Introduction to Game Theory and Multi-Agent Systems

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https://dicelab-rhul.github.io/Strategic-AI-Autoformalization

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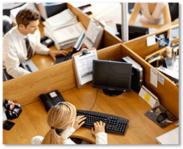
Outline

Introduction

Motivation

Why strategic interactions matter in AI

Many applications are naturally multi-agent:



Human teams and companies

Markets and economies

Transportation networks

Distributed software systems

Communication networks

Robotic teams

How should agents act in the presence of other agents?

Strategic interaction

Why reasoning is essential

One way to answer this question is by expecting agents to think strategically.

- **Interdependent Decisions:** Agent choices affect and depend on others — strategic reasoning enables intelligent interaction.
- **Lack of information:** Strategic reasoning allows agents to model beliefs about others and act under information that is incomplete or imperfect.
- **Dynamic environment:** Strategic reasoning allows agents respond effectively to these ongoing changes.
  - **Collaboration vs Competition**

Key Message

To build intelligent and interactive agents, strategic reasoning is indispensable.

Games as Microcosms of Intelligence

Games provide a natural setting for studying and evaluating strategy.

- Success in games captures key elements of **strategic reasoning**:
  - **Problem-solving** in dynamic, rule-based environments
  - **Plan** sequences of actions to achieve long-term objectives
  - **Adaptation** to opponents' tactics and evolving situations
  - **Learning** from feedback to improve future decisions
- Quantifiable performance in games enables a **comparative assessment of strategies** and, as a result, **intelligence**

Why Game Playing Matters to AI Research

Game Playing as a Testbed for strategic AI

- Games have long been used to test AI:

Deep Blue (Chess)

AlphaGo (Go)

- These systems were successful but **not general** - they only played one game.
- **Can we build AI that plays *any* game?**

- **Goal:** Build agents capable of **General Game Playing (GGP)** - excel in multiple games from their formal rules alone.
- **Key Idea:** Use a shared, declarative formalism for representing a game - the **Game Description Language (GDL)**.
- **Approach:**
  - Provide agents with new game rules expressed in GDL, without additional code or training.
  - When encountering a new game:
    - The agent parses and understands its rules and objectives.
    - It uses this understanding to plan, adapt, and compete strategically.
  - Shift away from game-specific logic toward **domain-general intelligence**.

Foundations of Strategic Interaction

Game Theory Foundations

A *game* models strategic interactions between decision-makers (players) where outcomes are interdependent on players' choices.[1]

[1] R. B. Myerson, "An introduction to game theory," Northwestern University, Center for Mathematical Studies in Economics and Management Science, Discussion Papers 623, 1984.

Games: Definition and Components

$$G = (N, \{A_i\}_{i \in N}, \{U_i\}_{i \in N})$$

- **Players** ($N = \{1, \ldots, n\}$): Strategic decision-makers involved in the game.
- **Actions** (A_i): The set of immediate choices available to player i.
- **Strategy Profile** ($o = (a_1, \ldots, a_n)$): A tuple representing the selected strategy of each player – the game's *outcome*.
- **Utility Functions** ($U_i : A_1 \times \cdots \times A_n \to \mathbb{R}$): An ordinal/cardinal representation of player i's preferences over outcomes.

Strategies

A strategy is a complete specification of play; a plan for a course of actions.

- A **pure strategy** for a player i is a single deterministic action $a_i \in A_i$.
- A **mixed strategy** is a probability distribution over pure strategies $\sigma_i \in \Delta(A_i), \quad \sum_{a_i \in A_i} \sigma_i(a_i) = 1$, where $\Delta(A_i)$ is the set of all probability distributions over A_i.

Representative Forms

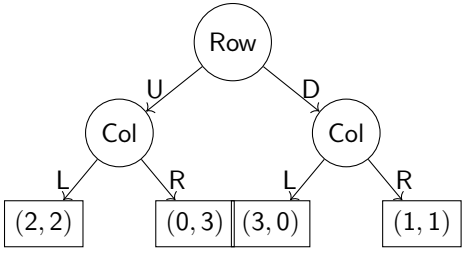
Normal Form

- Captures simultaneous actions
- Captures strategic structure
- $(\pi_{\text{row}}, \pi_{\text{col}})$ are the payoffs of an o

	L	R
U	(2,2)	(0,3)
D	(3,0)	(1,1)

Extensive Form

- Captures sequential moves
- Captures temporal/hierarchical structure



Non-cooperative Games[2,3]

Non-Cooperative Games

- Players act independently
- Focus on individual strategic behaviour
- Communication and binding agreements disallowed

Cooperative Games

- Players can form coalitions
- Focus on joint gains and payoff division
- Assumes binding agreements

[2] J. F. Nash, "Non-cooperative games," in *The Foundations of Price Theory Vol 4*, Routledge, 2024, pp. 329–340.

