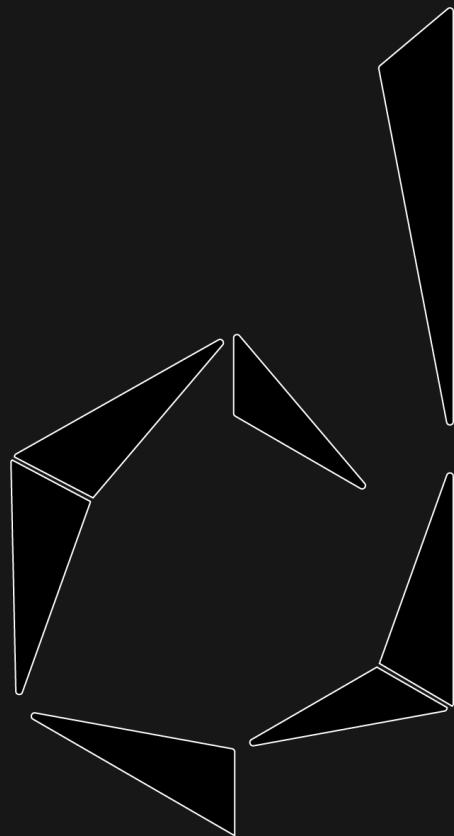


INGENIERÍA MECATRÓNICA



DI_CERO

DIEGO CERVANTES RODRÍGUEZ

INGENIERÍA ASISTIDA POR COMPUTADORA

COMSOL MULTIPHYSICS 5.6

16: Pandeo en Columnas

Contenido

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DESCRIPCIÓN DEL PROBLEMA:

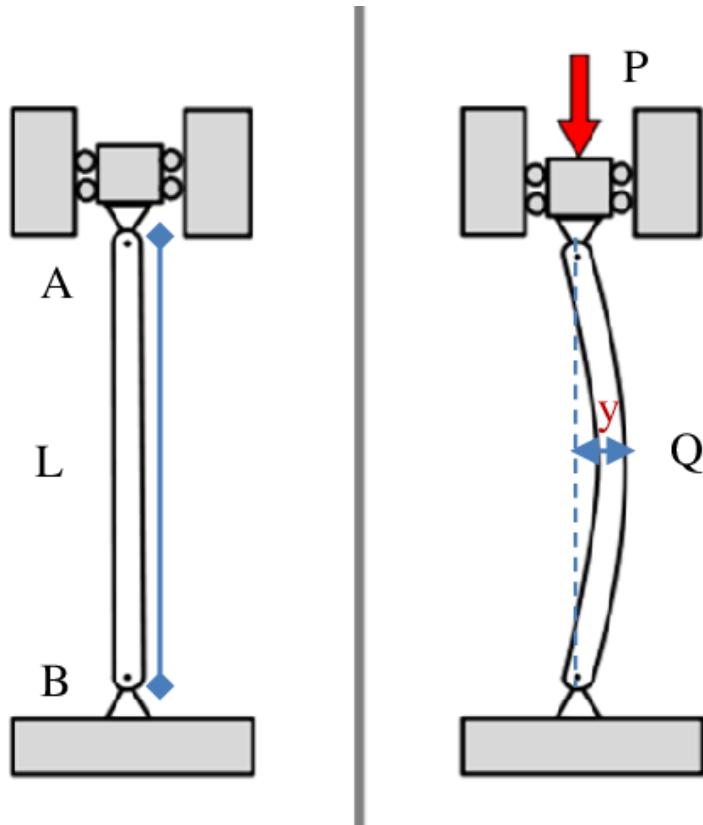
Presión crítica:

$$P_{cr} = \text{Factor de carga crítica} * \text{presión en la columna} = 1.686 \times 10^6 (100 \text{ N/m}^2)$$

$$P_{cr} = 168.6 \times 10^6 \text{ Pa} = 168.6 \text{ MPa}$$

*Fcr = Pcr * Área de la sección transversal de la columna*

$$F_{cr} = P_{cr} * A = (168.6 \times 10^6 \text{ N/m}^2) (0.033755 \text{ m}^2) = 5691093 \text{ N} = 5.7 \text{ MN}$$



Cálculo analítico usando la fórmula de Euler para condiciones de frontera en la columna empotrado – libre:

$$F_{cr} = \frac{C\pi^2 EA}{\left(\frac{L}{k_x}\right)^2}$$

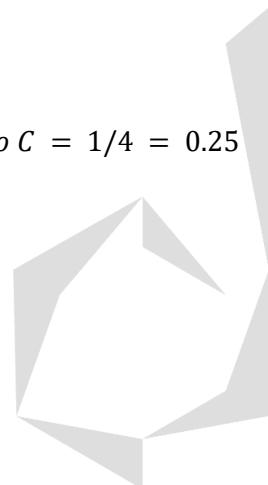
C = Constante que depende de las condiciones de frontera, en este caso $C = 1/4 = 0.25$

$$F_{cr} = \text{Carga crítica} = ?$$

$$E = \text{Módulo de elasticidad} = 69 \times 10^9 \text{ Pa}$$

$$A = 0.033755 \text{ m}^2$$

$$L = 2 \text{ m}$$



$$k = \sqrt{\frac{I}{A}}$$

I = el menor momento de inercia de I_x , I_y .

$$I_x = \left(\frac{bh^3}{36} \right) = \left[\frac{(0.3 \text{ m})(0.2598 \text{ m})^3}{36} \right] = 1.46129 \times 10^{-4} \text{ m}^4$$

$$I_y = \left(\frac{hb^3}{48} \right) = \left[\frac{(0.2598 \text{ m})(0.3 \text{ m})^3}{48} \right] = 1.461375 \times 10^{-4} \text{ m}^4$$

$$I_{\text{círculo}} = \frac{\pi}{64} d^4 = \frac{\pi}{64} (0.08)^4 = 2 \times 10^{-6} \text{ m}^4$$

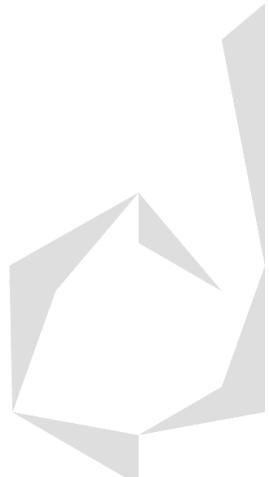
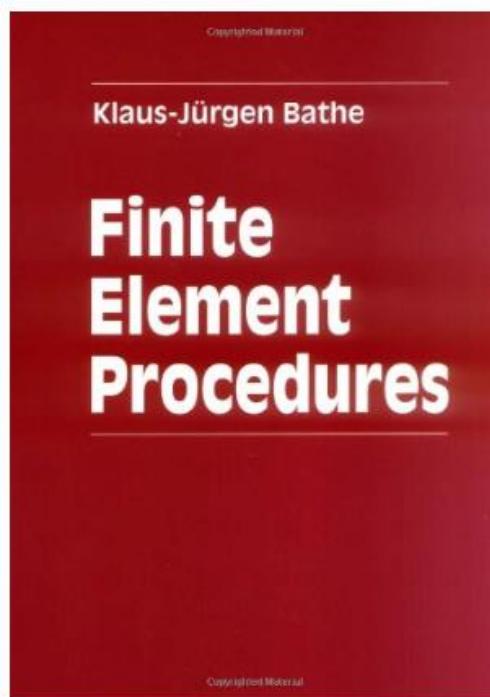
$$I = I_x - I_{\text{círculo}} = 1.46129 \times 10^{-4} \text{ m}^4 - 2 \times 10^{-6} \text{ m}^4 = 1.4411 \times 10^{-4} \text{ m}^4$$

$$k_x = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.4411 \times 10^{-4}}{0.033755}} = 0.06534 \text{ m}$$

$$F_{cr} = \frac{C\pi^2 EA}{\left(\frac{L}{k_x}\right)^2} = \frac{(0.25)\pi^2(69 \times 10^9)(0.033755)}{\left(\frac{2}{0.06534}\right)^2} = 6.13 \text{ MN}$$

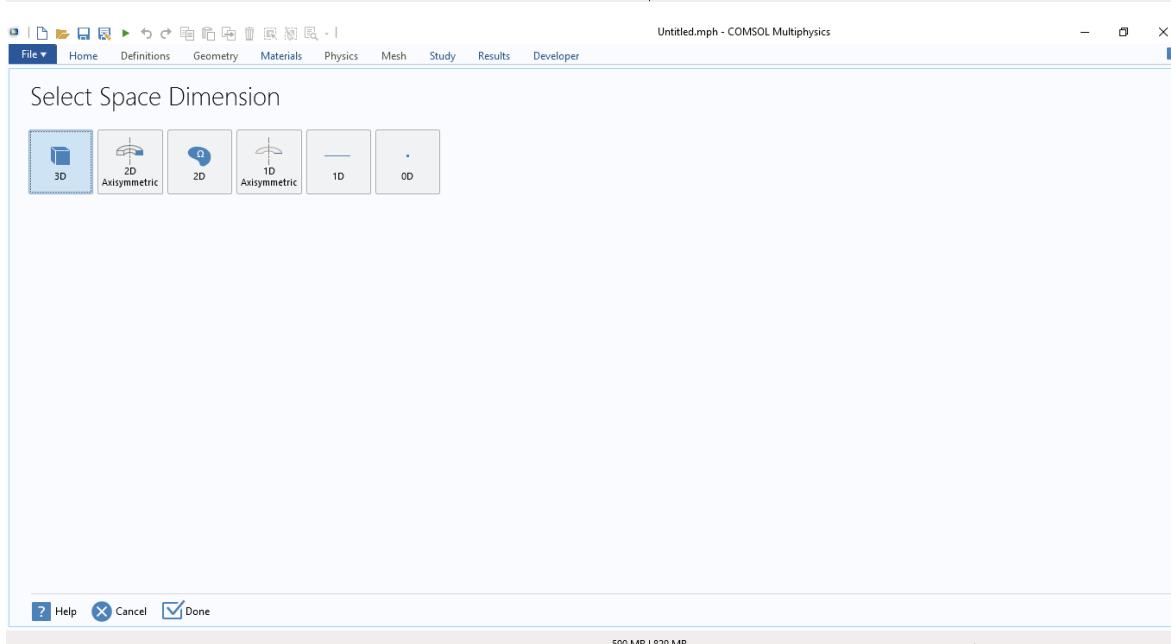
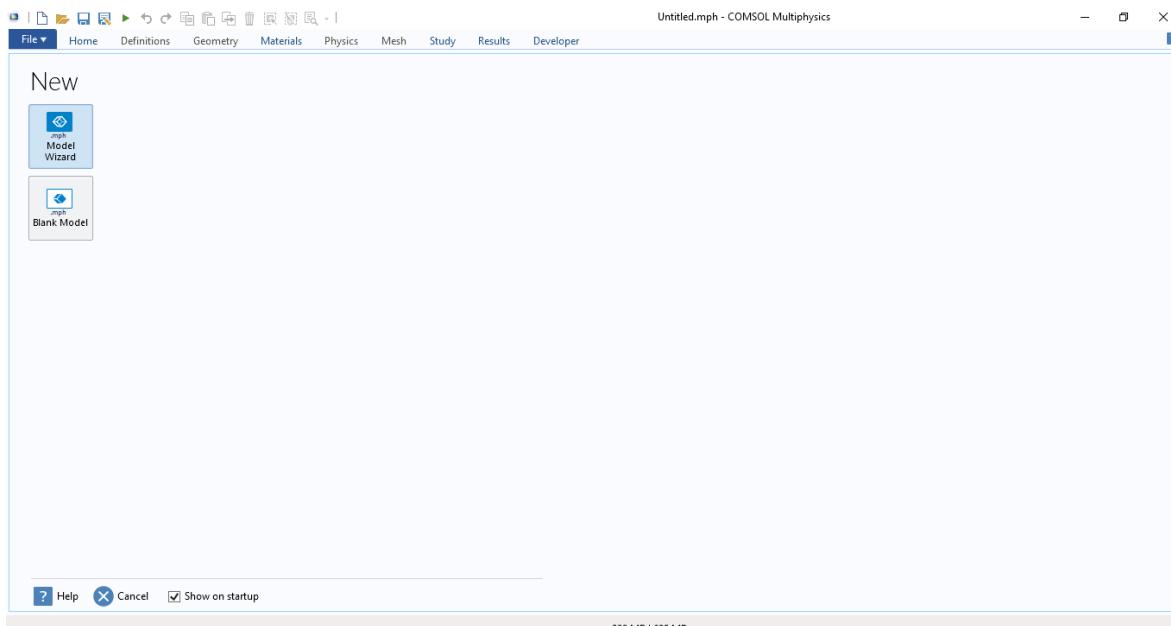
Cálculo del error relativo:

$$\epsilon = \frac{|F_{cr}^{\text{Analítica}} - F_{cr}^{\text{Numérica}}|}{F_{cr}^{\text{Analítica}}} \times 100 = \frac{|6.13 - 5.7|}{6.13} \times 100 = 7 \% < 11 \%$$



An important ingredient of a finite element analysis is the calculation of error estimates, that is, estimates of how closely the finite element solution approximates the exact solution of the mathematical model (see Section 4.3.6). These estimates indicate whether a specific finite element discretization has indeed yielded an accurate response prediction, and a designer can then rationally decide whether the given results should be used. In the case that unacceptable results have been obtained using unreliable finite element methods, the difficulty is of course *how* to obtain accurate results.

CREACIÓN DE LA PIEZA EN COMSOL:



Untitled.mph - COMSOL Multiphysics

Select Physics

Solid Mechanics

The Solid Mechanics interface is intended for general structural analysis of 3D, 2D, or axisymmetric bodies. In 2D, plane stress or plane strain assumptions can be used. The Solid Mechanics interface is based on solving Navier's equations, and results such as displacements, stresses, and strains are computed.

The Acoustics Module, MEMS Module, and Structural Mechanics Module add several features, for example geometric nonlinearity and advanced boundary conditions such as contact, follower loads, and nonreflecting boundaries.

With the Nonlinear Structural Materials Module or the Geomechanics Module, the interface is extended with, for example, material models for plasticity, hyperelasticity, creep, and concrete.

Recently Used

- Laminar Flow (spf)
- Solid Mechanics (solid)**
- Truss (truss)
- Beam (beam)
- AC/DC
- Acoustics
- Chemical Species Transport
- Electrochemistry
- Fluid Flow
- Heat Transfer
- Optics
- Plasma
- Radio Frequency
- Semiconductor

Added physics interfaces:

Add Remove

Space Dimension Study

Help Cancel Done

613 MB | 861 MB

Este es el cálculo elegido para el análisis mecánico de columnas.

Untitled.mph - COMSOL Multiphysics

Select Study

Linear Buckling

The Linear Buckling study is used for estimating the critical load at which a structure becomes unstable.

The Linear Buckling study consists of two study steps: a Stationary study step for applying an external load followed by a Linear Buckling study step. In the second study step, an eigenvalue solver is used to compute the buckling modes and the associated critical load factors.

Preset Studies

- Bolt Pre-Tension
- Eigenfrequency
- Frequency Domain
- Frequency Domain Modal
- Linear Buckling**
- Modal Reduced-Order Model
- Prestressed Analysis, Eigenfrequency
- Prestressed Analysis, Frequency Domain
- Stationary
- Time Dependent

Added study:

- Linear Buckling

Added physics interfaces:

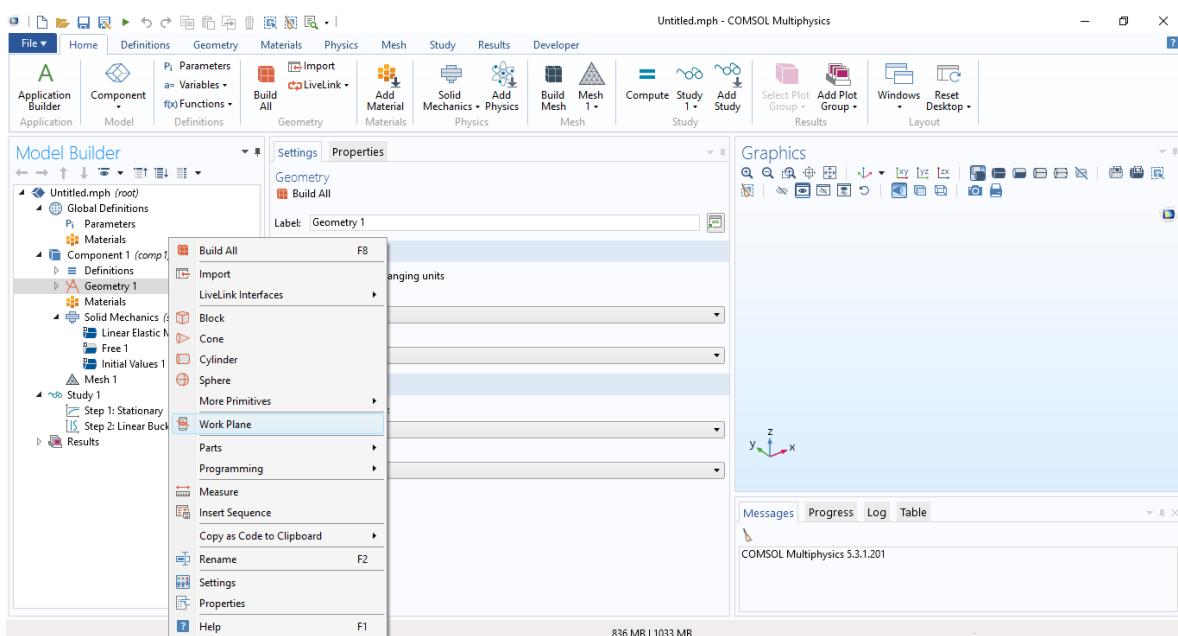
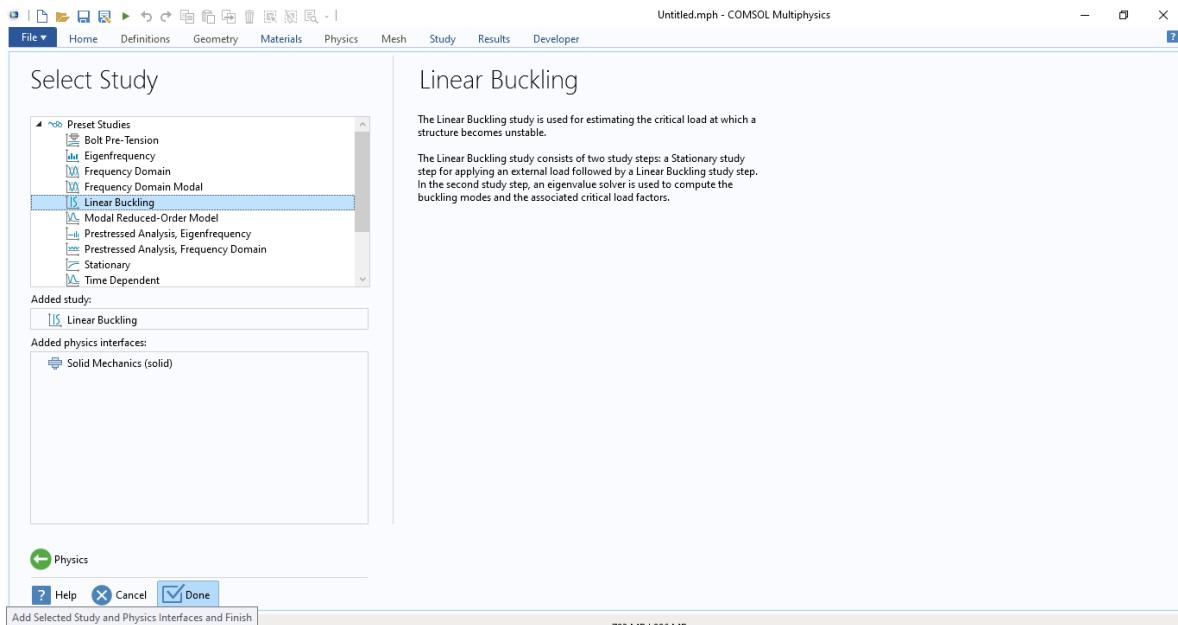
- Solid Mechanics (solid)

