1 Sensitivity Measure

Understanding how the value of a security changes relative to changes in a given parameter is key to hedging. Duration, for instance, measures the rate of change of bond value with respect to interest rate changes.$x = \left[\ln(s/x) + \left((r + \sigma^2/2)\right]/(\sigma\sqrt{t})\right.$ is Black scholes formula and $N'(y) = (1 + \sqrt{2\pi})e^{-y^2/2} > 0$ density function of standard normal distribution in this paper.

1.1 Delta

Delta Δ is defined as $\Delta = \frac{\partial f}{\partial s}$ where of is the price of the derivative and S is the price of underlying asset. European non-dividend paying stock $\frac{\partial C}{\partial S} = N(x) > 0$ and for put is $\frac{\partial P}{\partial S} = N(x) - 1 < 0$. Total delta 0 if delta-neutral.

1.2 Theta

Theta is defined as the rate of change of security value with respect to time. For European call on a non-dividend-paying stock

$$\theta = -\frac{SN'(x)\sigma}{2\sqrt{E}} - r \times e^{-r\tau} N(x - \sigma\sqrt{\tau}) < 0$$

and looses value for European put.

$$\theta = -\frac{SN'(x)\sigma}{2\sqrt{\tau}} + r \times e^{-r\tau} N(-x + \sigma\sqrt{\tau})$$

1.3 Gamma

Measures now sensitive tho delta is to changes in the price of underlying asset. A portfolio with a high gamma needs in practice to be re-balanced more often to maintain delta neutrality. For European call or put $N'(x)/(S\sigma\sqrt{\tau}) > 0$

1.4 Rho

The rho of a derivative is the rate of change in its value with respect to interest rates. the rhos of a European call and a European put on non-dividend-paying stocks are $x\tau e^{-r\tau} N(x - \sigma\sqrt{\tau}) > 0$ and $-x + e^{-r\tau} N(-x + \sigma\sqrt{\tau}) < 0$.

1.5 Vega

Volatility changes over time. The vega of a derivative is the rate of change of its value with respect to volatility of underlying asset. A security with a high vega is very sensitive to small changes in volatility for European put or call $S\sqrt{\tau}N'(x) > 0$