FOUNDATIONS OF INFERENCE

Parameters and confidence intervals

Research questions

Hypothesis test	Confidence interval
Under which diet plan will participants lose more weight on average?	How much should participants expect to lose on average?
Which of two car manufacturers are users more likely to recommend to their friends?	What percent of users are likely to recommend Subaru to their friends?
Are education level and average income linearly related?	For each additional year of education, what is the predicted average income?

Parameter

- A parameter is a *numerical value from the population*
- Examples (continued):
  - The true average amount all dieters will lose on a particular program
  - The proportion of individuals in a population who recommend Subaru cars
  - The average income of all individuals in the population with a particular education level

Confidence interval

- Range of numbers that (hopefully) captures the true parameter
- "95% confident that between 12% and 34% of the entire population recommends Subarus"

Let’s practice!

Bootstrapping

Hypothesis testing

- How do samples from the null population vary?

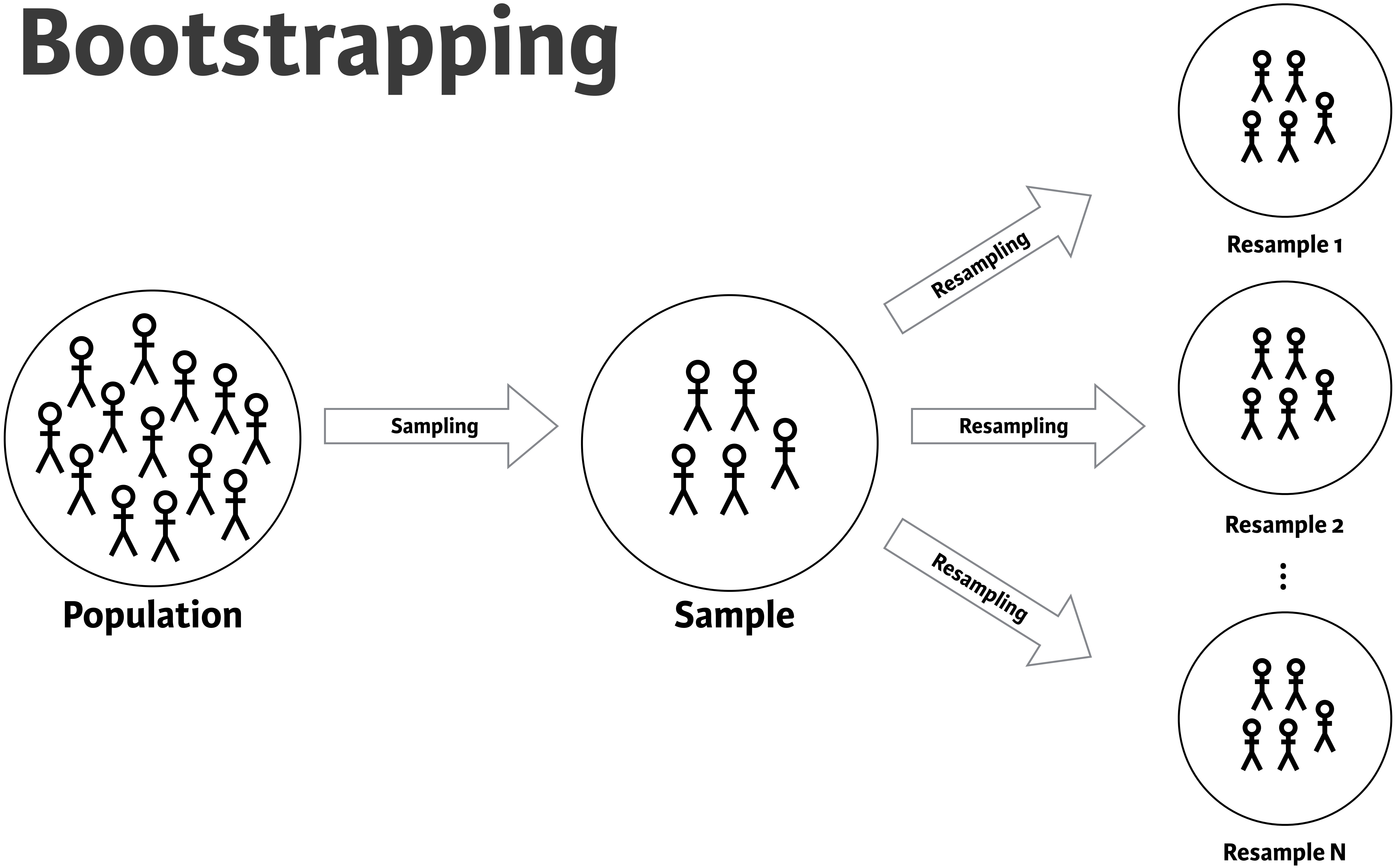
$\hat{p}$ → **statistic**, proportion of successes in ***sample***

p → **parameter,** proportion of successes in ***population***

Confidence intervals

- No null population, unlike in hypothesis testing
- How do p and p-hat vary?