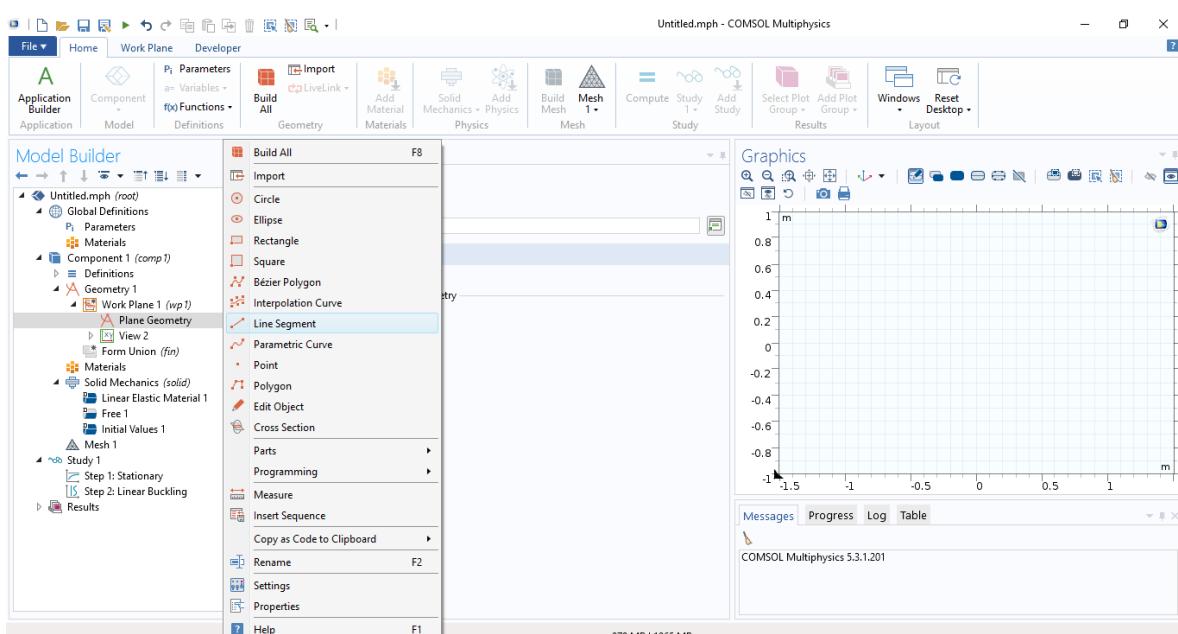
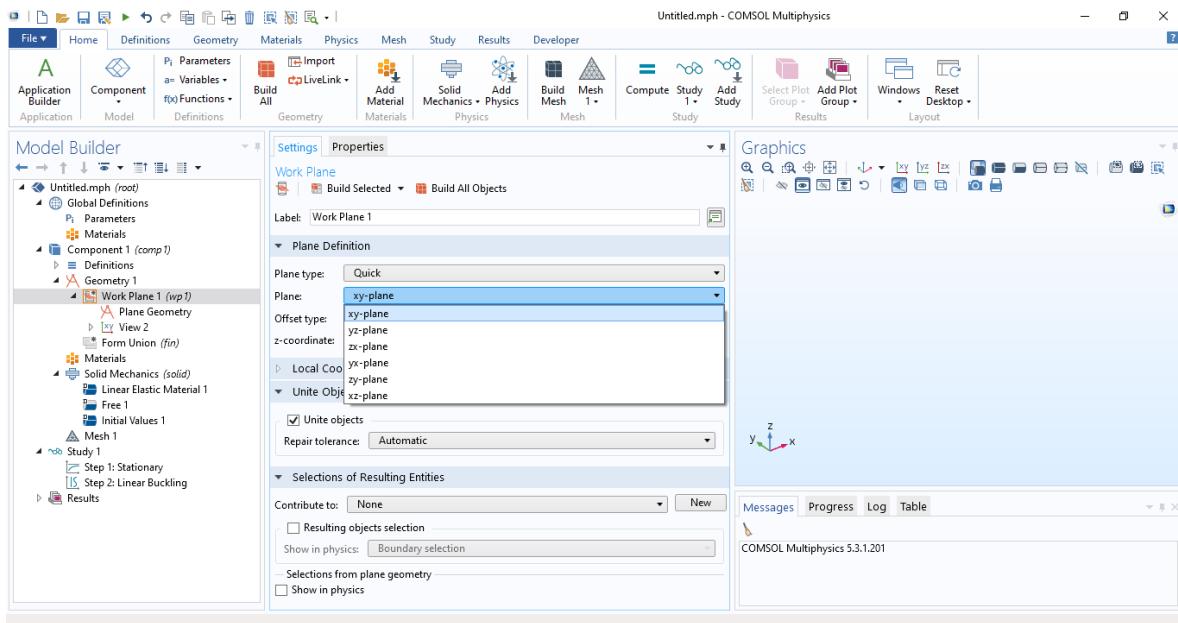
Physics

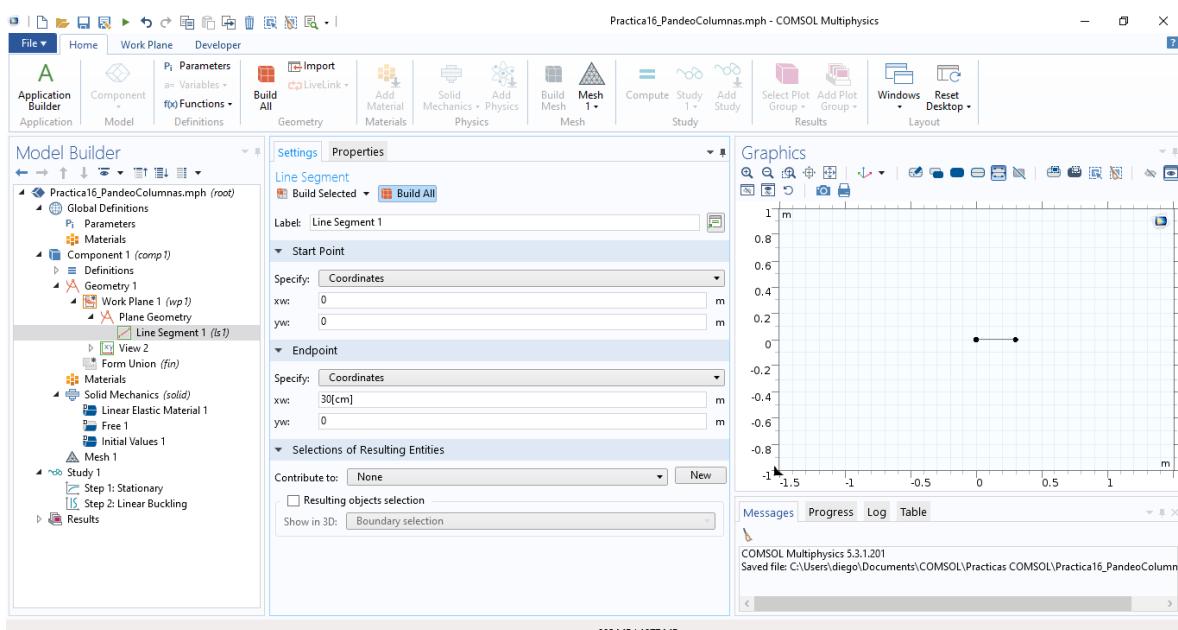
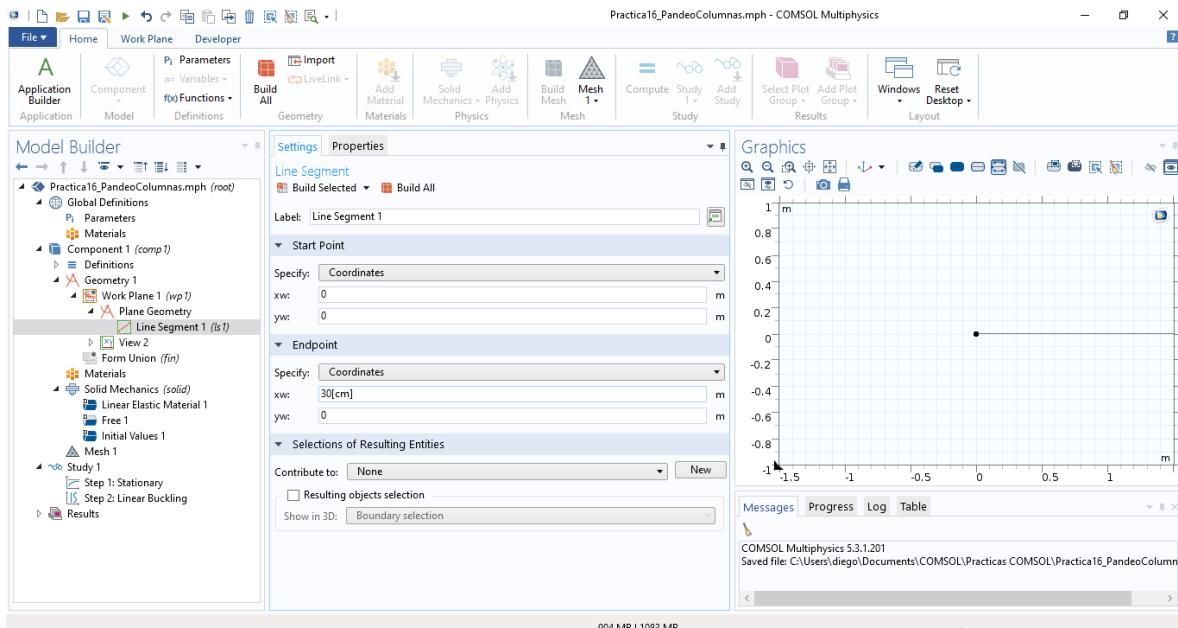
Help Cancel Done

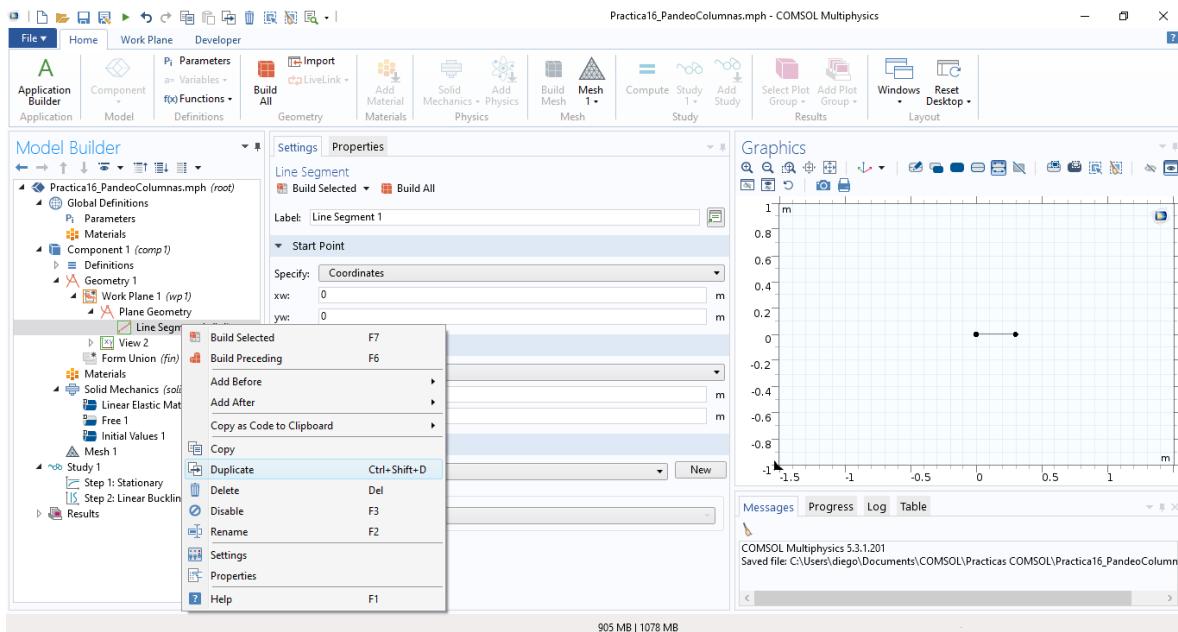
793 MB | 996 MB



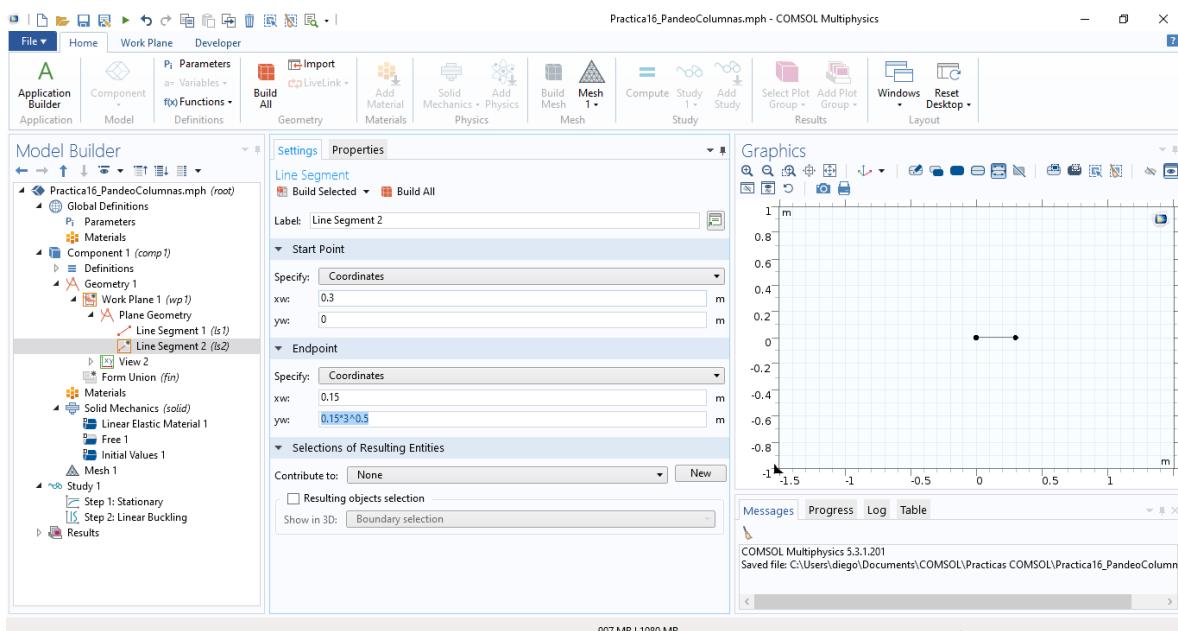


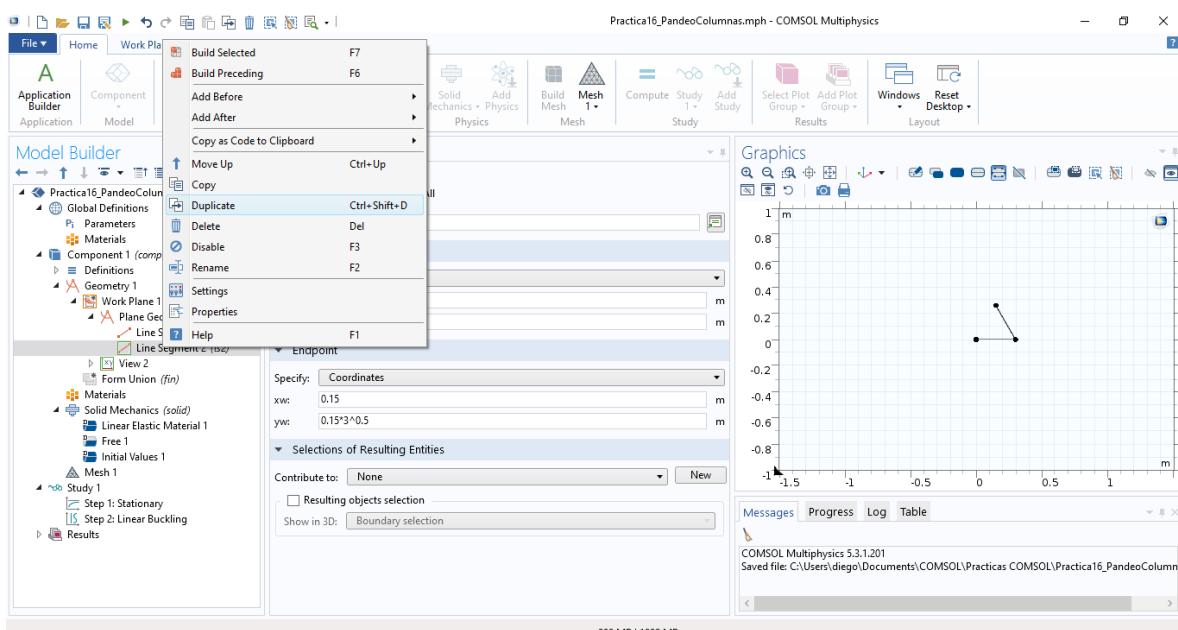
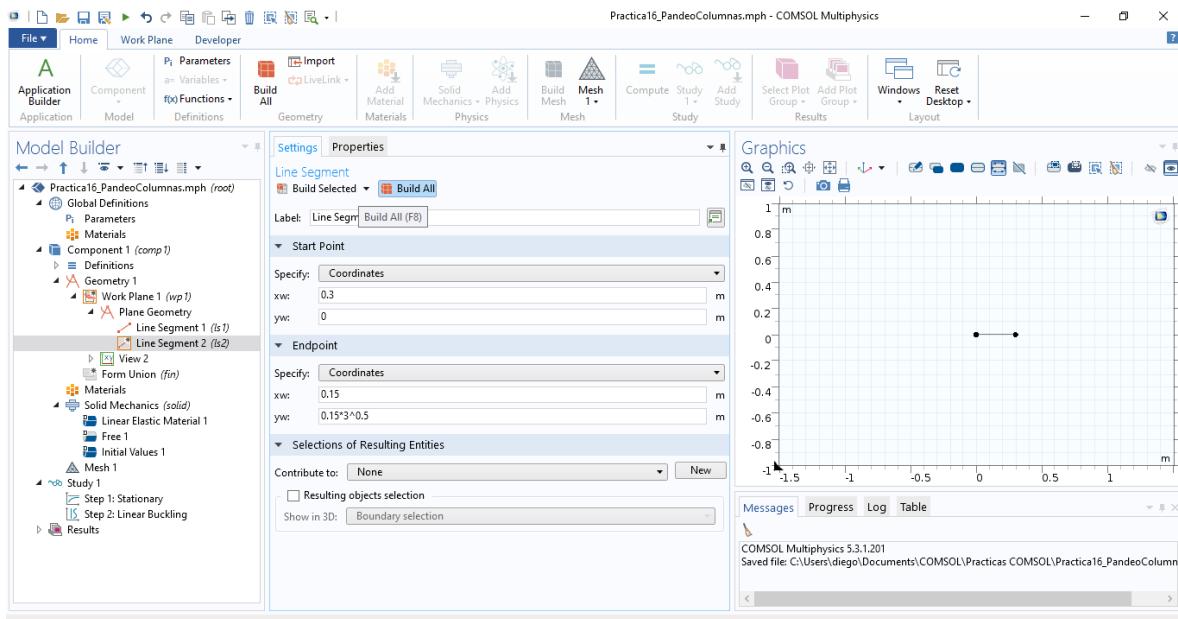


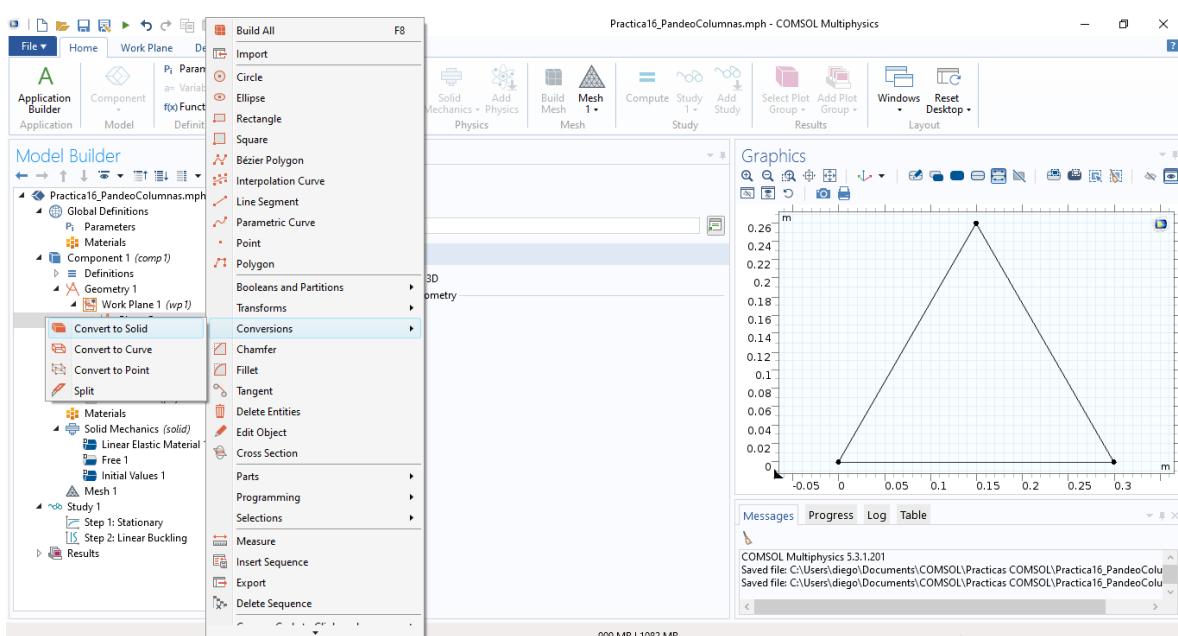
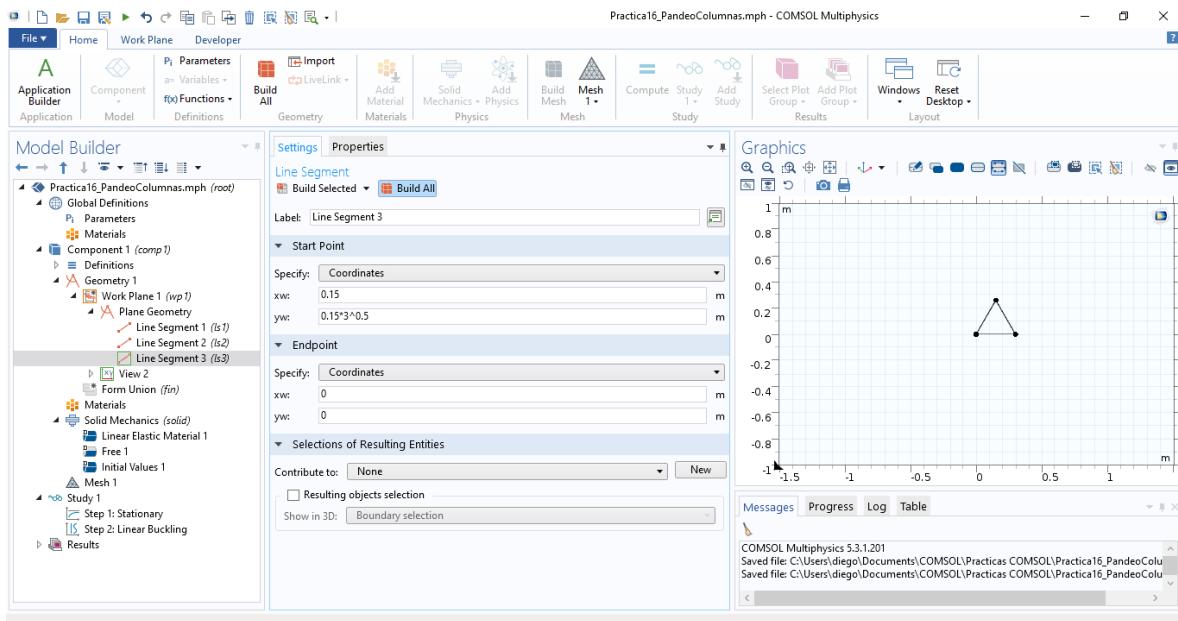


