



MULTIPLE AND LOGISTIC REGRESSION

Adding a numerical explanatory variable

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Adding a second numeric explanatory variable

- Mathematical:

$$\hat{bwt} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \text{gestation} + \hat{\beta}_2 \cdot \text{age}$$

- Syntactical:

```
lm(bwt ~ gestation + age, data = babies)
```



No longer a 2D problem

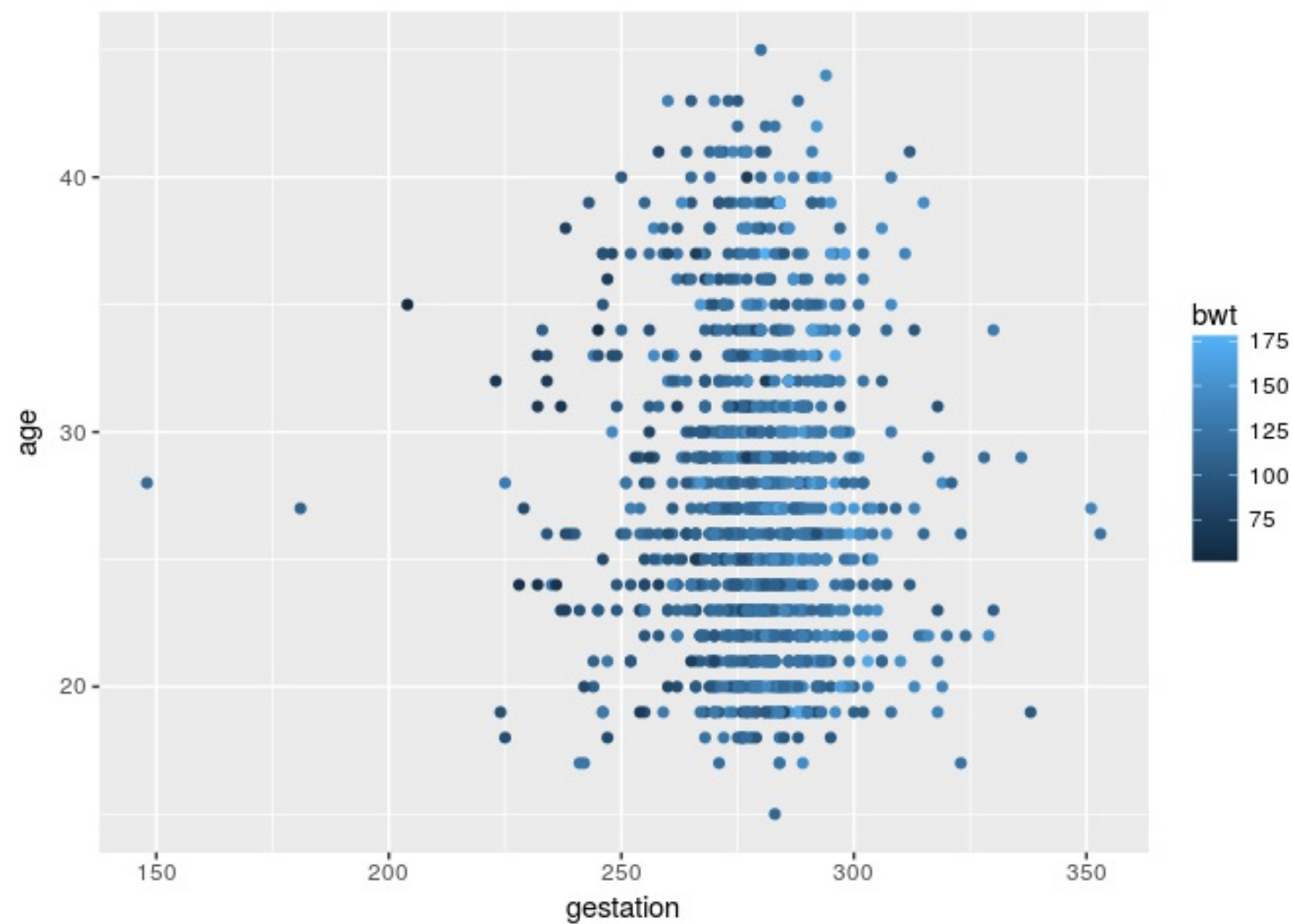
```
# doesn't work
ggplot(data = babies, aes(x = gestation, y = age, z = bwt)) +
  geom_point() +
  geom_smooth(method = "lm", se = 0)
```