Bootstrapping



Polling

```
# Original data
Source: local data frame [30 x 3]

     flip_num  flip
        <int> <chr>
1           1     H
2           2     H
3           3     H
4           4     T
5           5     H
6           6     H
7           7     H
8           8     T
9           9     T
10         10     T
# ... with 20 more rows
```

Original data

Candidate X	Total voters	Proportion X
17	30	0.5667

Polling

```
# First resample
Source: local data frame [30 x 3]

   replicate flip_num  flip
       <dbl>    <int> <chr>
1          1        7     H
2          1       17     T
3          1       13     H
4          1       14     H
5          1       24     H
6          1       28     T
7          1       13     T
8          1       29     H
9          1       24     H
10         1       20     T
# ... with 20 more rows
```

First resample

Candidate X	Total voters	Proportion X
17	30	0.5667
14	30	0.4667

Polling

```
# Second resample
Source: local data frame [30 x 3]

   replicate flip_num  flip
       <dbl>    <int> <chr>
1          2       21     H
2          2       19     T
3          2       25     H
4          2       24     T
5          2       21     H
6          2       28     T
7          2       13     H
8          2       23     H
9          2       24     T
10         2       24     T
# ... with 20 more rows
```

Second resample

Candidate X	Total voters	Proportion X
17	30	0.5667
14	30	0.4667
18	30	0.6

Polling

```
# Third resample
Source: local data frame [30 x 3]

   replicate flip_num  flip
       <dbl>    <int> <chr>
1          3        6     H
2          3       19     H
3          3        1     H
4          3       24     T
5          3       11     H
6          3       28     T
7          3       16     H
8          3       13     H
9          3       21     T
10         3       29     H
# ... with 20 more rows
```

Third resample

Candidate X	Total voters	Proportion X
17	30	0.5667
14	30	0.4667
18	30	0.6
12	30	0.4

Standard error

- Obtained standard error of 0.09 by resampling many times
- Describes how the statistic varies around parameter
- Bootstrap provides an approximation of the standard error

Variability of p-hat from the population

```
> # Compute p-hat for each poll
> ex1_props <- all_polls %>%
    group_by(poll) %>%
    summarize(prop_yes = mean(vote))

> # Select one poll from which to resample
> one_poll <- all_polls %>%
    filter(poll == 1) %>%
    select(vote)

> # Variability of p-hat
> ex1_props %>%
    summarize(sd(prop_yes))
# A tibble: 1 × 1
  `sd(prop_yes)`
           <dbl>
1     0.08523512
```