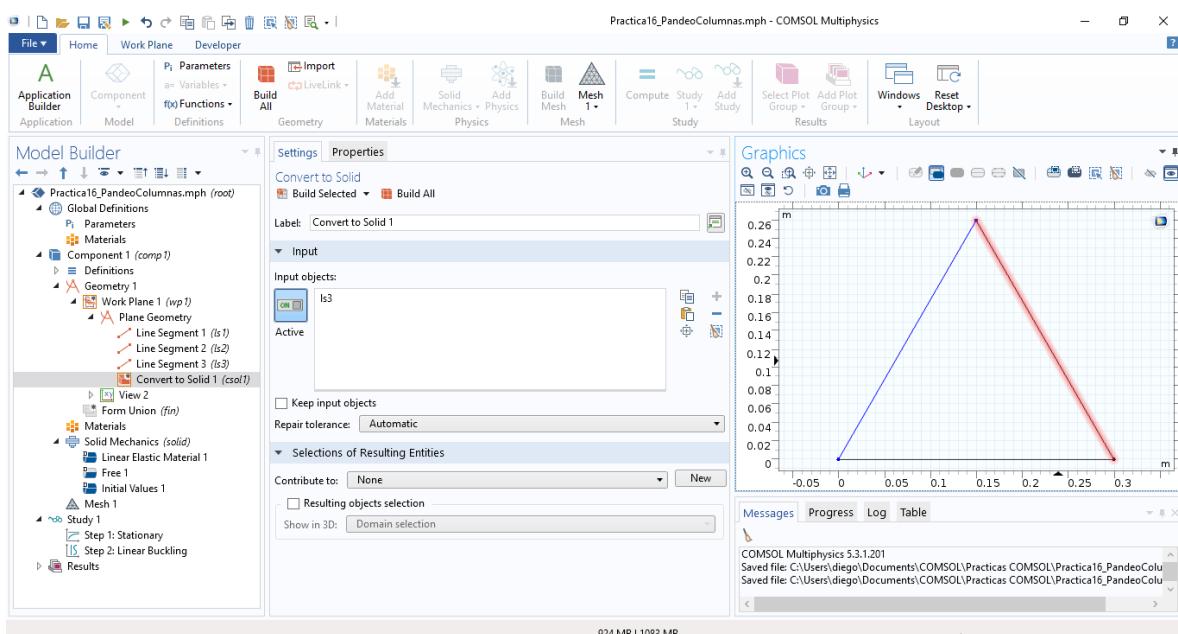
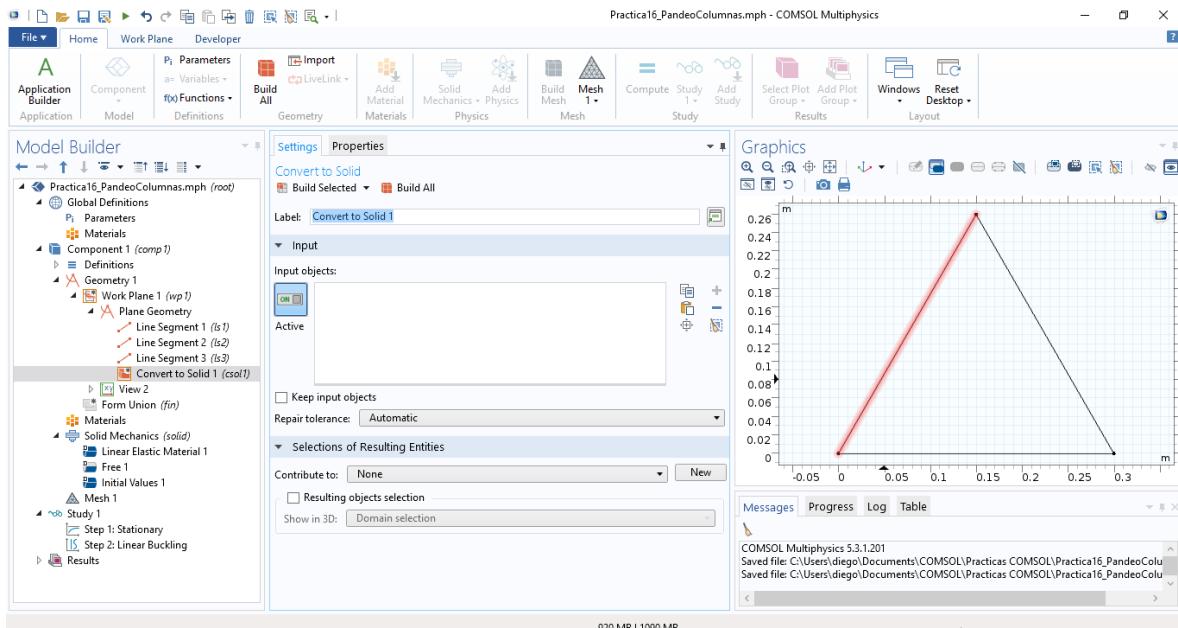


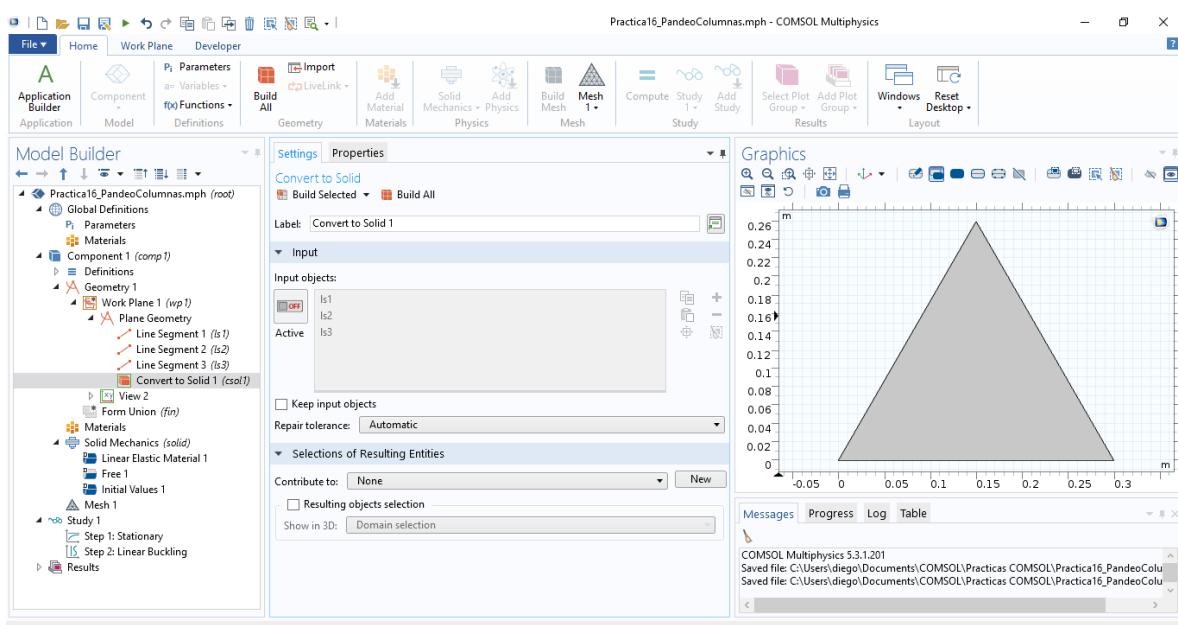
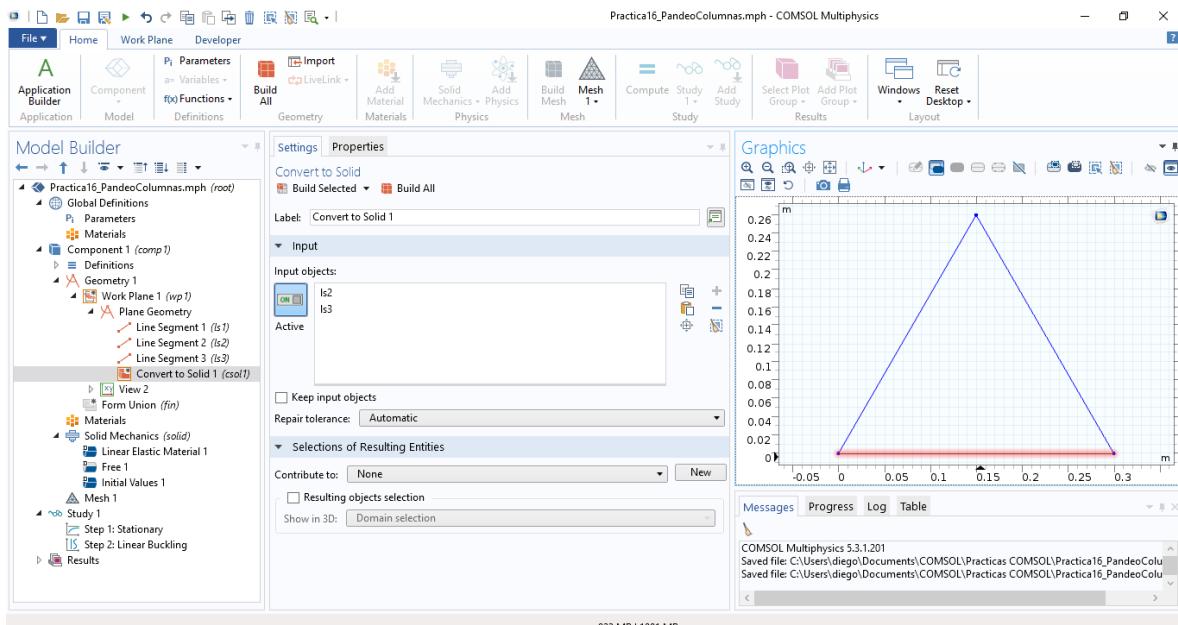
$0.15 * 3^{0.5}$ Es la ecuación del seno para que se cree un triángulo que es la base de la columna.

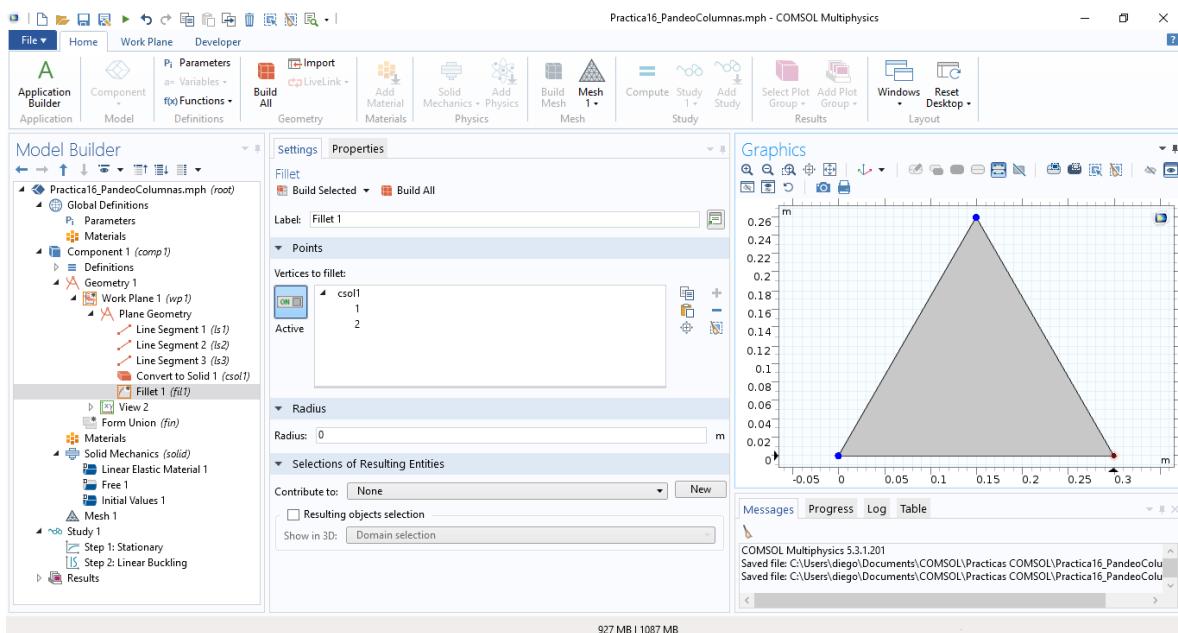
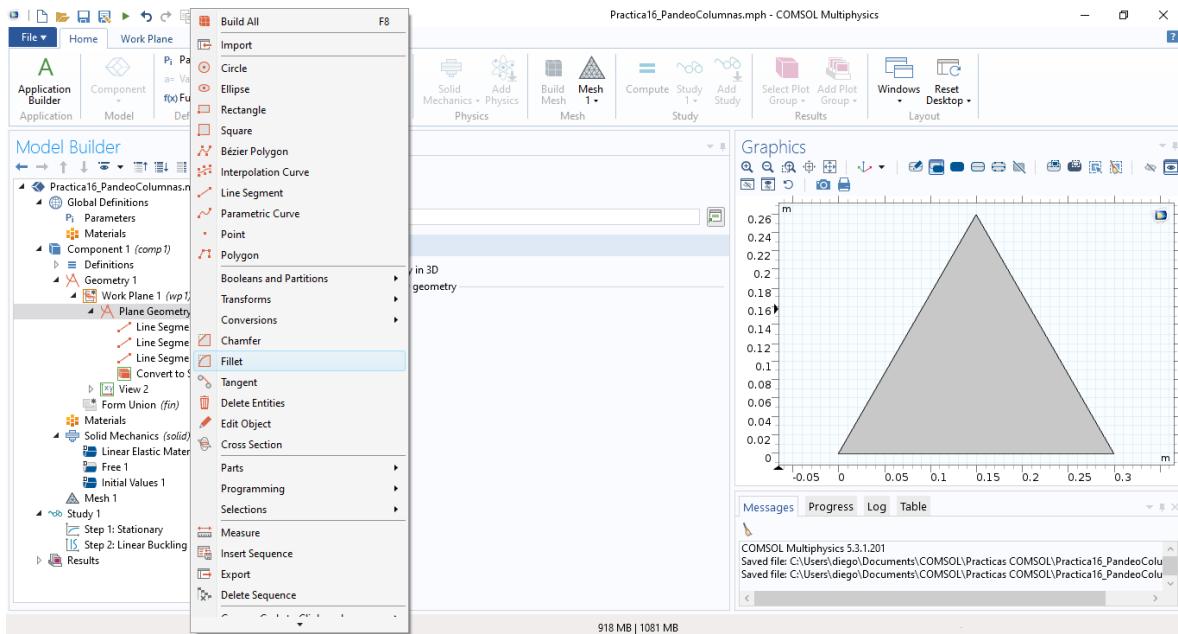


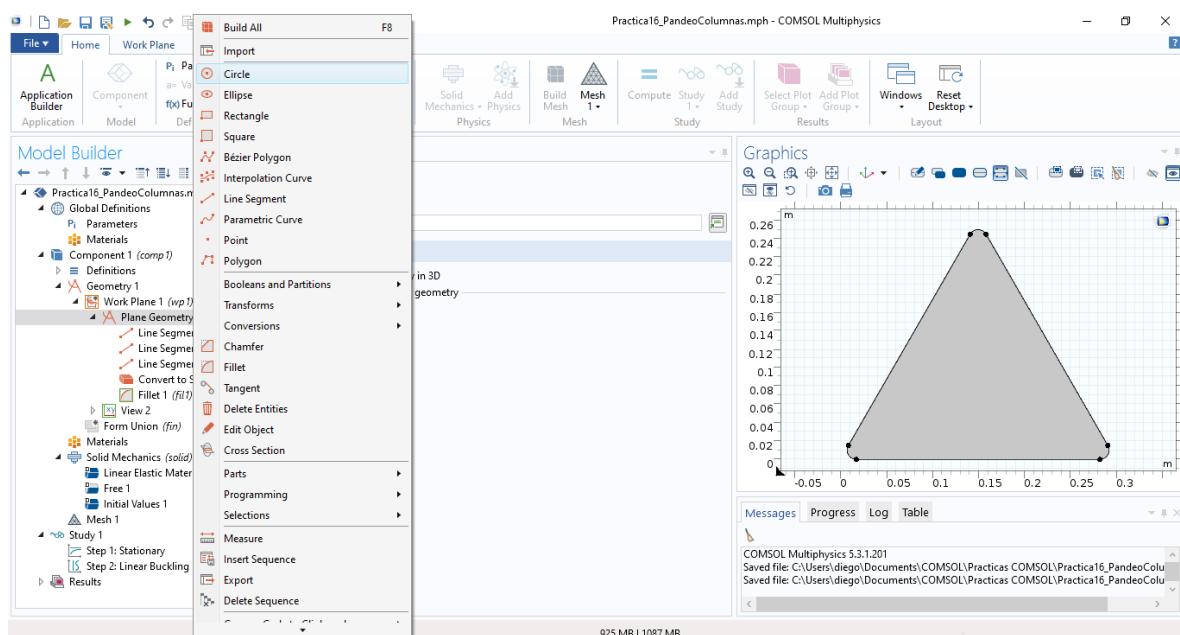
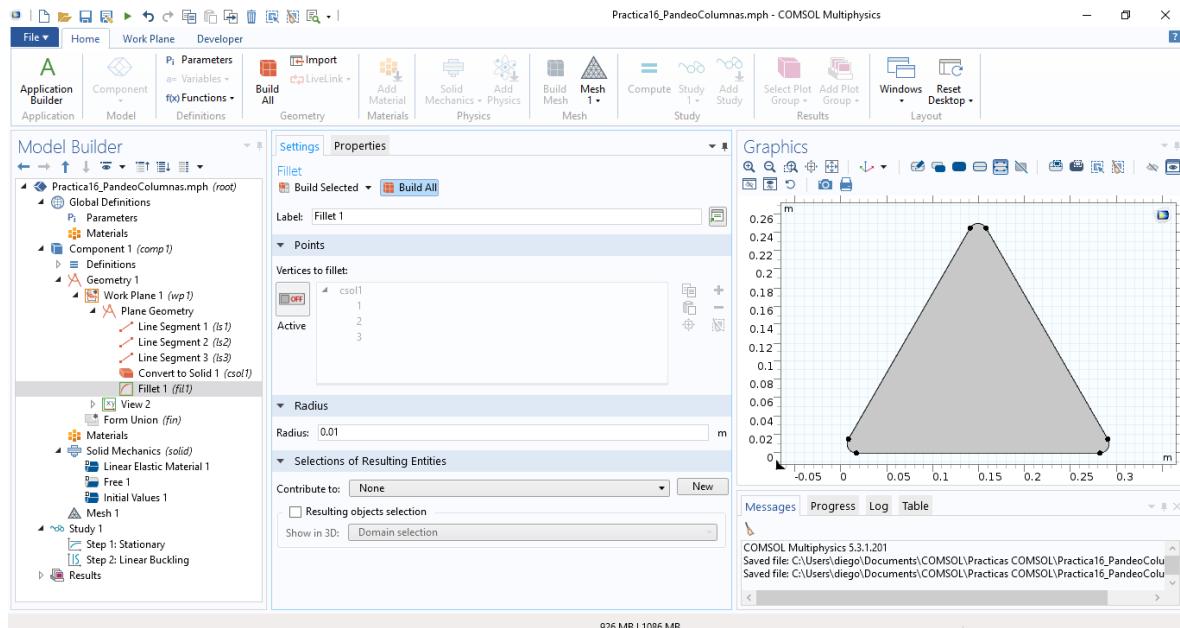