Data space is 3D

```
data_space <- ggplot(babies, aes(x = gestation, y = age)) +  
  geom_point(aes(color = bwt))
```

```
data_space
```





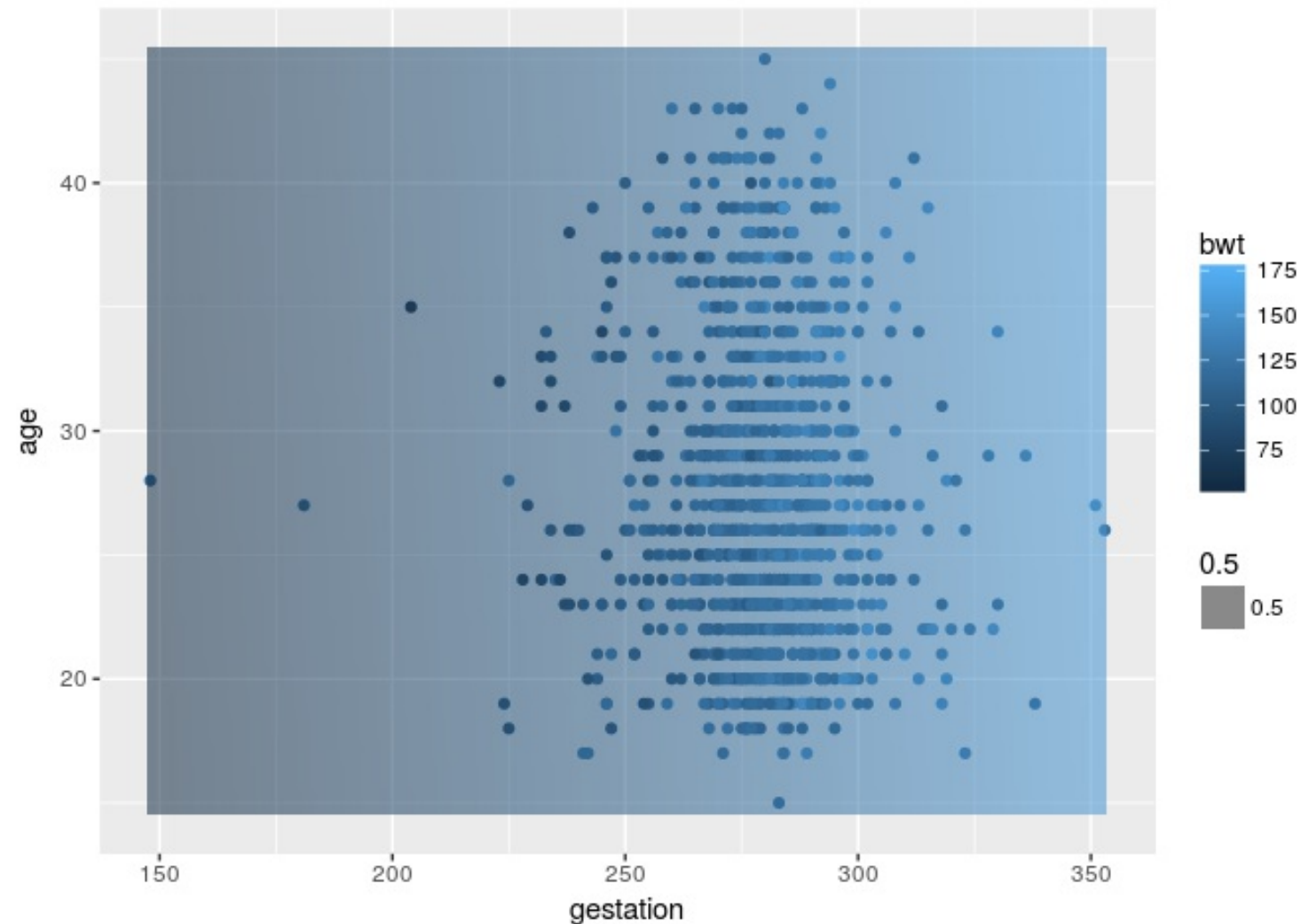
Tiling the plane

```
grid <- babies %>%  
  data_grid(  
    gestation = seq_range(gestation, by = 1),  
    age = seq_range(age, by = 1)  
  )  
  
mod <- lm(bwt ~ gestation + age, data = babies)  
  
bwt_hats <- augment(mod, newdata = grid)
```