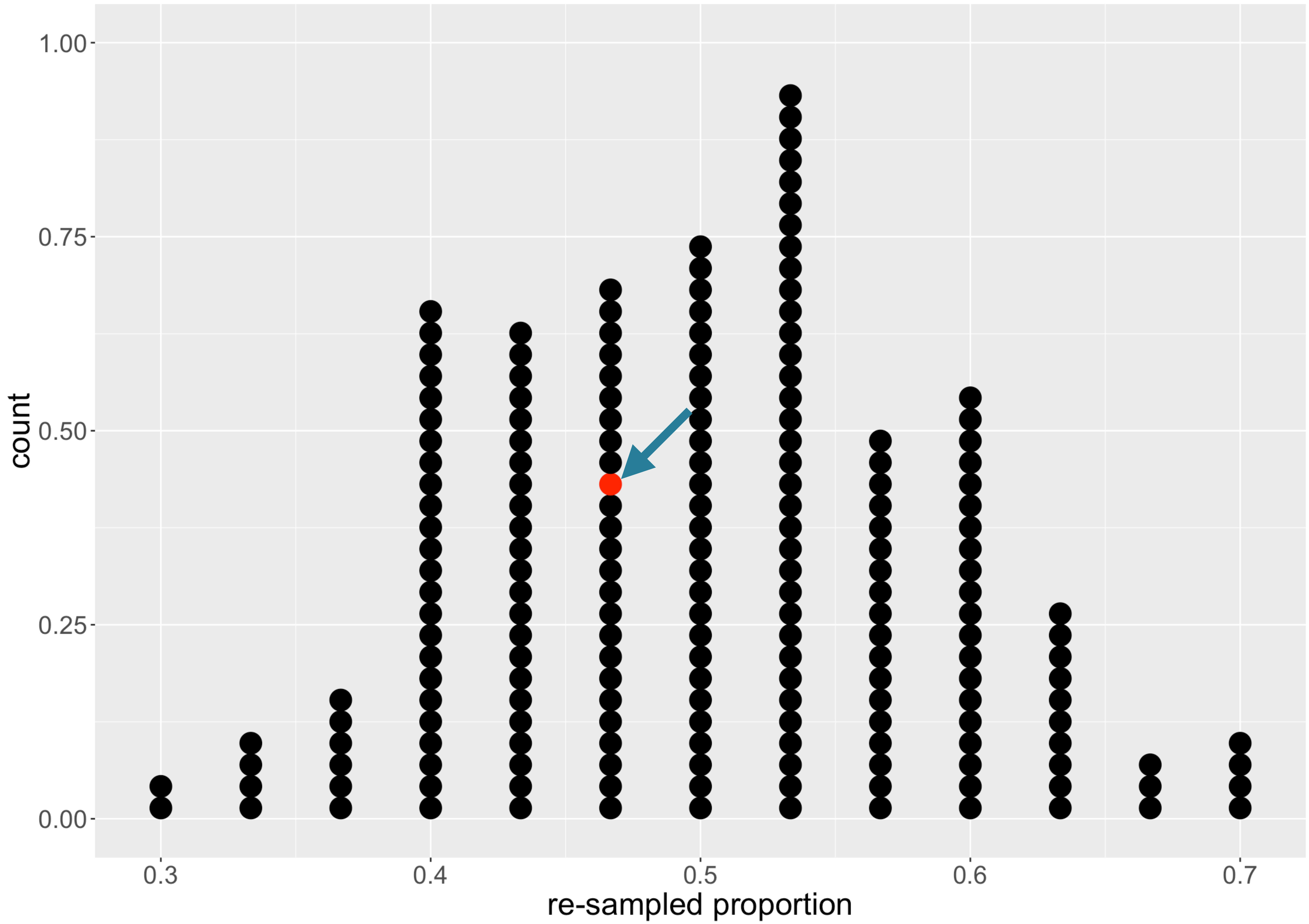
Variability of p-hat from the sample (bootstrapping)

```
> # Generate 1000 resamples of one poll
> one_poll_boot_30 <- one_poll %>%
    oilabs::rep_sample_n(30, replace = TRUE, reps = 1000)

> # Compute p-hat for each resampled poll
> ex2_props <- one_poll_boot_30 %>%
    summarize(prop_yes = mean(vote))

> # Variability of p-hat
> ex2_props %>%
    summarize(sd(prop_yes))
# A tibble: 1 × 1
  `sd(prop_yes)`
           <dbl>
1     0.08691885
```

