

Solving a recurrence by expansion

Example:

$$T(n) = T(n-3) + 2 \quad \text{for } n \geq 0 \quad ; \quad T(0) = T(1) = T(2) = 5.$$

Step 1: Calculate the first few values to get some insight and understanding

n	$T(n)$
0	5
1	5
2	5
3	7
4	7
5	7
6	9
7	9
8	9
9	11

Step 2: Expand the definition and look for a pattern

$$\begin{aligned} T(n) &= T(n-3) + 2 \\ &= T(n-6) + 2 + 2 \\ &= T(n-9) + 2 + 2 + 2 \\ &\vdots \end{aligned}$$

Here we assumed n is a multiple of 3, for simplicity

$$\begin{aligned} &= T(0) + \underbrace{2 + 2 + 2 + \dots + 2 + 2}_{n/3 \text{ copies of } 2} \\ &= 5 + 2 \cdot \frac{n}{3} \end{aligned}$$

So we conclude $T(n) = 5 + \frac{2n}{3}$, when n is a multiple of 3.

Step 3: check agreement with step 1. Also use an online tool like Wolfram Alpha to check the answer. *(but it may not help!!)*