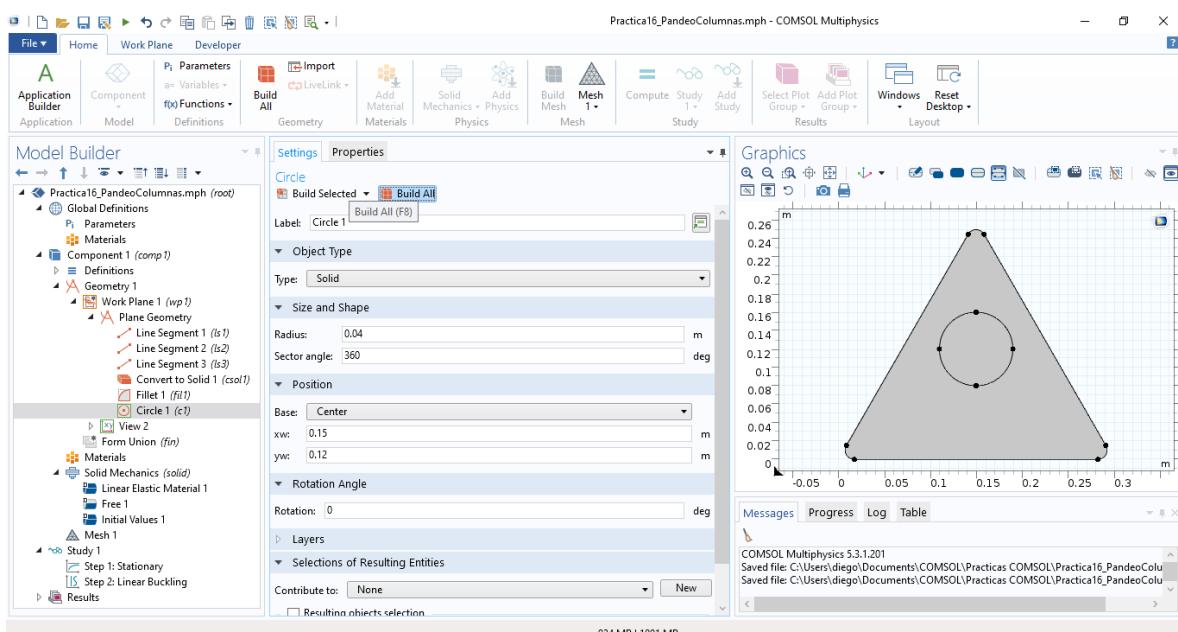
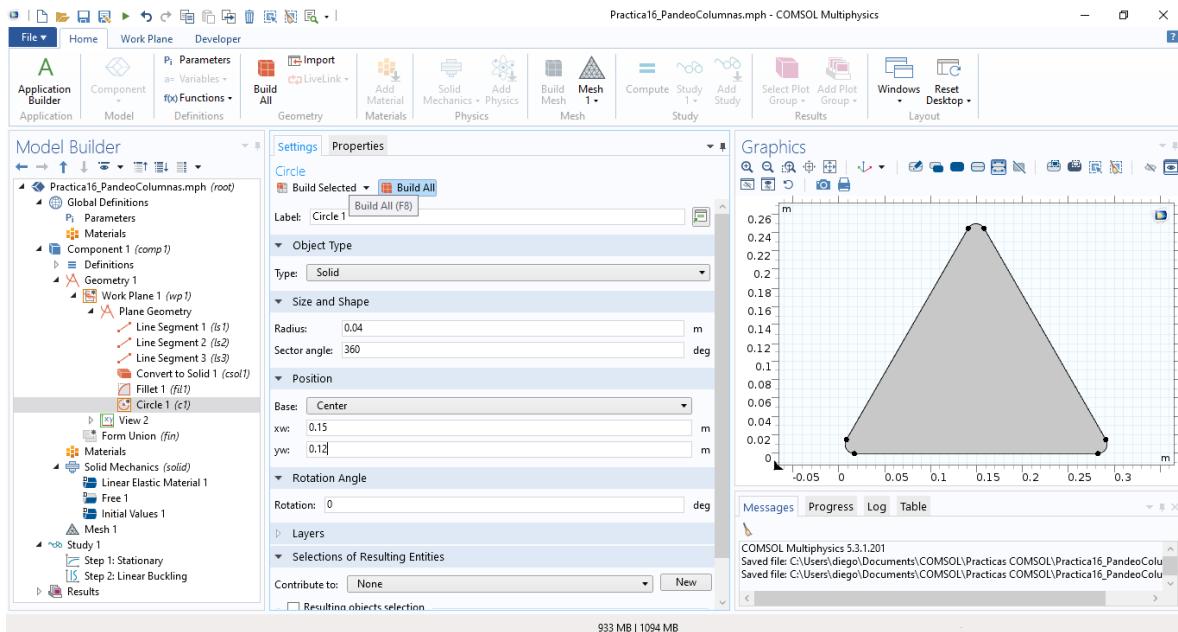


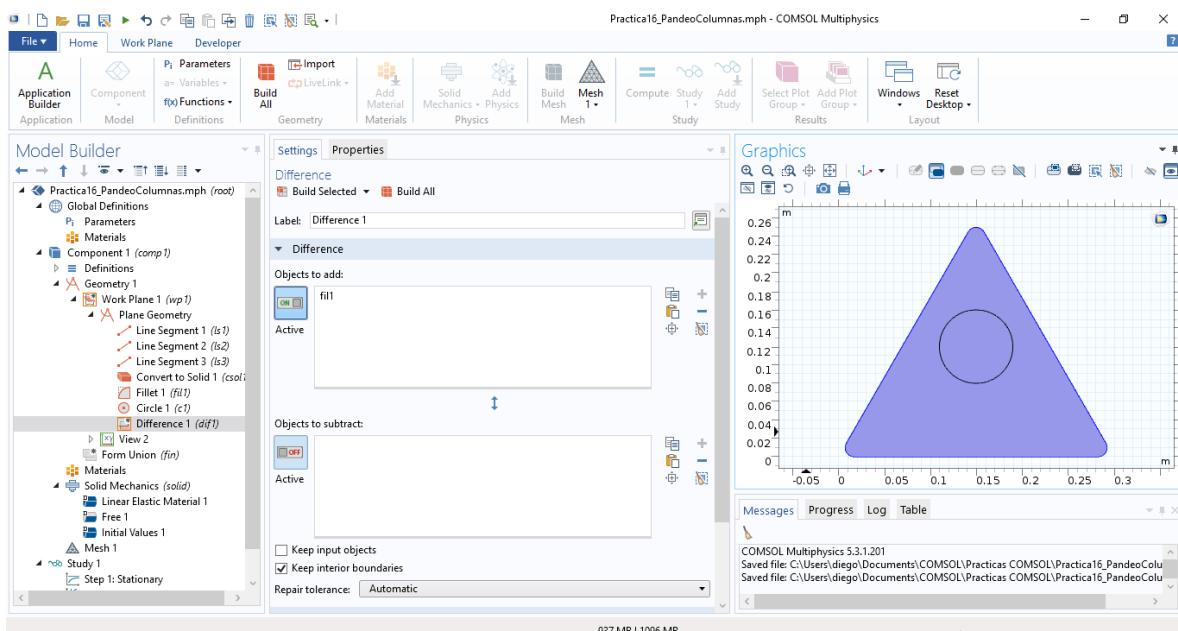
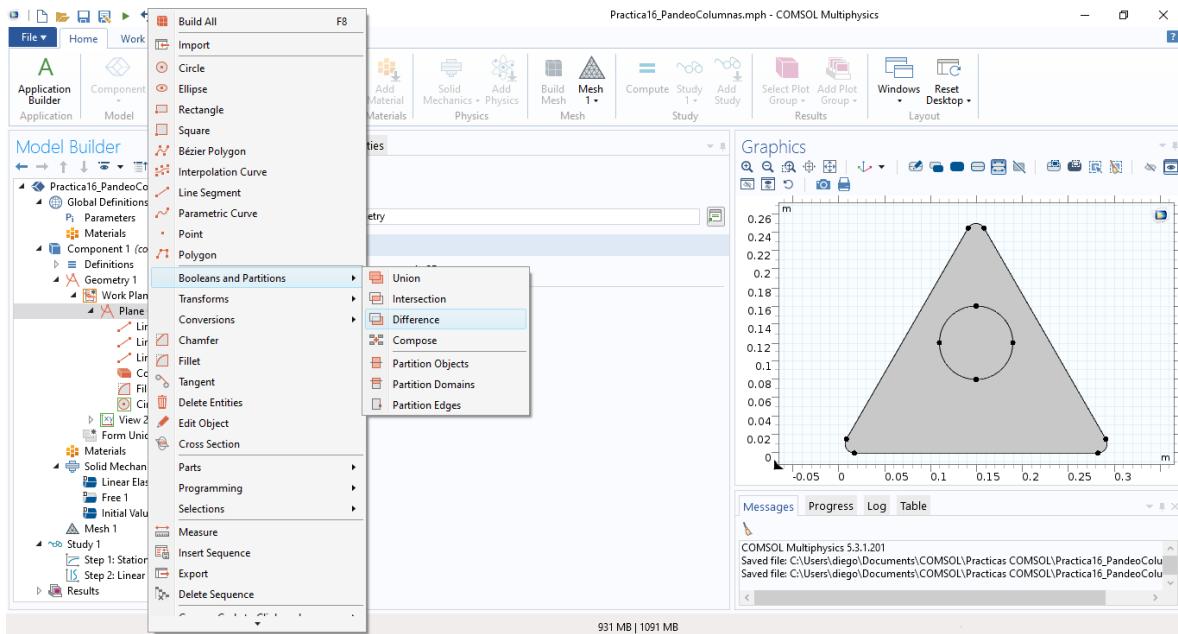


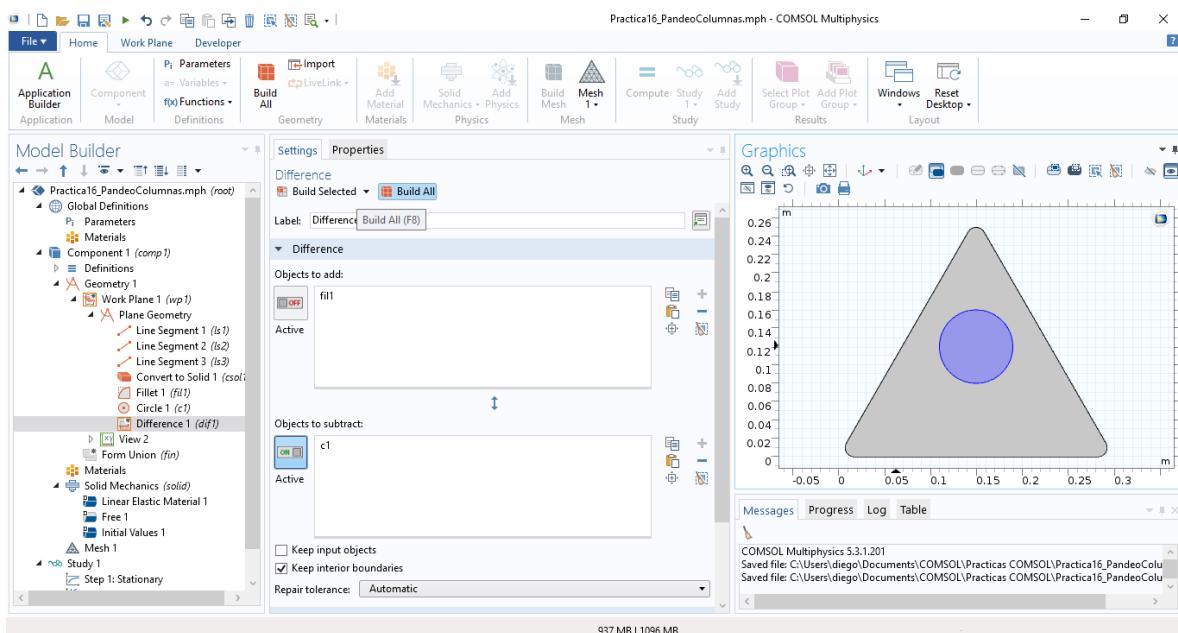
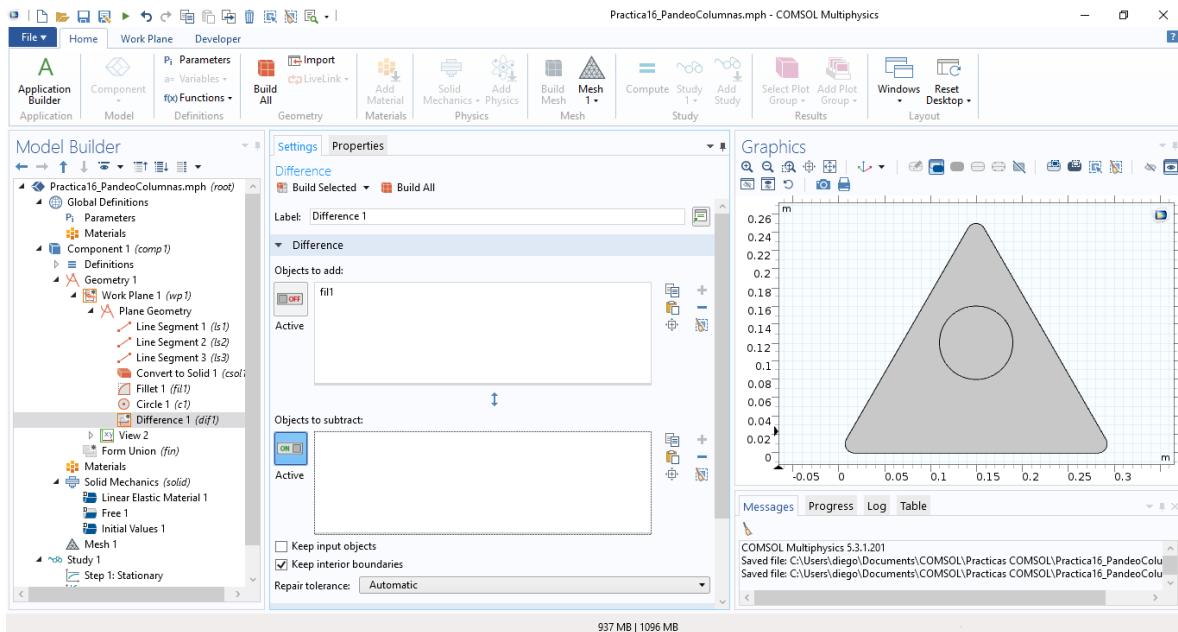






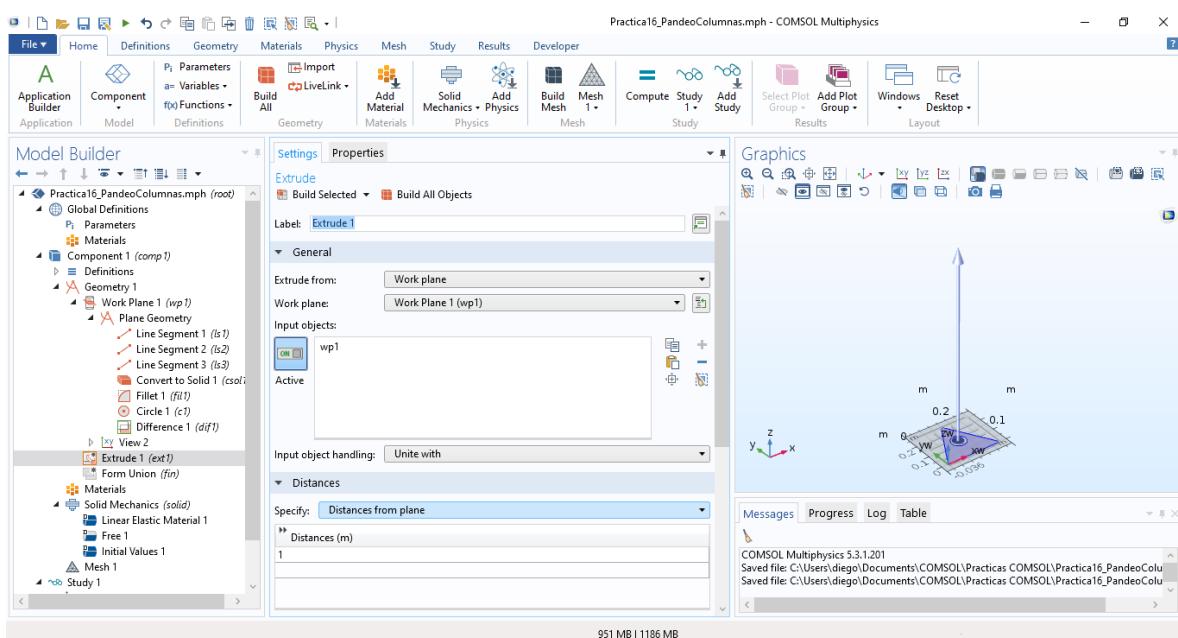
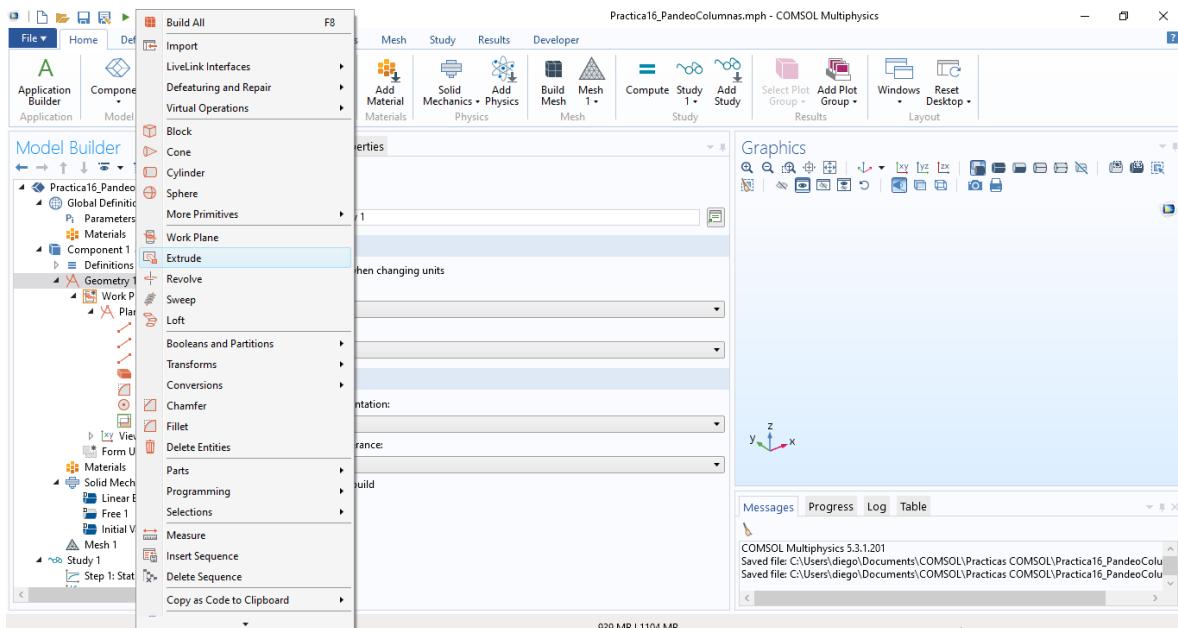


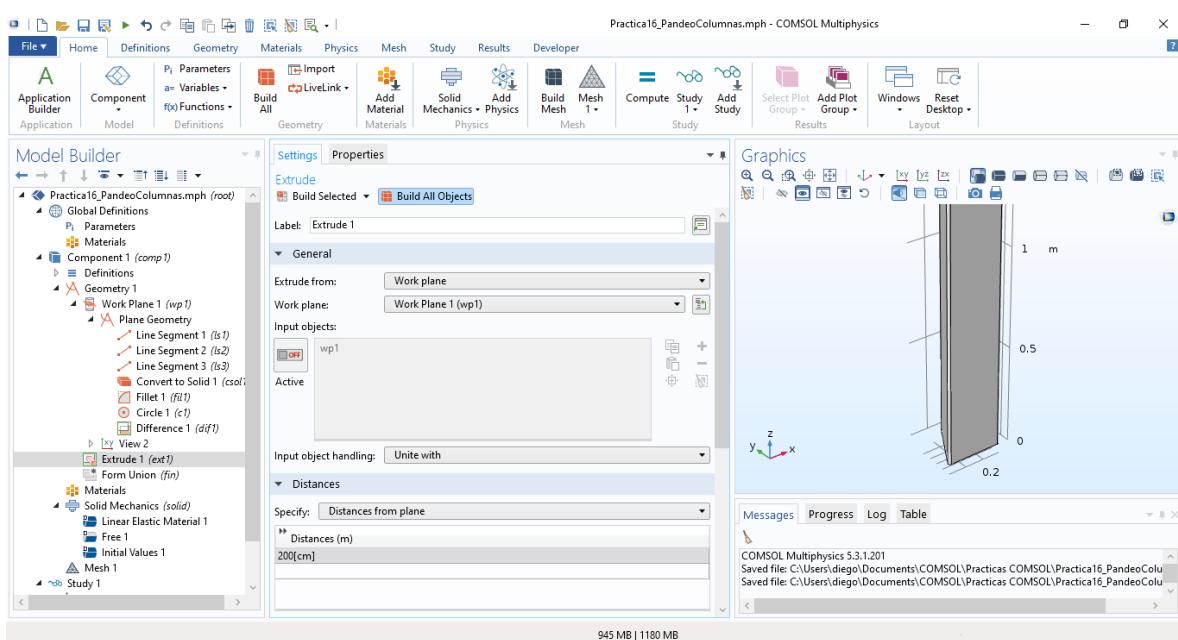
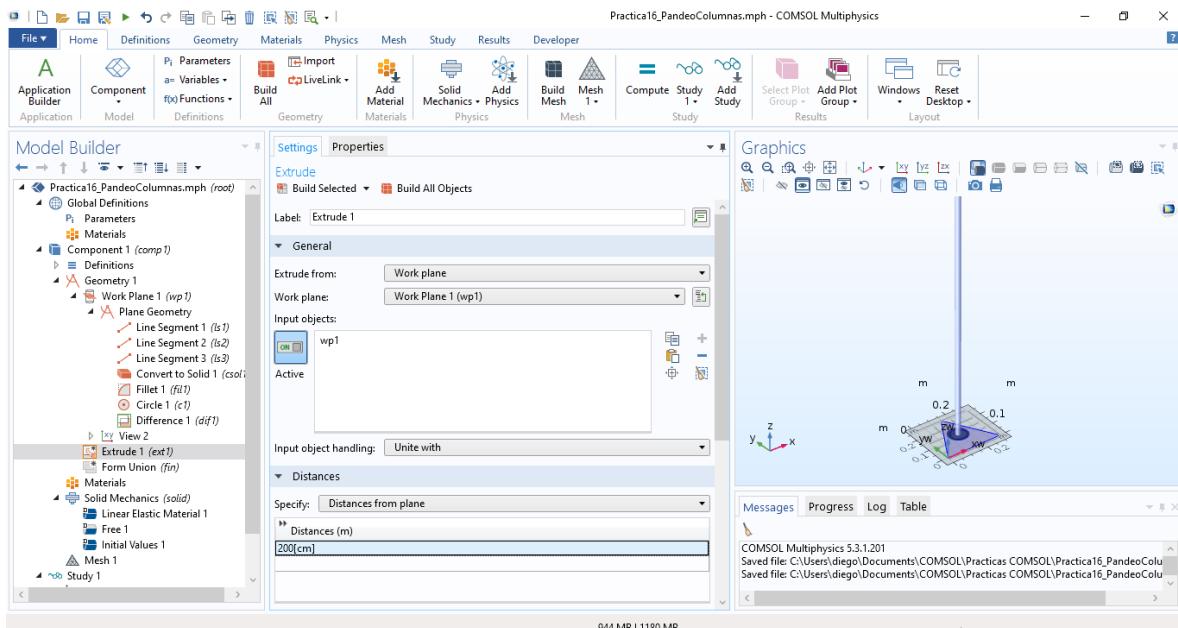


