

# CS1632, Lecture 13: Pairwise and Combinatorial Testing

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# Let's Test A Word Processor

- › Specifically, its ten possible font effects
  - Italic
  - Bold
  - Underline
  - Strikethrough
  - Superscript
  - Shadow
  - Embossed
  - 3-D
  - Outline
  - Inverse

These can be combined

- › Plain text
- › Superscript
- › Bold
- › ~~Italic and strikethrough~~
- › Bold and underlined
- › ~~***Bold italic strikethrough shadowed superscript***~~

How many tests would you need to test all the possible font combinations?

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$2^{10}$

1,024 tests!



# That's quite a few tests...

[illegible]

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But it's necessary! What if...

... a problem only occurs with 3-D shadowed  
bold italic superscript text?

That's going to be hard to find.

## Turns Out Other People Have Thought About This!

The National Institute of Standards and Technology did a study on the topic.

See: "Practical Combinatorial Testing", [http://  
nvlpubs.nist.gov/nistpubs/Legacy/SP/  
nistspecialpublication800-142.pdf](http://nvlpubs.nist.gov/nistpubs/Legacy/SP/nistspecialpublication800-142.pdf)



## Turns Out That's Unlikely!

- › Think of each font effect as a Boolean variable (e.g. bold vs not bold, italic vs non-italic)
- › Most (50 - 90%, depending on the project) defects come from combinations of one or two interactions (variables).
- › In other words, most defects would be found if you just tested, e.g., "bold 3-D" (two interactions) text or just "bold text" (one interactions).

# Similar Distribution Found In Many Domains

- › Web browser
- › Avionics software
- › Telecommunications software
- › Flight Traffic Control
- › Network security software

## The Interaction Rule

"Most failures are triggered by one or two parameters, and progressively fewer by three, four, or more parameters, and the maximum interaction degree is small." -Eric Kuhn, NIST

## The Interaction Rule

- › The maximum number of interactions found to cause a defect was SIX, no matter the value of  $n$  (# of variables).
- › This was after an analysis of dozens of software projects.

So...

- › So we can find a large percentage of defects with minimal work by making sure we test all possible pairs of values.

# Pairwise Testing

- › This is called “pairwise”, or “all-pairs” testing.
- › We are testing all possible pairs of interactions, e.g.:
  - Not-Bold / Not-Italic
  - Bold / Not-Italic
  - Not-Bold / Italic
  - Bold / Italic

# Remember our exhaustive 10-font-effect testing plan?

- › It was  $2^n$ , thus 1,024 ( $2^{10}$ ) tests.
- › How many tests would it require to test all pairs of interactions?
  - That is, all possible combinations of:
    - › bold/italic,
    - › subscript/bold
    - › underline/strikethrough
    - › 3-D / italic
    - › Every possible pairing of two variables

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Answer: 10

	BOLD	ITALIC	STRIKETHROUGH	UNDERLINE	THREED	SHADOW	SUPERSCRIPT	SUBSCRIPT	EMBOSSSED	ENGRAVED
1	true	true	false	false	false	false	false	false	false	false
2	true	false	true	true	true	true	true	true	true	true
3	false	true	true	false	true	false	true	false	true	false
4	false	false	false	true	false	true	false	true	false	true
5	false	true	false	true	true	false	true	true	false	false
6	false	false	true	false	false	true	false	false	true	true
7	true	true	false	false	false	true	true	true	true	false
8	false	false	true	true	true	false	false	false	false	true
9	false	true	true	false	true	false	false	true	true	true
10	true	false	false	false	false	false	true	false	true	false



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# Reduce Number of Tests By Two Orders Of Magnitude



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# Is This Always Good Enough?



## Of course not

- › But we can “dial up” the number of possible interactions to check for any  $t$
- › For example, check every three-way combination ( $t = 3$ ):
  - Bold / Italic / Underline
  - Italic / Underline / Superscript
  - Shadow / Italic / Bold
- › Or four-way ( $t = 4$ )
  - Bold / Italic / Underline / Superscript
  - Embossed / 3-D / Outline / Strikethrough
  - Shadow / Bold / Inverse / Outline
- › Up to  $n$  (the number of interactions) - This would be the same as exhaustive testing

# Combinatorial Testing

- › This generalized version of pairwise testing is known as “combinatorial testing”
- › Note that pairwise testing is technically just a specific kind of combinatorial testing where  $t = 2$

## Combinatorial Testing Example

- › The maximum number of interactions causing a defect found in the NIST studies was six. So let's test all six-way combinations of our font effects.
- › Recall that:
  - # tests required for full pairwise testing was 10
  - # tests required for exhaustive testing was 1,024
  - How many to test all six-way interactions?