Tiles in the data space

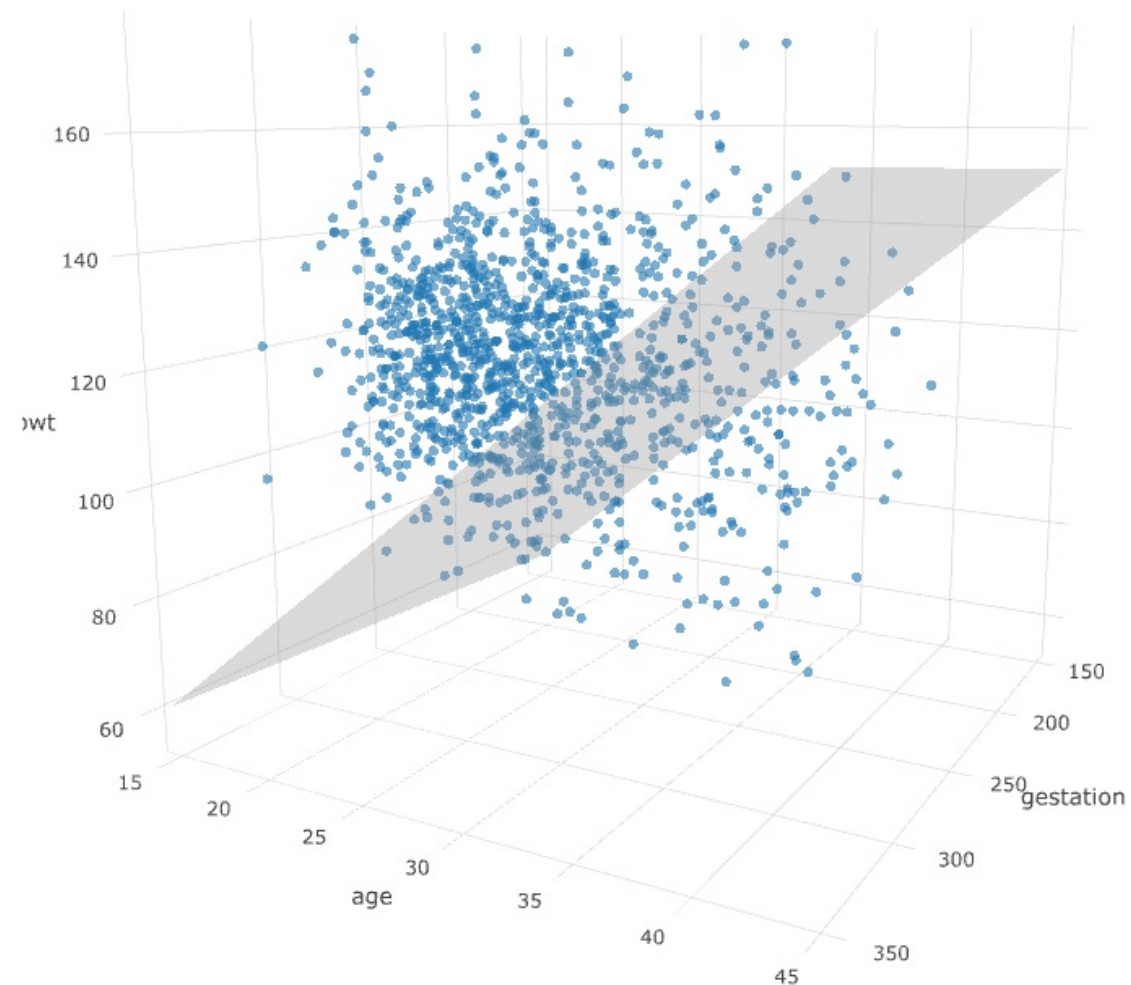
```
data_space +  
  geom_tile(data = bwt_hats, aes(fill = .fitted, alpha = 0.5)) +  
  scale_fill_continuous("bwt", limits = range(babies$bwt))
```