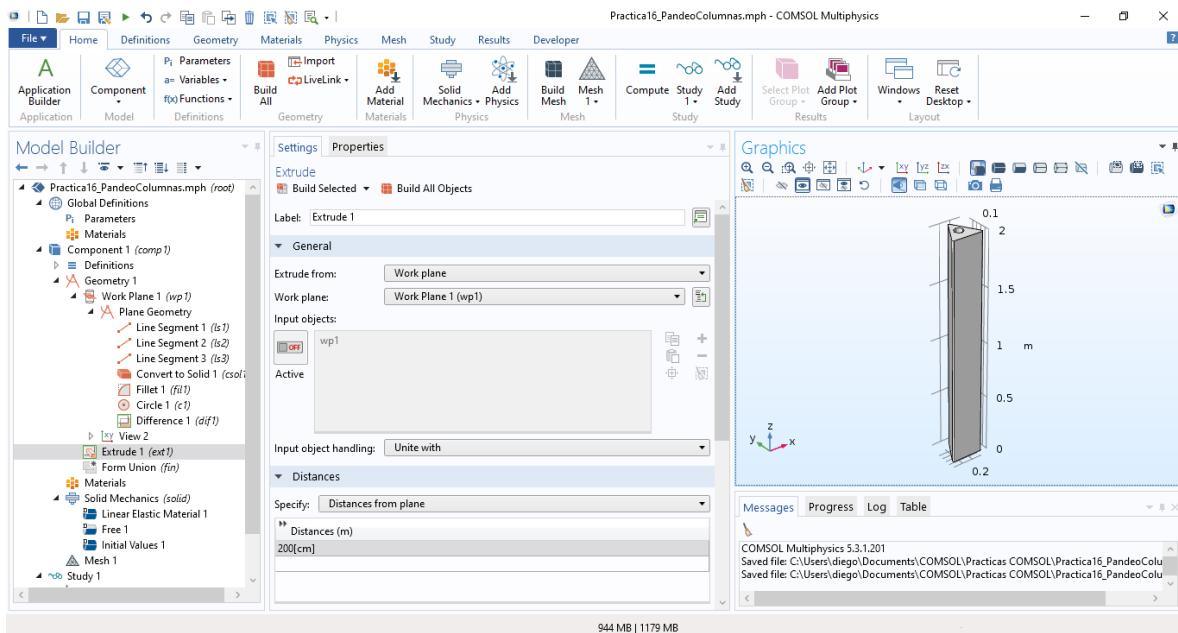
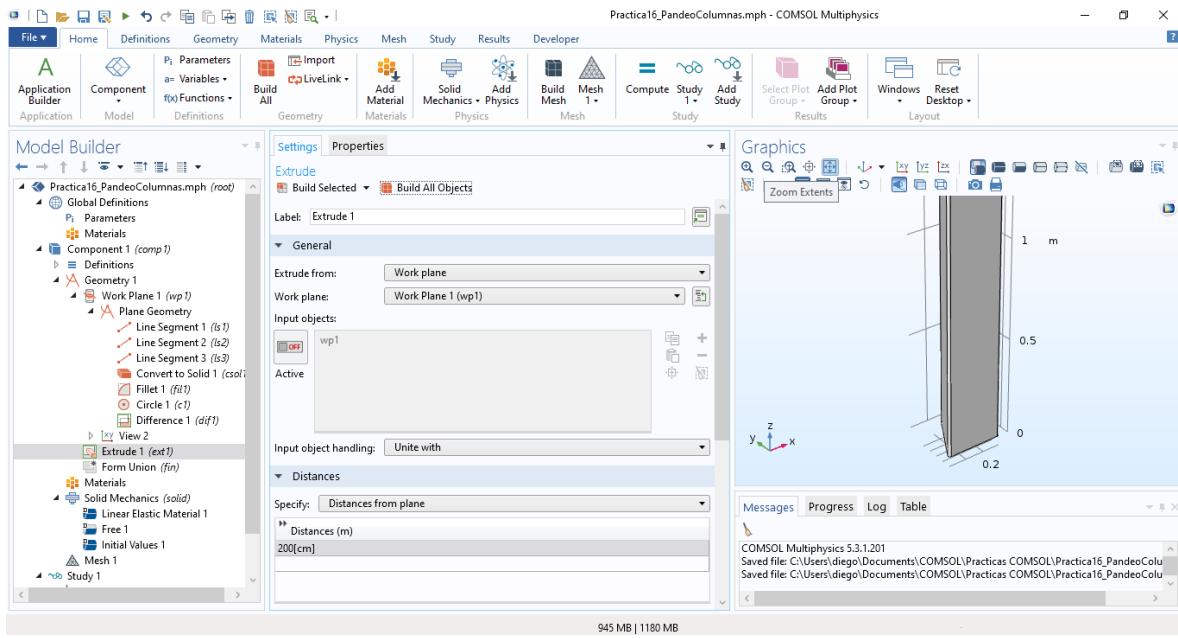


Ya no voy a estar en Plane Geometry porque ya no voy a hacer nada en este plano, ahora lo voy a extruir, por eso me voy a Geometry 1 que hace referencia al 3D.





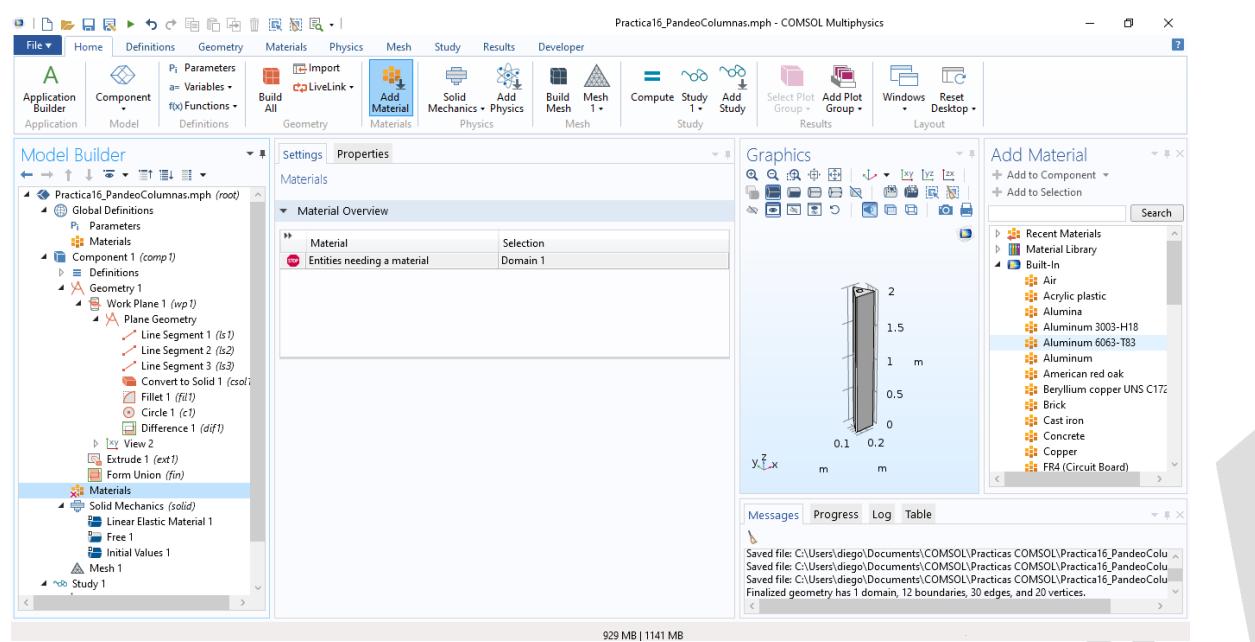
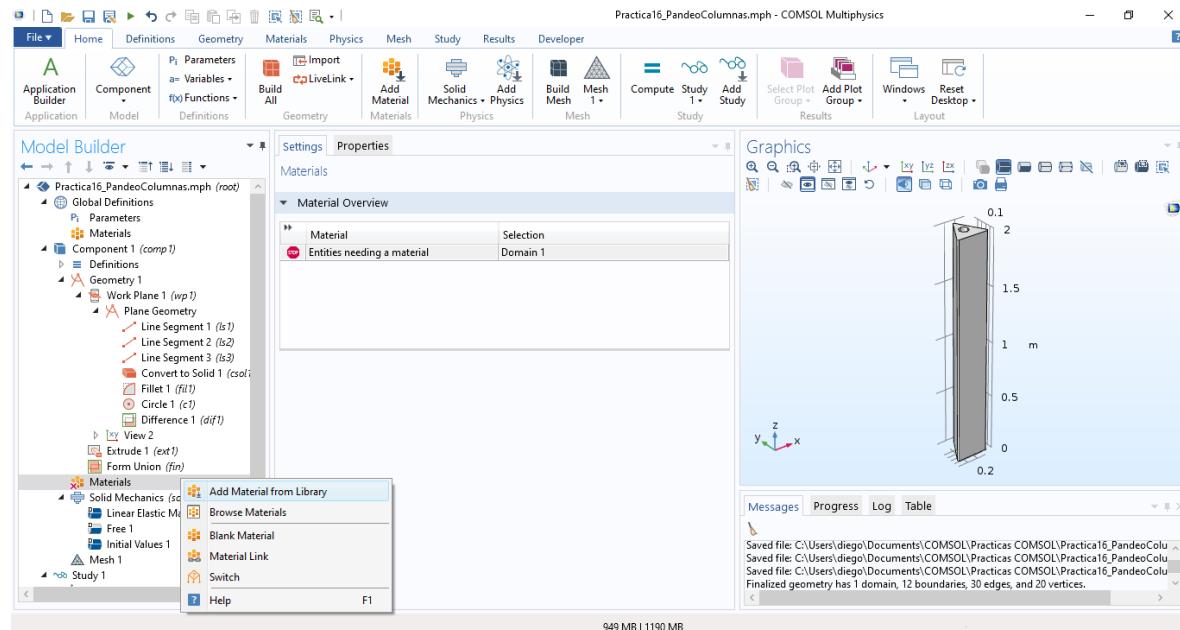


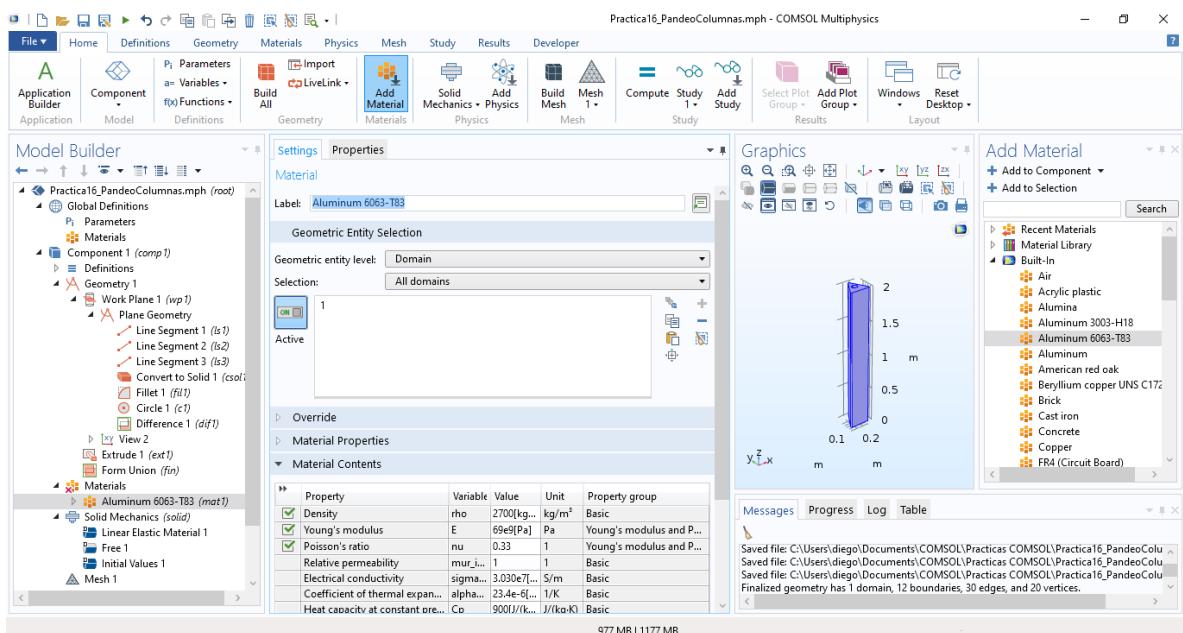
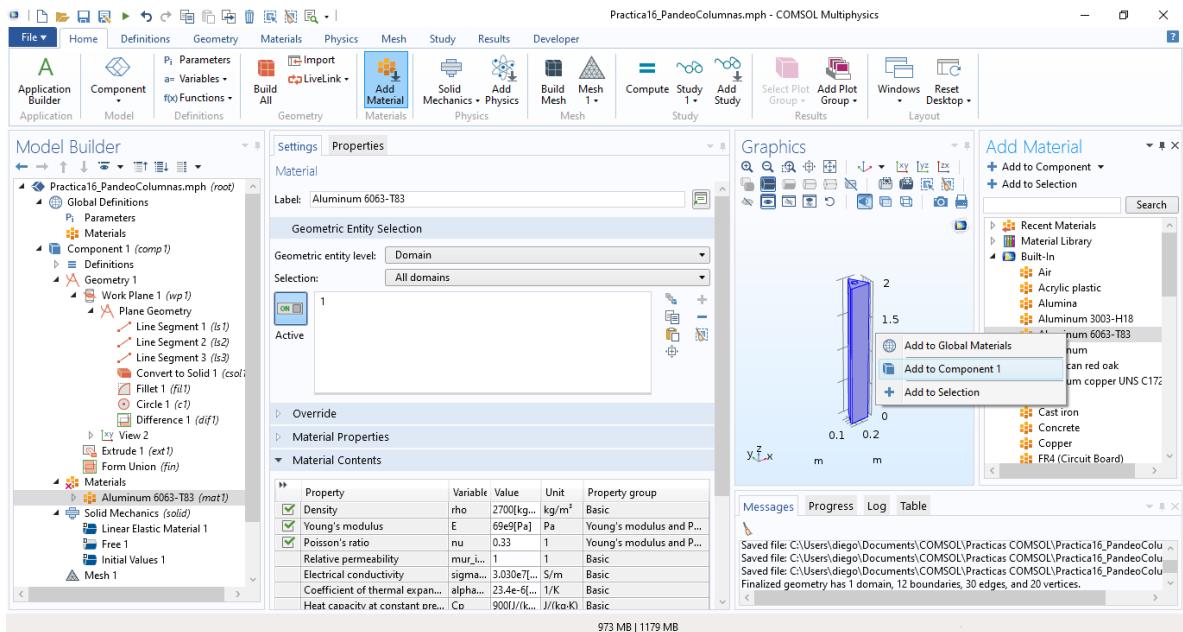


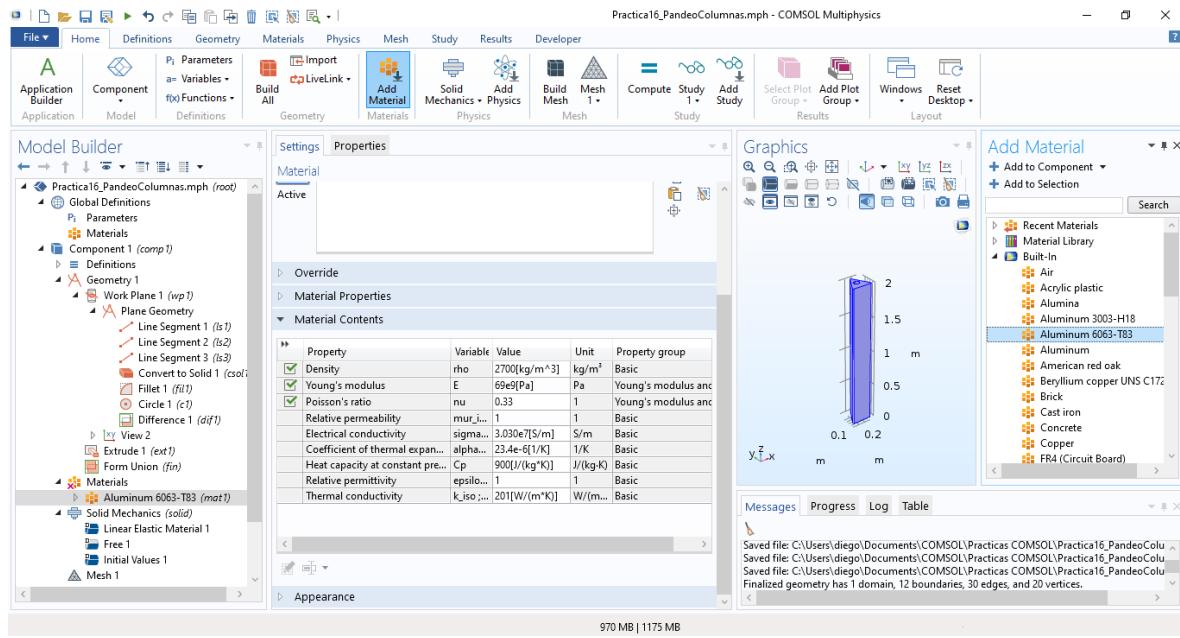
ANÁLISIS MECÁNICO EN COMSOL:

El material debe ser aluminio con tratamiento térmico y debe tener 100 Pascales de presión en la parte de arriba y la parte de abajo debe estar fija.

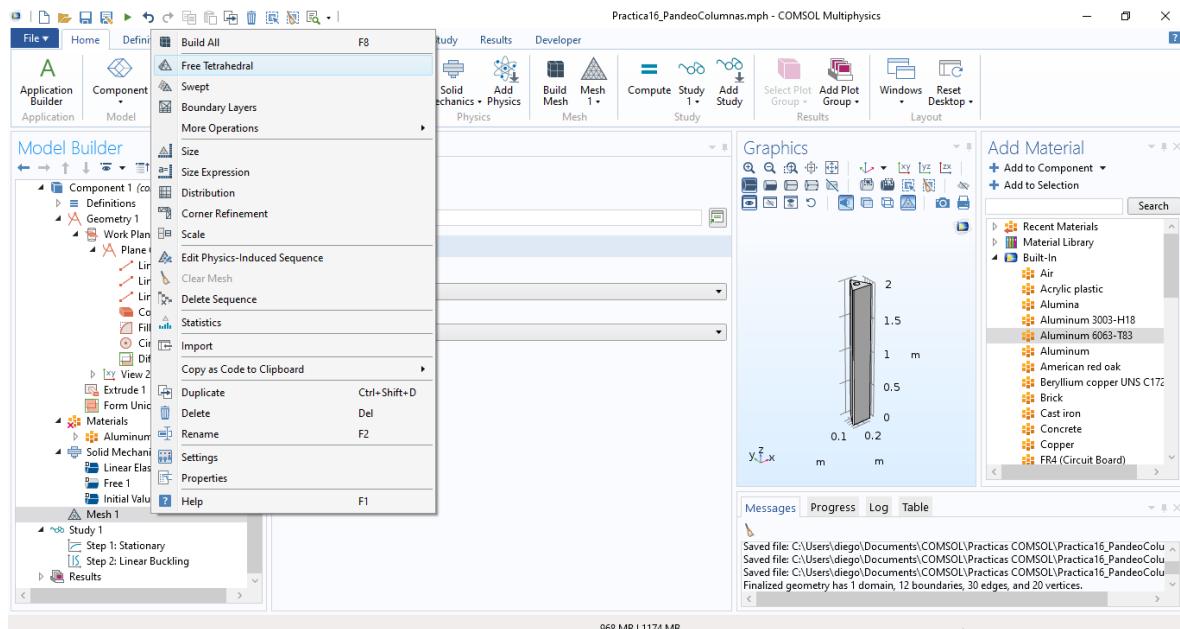
- El aluminio 6063 (se refiere a que es de aluminio).
- T83 (se refiere a que tiene un tratamiento térmico).