[3] A. Rubinstein, H. W. Kuhn, O. Morgenstern, *et al.*, *Theory of Games and Economic Behavior: 60th Anniversary Commemorative Edition*. Princeton university press, 2007.

Non-cooperative Participatory Game

A group of hunters awaits a stag known to follow a certain path. If they all cooperate, they can kill it and feast. If they act alone or are discovered, the stag flees, and all go hungry.

Hours pass without a sign. A day goes by. The stag is not guaranteed to come today, but the hunters believe it will. Suddenly, a hare appears.

If one hunter breaks cover to catch the hare, he eats – but ruins the chance at the stag. The others starve.

Do you shoot the Hare or wait for the Stag?
menti.com : 7649 2610

Payoff Structures

Matching Pennies

	Head	Tails
Head	(1, -1)	(-1, 1)
Tails	(-1, 1)	(1, -1)

Stag Hunt

	Stag	Hare
Stag	(5, 5)	(0, 3)
Hare	(3, 0)	(1, 1)

Prisoner's Dilemma

	C	D
C	(3, 3)	(0, 5)
D	(5, 0)	(1, 1)

Battle of the Sexes

	Ballet	Football
Ballet	(2, 1)	(0, 0)
Football	(0, 0)	(1, 2)

Payoff Structures

Zero-Sum Game

	L	R
U	(A, -A)	(-C, C)
D	(-B, B)	(D, -D)

Non-Zero-Sum Game

	L	R
U	(A, A)	(C, C)
D	(B, B)	(D, D)

Symmetric Payoffs

	L	R
U	(A, A)	(C, C)
D	(B, B)	(D, D)

Asymmetric Payoffs

	L	R
U	(A_i, A_j)	(C_i, C_j)
D	(B_i, B_j)	(D_i, D_j)

Solution Concepts: Dominant Strategies

A **dominant strategy** is one that always yields a higher payoff, regardless of what the opponent does.

Strict dominance:
$a_i \in A_i$ is strictly dominant for player i if

$$\forall a_{-i} \in A_{-i},\ \forall a'_i \neq a_i :$$
$$U_i(a_i, a_{-i}) > U_i(a'_i, a_{-i})$$

Prisoner's Dilemma

	C	**D**
C	(3, 3)	(0, **5**)
D	(**5**, 0)	(**1**, **1**)

Stag Hunt

	S	H
S	(4, 4)	(0, 3)
H	(3, 0)	(3, 3)

Solution Concepts: Nash Equilibrium

A **Nash equilibrium** is a strategy profile where no player can benefit by unilaterally deviating.

$(a_1^*, \ldots, a_n^*)$ is a Nash equilibrium if $\forall i \in N$:

$$U_i(a_i^*, a_{-i}^*) \geqslant U_i(a_i, a_{-i}^*) \quad \forall a_i \in A_i$$

Prisoner's Dilemma

	C	**D**
C	(3, 3)	(0, 5)
D	(5, 0)	(**1**, **1**)

Stag Hunt

	S	**H**
S	(**4**, **4**)	(0, 3)
H	(3, 0)	(**3**, **3**)

Solution Concepts: Pareto Efficiency

An outcome is **Pareto efficient** if no player can be made better off without making another worse off.

$o \in A_1 \times \cdots \times A_n$ is Pareto efficient if

$$\nexists o' \text{ such that } U_i(o') \geqslant U_i(o)\ \forall i \text{ and } \exists j,\ U_j(o') > U_j(o)$$

Prisoner's Dilemma

	C	D
C	(**3**, **3**)	(0, 5)
D	(5, 0)	(1, 1)

Stag Hunt

	S	**H**
S	(**4**, **4**)	(0, 3)
H	(3, 0)	(**3**, **3**)

Solution Concepts: Social Welfare

Social welfare is the total utility across all players.

