**Homework #2** (updated 9/15/2021)

**Algorithm Analysis**

total points: 87

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All of the code snippets used in this assignment are available in the hw02 package, within the assignment’s GitHub repository which you should create in the normal way via the invitation link on the homework webpage. However, none of the questions in this assignment requires you to edit or submit code. All answers will be submitted as a single PDF file to the noncode-answers folder of the GitHub repository.

Note that it is fine to provide handwritten solutions to any or all of these questions. You must scan or photograph the solutions and insert the scans into the relevant places of your document, submitting the final version as a single PDF in the noncode-answers folder.

1. Consider the algorithm encoded as a Java method given below. Assume that the array referred to by vals is square and that the values stored in vals are uniformly distributed, which means that any integer is equally likely to occur in any element of the array. Let denote the number of rows in the vals array.

**public** **static** **int** sumRowsThatStartWithEven(**int**[][] vals) {

**int** total = 0;

**for** (**int** row=0; row < vals.length; row++) {

// if first element of row is even…

**if** (vals[row][0] % 2 == 0) {

**for** (**int** col=0; col < vals[row].length; col++) {

total = total + vals[row][col];

}

}

}

**return** total;

}

a. 5 5(5 points) Give and briefly justify a bound for the best-case running time this algorithm, in terms of .

b. 5 10(5 points) Give and briefly justify a bound for the worst-case running time of this algorithm, in terms of .

c. 5 15(5 points) Give and briefly justify a bound for the average-case running time of this algorithm, in terms of .

2. Consider the following recursive algorithm implemented as a Java method:

**public** **static** **boolean** isPalindrome(String s) {

**if** (s.length() == 0) {

**return** **true**;

}

**else** **if** (s.length() == 1) {

**return** **false**;

}

**else** **if** (s.charAt(0) != s.charAt(s.length()-1)) {

**return** **false**;

}

**else** {

**return** *isPalindrome*(s.substring(1, s.length()-2));

}

}

a. 5 20(5 points) Briefly describe an input that will cause the worst-case performance for this algorithm.

b. 5 25(5 points) Assuming that the recursive call takes 15 basic operations, a return statement takes 3 basic operations and that any of the if statements takes 7 basic operations, give a recurrence describing the worst-case running time for this algorithm.

3. Use the technique of expansion to find closed-form solutions for the following recurrences:

a. 5 30(5 points)

and

b. 5 35(5 points)

and

4. Use the formal definitions of and to show that:

a. 20 55(20 points) is in and in .

b. 20 75(20 points) is in

5. (Ungraded optional question.) Repeat #5b using the following formal definition of Θ. A function mapping positive integers to positive integers is in if there exist three positive constants , , and such that for all .

6. 12 87(12 points) Assuming A and B are two algorithms that solve the same problem, indicate whether each of the following statements is true or false:

a. T / F If A O(n) then A O(n2).

b. T / F If A O(n lg n) then A Ο(lg n).

c. T / F If A Ω(n lg n) then A Ω(lg n).

d. T / F If A Ω(n2) then A Ο(n).

e. T / F If A O(n) then A Ω(lg n).

f. T / F A Ω(1).

g. T / F If A Θ(nlg n) then A Ω(n) Ο(n2).

h. T / F If A O(n) it will always be slower than B O(lg n) when run on the same input.

i. T / F If A Θ(n) and B Ω(n2) then there is guaranteed to be an instance of the problem for which A is faster than B.

j. T / F If A Θ(n) and B Ω(lg n) then there is guaranteed to be an instance of the problem for which B is faster than A.

k. T / F If A Ω(nlg n) and B O(n) then there is guaranteed to be an instance of the problem for which B is faster than A.

l. T / F If A Θ(n2) and B Θ(n) then B will always be faster than A.