

# High-Cycle Fatigue Analysis of a Cylindrical Test Specimen

This is a benchmark model for the Fatigue Module. A cylindrical test specimen is subjected to nonproportional loading. Three stress based models — Findley, Matake, and Normal stress — are compared to analytical values and to each other. The non-smooth behavior of the Matake model is captured and discussed.

# Model Definition

A cylindrical test specimen with a geometry according to Figure 1 is subjected to fatigue testing. A combination of axial force and twisting moment are applied in such a way that the stress scenario shown in Figure 2 is affecting in the central thin part of the specimen. The maximum shear stress is experienced on the outer radius of the bar.

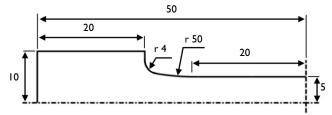


Figure 1: Geometry of the test specimen.

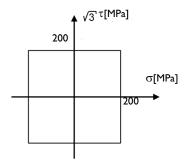


Figure 2: Stress history at the outer radius in the central part of the specimen.

This stress state can be simulated with cyclic normal force of 15708 N and cyclic twisting moment of 22.672 Nm.

The specimen is made of mild steel with Young's modulus *E* of 210 GPa and a Poisson's ratio v of 0.3.

The examined fatigue criteria are Findley, Matake, and Normal stress. Material parameters are calculated from uniaxial fatigue tests. Results of two tests are available.

In the first test, the material is tested in reversed tension-compression, which means that the stress amplitude is alternating around zero stress, and the endurance limit is found to be  $\sigma_{R=-1} = 350$  MPa.

In the second test, the material is tested in pure tension, where the stress is pulsating between zero to two times the stress amplitude. The endurance limit is then  $\sigma_{R=0}$  = 288 MPa. The subscript of the endurance limit shows the R-value of the test.

Findley parameters are related to the fatigue test data via

$$f_{\rm F} = \frac{\sigma_{R=-1}}{2} (k_{\rm F} + \sqrt{1 + k_{\rm F}^2})$$
$$f_{\rm F} = \frac{\sigma_{R=0}}{2} (2k_{\rm F} + \sqrt{1 + 4k_{\rm F}^2})$$

and can thus be computed as  $f_{
m F}$  = 213 MPa and  $k_{
m F}$  = 0.20.

Matake parameters are related to the fatigue test data via

$$f_{\rm M} = \frac{\sigma_{R=-1}}{2} (1 + k_{\rm M})$$

$$f_{\rm M}=\frac{\sigma_{R=0}}{2}(1+2k_{\rm M})$$

which gives  $f_{\rm M}$  = 223 MPa and  $k_{\rm F}$  = 0.27. In the normal stress model, the stress limit equals twice the stress amplitude of the fatigue test. The model does not take into account the R-value dependency, and therefore two different limits are obtained. In order to give conservative results, the lower limit,  $\sigma_{R=0}$  is chosen and thus the model parameter is  $f_N$ = 576 MPa. This actually indicates that normal stress may not be a good criterion for this material.

The load cycle is obtained in four load steps. The resulting stress at the outer radius of the thin section follows the specification, see Figure 3.

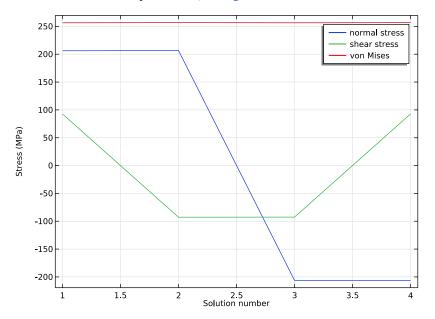


Figure 3: Resulting stresses at the outer radius of the smallest section of the specimen.

The three fatigue criteria evaluated on the boundary of the specimen are presented in Figure 4, Figure 5, and Figure 6.

In this example, it is the central part of the specimen which is in focus for the evaluation, even though the maximum fatigue usage is slightly higher in the fillets.