An outcome is *socially optimal* if it maximizes:

$$o^* = \arg\max_{o \in A_1 \times \cdots \times A_n} \sum_{i \in N} U_i(o)$$

Prisoner's Dilemma

	C	D
C	(**3**, **3**)	(0, 5)
D	(5, 0)	(1, 1)

Stag Hunt

	S	H
S	(**4**, **4**)	(0, 3)
H	(3, 0)	(3, 3)

Participatory Guessing Game

2 person game instruction:

"Your task is to guess how many balls are hidden in this black box. The box can be empty and can hold up to 100 balls. However, there is a twist! The winning guess is not the one closest to the actual number of balls hidden in the box. The closest guess to the two players' choices' mean multiplied by 0.66 will be the winner."

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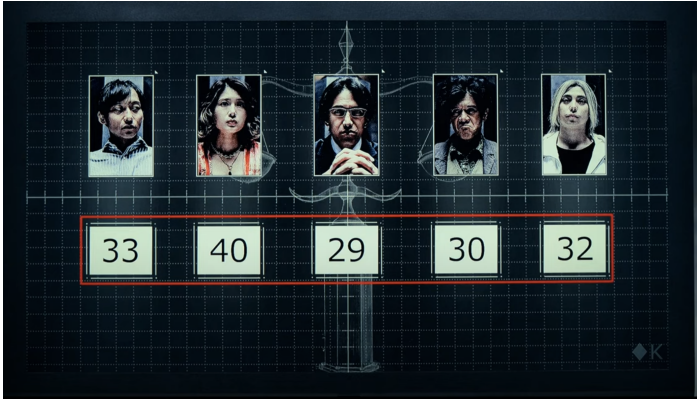
Strategic Reasoning in Guessing Games



Strategic Reasoning in the p-Beauty Contest

The p-Beauty Contest[4,5]

- Each player selects a number in a known interval (e.g., [0, 100]).
- The winner is the player whose guess is closest to p times the average of all guesses.

Why is it interesting?

- Models *strategic anticipation* through **recursive reasoning**:
  - "What others think I will think..."
  - Formally captured by k-level theory
- No strictly dominant strategy
- Nash equilibrium: all players choose 0
- Special case[6] with $n = 2$

[4] J. M. Keynes, "The general theory of employment," *The quarterly journal of economics*, vol. 51, no. 2, pp. 209–223, 1937.

[5] R. Nagel, "Unraveling in guessing games: An experimental study," *The American Economic Rev.*, vol. 85, no. 5, pp. 1313–1326, 1995.

[6] B. Grosskopf and R. Nagel, "The two-person beauty contest," *Games and Economic Behavior*, vol. 62, no. 1, pp. 93–99, 2008.

Information Structures

Complete vs Incomplete Information

- Can players fully observe others' payoffs?

Perfect vs Imperfect Information

- Can players fully observe others' actions?

Symmetric vs Asymmetric Information

- Can players access the same quality/quantity of information?

Multi-agent Systems

Intelligent Agents and Strategic Reasoning

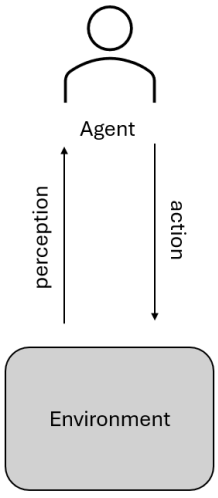
Real-world conflicts entail practical scenarios, where societies of actors are involved in cooperative or competitive interactions. For such cases, agent-based approaches are deemed a natural metaphor[7].

[7] M. Wooldridge, *An introduction to multiagent systems*. John wiley & sons, 2009.

Agents and the Environment

What is an agent?

An agent perceives the environment and produces actions that affect it.

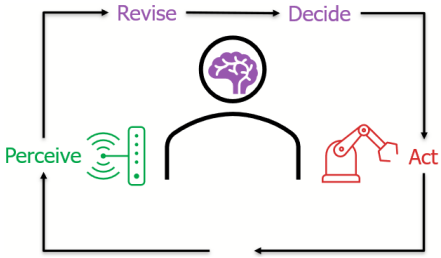


Agents and the Environment

What is an agent?