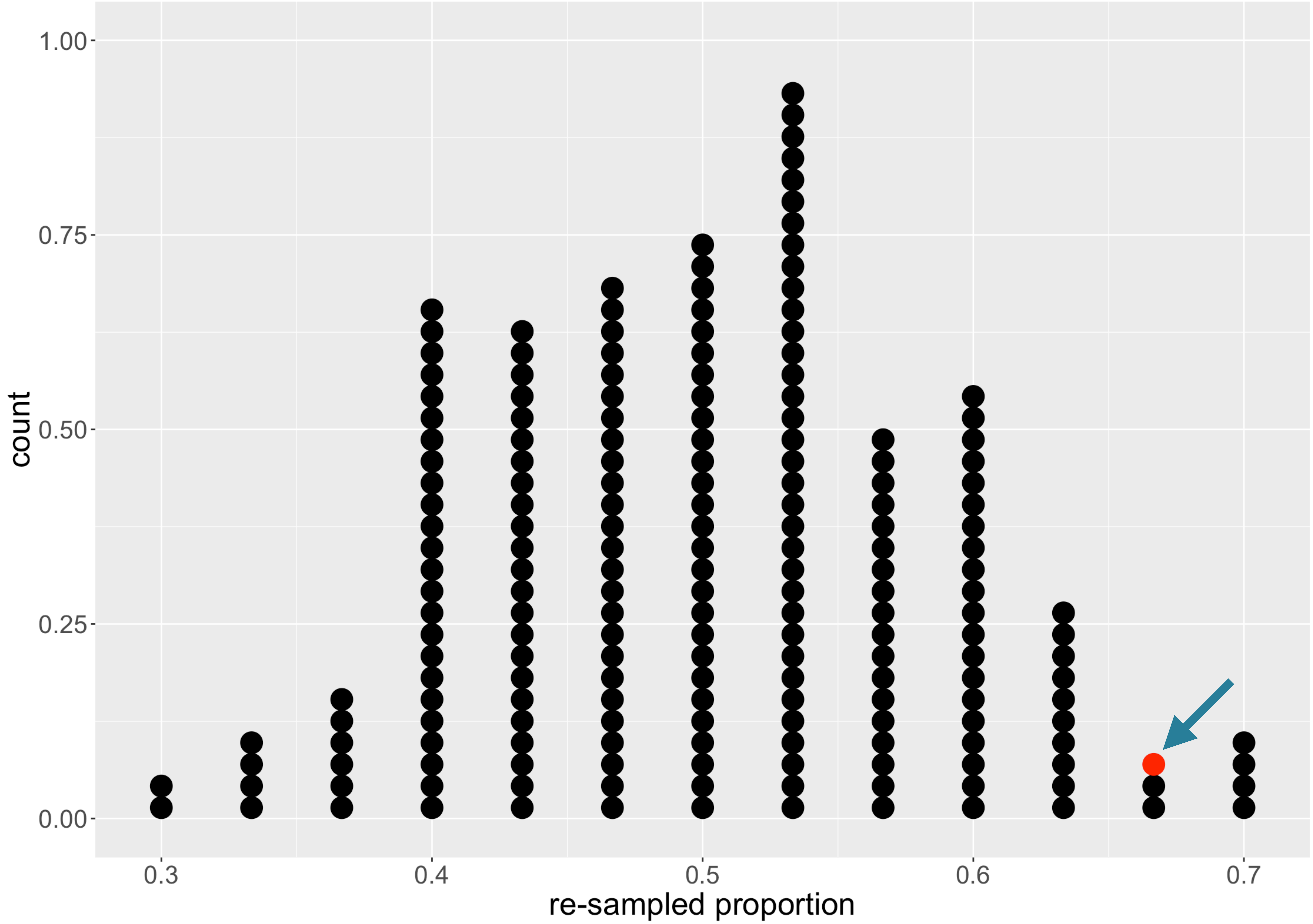
Let's practice!

