

05 – BITS, BITES, WORDS & INTEGERS

COMP256 – COMPUTING ABSTRACTIONS

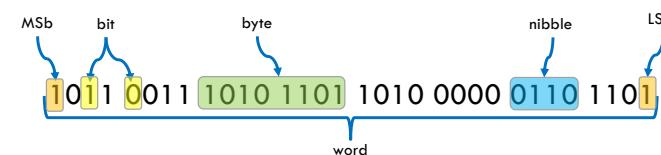
DICKINSON COLLEGE

WHAT DO ALL THOSE 1'S AND 0'S MEAN?

- It depends...
 - Consider the byte: 1001 0100
 - It could be:
 - 148, 94, -20, -107
 - 9.25, 4.5×2^{-3}
 - 'm'
 - An instruction the computer to perform an addition
- How do we know which it is?
 - We need to know what representation is being used.
 - E.g. What is pie?

TERMINOLOGY

- *Binary* (base 2/radix 2) is a representation of numeric values using only the values 0 and 1.



SOME BINARY REPRESENTATIONS

- Non-Negative Integers
 - Unsigned Binary
 - Binary Coded Decimal
- Signed Integers
 - Sign Magnitude
 - Two's Complement
- Decimal Numbers
 - Fixed Point
 - Floating Point
- Text Data
 - EBCDIC
 - ASCII
 - Unicode
- Colors
 - RGB
 - CMYK
 - HSV
- Instructions
 - Processor dependent

BINARY (BASE 2/RADIX 2) NUMBERS

- It's all about that base, no trouble:

$$\begin{array}{lll}
 \text{• Base 10: } 501 & = 5*100 + & \text{Hundreds} \\
 & = 5*10^2 + & \\
 & 0*10 + & \text{Tens} \\
 & 0*10^1 + & \\
 & 1*1 & \text{Ones} \\
 & 1*10^0 & \\
 & & \text{Base 10}
 \end{array}$$

$$\begin{array}{lll}
 \text{• Base 2: } 110 & = 1*2^2 + & \text{Fours} \\
 & = 1*4 + & \\
 & 1*2 + & \text{Twos} \\
 & 1*2^1 + & \\
 & 0*2^0 & \text{Ones} \\
 & 0*1 & \\
 & & \text{Base 2}
 \end{array}$$

BINARY ADDITION

- Adding unsigned binary numbers.

$$\begin{array}{r}
 100 \\
 + 001 \\
 \hline
 4
 \end{array}$$

$$\begin{array}{r}
 011 \\
 + 010 \\
 \hline
 3
 \end{array}$$

$$\begin{array}{r}
 011 \\
 + 011 \\
 \hline
 3
 \end{array}$$

$$\begin{array}{r}
 0111 \ 1001 \\
 + 0010 \ 1010 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 121 \\
 + 42 \\
 \hline
 \end{array}$$

UNSIGNED BINARY

- Unsigned binary is used to represent non-negative integers.

- Unsigned Binary to Decimal:

- What is the base 10 value of the following 8-bit unsigned binary value:

$1010\ 1101_2$

- Decimal to Unsigned Binary

- What is the 8-bit unsigned binary representation of the following base 10 value:

99_{10}

SIGN MAGNITUDE

- Sign Magnitude can be used to represent signed (positive and negative) integers.

- The MSb is used as a sign bit.

• 0 → positive 1 → negative

- The remaining bits give the value using unsigned binary.

- Perform the following conversions using 4-bit Sign Magnitude representation:

$0101_{2SM} = \underline{\hspace{2cm}}_{10}$

$1001_{2SM} = \underline{\hspace{2cm}}_{10}$

$6_{10} = \underline{\hspace{2cm}}_{2SM}$

$-3_{10} = \underline{\hspace{2cm}}_{2SM}$

TWO'S COMPLEMENT

- Two's complement is used by modern computers to represent signed integers.
 - The MSb indicates the sign.
 - 0 → positive 1 → negative

- Positive two's complement values are the same as unsigned binary.
 - For example in 8-bit two's complement:

$$\text{Positive} \rightarrow 010\ 0110_{2\text{TC}} = 38_{10}$$

$$18_{10} = \text{Positive} \rightarrow 001\ 0010_{2\text{TC}}$$

TWO'S COMPLEMENT

- The negative of a value in two's complement **is found from its positive** by:
 - "Flipping" all the bits
 - Adding 1

Positive

Flip the bits

1101 1001

+ 1

1101 1010_{2TC}

→ -38₁₀

Negative

Add 1

TWO'S COMPLEMENT

- The value of a negative two's complement number **is found from its positive** by:
 - "Flipping" all the bits
 - Adding 1

Negative

$$\begin{array}{r} \text{Flip the bits} \\ \text{Add 1} \\ \hline 0010\ 0101 \\ + 1 \\ \hline 010\ 0110_{2\text{TC}} = 38_{10} \end{array}$$

$$\text{So } 1101\ 1010_{2\text{TC}} \rightarrow -38_{10}$$

TWO'S COMPLEMENT EXAMPLES

- What is the base 10 value of the following four-bit two's complement numbers?

- 0101_{2TC}
- 1011_{2TC}

- What is the four-bit two's complement representation of the following base 10 numbers?

- 6₁₀
- 3₁₀

MORE TWO'S COMPLEMENT EXAMPLES

- What is the base 10 value of the following 8-bit two's complement numbers?
 - $0001\ 1001_{2TC}$
 - $1010\ 1101_{2TC}$
- What is the 8-bit two's complement representation of the following base 10 numbers?
 - 11_{10}
 - -11_{10}

HEXADECIMAL REPRESENTATION

- Hexadecimal (base 16) representation is commonly found in computer science.

$$\bullet \ 0x3AF9 = 3*16^3 + A*16^2 + F*16^1 + 9*16^0$$

$$3*4096 + A*256 + F*16 + 9*1$$

$$3*4096 + 10*256 + 15*16 + 9*1$$

A → 10 D → 13
B → 11 E → 14
C → 12 F → 15

HEXADECIMAL SHORTHAND

- Hexadecimal values are often used as a shorthand for expressing long binary strings.

- Conveniently every hex digit can be represented in 4 bits.

0 → 0000	6 → 0110	C → 1100
1 → 0001	7 → 0111	D → 1101
2 → 0010	8 → 1000	E → 1110
3 → 0011	9 → 1001	F → 1111
4 → 0100	A → 1010	
5 → 0101	B → 1011	

- So long binary strings can be easily represented as hex values:

1110 0011 0100 1111 1011 1010 1100 0010
E 3 4 F B A C 2

0xE34FBAC2