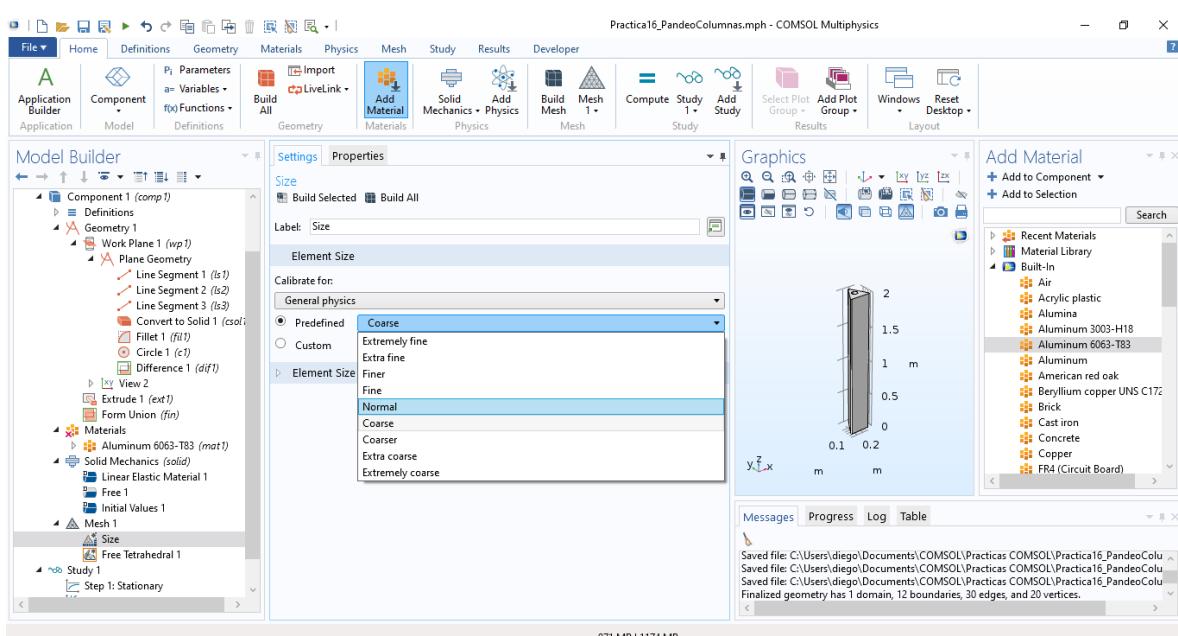
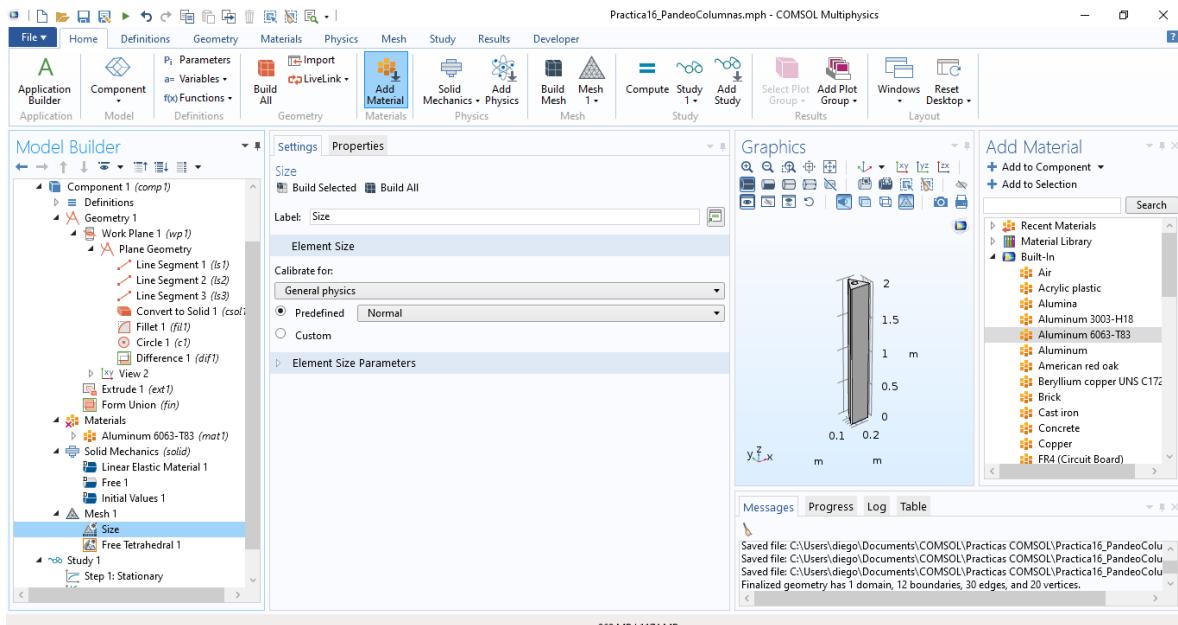


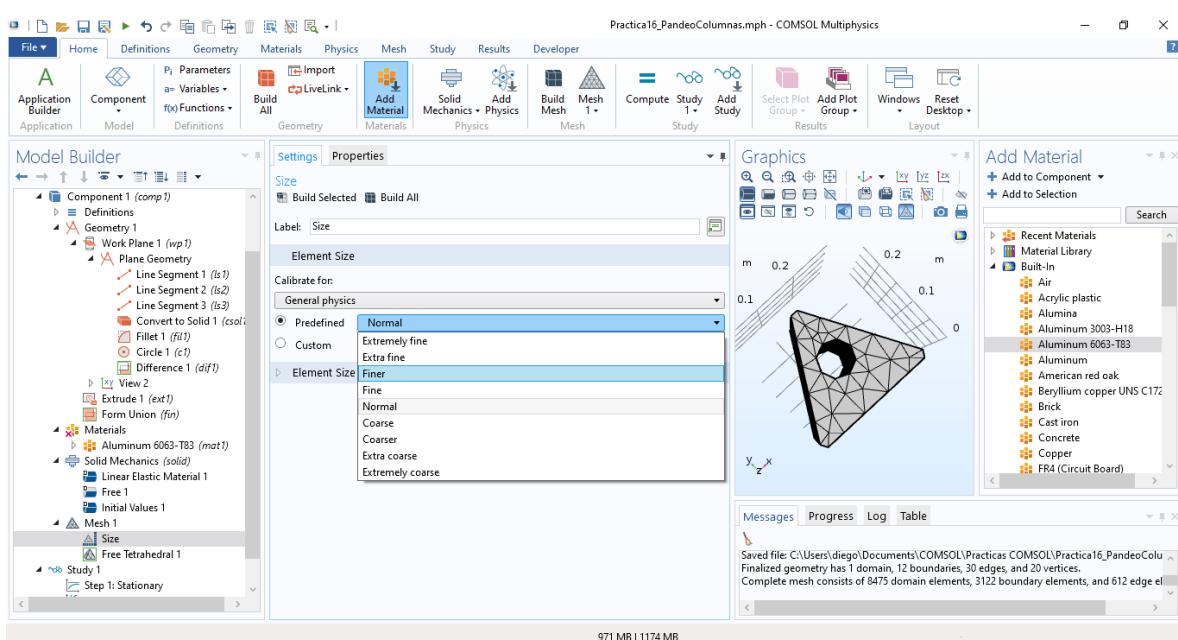
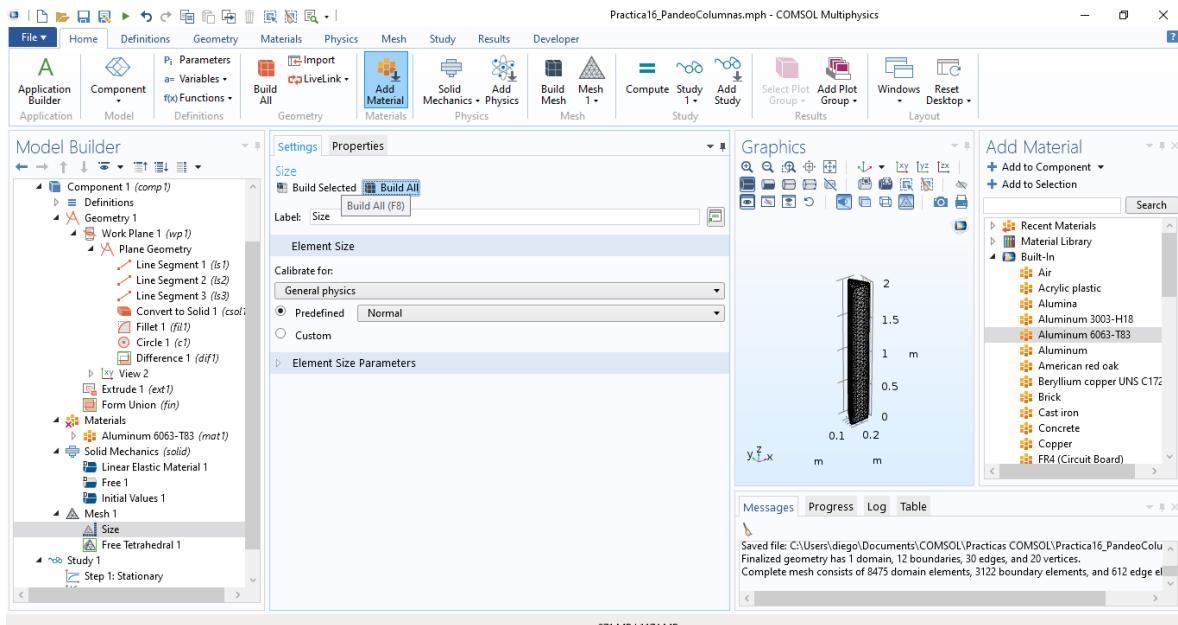


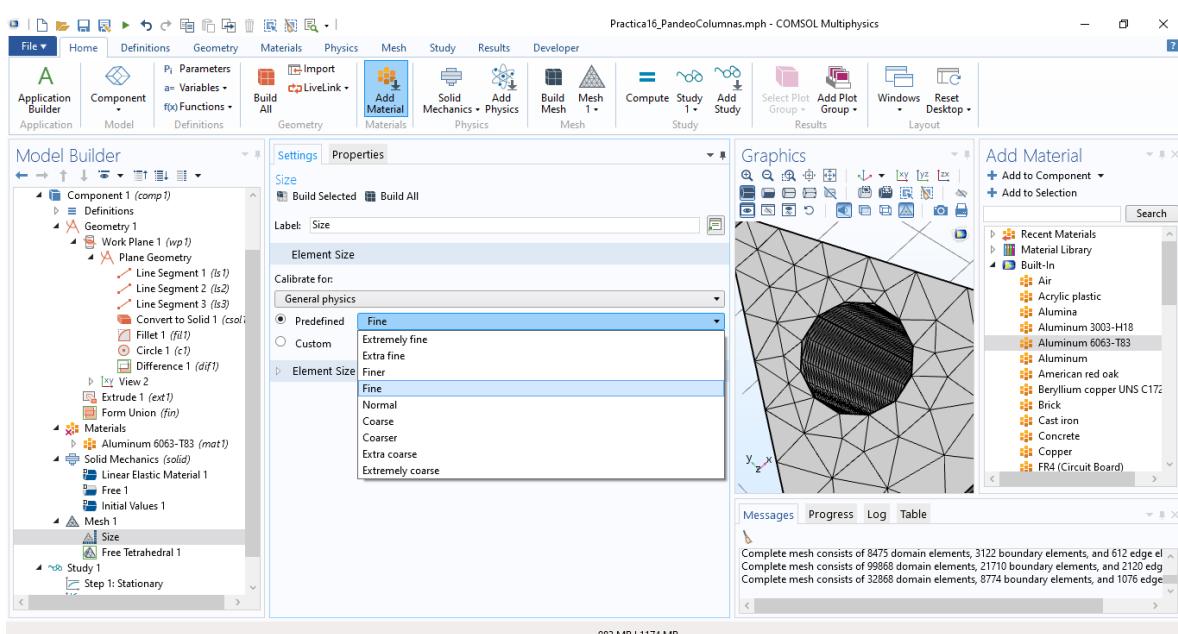
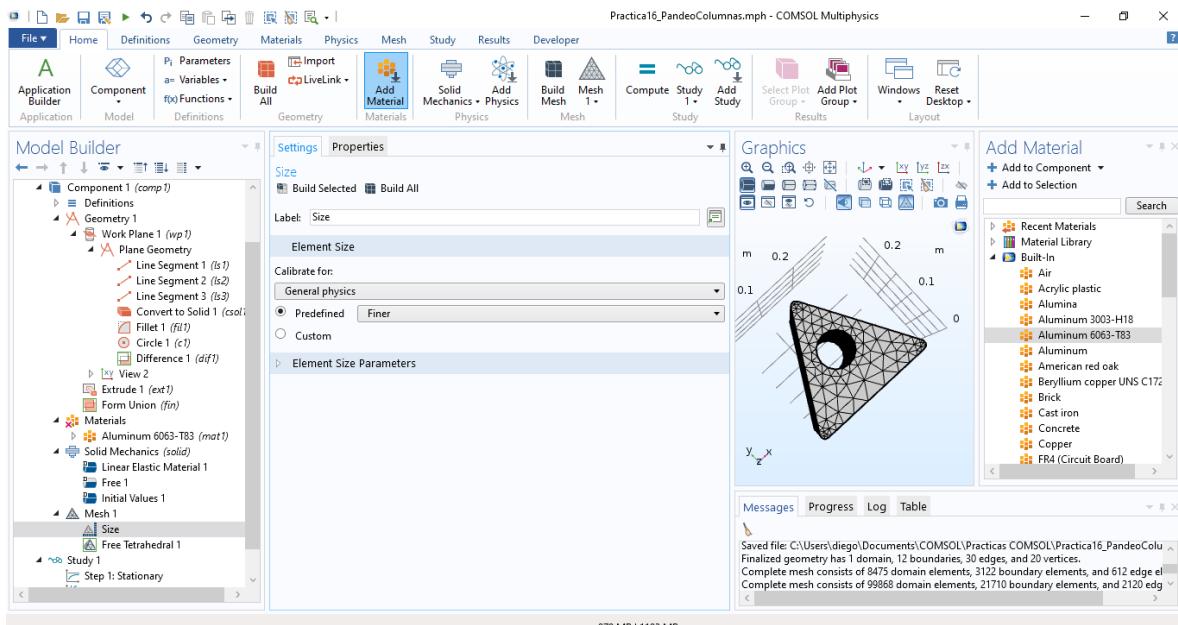


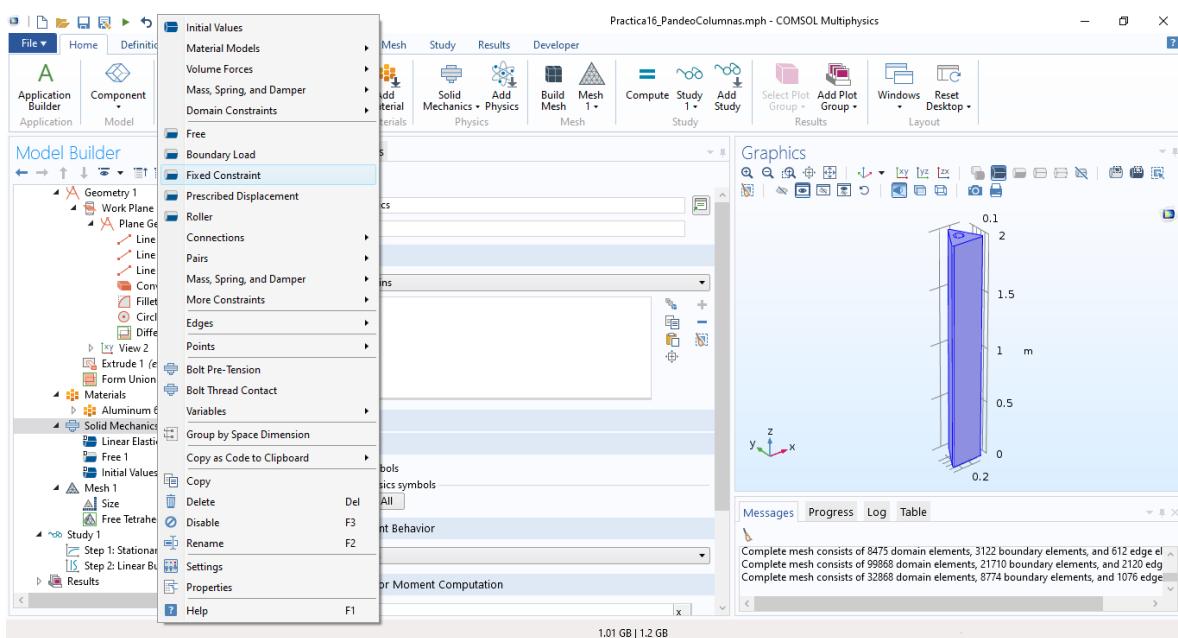
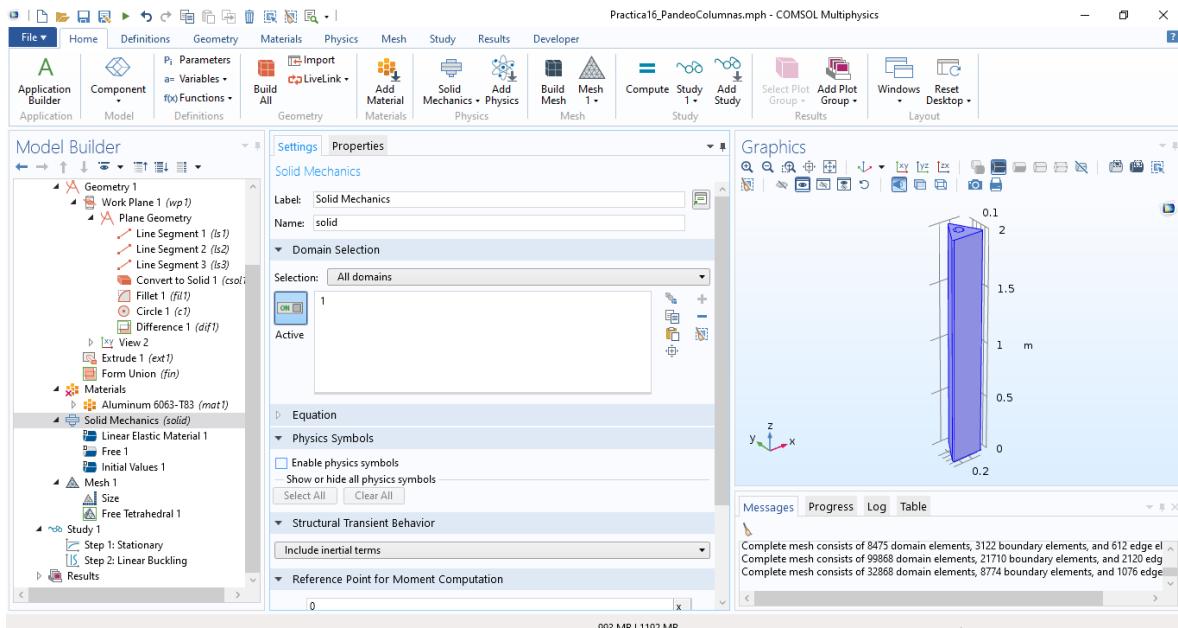
Si no se encuentra el material en la biblioteca, solo debo saber su coeficiente de elasticidad (E), su número de poisson y densidad.