# Actually a difficult question to answer off the top of your head

- › Determining the exact number necessary is an NP-Hard problem.
- › But there are some good algorithms out there that approximate it (e.g. IPOG).
- › See “IPOG: A General Strategy for T-Way Software Testing”  
<http://csrc.nist.gov/acts/ecbs-cr-final.pdf>

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... and the answer is...

- › The best answer my software could come up with is 178.
- › Approximately an order of magnitude less than exhaustive testing!
- › But in any piece of software tested by NIST, would have found the same number of defects

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## Interesting!

- › 10 tests catch 90% of defects
- › 178 tests catch ~99.9999999% of defects
- › 1024 tests catch ~100% of defects

IF THEY ARE DONE RIGHT!



## Sidenote: The Pareto Principle

- › "80% of effects come from 20% of causes."
- › Examples:
  - 80% of your sales come from 20% of your customers.
  - 80% of your code execution time is in 20% of your code.
- › Specific Testing Examples
  - 80% of your defects will be found with 20% of your tests
  - 80% of your defects will be found in 20% of the code

## Recap

- › 10 tests catch 90% of defects
- › 178 tests catch ~99.9999999% of defects
- › 1024 tests catch ~100% of defects

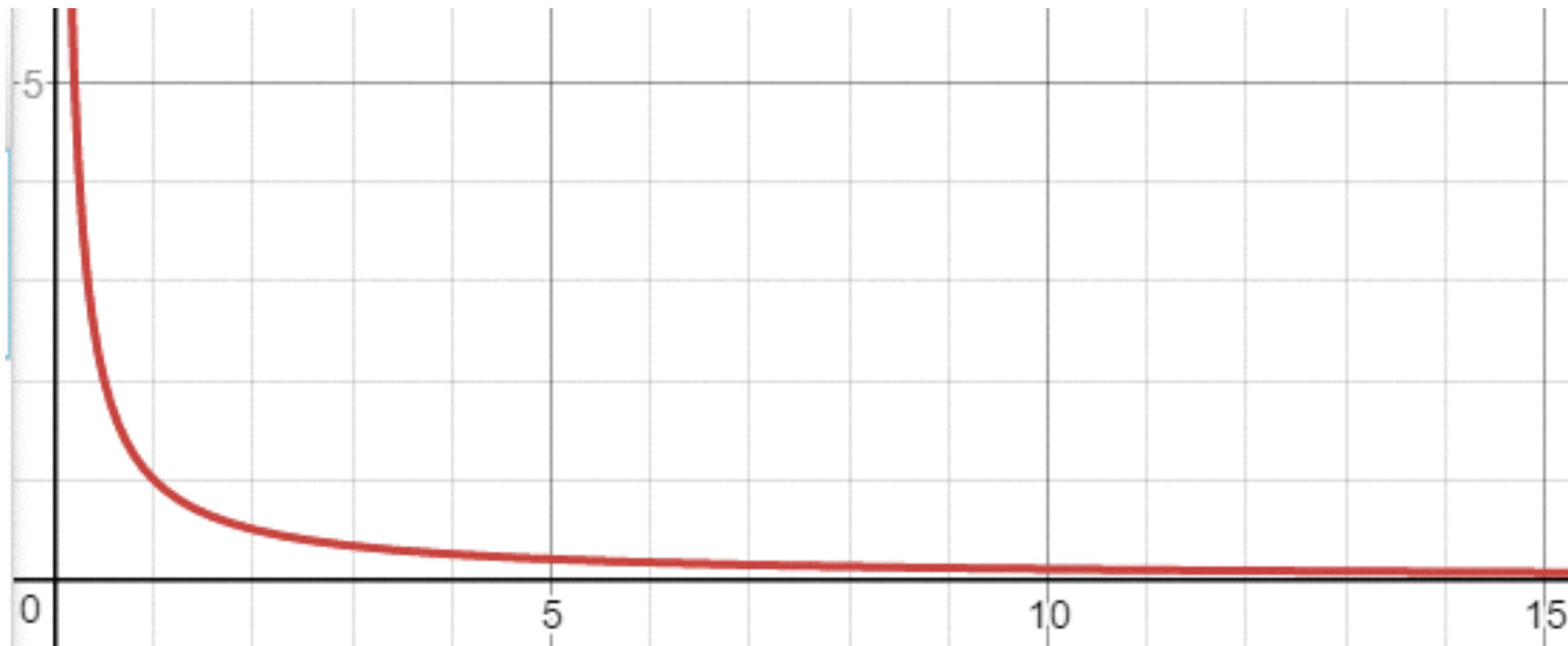
IF THEY ARE DONE RIGHT!