The Findley model shows that the highest fatigue usage factor is found at the transition between the thin cylinder and the fillet, with the value being 1.00 against 0.95 in the

center. A smooth transition in the results indicates that the specified search resolution for the critical plane is sufficient to correctly capture the fatigue response.

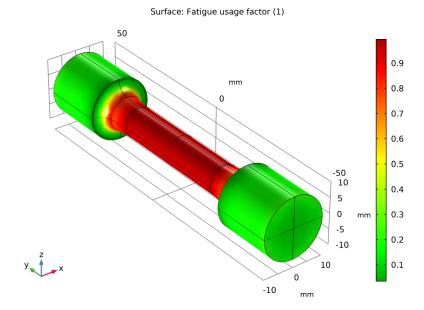


Figure 4: Fatigue usage factor Findley criterion.

The Matake criterion gives less smooth results. The reason can be found in the definition of the criterion, which selects the critical plane using the shear stresses only and then adds the normal stress. This is discussed in more detail on page 10. The analysis of the test

specimen predicts a fatigue usage factor in the range from 0.79 to 0.99 in the central part, as compared to 1.04 at some points at the transition from the thin cylinder to the notch.

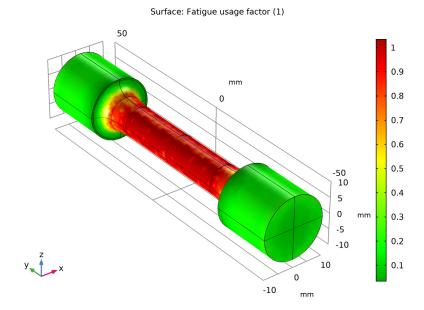


Figure 5: Fatigue usage factor Matake criterion.

The Normal Stress criterion predicts yet another fatigue usage factor. Again, the highest fatigue usage factor is found on the transition from the thin section to the notch, 0.94, as compared to the center of the specimen, where it is 0.88.

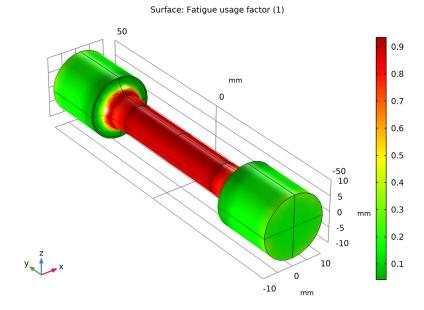


Figure 6: Fatigue usage factor Normal stress criterion.

The stress state on the surface in the central part of the specimen can be evaluated using analytical expressions. As there are no tractions on the boundary, it is in a state of plane stress. Therefore, the search for the critical plane can be reduced to two dimensions. In that case, the normal and shear stresses on a plane can be obtained as

$$\begin{split} \sigma(\phi) &= \sigma_a \cos \phi^2 + \sigma_b \sin \phi^2 + \tau_{ab} \sin \phi \cos \phi \\ \tau(\phi) &= \frac{\sigma_b - \sigma_a}{2} \sin 2\phi + \tau_{ab} \cos 2\phi \end{split} \tag{1}$$

where  $\sigma_a$  and  $\sigma_b$  are orthogonal normal stresses, and  $\tau_{ab}$  is the shear stress. The orientation angle  $\varphi$  is calculated from the axis of  $\sigma_a$  towards  $\sigma_b$ . In the current example, the  $\sigma_a$  is the axial stress, and  $\tau_{ab}$  as the shear stress caused by the twisting moment.

In the current example, a search resolution of Q = 11 is used. This gives a  $9^{\circ}$  angular increment in the critical plane evaluation. Using Equation 1, the normal and shear stresses

evaluated on discrete planes are given in Table 1 and Table 2. The number in the subscripts indicates the load case.

TABLE I: NORMAL STRESS ON DISCRETE PLANES.