3D visualization

```
plot_ly(data = babies, z = ~bwt, x = ~gestation, y = ~age, opacity = 0.6) %>%  
  add_markers(text = ~case, marker = list(size = 2)) %>%  
  add_surface(x = ~x, y = ~y, z = ~plane, showscale = FALSE,  
             cmap = 1, surfacecolor = color1, colorscale = col1)
```





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Let's practice!



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Conditional interpretation of coefficients

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Two slope coefficients

```
lm(bwt ~ gestation + age, data = babies)
```

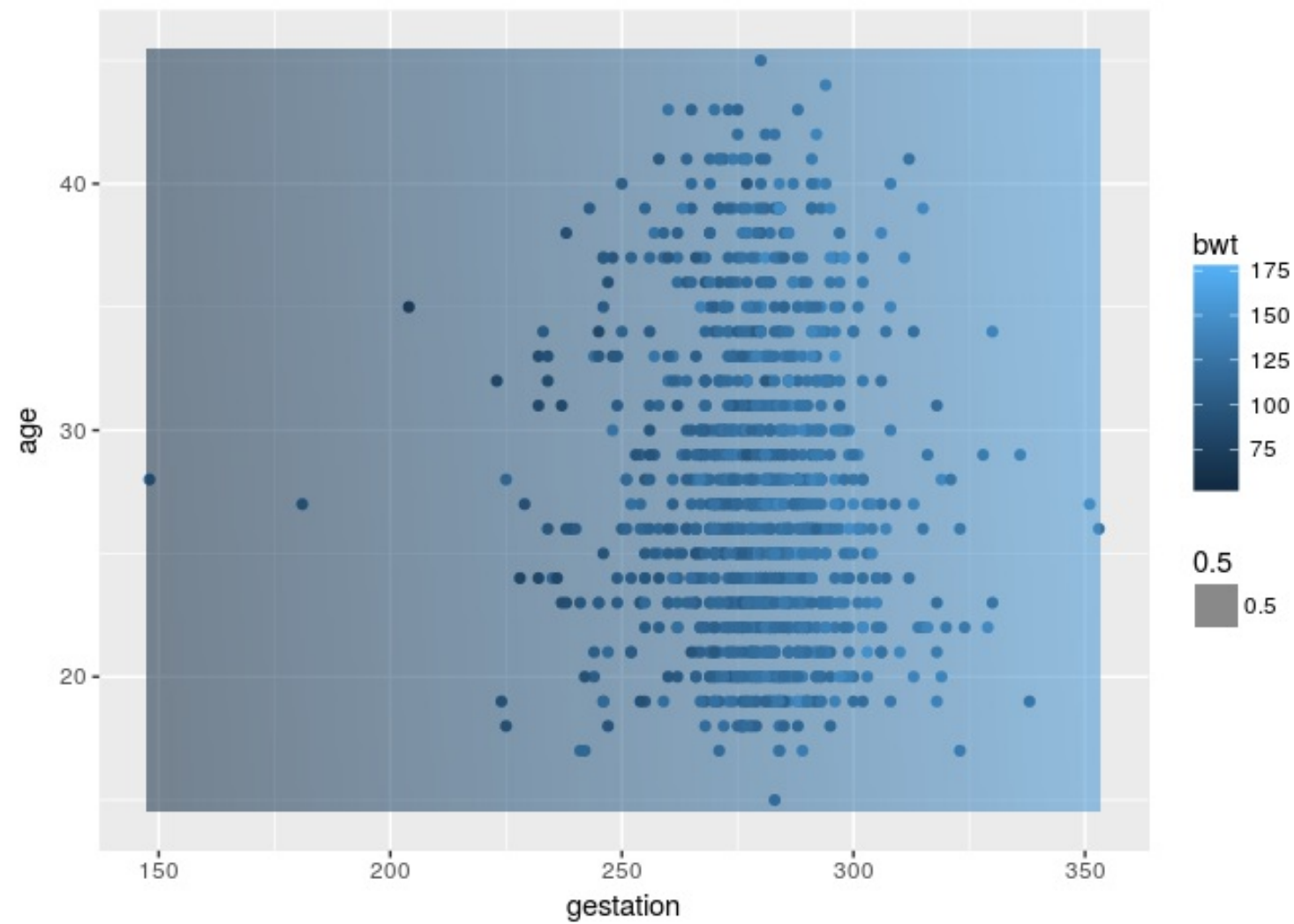
```
## Coefficients:
```

```
## (Intercept)    gestation         age
```

```
##    -15.5226         0.4676         0.1657
```