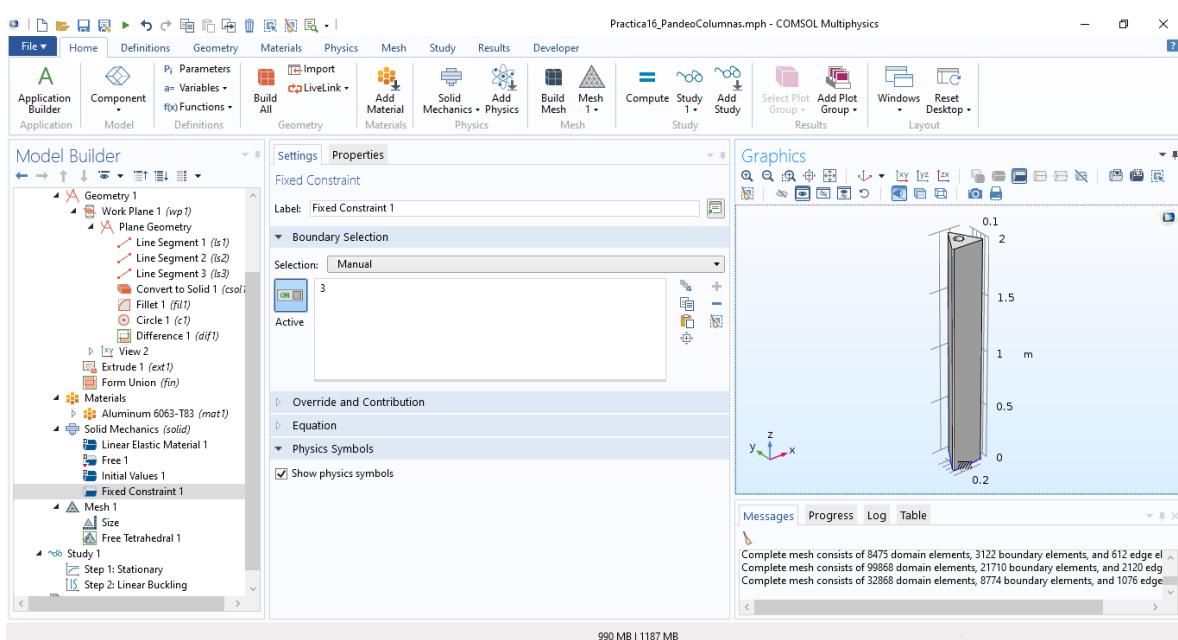
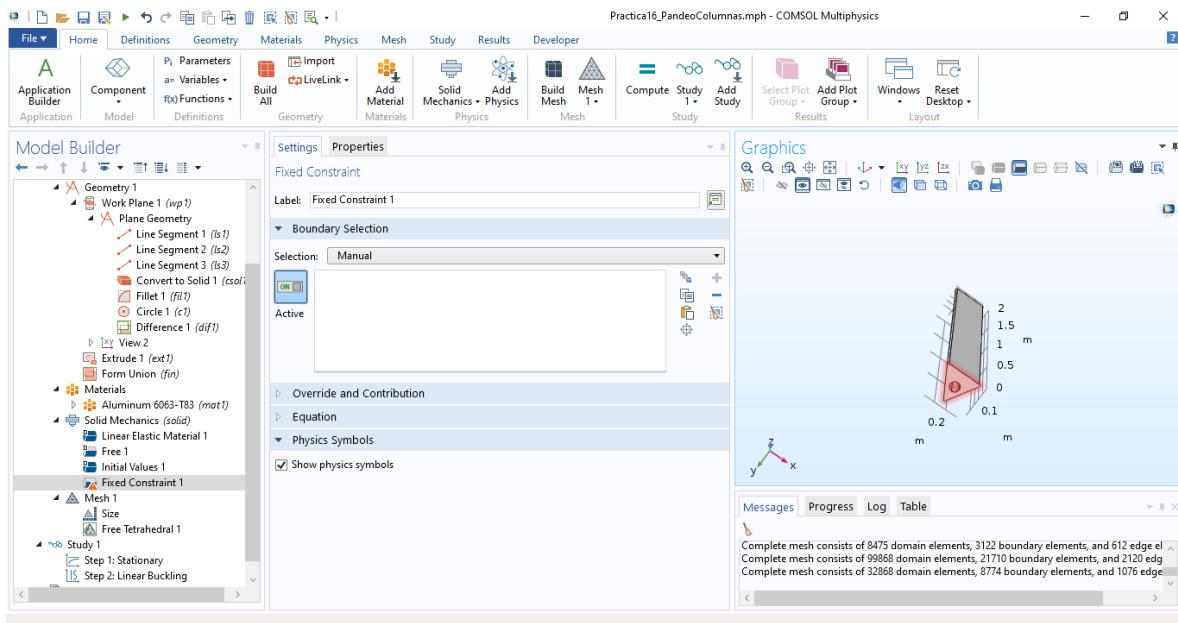


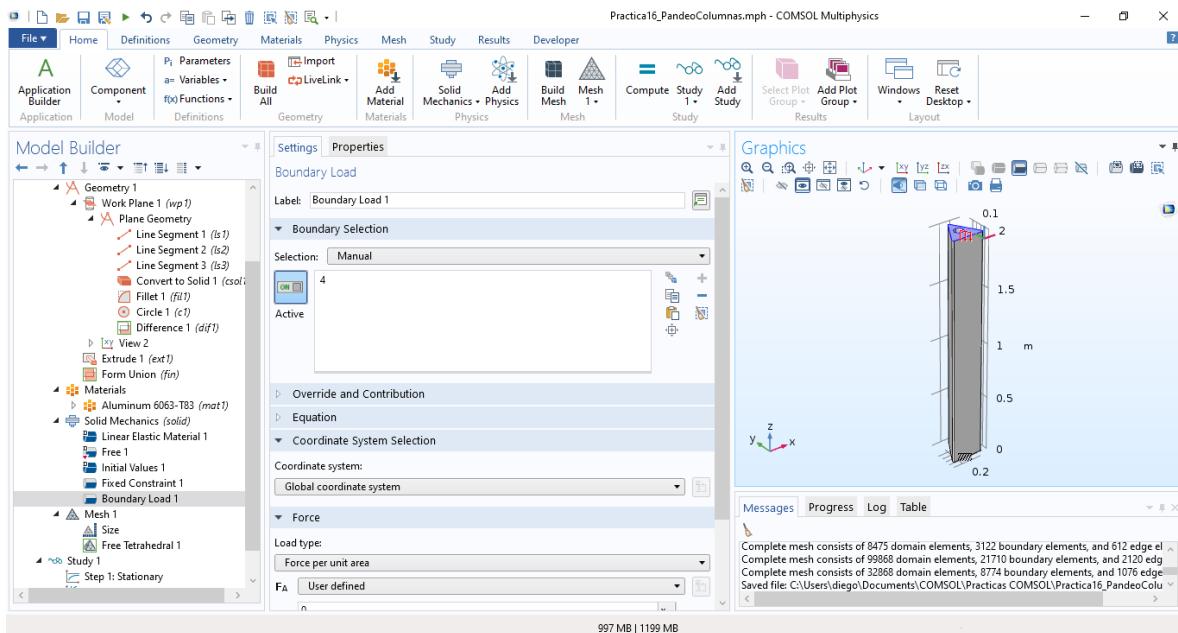
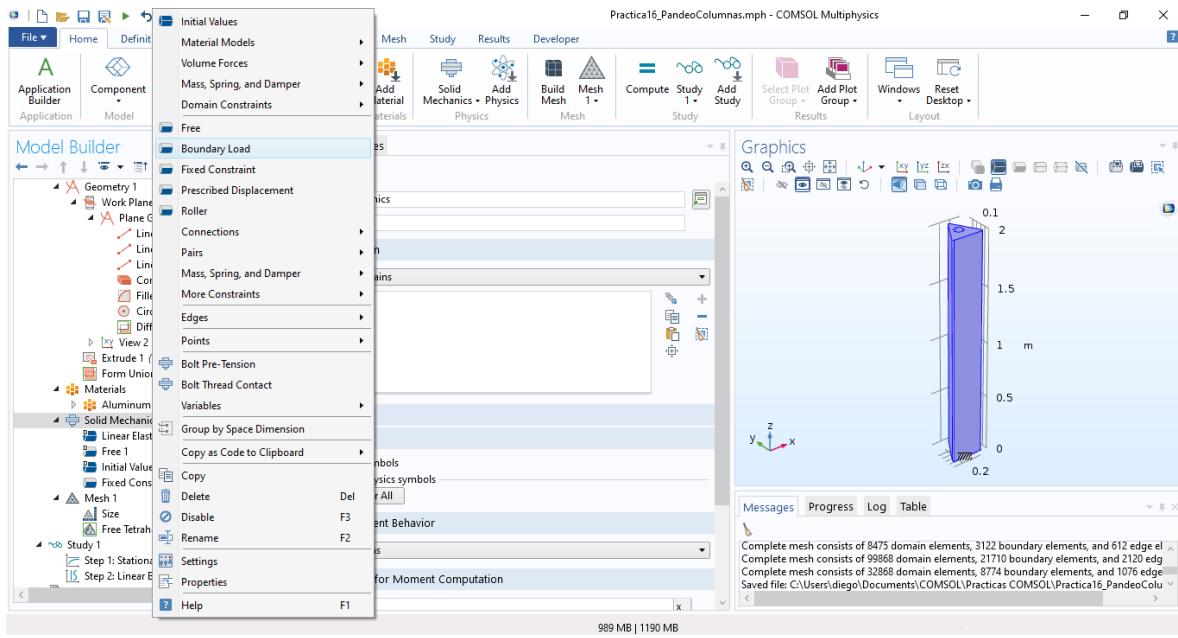


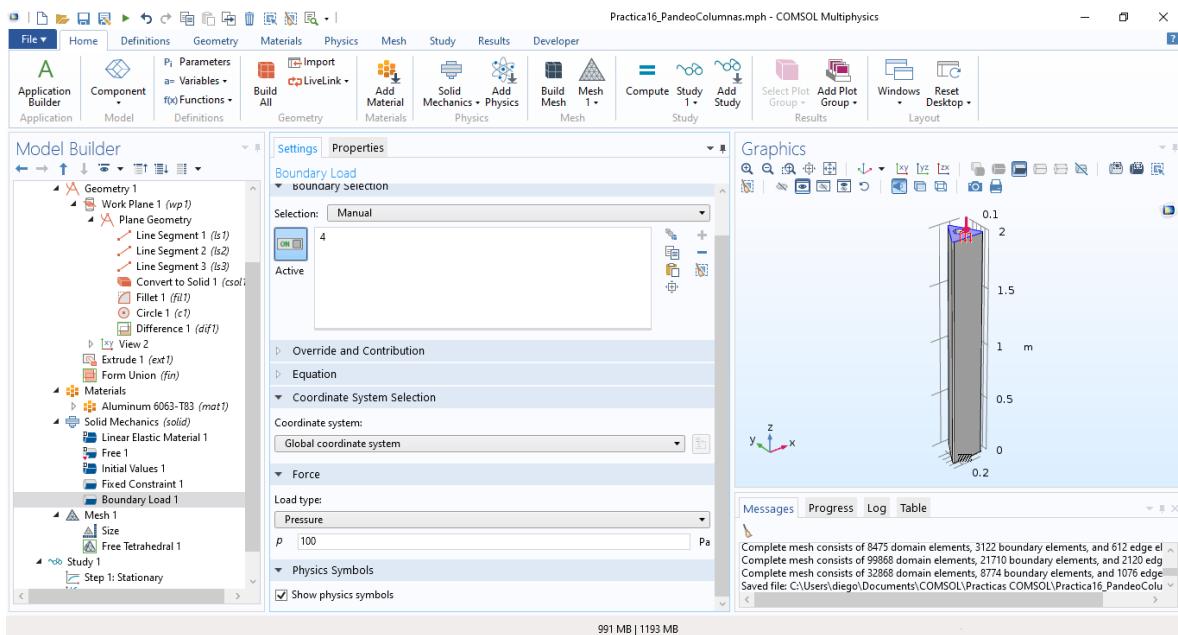
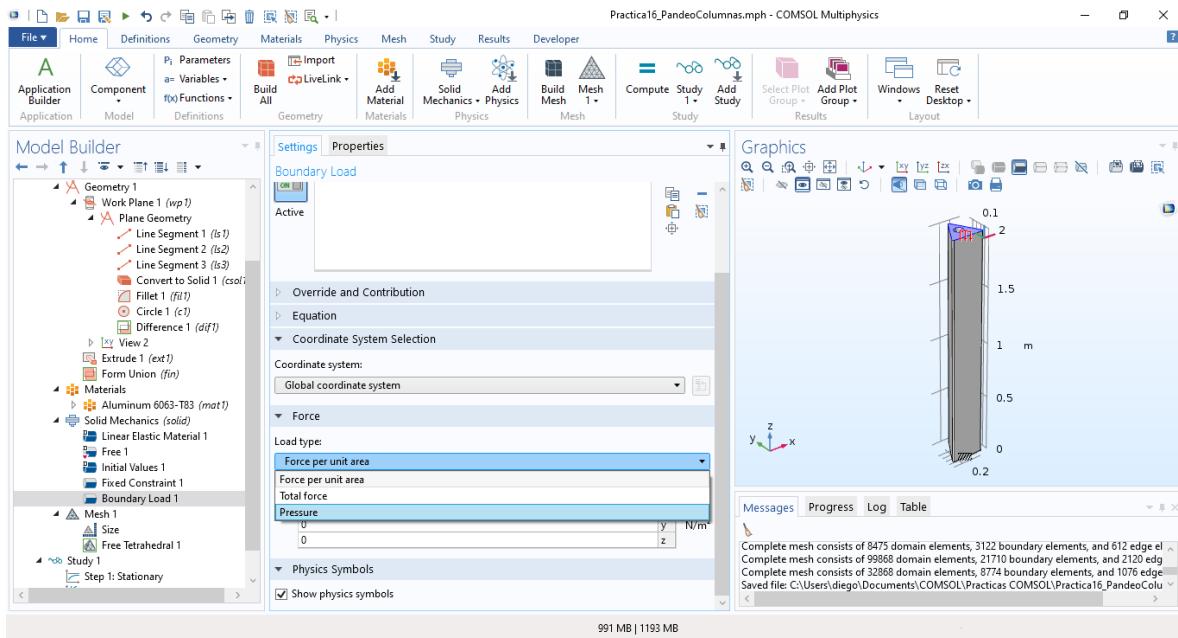




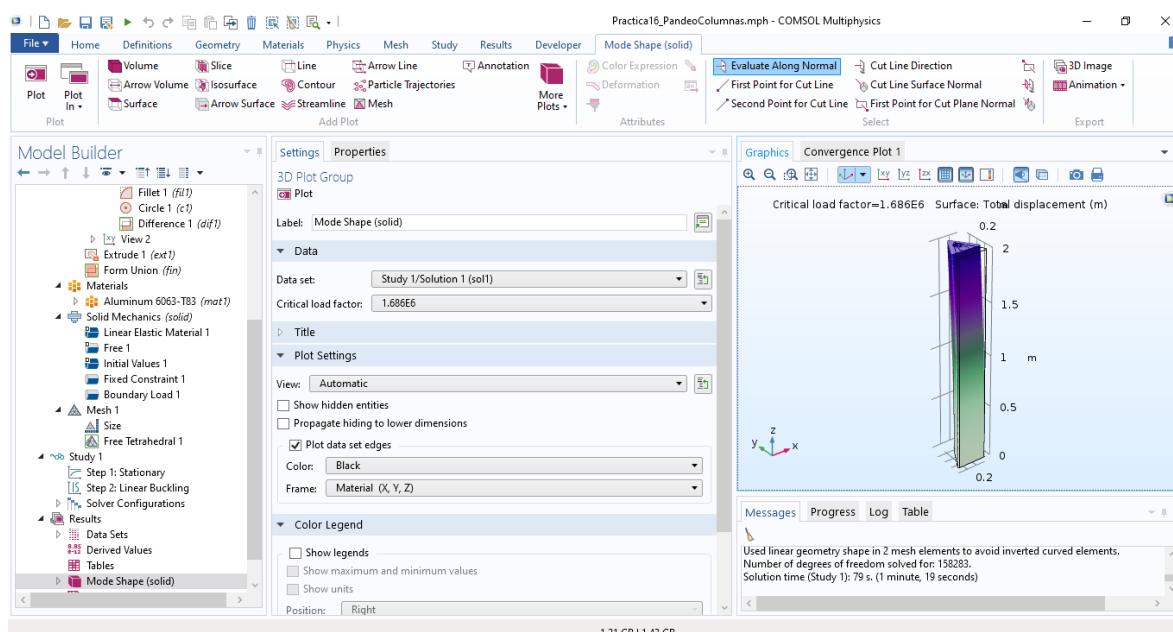
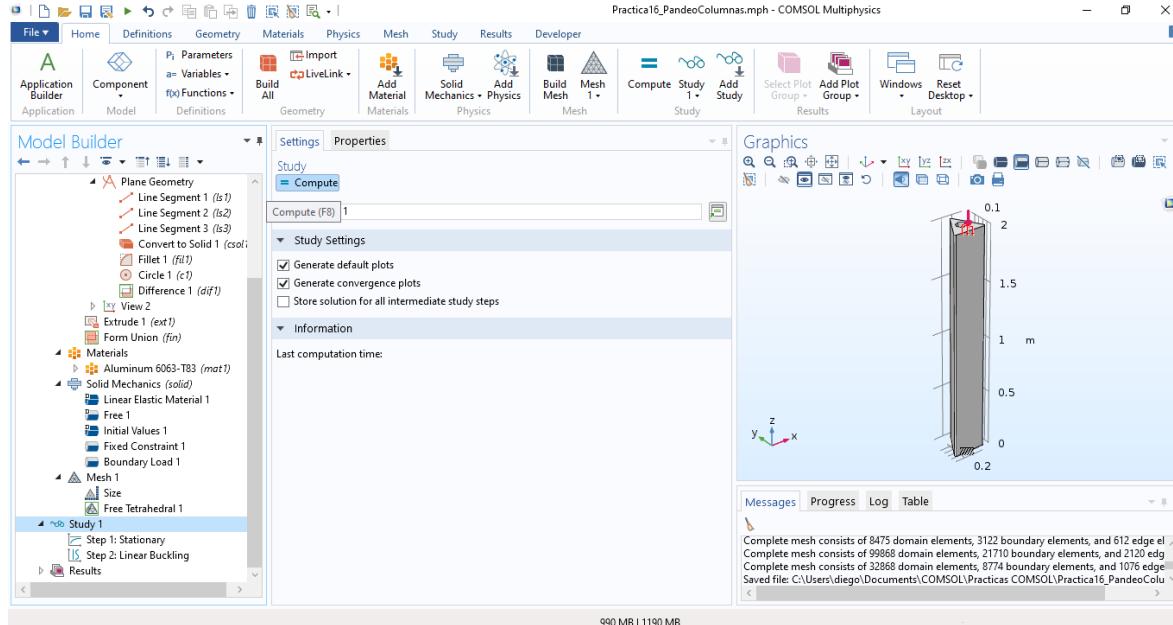


