For each bond its price:-

$$P = \sum_{i=1}^{n} \frac{C}{(1+y)^i} + \frac{F}{(1+y)^n} \quad \text{---} (i)$$

y = interest rate

So;

$$FV = \left(\sum_{i=1}^{n_1} \frac{C_1}{(1+y)^i} + \frac{F_1}{(1+y)^{n_1}} \right) (1+y)^D +$$

$$\left(\sum_{i=1}^{n_2} \frac{C_2}{(1+y)^i} + \frac{F_2}{(1+y)^{n_2}} \right) (1+y)^D$$

$$\Rightarrow \sum_{i=1}^{n_1} C_1 (1+y)^{D-i} + F_1 (1+y)^{D-n_1} +$$

$$\sum_{i=1}^{n_2} C_2 (1+y)^{D-i} + F_2 (1+y)^{D-n_2}$$

At horizon

$$\frac{\partial FV}{\partial y} = 0$$

So;

$$\frac{\partial FV}{\partial y} = \frac{\partial FV_1}{\partial y} + \frac{\partial FV_2}{\partial y} = 0$$

$$\frac{\partial FV}{\partial y} = \sum_{i=1}^{n_1} (D-i) C_1 (1+y)^{D-i-1} + (D-n_1) F_1 (1+y)^{D-n_1-1}$$

$$+ \sum_{i=1}^{n_2} (D-i) C_2 (1+y)^{D-i-1} + (D-n_2) F_2 (1+y)^{D-n_2-1} = 0$$

$$(1+y)^{D-1} \left[\sum_{i=1}^{n_1} \frac{(D-i) C_i}{(1+y)^i} + \frac{(D-n_1) F_1}{(1+y)^{n_1}} + \sum_{i=1}^{n_2} \frac{(D-i) C_2}{(1+y)^i} + \frac{(D-n_2) F_2}{(1+y)^{n_2}} \right] = 0$$

$$\Rightarrow \sum_{i=1}^{n_1} \frac{(D-i) C_i}{(1+y)^i} + \frac{(D-n_1) F_1}{(1+y)^{n_1}} + \sum_{i=1}^{n_2} \frac{(D-i) C_2}{(1+y)^i} + \frac{(D-n_2) F_2}{(1+y)^{n_2}} = 0$$

$$\Rightarrow \sum_{i=1}^{n_1} \left[\frac{C_1 D}{(1+y)^i} - \frac{C_1 i}{(1+y)^i} \right] + \frac{F_1 D}{(1+y)^{n_1}} - \frac{F_1 n_2}{(1+y)^{n_1}} +$$

$$\sum_{i=1}^{n_2} \left[\frac{C_2 D}{(1+y)^i} - \frac{C_2 i}{(1+y)^i} \right] + \frac{F_2 D}{(1+y)^{n_2}} - \frac{F_2 n_2}{(1+y)^{n_2}} = 0$$

$$-\left[\sum_{i=1}^{n_1} \frac{C_1 i}{(1+y)^i} + \frac{F_1 n_1}{(1+y)^{n_1}}\right] + D\left[\sum_{i=1}^{n_1} \frac{C_1}{(1+y)^i} + \frac{F_1}{(1+y)^{n_1}}\right]$$

$$-\left[\sum_{i=1}^{n_2} \frac{C_2 i}{(1+y)^i} + \frac{F_2 n_2}{(1+y)^{n_2}}\right] + D\left[\sum_{i=1}^{n_2} \frac{C_2}{(1+y)^i} + \frac{F_2}{(1+y)^{n_2}}\right]$$

$$= 0 \quad \text{—(ii)}$$

$$MD = \frac{1}{P} \sum_{i=1}^{n} \left[\frac{C i}{(1+y)^i} + \frac{F n}{(1+y)^n}\right] \quad \text{—(iii)}$$

Substitute (iii) into (i) and (ii)

$$-P_1 D_1 + D P_1 - P_2 D_2 + D P_2 = 0$$

$$D(P_1 + P_2) = P_1 D_1 + P_2 D_2$$

$$\underbrace{\frac{P_1}{P_1 + P_2}}_{\omega_1} D_1 + \underbrace{\frac{P_2}{P_1 + P_2}}_{\omega_2} D_2 = D$$

$$\omega_1 + \omega_2 = 1$$

or $\omega_1 D_1 + \omega_2 D_2 = D$ $\square$ //