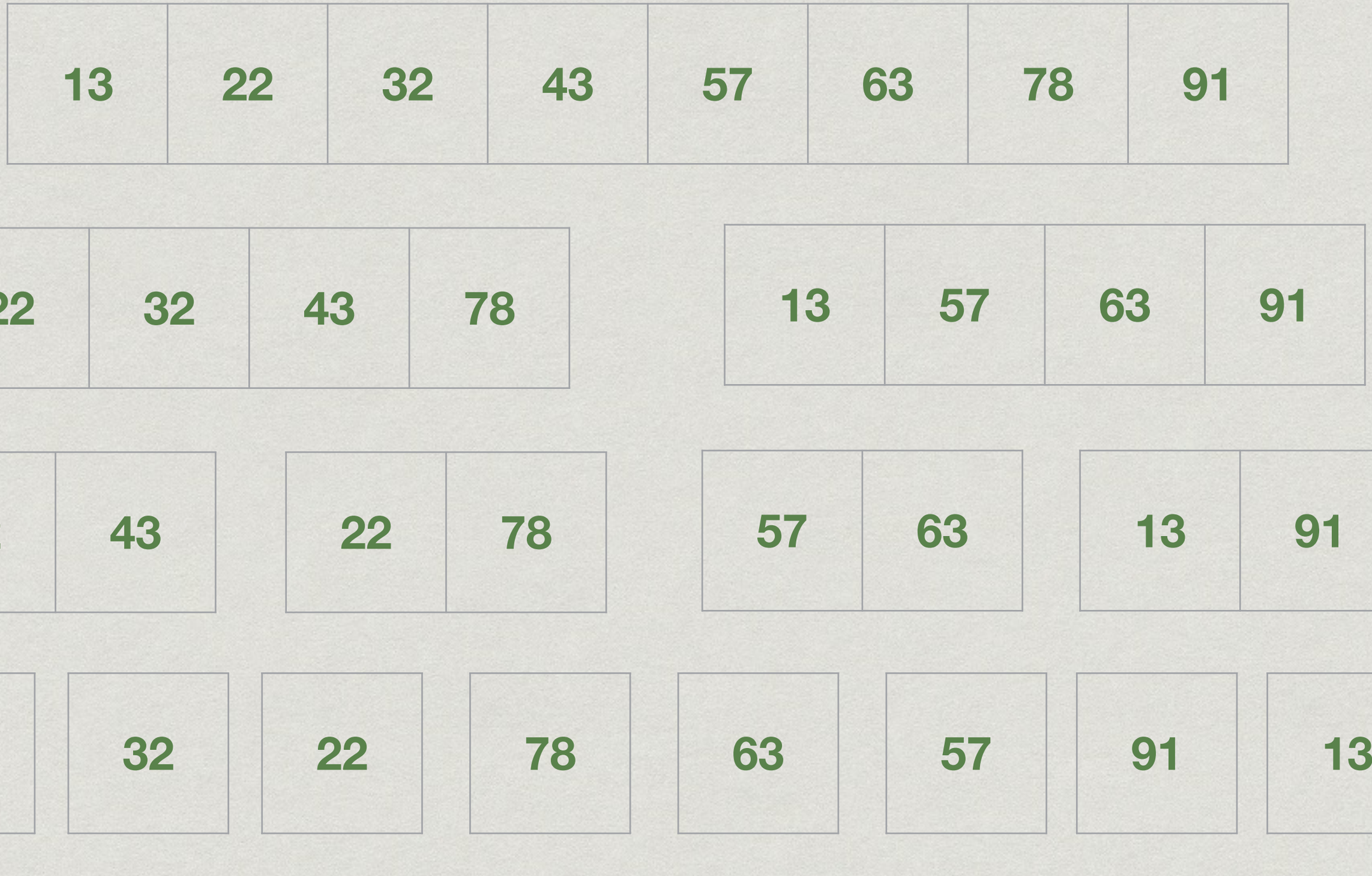
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Divide and conquer

- Break up problem into disjoint parts
- Solve each part separately
- Combine the solutions efficiently

Merging sorted lists

Combine two sorted lists A and B into C

- If A is empty, copy B into C
- If B is empty, copy A into C
- Otherwise, compare first element of A and B and move the smaller of the two into C
- Repeat until all elements in A and B have been moved

Merging

```
function Merge(A,m,B,n,C)
   // Merge A[0..m-1], B[0..n-1] into C[0..m+n-1]

   i = 0; j = 0; k = 0;
   // Current positions in A,B,C respectively

   while (k < m+n)
   // Case 0: One of the two lists is empty
      if (i==m) {j++; k++;}
       if (j==n) {i++; k++;}
   // Case 1: Move head of A into C
      if (A[i] <= B[j]) { C[k] = B[j]; j++; k++;}
   // Case 2: Move head of B into C
      if (A[i] > B[j]) {C[k] = B[j]; j++; k++;}
```

Merge Sort

To sort A[0..n-1] into B[0..n-1]

- If n is 1, nothing to be done
- Otherwise
  - Sort A[0..n/2-1] into L (left)
  - Sort A[n/2..n-1] into R (right)
  - Merge L and R into B

Merge Sort

```
function MergeSort(A,left,right,B)
   // Sort the segment A[left..right-1] into B

   if (right - left == 1) // Base case
      B[0] = A[left]

   if (right - left > 1)  // Recursive call

      mid = (left+right)/2

      MergeSort(A,left,mid,L)
      MergeSort(A,mid,right,R)

      Merge(L,mid-left,R,right-mid,B)
```