## It Gets Harder the Closer You Get

- › You can see how much more expensive it becomes to test depending on how arbitrarily close to "100% free of defects" you want to be.
- › It is NOT a linear relationship.
- › It is asymptotic.

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Example:  $f(x) = 1 / x$



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# Covering Arrays

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	0	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	1	0	1	0
0	0	0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	1	1	1	0	1

# Steps To Make Your Own Covering Array

- › Make a truth table with all variables
  - Each line in truth table indicates a test
  - Running all these tests would be an exhaustive test
- › Make a list of all t-way interactions for desired t
  - Example: Bold, Italic, Underline.  $t = 2$ 
    - › Bold / Italic
    - › Bold /Underline
    - › Italic/Underline

## Generating Covering Arrays

- › Look for tests which make a complete truth table for each t-way interaction
- › Mark these tests as “Tests To Be Executed”
- › Continue adding t-way interactions tests
  - Prefer using tests which are already scheduled to be executed
- › When all t-way interaction “mini truth tables” have been completed, put together all tests to be executed

# Covering Array Example

Bold	Italic	Underline		Mini-Truth	
F	F	F		F	F
F	F	T		F	T
F	T	F		T	F
F	T	T		T	T
T	F	F			
T	F	T			
T	T	F			
T	T	T			



# Covering Array Example

Test	Bold	Italic	Underline		
1	F	F	F		<b>Bold / Italic</b>
2	F	F	T		<b>Bold / Underline</b>
3	F	T	F		<b>Italic / Underline</b>
4	F	T	T		
5	T	F	F		
6	T	F	T		
7	T	T	F		
8	T	T	T		

## Covering Array Example – Bold / Italic

Test	Bold	Italic	Underline		
1	F	F	F		<b>Bold / Italic</b>
2	F	F	T		<b>Bold / Underline</b>
3	F	T	F		<b>Italic / Underline</b>
4	F	T	T		
5	T	F	F		
6	T	F	T		
7	T	T	F		
8	T	T	T		

## Covering Array Example – Bold / Underline

Test	Bold	Italic	Underline		
1	F	F	F		<b>Bold / Italic</b>
2	F	F	T		<b>Bold / Underline</b>
3	F	T	F		<b>Italic / Underline</b>
4	F	T	T		
5	T	F	F		
6	T	F	T		
7	T	T	F		
8	T	T	T		

## Covering Array Example – Italic / Underline

Test	Bold	Italic	Underline		
1	F	F	F		<b>Bold / Italic</b>
2	F	F	T		<b>Bold / Underline</b>
3	F	T	F		<b>Italic / Underline</b>
4	F	T	T		
5	T	F	F		
6	T	F	T		
7	T	T	F		
8	T	T	T		

# Run a Subset of Tests

Test	Bold	Italic	Underline		
1	F	F	F		Bold / Italic
2	F	F	T		Bold / Underline
3	F	T	F		Italic / Underline
4	F	T	T		
5	T	F	F		Necessary Tests
6	T	F	T		Unnecessary Tests
7	T	T	F		
8	T	T	T		

# Can Minimize Further Using “Intuition” Or Better Algorithms

Test	Bold	Italic	Underline		
1	<b>F</b>	<b>F</b>	<b>F</b>		<b>Bold / Italic</b>
2	<b>F</b>	<b>F</b>	<b>T</b>		<b>Bold / Underline</b>
3	<b>F</b>	<b>T</b>	<b>F</b>		<b>Italic / Underline</b>
4	<b>F</b>	<b>T</b>	<b>T</b>		
5	<b>T</b>	<b>F</b>	<b>F</b>		<b>Necessary Tests</b>
6	<b>T</b>	<b>F</b>	<b>T</b>		<b>Unnecessary Tests</b>
7	<b>T</b>	<b>T</b>	<b>F</b>		
8	<b>T</b>	<b>T</b>	<b>T</b>		

OK, this works for small numbers of variables, but what about big ones?

- › Imagine a 34-variable system
  - Exhaustive testing: 17 billion tests
  - All 3-way interactions: 33 tests
  - All 4-way interactions: 85 tests
- › Actually gets BETTER the higher the number of variables
- › Not just a little better – many orders of magnitude better

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Remember at the beginning of the term when I talked about the impossibility of testing every combination of inputs?

This is a possible amelioration.

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Won't It Take a Long Time To Make Covering  
Arrays For Large Values of  $n$  or  $t$ ?



# YES

- › These are not artisanal, hand-crafted arrays, carved by the European masters high in their Swiss valleys
- › Let's use a program to do it
- › Example: NIST ACTS