Agents are characterized by[6]:

- Autonomy (operating without direct intervention)
- Social ability (communicating with other agents)
- Reactivity (responding to environmental changes)
- Proactivity (exhibiting goal-directed behaviour)



OODA Agent[7]

[6] M. Wooldridge and N. R. Jennings, "Intelligent agents: Theory and practice," *The Knowledge Engineering Review*, vol. 10, no. 2, pp. 115–152, 1995

[7] V. Trencsenyi, A. Mensfelt, and K. Stathis, "Approximating human strategic reasoning with llm-enhanced recursive reasoners leveraging multi-agent hypergames," *arXiv preprint arXiv:2502.07443*, 2025

Agent Theories

How do agents work internally?

- **Reactive Architectures** Rely on simple stimulus-response mechanisms without internal models or planning.
- **Deliberative Architectures** Maintain internal representations, and support reasoning, memory, and planning, often inspired by cognitive processes.
- **Hybrid Architectures** Combine reactive and deliberative layers to balance reactive responsiveness with symbolic representations and goal-driven behaviour.

BDI Agents: Human-like Practical Reasoning

Belief–Desire–Intention (BDI) agents model human-like deliberative processes by decoupling reasoning into:

- **Beliefs (B)**: informational state (agent's internal model of the world)
- **Desires (D)**: motivational state (goals the agent might want to achieve)
- **Intentions (I)**: deliberative commitments to plans of action

BDI architecture functions:

- $\mathcal{B}$: Belief revision from new percepts
- $\mathcal{O}$: Option generation based on B, I
- $\mathcal{F}$: Intention filtering from B, D, I
- $\mathcal{P}$: Planning over B, I to action sequence Π

BDI agents cycle through these functions to continuously update beliefs, deliberate, and act — paralleling human practical reasoning.

Are LLMs Agents?

Are LLMs agents?

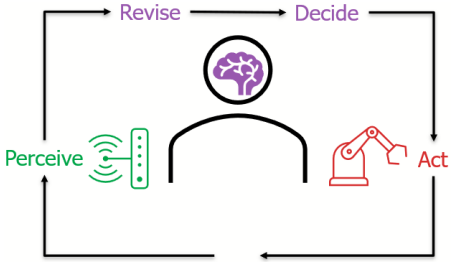
What kind of agent are they?

Are LLMs agents?

An agent perceives the environment and produces actions that affect it.

LLM-as-agent[8] builds on this weaker notion, where the prompt is a perception and the response is the action.

LLM-as-mind[9]



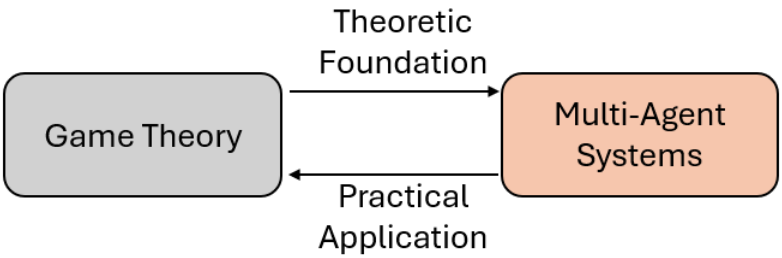
[8] X. Liu, H. Yu, H. Zhang, *et al.*, "Agentbench: Evaluating llms as agents," *arXiv preprint arXiv:2308.03688*, 2023

[9] V. Trencsenyi, A. Mensfelt, and K. Stathis, "Approximating human strategic reasoning with llm-enhanced recursive reasoners leveraging multi-agent hypergames," *arXiv preprint arXiv:2502.07443*, 2025

Multi-Agent Systems

Multi-Agent Systems (MAS) are societies of autonomous agents interacting in a shared environment.

- Each agent pursues individual or shared goals
- Agents coordinate, compete, and communicate
- Strategic interactions naturally emerge



Uncertainty in MAS

Realistic MAS must handle uncertainty beyond ideal game-theoretic assumptions.

- **Environmental uncertainty**: Partial observability, dynamic environments, non-stationarity
- **Action and Strategic uncertainty**: Non-deterministic effects of actions, unknown opponent strategies
- **Bounded rationality and belief misalignment**: Cognitive limits, mismatched models of the world or other agents