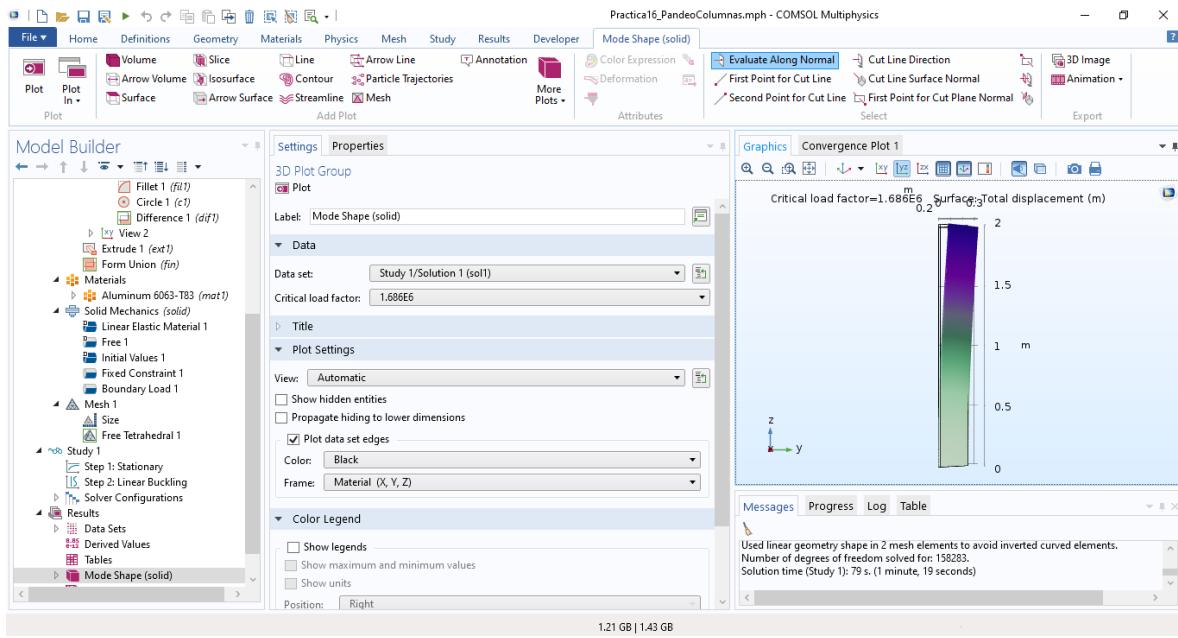




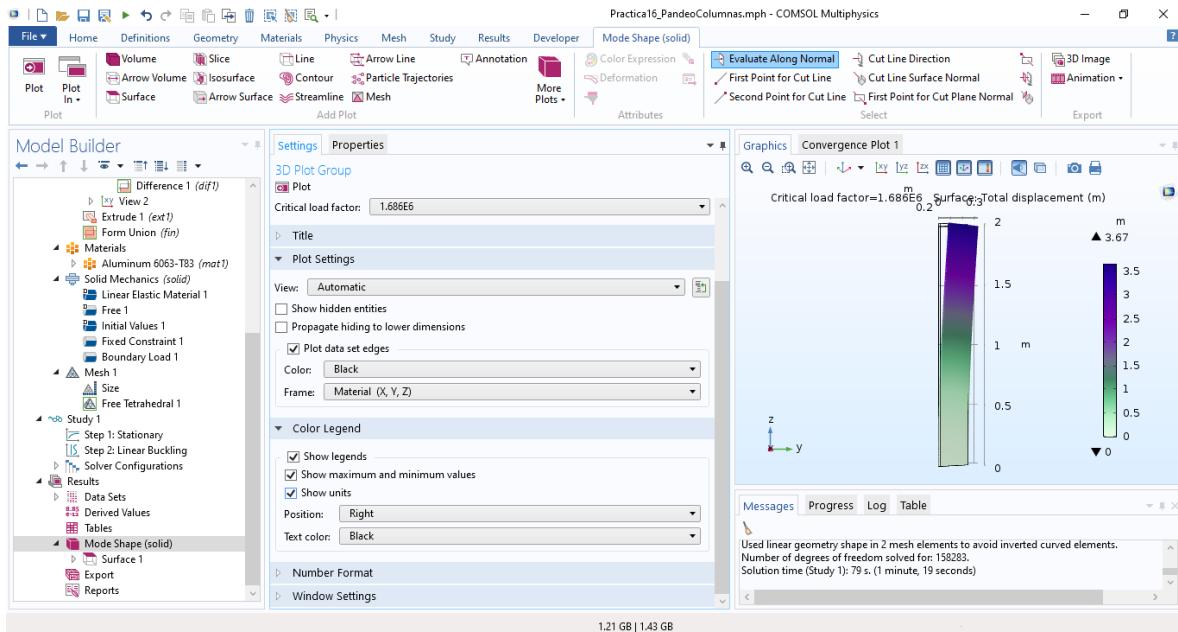


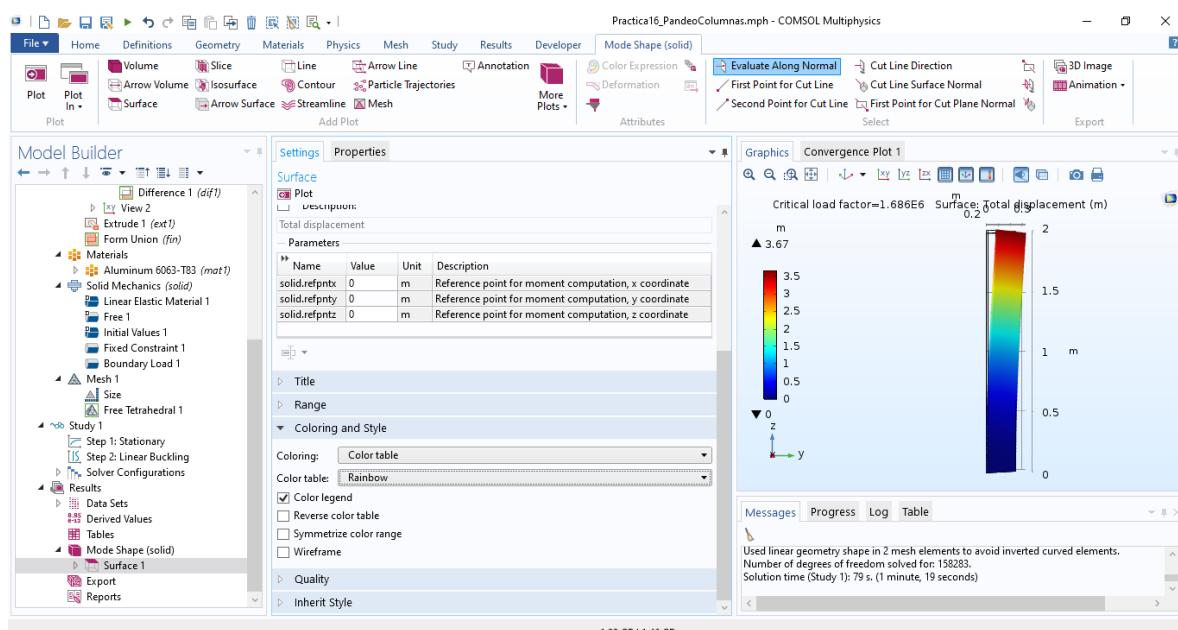
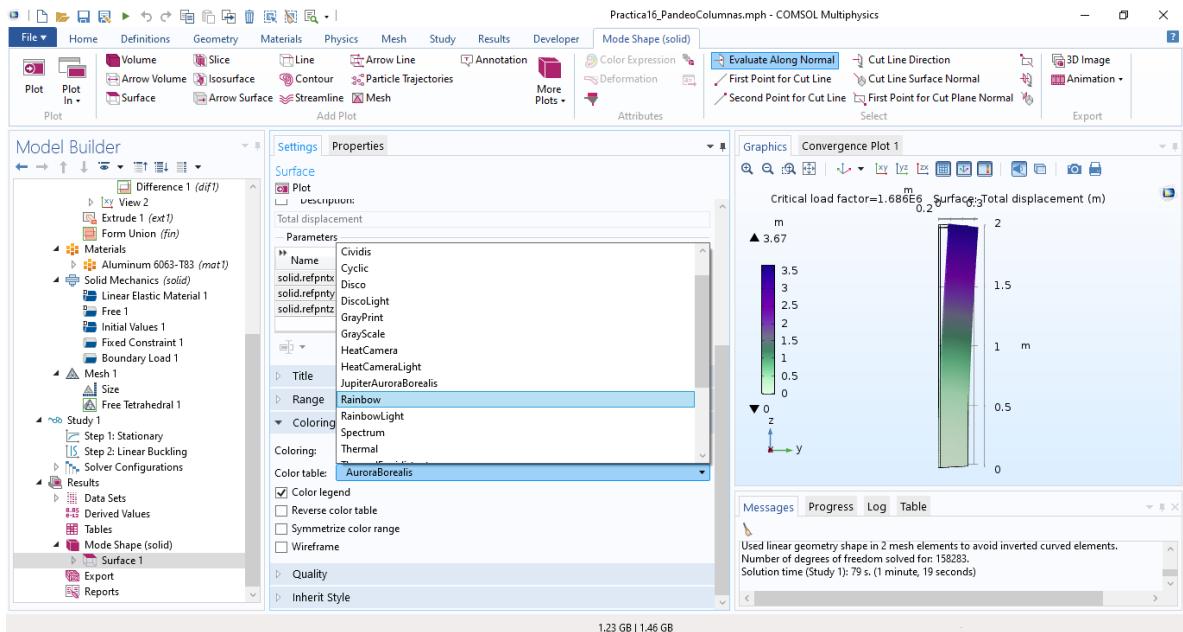
RESULTADO DEL ELEMENTO FINITO EN COMSOL:

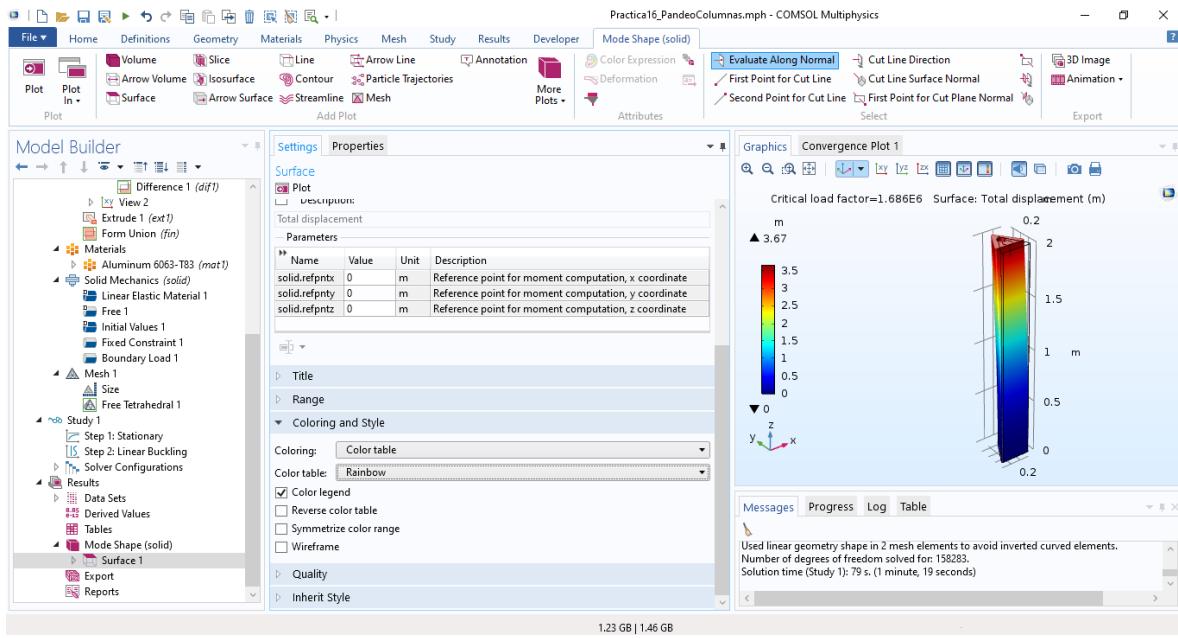




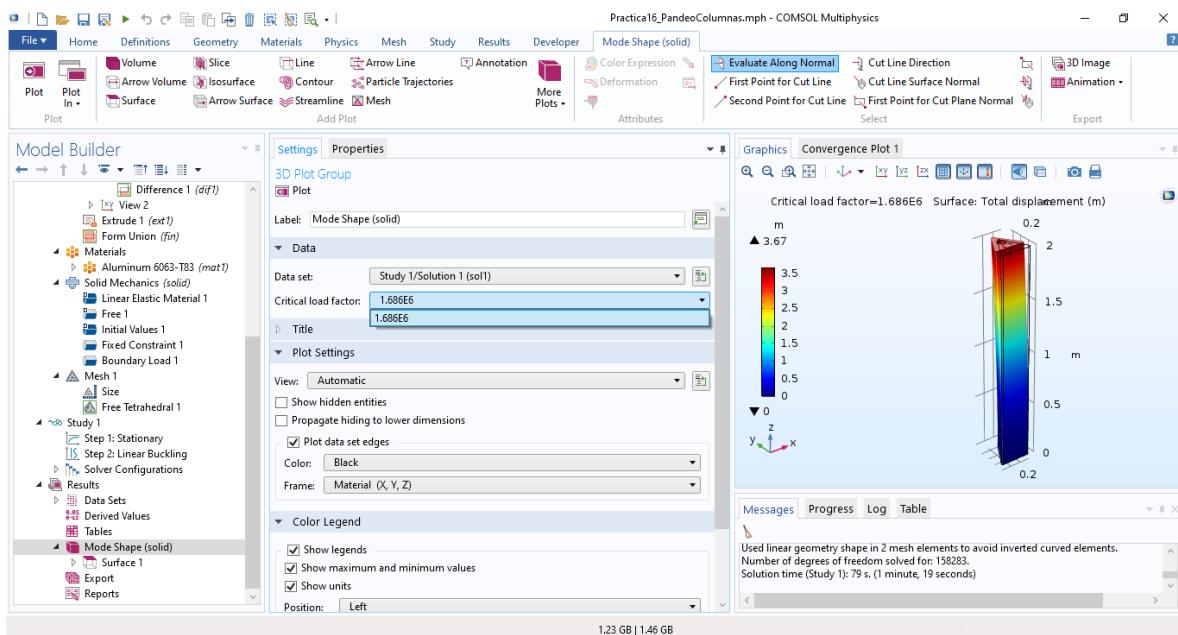
El problema de pandeo en columnas es de eigenvalores, por eso sale el resultado de Mode Shape en el programa.







Los valores de pandeo están exagerados, no puede ser que la barra sea de 2 metros y el pandeo sea de 3.67 metros



El factor de carga crítica es de $1.686e6 = 1.686 \times 10^6$, esto lo uso para multiplicarlo por la carga que está soportando la columna.

Presión crítica = Factor de carga crítica * carga en la columna = $100\text{Pa} \times 1.686e6 = 168.6 \text{ MPa}$.

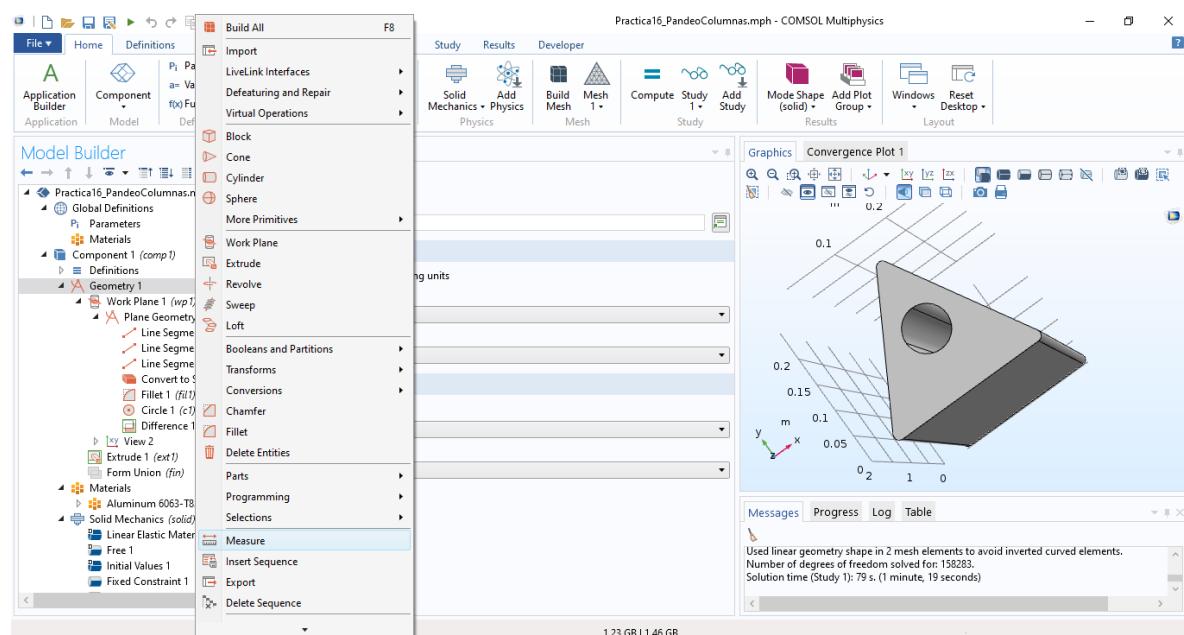
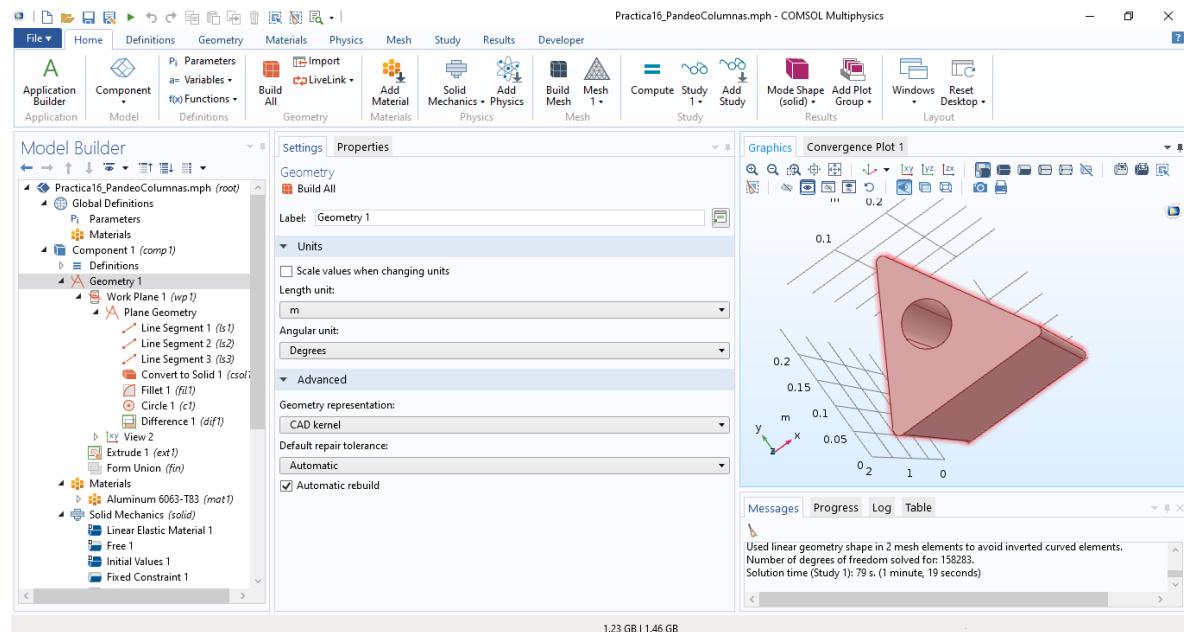
Pero como es una presión lo que tengo, lo obtenido es Presión Crítica.

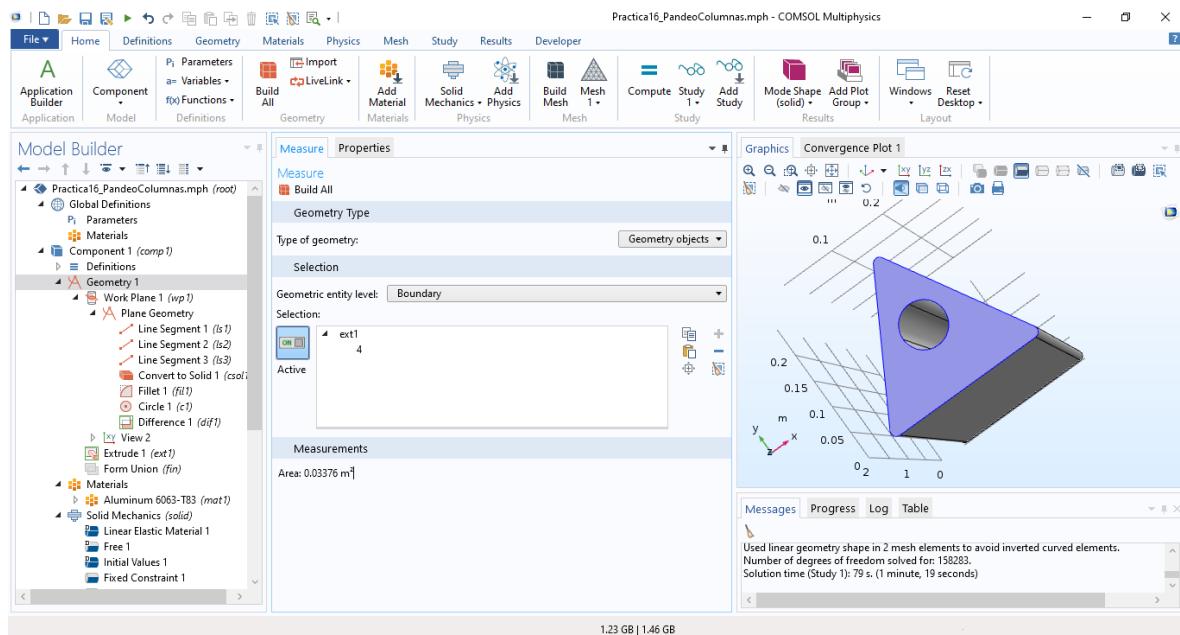
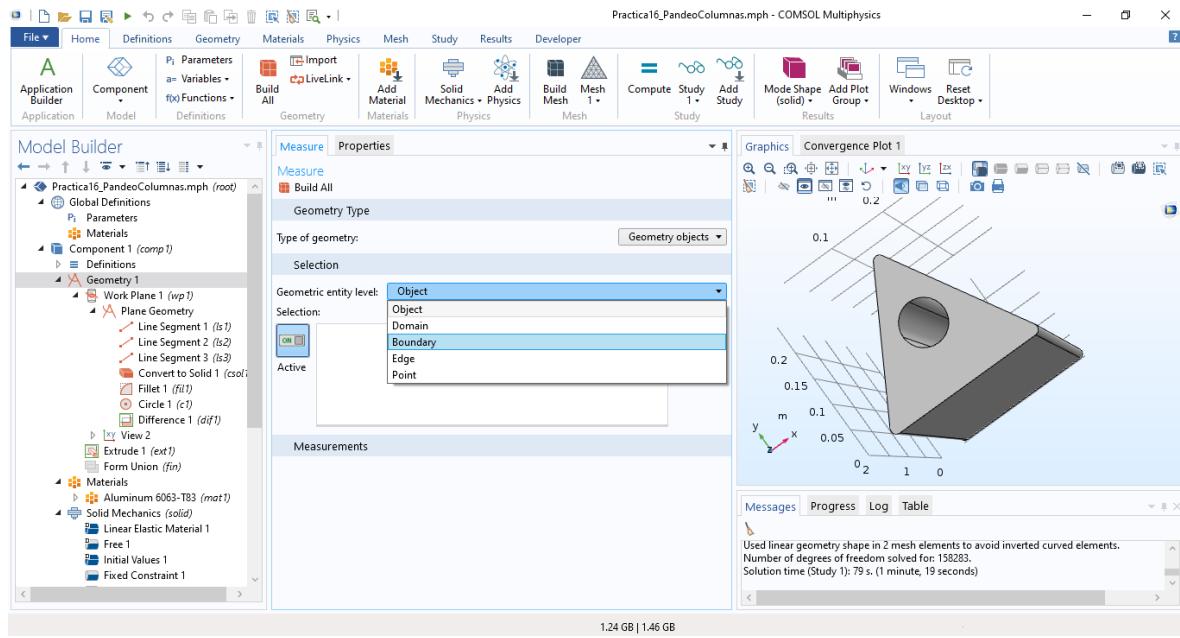
$P_c = 168.6 \text{ MPa}$.

Carga crítica = $C_c = P$ (Área de sección transversal) = $168.6e6/A$



Para medir el área puedo usar el programa.





BIBLIOGRAFÍA:

INGENIERÍA MECÁNICA ESTÁTICA (12VA EDICIÓN) – RUSSELL C. HIBBELER.



CONCLUSIÓN:

Área que se busca = 0.03376 m²

$$\text{Carga crítica} = C_c = P \left(\text{Área de sección transversal} \right) = 168.6e6 \left[\frac{N}{m^2} \right] * 0.03376 [m^2] = 5.691936 e6 [N]$$

No es un hecho que esta sea la carga crítica, para que esto salga es mejor. Para que se pandee críticamente se debe aplicar la siguiente carga a la columna.

Fórmula de Euler:

$$F_{cr} = \frac{\pi^2}{L^2} * E * I$$

$$F_{crx} = \frac{\pi^2}{L^2} * E * I_x = \frac{\pi^2}{L^2} * E * \frac{b * h^3}{36}$$

$$F_{crx} = \frac{\pi^2}{2[m]^2} * 69e9 \left[\frac{N}{m^2} \right] * \frac{0.3[m] * 0.2598[m]^3}{36} = 24.87 [\text{Mega Newtons}]$$

$$Fc_{ry} = \frac{\pi^2}{L^2} * E * I_y = \frac{\pi^2}{L^2} * E * \frac{h * b^3}{48}$$

$$Fc_{ry} = \frac{\pi^2}{2[m]^2} * 69e9 \left[\frac{N}{m^2} \right] * \frac{0.2598[m] * 0.3[m]^3}{48} = 24.88 [\text{Mega Newtons}]$$