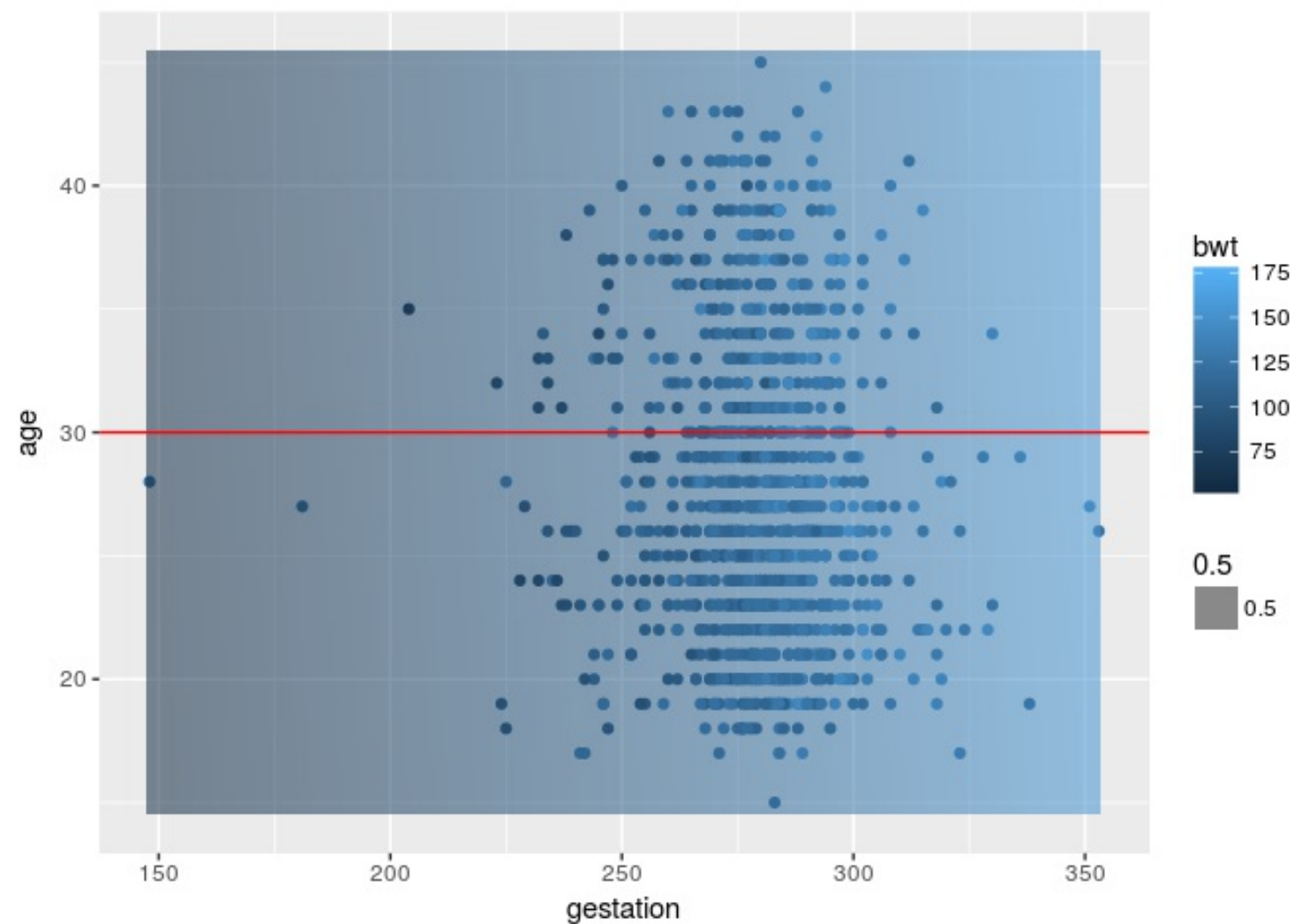
Tiled plane

`model_space`



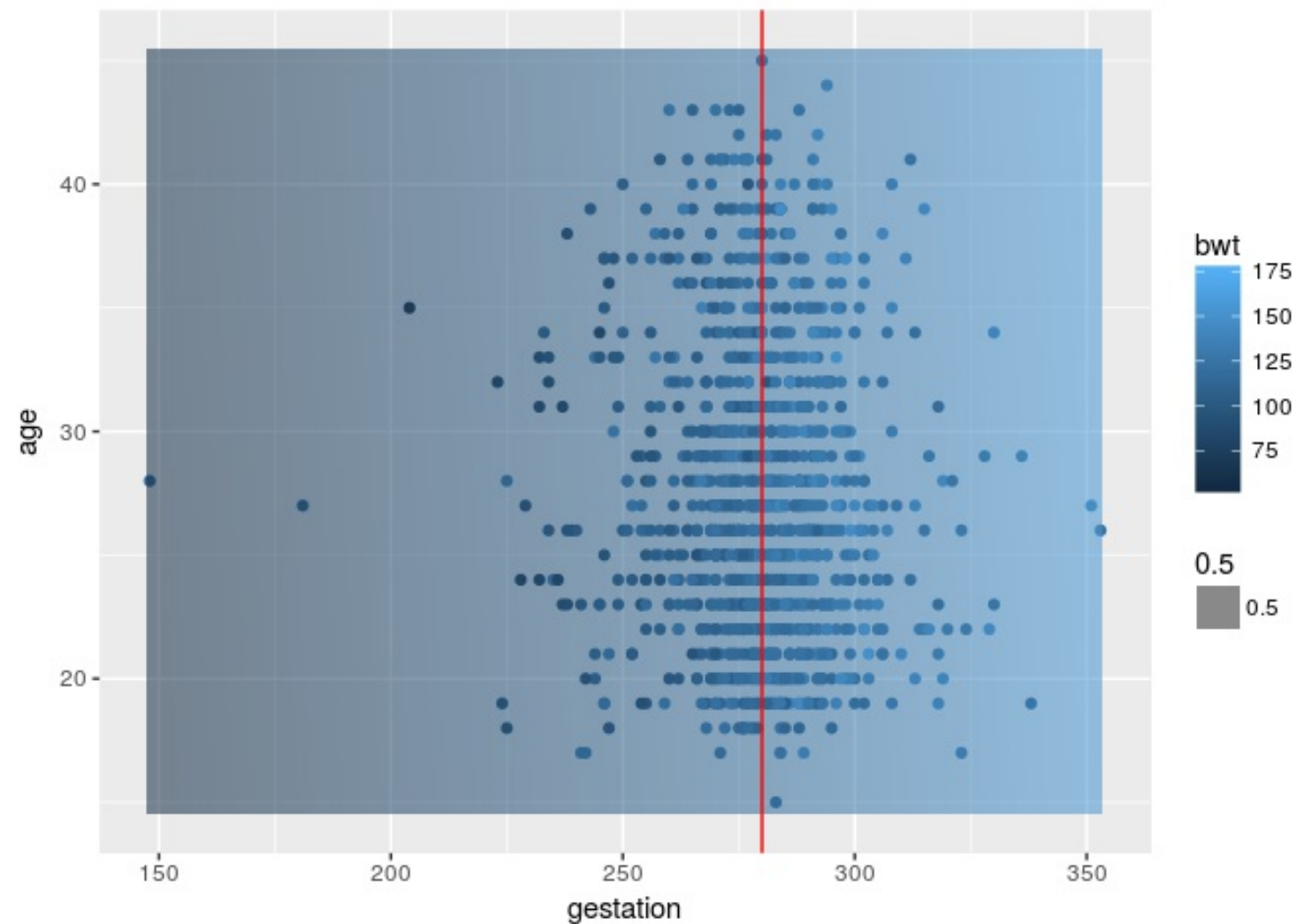
Tiled plane plus first slope

```
model_space +  
  geom_hline(yintercept = 30, color = "red")
```