Variability in p-hat

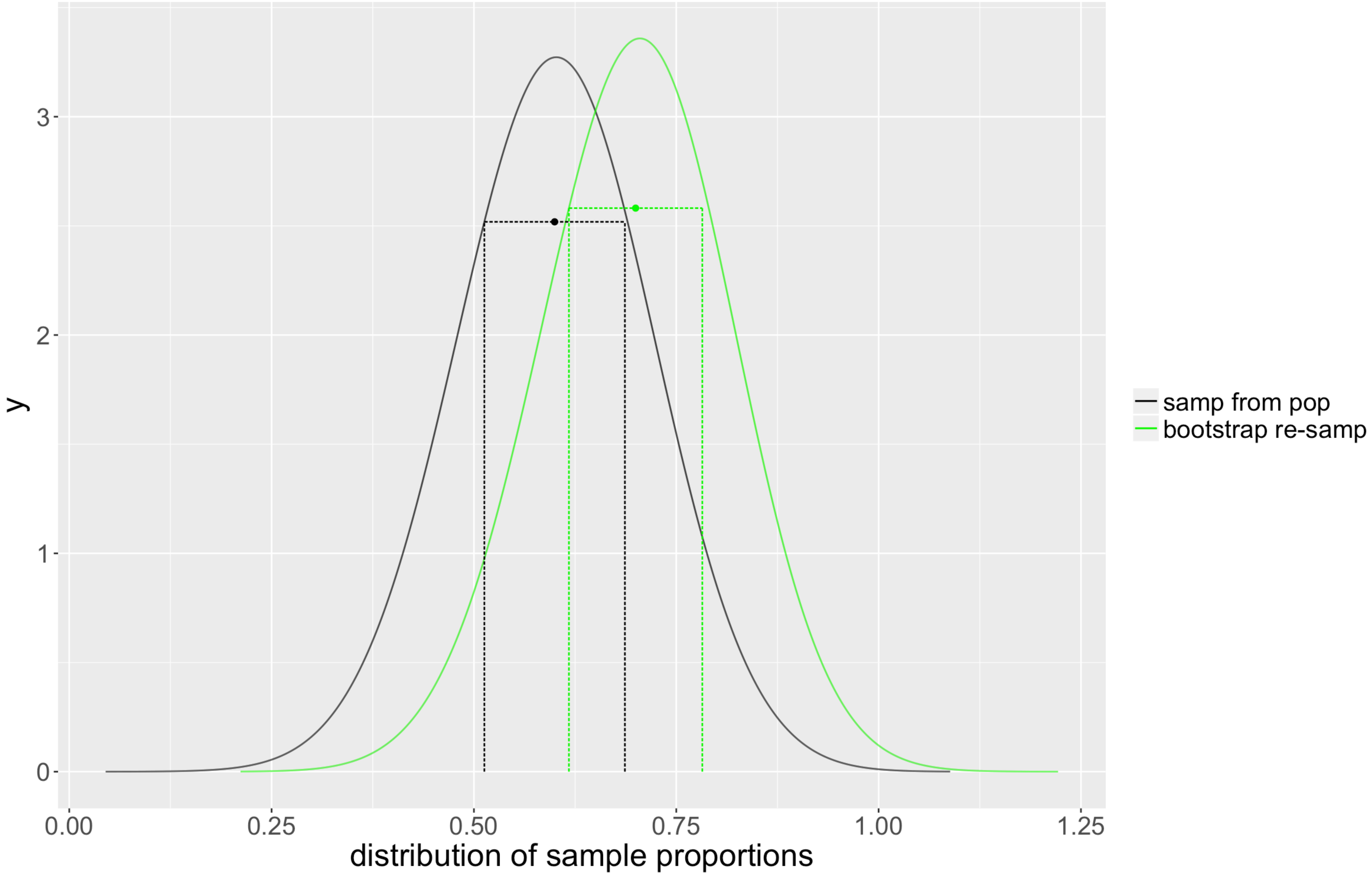
How far are the data from the parameter?