φ <b>(°)</b>	$\sigma_{n,1}$ (MPa)	$\sigma_{n,2}$ (MPa)	$\sigma_{n,3}$ (MPa)	$\sigma_{n,4}$ (MPa)	$\sigma_{n,max}$ (MPa)	$\Delta\sigma_n$ (MPa)
0	200	200	-200	-200	200	400
9	231	159	-231	-159	231	462
18	249	113	-249	-133	249	498
27	252	65	-252	-65	252	504
36	241	21	-241	-21	241	481
45	215	-15	-215	15	215	431
54	179	-41	-17	41	179	358
63	135	-52	-135	52	135	269
72	87	-49	-87	49	87	174
81	41	-31	-41	31	41	81
90	0	0	0	0	0	0
99	-31	41	31	-41	41	81
108	-49	87	49	-87	87	174
117	-52	135	52	-135	135	269
126	-41	179	41	-179	179	358
135	-15	215	15	-215	215	431
144	21	241	-21	-241	230	481
153	65	252	-65	-252	252	504
162	113	249	-113	-249	249	498
171	159	231	-159	231	231	462

TABLE 2: SHEAR STRESS ON DISCRETE PLANES.

φ <b>(°)</b>	τ <sub>1</sub> (MPa)	τ <sub>2</sub> (MPa)	τ <sub>3</sub> (MPa)	τ <sub>4</sub> (MPa)	<b>Δτ (MPa)</b>
0	115	-115	-115	115	230
9	79	-141	-79	141	281
18	35	-152	-35	152	304
27	-13	-149	13	149	298
36	-59	-131	59	131	262
45	-100	-100	100	100	200

TABLE 2: SHEAR STRESS ON DISCRETE PLANES.

φ <b>(°)</b>	τ <sub>1</sub> (MPa)	τ <sub>2</sub> (MPa)	τ <sub>3</sub> (MPa)	τ <sub>4</sub> (MPa)	<b>Δτ (MPa)</b>
54	-131	-59	131	59	262
63	-149	-13	149	13	298
72	-152	35	152	-35	304
81	-141	79	141	-79	281
90	-115	115	115	-115	231
99	-79	141	79	-141	281
108	-35	152	35	-152	304
117	13	149	-13	-149	298
126	59	131	-59	-131	262
135	100	100	-100	-100	200
144	131	59	-131	-59	262
153	149	13	-149	-13	298
162	152	-35	-152	35	304
171	141	-79	-141	79	281

The Findley criterion searches for a critical plane where a combination between the shear stress range and the normal stress is highest. This stress is shown in the second column of Table 3, where it can be seen that 202 MPa is found at planes oriented at 18° and 162°. This results into a usage factor of 202/213 = 0.95, which is in good agreement with the computed results.

TABLE 3: FATIGUE STRESS.

φ <b>(°)</b>	$\Delta \tau/2 + 0.20\sigma_n$ (MPa)	$\Delta \tau/2 + 0.27 \sigma_n$ (MPa)	$\Delta\sigma_{\rm n}$ (MPa)
0	155	169	400
9	187	203	462
18	202	219	498
27	199	217	504
36	179	196	481
45	143	158	431
54	167	179	358
63	176	185	269
72	170	175	174
81	149	152	81
90	115	115	0

TABLE 3: FATIGUE STRESS.

φ <b>(°)</b>	$\Delta \tau/2 + 0.20\sigma_n$ (MPa)	$\Delta \tau/2 + 0.27 \sigma_n$ (MPa)	$\Delta\sigma_{\rm n}$ (MPa)
99	149	152	81
108	170	175	174
117	176	185	269
126	167	179	358
135	143	158	431
144	179	196	481
153	199	217	504
162	202	219	498
171	187	203	462

The Matake criterion selects the critical plane as the one with the largest shear stress range. This occurs at orientations 18°, 72°, 108°, and 162° where the range is 304 MPa, see Table 2. The maximum normal stress on those planes is either 87 MPa or 249 MPa, see Table 1 and the Matake stress is either 219 MPa or 175 MPa, see Table 3. Since the Matake criterion does not contain the normal stress in the selection of the critical plane, the fatigue criteria is calculated with either one of them. In Figure 5, this feature is demonstrated by the nonsmooth results. The analytical values at the critical planes for the Matake usage factor are 219/223 = 0.98 and 175/223 = 0.78, which is in good agreement with the computed results.