Tiled plane plus second slope

```
model_space +  
  geom_vline(xintercept = 280, color = "red")
```





Coefficient interpretation

```
lm(bwt ~ gestation + age, data = babies)
```

```
## Coefficients:
```

```
## (Intercept)    gestation         age
```

```
##    -15.5226         0.4676         0.1657
```



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Adding a third (categorical) variable

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How could we forget about smoking?

- Mathematical:

$$\hat{bwt} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \text{gestation} + \hat{\beta}_2 \cdot \text{age} + \hat{\beta}_3 \cdot \text{smoke}$$

- Syntactical:

```
lm(bwt ~ gestation + age + smoke, data = babies)
```



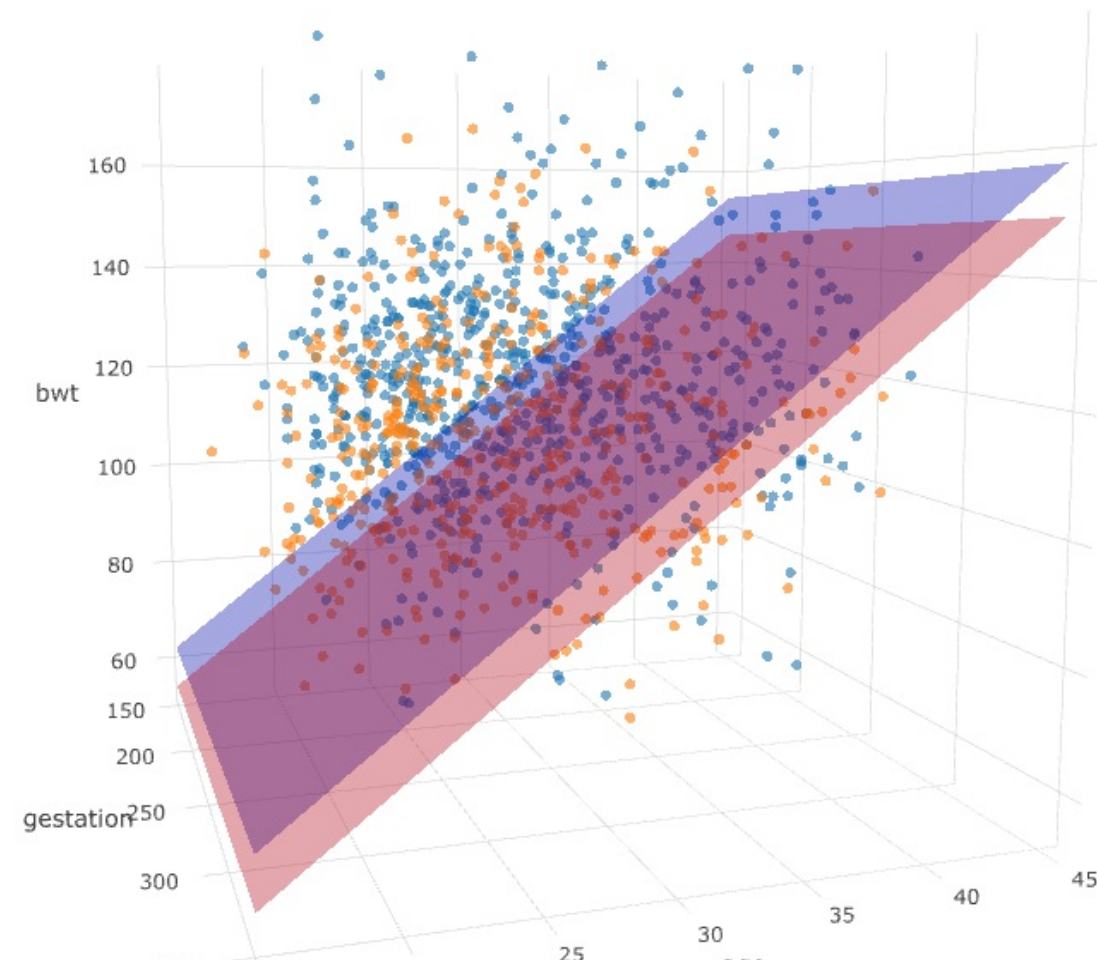
Geometry

- 1 numeric + 1 categorical:
 - parallel lines
- 2 numeric:
 - a plane
- 2 numeric + 1 categorical:
 - parallel planes!



Drawing parallel planes in 3D

```
plot_ly(data = babies, z = ~bwt, x = ~gestation, y = ~age, opacity = 0.6) %>%  
  add_markers(color = ~factor(smoke), text = ~case, marker = list(size = 2)) %>%  
  add_surface(x = ~x, y = ~y, z = ~plane0, showscale = FALSE,  
             cmin = 0, cmax = 1, surfacecolor = color1, colorscale = col1) %>%  
  add_surface(x = ~x, y = ~y, z = ~plane1, showscale = FALSE,  
             cmin = 0, cmax = 1, surfacecolor = color2, colorscale = col1)
```





Coefficient interpretation

```