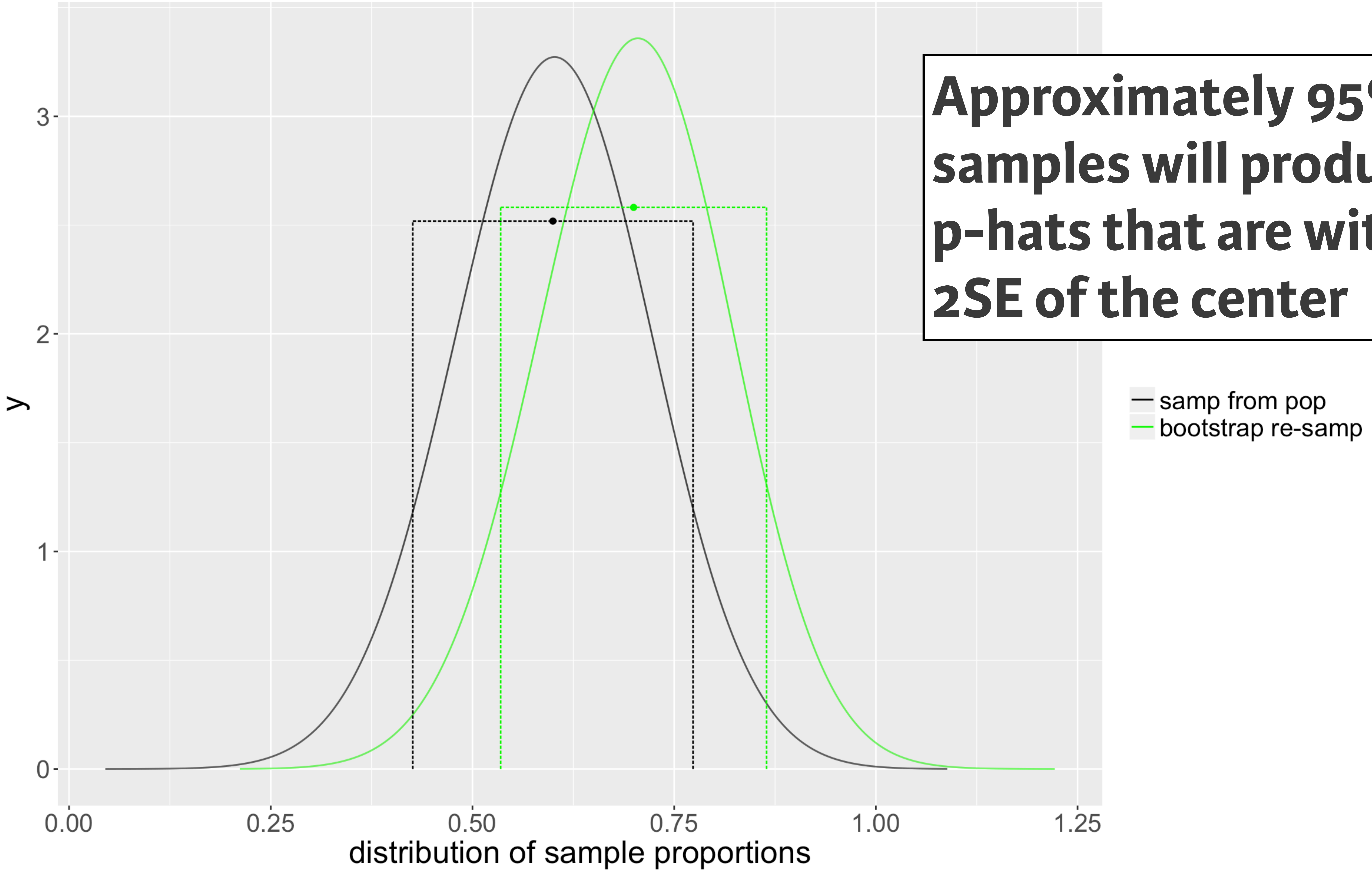
How far are the data from the parameter?