The Normal stress criterion considers the plane with the largest normal stress range. The last column of Table 3 shows that this is found on planes with orientations 27° and 153° where the range is 504 MPa. This results into a fatigue usage factor of 504/576 = 0.88, which is in good agreement with the computed results.

# Notes About the COMSOL Implementation

In the critical plane evaluation, a search resolution Q indicates the number of evaluation point along 90° of a unit circle. The angle between two evaluation points is then at most  $(90^{\circ})/(Q-1)$ .

Application Library path: Fatigue Module/Verification Examples/ cylindrical test specimen

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

### MODEL WIZARD

- I In the Model Wizard window, click 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click Done.

### GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

### **GLOBAL DEFINITIONS**

### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
Moment	22.672 [N*m]	22.672 N·m	Twisting moment
Force	15708 [N]	15708 N	Normal force

### GEOMETRY I

Work Plane I (wpl)

In the Geometry toolbar, click Work Plane.

Work Plane I (wp I)>Plane Geometry

Right-click Work Plane I (wpI) and choose Show Work Plane.

Work Plane I (wpl)>Polygon I (poll)

- I In the Work Plane toolbar, click Polygon.
- 2 In the Settings window for Polygon, locate the Object Type section.
- **3** From the **Type** list, choose **Open curve**.
- **4** Locate the **Coordinates** section. From the **Data source** list, choose **Vectors**.
- **5** In the **xw** text field, type 5 5 0 0 10 10 6.01.
- 6 In the yw text field, type 20 0 0 50 50 30 30.
- 7 Click the **Zoom Extents** button in the **Graphics** toolbar.

Work Plane I (wp I)>Quadratic Bézier I (qb I)

- I In the Work Plane toolbar, click More Primitives and choose Quadratic Bézier.
- 2 In the Settings window for Quadratic Bézier, locate the Control Points section.
- 3 In row 1, set xw to 6.01.
- 4 In row 2, set xw to 5.25.
- **5** In row **3**, set **xw** to **5**.
- 6 In row 1, set yw to 30.
- 7 In row 2, set yw to 25.
- 8 In row 3, set yw to 20.

Work Plane I (wbl)>Convert to Solid I (csoll)

- I In the Work Plane toolbar, click Conversions and choose Convert to Solid.
- 2 Click in the **Graphics** window and then press Ctrl+A to select both objects.
- 3 In the Settings window for Convert to Solid, click Build Selected.

Work Plane I (wbl)>Fillet I (fill)

- I In the Work Plane toolbar, click Fillet.
- **2** On the object **csoll**, select Point 5 only.
- 3 In the Settings window for Fillet, locate the Radius section.
- 4 In the Radius text field, type 4.

Work Plane I (wpl)>Mirror I (mirl)

- I In the Work Plane toolbar, click Transforms and choose Mirror.
- **2** Select the object **fill** only.
- 3 In the Settings window for Mirror, locate the Normal Vector to Line of Reflection section.
- 4 In the xw text field, type 0.

- **5** In the **yw** text field, type 1.
- **6** Locate the **Input** section. Select the **Keep input objects** check box.
- 7 Click Build Selected.
- 8 Click the Zoom Extents button in the Graphics toolbar.

Work Plane I (wpl)>Union I (unil)

- I In the Work Plane toolbar, click Booleans and Partitions and choose Union.
- **2** Click in the **Graphics** window and then press Ctrl+A to select both objects.
- 3 In the Settings window for Union, locate the Union section.
- 4 Clear the Keep interior boundaries check box.
- 5 In the Work Plane toolbar, click Build All.

Work Plane I (wbl)

In the Model Builder window, click Work Plane I (wpl).

Revolve I (rev I)

- I In the Geometry toolbar, click Revolve.
- 2 In the Settings window for Revolve, locate the Revolution Angles section.
- 3 Clear the **Keep original faces** check box.
- 4 Click Build All Objects.
- 5 Click the **Zoom Extents** button in the **Graphics** toolbar.

