Topic: Bayes' Theorem

Question: When should you use Bayes’ Theorem?

Answer choices:

A When you have $P(A \cap B)$ but want to find $P(A)$.

B When you have $P(A|B)$ but want to find $P(B|A)$.

C When you have $P(A)$ but want to find $P(B)$.

D When you have $P(A|B)$ but want to find $P(A)$.

Solution: B

Bayes' Theorem is used when you have a conditional probability of two events, and you're interested in the reversed conditional probability. For example, when you have $P(A \mid B)$ but want to find $P(B \mid A)$.

Topic: Bayes' Theorem

Question: Three factories A, B, and C produce car seats. What is the probability that a defective car seat comes from factory C, given that factory C produces 40% of all the car seats, that there's a 1% chance that any given car seat is defective, and that the defective rate at factory C is 0.8%?

Answer choices:

A 28%

B 32%

C 36%

D 40%

Solution: B

We could name these events.

A represents a car seat from factory A

B represents a car seat from factory B

C represents a car seat from factory C

D represents a defective car seat

We're looking for $P(C|D)$, the probability that a car seat came from factory C, given that it was defective. We know

$$P(C) = 0.4$$

$$P(D) = 0.01$$

$$P(D|C) = 0.008$$

Bayes' Theorem therefore tells us that the probability of $P(C|D)$ is given by

$$P(C|D) = \frac{P(D|C) \cdot P(C)}{P(D)}$$

$$P(C|D) = \frac{(0.008)(0.4)}{0.01}$$

$$P(C|D) = \frac{0.0032}{0.01}$$

$$P(C|D) = 0.32$$

Topic: Bayes' Theorem

Question: Which choice is equivalent to $P(C|D)$?

Answer choices:

A $\dfrac{P(D|C) \cdot P(C)}{P(D)}$

B $\dfrac{P(C \cap D)}{P(D)}$

C $\dfrac{P(C \cup D)}{P(D)}$

D Both A and B

Solution: D

Bayes' Theorem is

$$P(A\,|\,B) = \frac{P(B\,|\,A) \cdot P(A)}{P(B)}$$

This problem uses different variables. If you replace A with C and B with D, then Bayes' Theorem is

$$P(C\,|\,D) = \frac{P(D\,|\,C) \cdot P(C)}{P(D)}$$

For dependent events, the multiplication rule says that $P(C \cap D) = P(C) \cdot P(D\,|\,C)$, which means we could also write Bayes' Theorem as

$$P(C\,|\,D) = \frac{P(D\,|\,C) \cdot P(C)}{P(D)} = \frac{P(C \cap D)}{P(D)}$$