Standard error of p-hat

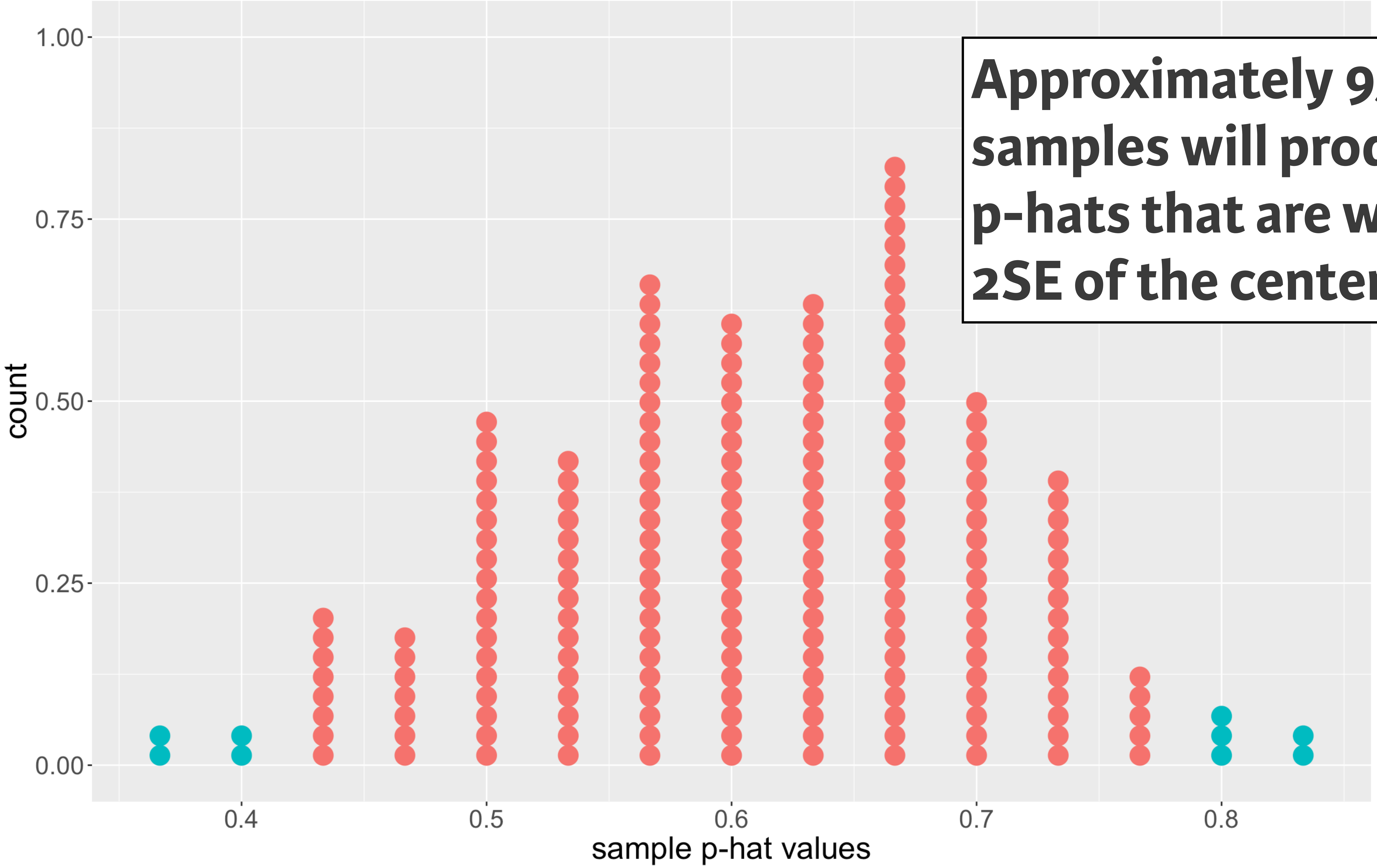


Empirical rule



Approximately 95% of samples will produce p-hats that are within 2SE of the center

Empirical rule



Approximately 95% of samples will produce p-hats that are within 2SE of the center

Let's practice!

Interpreting CIs and technical conditions

Creating CIs

```
> # Look at the bootstrap t-confidence interval for comparison
> one_poll_boot %>%
    summarize(lower = p_hat - 2 * sd(prop_yes_boot),
              upper = p_hat + 2 * sd(prop_yes_boot))
# A tibble: 1 × 2
     lower     upper
     <dbl>     <dbl>
1 0.536148 0.863852

> # Find the 2.5% and 97.5% of the p-hat values
> one_poll_boot %>%
    summarize(q025_prop = quantile(prop_yes_boot, p = .025),
              q975_prop = quantile(prop_yes_boot, p = .975))
# A tibble: 1 × 2
  q025_prop q975_prop
      <dbl>     <dbl>
1 0.5333333 0.8333333
```

Motivating CIs

- Goal is to find the parameter when all we know is the statistic
- Never know whether the sample you collected actually contains the true parameter

Interpreting the CIs

- Bootstrap t-CI: (0.536, 0.864)
- Percentile interval: (0.533, 0.833)

We are 95% confident that the true proportion of people planning to vote for candidate X is between 0.536 and 0.864 (or 0.533 and 0.833)