### MATERIALS

Material I (mat I)

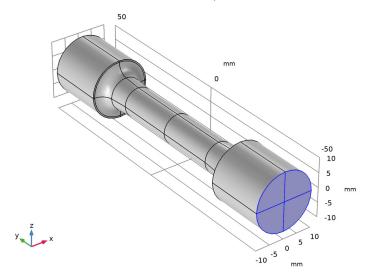
- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	210e9	Pa	Basic
Poisson's ratio	nu	0.3	I	Basic
Density	rho	7800	kg/m³	Basic

## SOLID MECHANICS (SOLID)

### Fixed Constraint I

- I In the Model Builder window, under Component I (compl) right-click Solid Mechanics (solid) and choose Fixed Constraint.
- 2 Select Boundaries 3, 4, 25, and 27 only.



# Rigid Connector I

- I In the Physics toolbar, click Boundaries and choose Rigid Connector.
- 2 Select Boundaries 11, 12, 47, and 48 only.

### Applied Force 1

- I Right-click Rigid Connector I and choose Applied Force.
- 2 In the Settings window for Applied Force, locate the Applied Force section.
- **3** Specify the  $\mathbf{F}$  vector as

0	x
Force	у
0	z

4 In the Physics toolbar, click Load Group and choose New Load Group.

### Applied Moment 1

I In the Model Builder window, right-click Rigid Connector I and choose Applied Moment.

- 2 In the Settings window for Applied Moment, locate the Applied Moment section.
- **3** Specify the **M** vector as

0	x
Moment	у
0	z

4 In the Physics toolbar, click Load Group and choose New Load Group.

### **GLOBAL DEFINITIONS**

Load Group 1

- I In the Model Builder window, under Global Definitions>Load and Constraint Groups click Load Group 1.
- 2 In the Settings window for Load Group, type 1gf in the Parameter name text field.

Load Group 2

- I In the Model Builder window, click Load Group 2.
- 2 In the Settings window for Load Group, type 1gm in the Parameter name text field. Create a load cycle consisting of four load cases.

### STUDY I

Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, click to expand the Study Extensions section.
- 3 Select the Define load cases check box.
- 4 Click Add four times.
- **5** In the table, enter the following settings:

Load case	lgf	Weight	lgm	Weight
Load case 1		1.0	V	1.0
Load case 2	$\sqrt{}$	1.0	<b>V</b>	-1.0
Load case 3	$\checkmark$	-1.0	V	-1.0
Load case 4	V	-1.0	V	1.0

6 In the Home toolbar, click Compute.

### RESULTS

Verify stresses at the center of the test specimen.

ID Plot Group 4

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Stresses in the Label text field.

Point Graph 1

- I Right-click Stresses and choose Point Graph.
- **2** Select Point 20 only.

It might be easier to select the correct point by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)

3 In the Settings window for Point Graph, click Replace Expression in the upper-right corner of the y-axis data section. From the menu, choose Component I>Solid Mechanics> Stress (Gauss points)>Second Piola-Kirchhoff stress,

Gauss point evaluation (material and geometry frames) - N/m2>solid.SGpY - Second Piola-Kirchhoff stress, Gauss point evaluation, Y component.

- 4 Locate the y-Axis Data section. From the Unit list, choose MPa.
- **5** Click to expand the **Legends** section. Select the **Show legends** check box.
- 6 From the Legends list, choose Manual.
- **7** In the table, enter the following settings:

# Legends normal stress

Point Graph 2

- I Right-click Point Graph I and choose Duplicate.
- 2 In the Settings window for Point Graph, locate the y-Axis Data section.
- **3** In the **Expression** text field, type solid.SGpXY.
- **4** Locate the **Legends** section. In the table, enter the following settings:

# Legends shear stress

Point Graph 3

- I Right-click Point Graph 2 and choose Duplicate.
- 2 In the Settings window for Point Graph, locate the y-Axis Data section.

- 3 In the Expression text field, type solid.misesGp.
- **4** Locate the **Legends** section. In the table, enter the following settings:

### Legends

von Mises

### Stresses

- I In the Model Builder window, click Stresses.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- **3** From the **Title type** list, choose **None**.
- **4** Locate the **Plot Settings** section. Select the **y-axis label** check box.
- 5 In the associated text field, type Stress (MPa).
- 6 In the Stresses toolbar, click Plot.

