(P1)

Given eqn:-

$$F(x,t) = (1+t)^2 \cos(x) \quad \text{—(i)}$$

$$dx = \mu x \, d(t) + \sigma x \, dw(t) \quad \text{—(ii)}$$

Itô lemma

$$dF = \left[\frac{\partial F}{\partial t} + \frac{\partial F}{\partial x} F + \frac{1}{2}\frac{\partial^2 F}{\partial^2 x} g^2\right] dt + g \frac{\partial F}{\partial x} dw \quad \text{—— (iii)}$$

Applying Itô's lemma

$$dF = [2\cos(x)(1+t) - \sin(x)(1+t)^2 \mu x - \frac{1}{2}\cos(x)(1+t)^2 \sigma^2 x^2] dt - \sigma x \sin(x)(1+t)^2 dw$$

From eqn (i)

$$\cos(x) = \frac{F}{(1+t)^2}$$

$$\Rightarrow x = \cos^{-1}\left(\frac{F}{(1+t)^2}\right)$$

$$\& \quad \sin(x) = \frac{\sqrt{(1+t)^4 - F^2}}{(1+t)^2}$$

$$dF = \left[2 \frac{F}{(1+t)^2}(1+t) - 4(1+t)^3 \mu x \left(\frac{\sqrt{(1+t)^4 - F^2}}{(1+t)^2} \right) - \frac{1}{2}(1+t)^2 \sigma^2 x^2 \left(\frac{F}{(1+t)^2} \right) \right] dt$$

$$- \sigma x (1+t)^2$$

$$\hookrightarrow \left(\frac{\sqrt{(1+t)^4 - F^2}}{(1+t)^2} \right) dw$$

Changing x in F.

$$dF = \left[\frac{2F}{(1+t)} - \mu \cos^{-1}\left(\frac{F}{(1+t)^2} \right) \left(\sqrt{(1+t)^4 - F^2} \right) - \frac{\sigma^2}{2} F \left(\cos^{-1}\left(\frac{F}{(1+t)^2} \right) \right)^2 \right] dt$$

$$- \sigma \cos^{-1}\left(\frac{F}{(1+t)^2} \right) \left(\sqrt{(1+t)^4 - F^2} \right.$$