Technical conditions

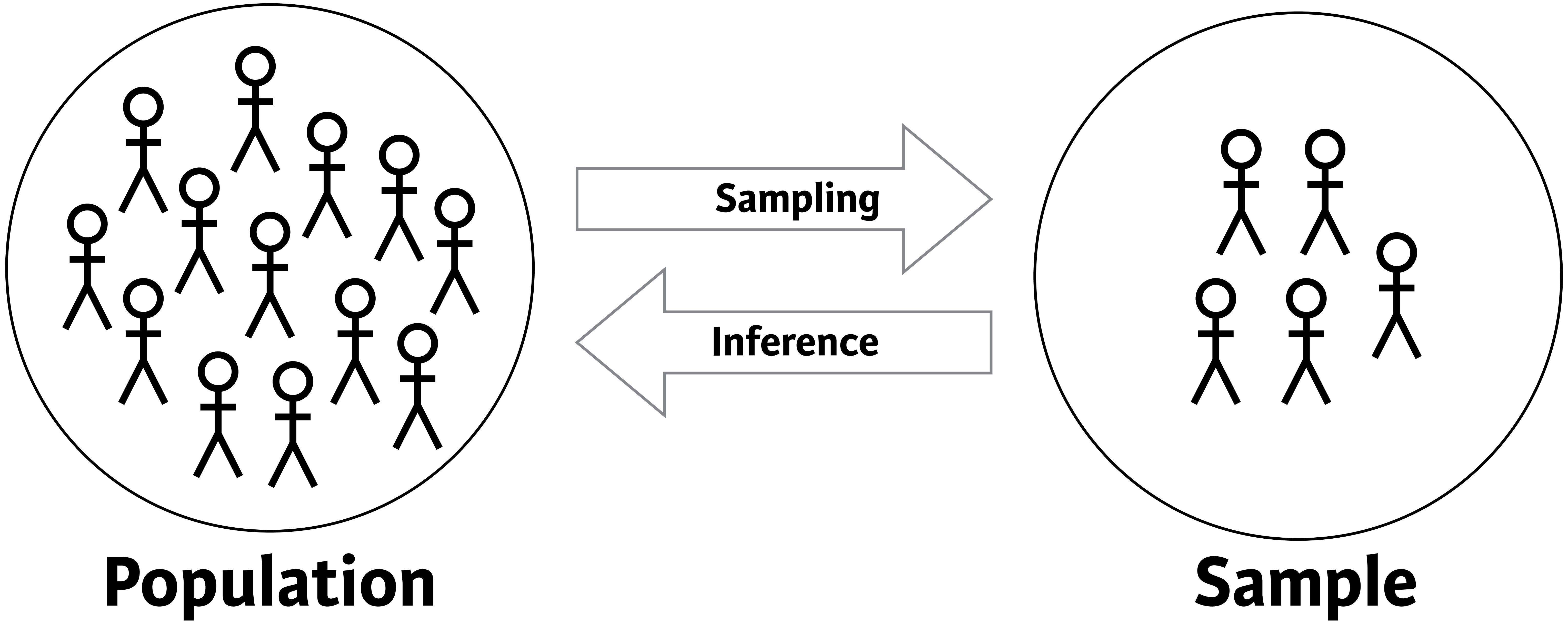
- Sampling distribution of the statistic is reasonably symmetric and bell-shaped
- Sample size is reasonably large
- Variability of resampled proportions

Let’s practice!

FOUNDATIONS OF INFERENCE

Summary of statistical inference

Inference



Testing

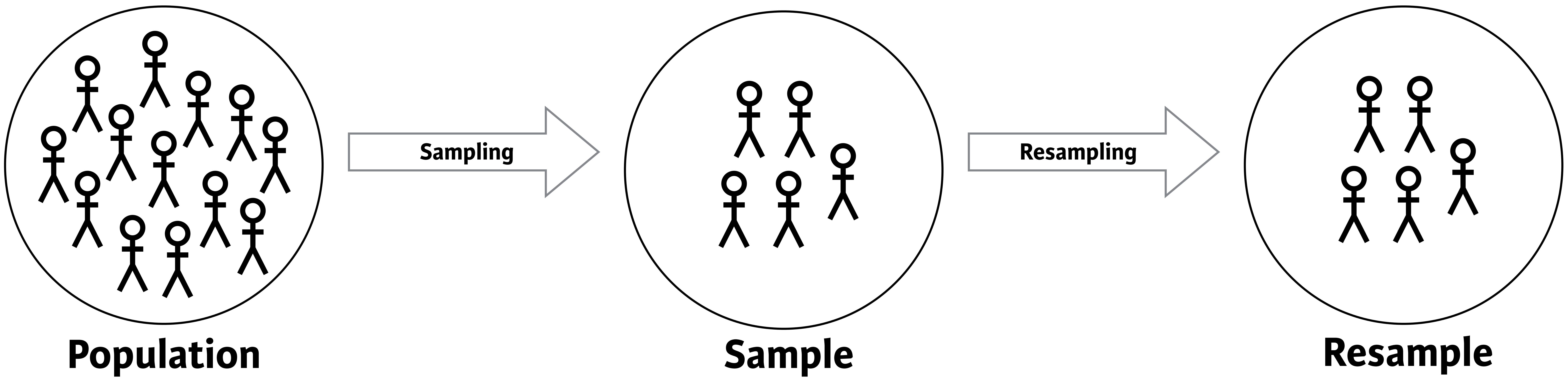
- **H_0**: There is no gender discrimination in hiring
- **H_A**: Men are more likely to be promoted than women

		Test	
		Do not reject H_0	Reject H_0 in favor of H_A
Truth	H_0 true	✓	Type I error
	H_A true	Type II error	✓

Estimation

What proportion of the voters will select candidate X?

Bootstrapping



Let’s practice!