### ADD PHYSICS

- I In the Home toolbar, click Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Structural Mechanics>Fatigue (ftg).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Study 1.
- **5** Click **Add to Component I** in the window toolbar.

### FATIGUE (FTG)

In the Settings window for Fatigue, type Fatigue Findley in the Label text field.

### Stress-Based I

- I Right-click Component I (compl)>Fatigue Findley and choose the boundary evaluation Stress-Based
- 2 In the Settings window for Stress-Based, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Locate the Solution Field section. From the Physics interface list, choose Solid Mechanics (solid).

### ADD PHYSICS

- I Go to the Add Physics window.
- 2 In the tree, select Structural Mechanics>Fatigue (ftg).

- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for Study 1.
- **4** Click **Add to Component I** in the window toolbar.

### FATIGUE 2 (FTG2)

- I In the Settings window for Fatigue, type Fatigue Matake in the Label text field.
- 2 Right-click Component I (compl)>Fatigue Matake and choose the boundary evaluation Stress-Based.

### Stress-Based 1

- I In the Settings window for Stress-Based, locate the Boundary Selection section.
- 2 From the Selection list, choose All boundaries.
- 3 Locate the Fatigue Model Selection section. From the Criterion list, choose Matake.
- 4 Locate the Solution Field section. From the Physics interface list, choose Solid Mechanics (solid).

### ADD PHYSICS

- I Go to the Add Physics window.
- 2 In the tree, select Structural Mechanics>Fatigue (ftg).
- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for Study 1.
- 4 Click Add to Component I in the window toolbar.
- 5 In the Model Builder window, click Component I (compl).
- 6 In the Home toolbar, click Add Physics to close the Add Physics window.

### FATIGUE 3 (FTG3)

In the Settings window for Fatigue, type Fatigue Normal Stress in the Label text field.

### Stress-Based 1

- I In the Physics toolbar, click Boundaries and choose Stress-Based.
- 2 In the Settings window for Stress-Based, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Locate the Fatigue Model Selection section. From the Criterion list, choose Normal stress.
- 5 Locate the Solution Field section. From the Physics interface list, choose Solid Mechanics (solid).

### MATERIALS

Material 2 (mat2)

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose All boundaries.
- **5** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Normal stress sensitivity coefficient	k_Findley	0.20	I	Findley
Limit factor	f_Findley	213[MPa]	Pa	Findley
Normal stress sensitivity coefficient	k_Matake	0.27	I	Matake
Limit factor	f_Matake	223[MPa]	Pa	Matake
Limit factor	f_NormalStress	576[MPa]	Pa	Normal stress

### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for the Solid Mechanics (solid) interface.
- 4 Find the **Studies** subsection. In the **Select Study** tree, select Preset Studies for Selected Physics Interfaces>Fatigue.
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click Add Study to close the Add Study window.

### STUDY 2

Step 1: Fatigue

- I In the Settings window for Fatigue, locate the Values of Dependent Variables section.
- 2 Find the Values of variables not solved for subsection. From the Settings list, choose User controlled.

- **3** From the **Method** list, choose **Solution**.
- 4 From the Study list, choose Study I, Stationary.
- 5 In the Home toolbar, click Compute.

### RESULTS

Fatigue Usage Factor (ftg)

In the Settings window for 3D Plot Group, type Fatigue Usage Factor (Findley) in the Label text field.

Fatigue Usage Factor (ftg2)

- I In the Model Builder window, under Results click Fatigue Usage Factor (ftg2).
- 2 In the Settings window for 3D Plot Group, type Fatigue Usage Factor (Matake) in the Label text field.

Fatigue Usage Factor (ftg3)

- I In the Model Builder window, under Results click Fatigue Usage Factor (ftg3).
- 2 In the Settings window for 3D Plot Group, type Fatigue Usage Factor (Normal Stress) in the Label text field.