lm(bwt ~ gestation + age, data = babies)
```

```
## Coefficients:
```

```
## (Intercept)    gestation        age  
##    -15.5226         0.4676        0.1657
```

```
lm(bwt ~ gestation + age + smoke, data = babies)
```

```
## Coefficients:
```

```
## (Intercept)    gestation        age        smoke  
##    -4.6037         0.4455        0.1069       -8.0143
```



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Higher dimensions

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Adding more variables

- Mathematical:

$$\hat{bwt} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \text{gestation} + \hat{\beta}_2 \cdot \text{age} + \hat{\beta}_3 \cdot \text{smoke} + \hat{\beta}_4 \cdot \text{height} + \hat{\beta}_5 \cdot \text{weight} + \hat{\beta}_6 \cdot \text{parity}$$

- Syntactical:

```
lm(bwt ~ gestation + age + smoke + height + weight + parity, data = babies)
```

- Syntactical (same model, but note order of coefficients)

```
lm(bwt ~ . - case, data = babies)
```



Higher dimensional geometry

- (Parallel) hyperplanes, etc.



Interpretation in large models

```
lm(bwt ~ gestation + age + smoke + height + weight + parity, data = babies)
```

```
## Coefficients:
```

```
## (Intercept)    gestation         age         smoke         height
##   -80.41085         0.44398    -0.00895    -8.40073         1.15402
##      weight         parity
##    0.05017        -3.32720
```



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