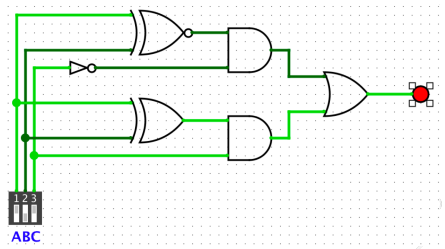
03 –GATES TO CIRCUITS

COMP256 – COMPUTING ABSTRACTIONS

DICKINSON COLLEGE

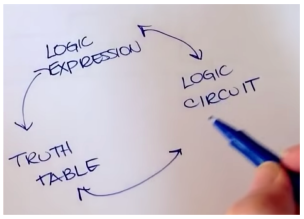
LOGIC CIRCUITS & BOOLEAN FUNCTIONS

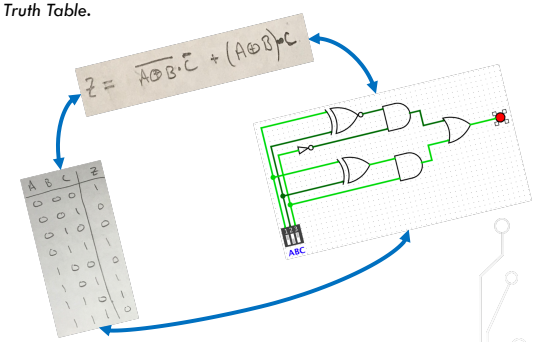
- A *Logic Circuit* is a combination of *logic gates* that perform a computation.
- Logic circuits compute *Boolean Functions*.
  - A Boolean Function is a function that maps Boolean (true/false, 1/0, +5V/ground) input(s) to a Boolean output.



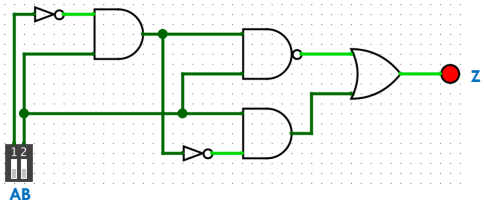
REPRESENTING BOOLEAN FUNCTIONS

- Boolean Functions have three equivalent representations:
  - *Logic Expression*, *Logic Circuit* and *Truth Table*.



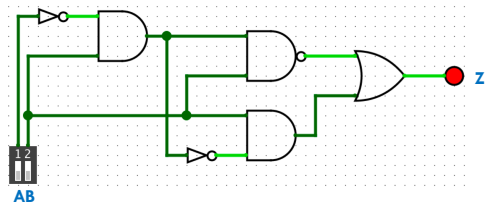


GIVEN A LOGIC CIRCUIT …



- … generate a logic expression.
  - Label intermediate gate outputs
  - Write expressions for intermediate gate outputs
  - Use substitution to propagate through to output
- … generate a truth table.
  - Apply an input pattern
  - Propagate values through gates to output
  - Repeat for all input patterns

GIVEN A LOGIC CIRCUIT …



- … generate a logic expression.

$$Z = \overline{\bar{A} \cdot B \cdot \bar{B}} + B \cdot \overline{\bar{A}B}$$

- … generate a truth table.

A	B	Z
0	0	1
0	1	0
1	0	1
1	1	1

GIVEN A LOGIC EXPRESSION …

$$Z = \overline{A\bar{B}} \cdot (\bar{A} + B)$$

- … generate a truth table.
  - Rows for all possible inputs
  - Columns for intermediate values
  - Continue until output is reached
- … generate a logic circuit.
  - Create sub-circuits for terms
  - Combine sub-circuits with additional gates

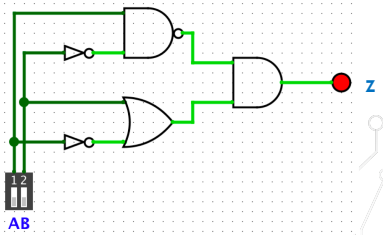
GIVEN A LOGIC EXPRESSION …

$$Z = \overline{A\bar{B}} \cdot (\bar{A} + B)$$

- … generate a truth table.

A	B	$\bar{A}$	$\bar{B}$	$A \cdot \bar{B}$	$\overline{A \cdot \bar{B}}$	$\bar{A} + B$	Z
0	0	1	1	0	1	1	1
0	1	1	0	0	1	1	1
1	0	0	1	1	0	0	0
1	1	0	0	0	1	1	1

- … generate a logic circuit.



GIVEN A TRUTH TABLE …

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- … generate a logic expression.
  - Add one AND term for each 1 in output
  - Add negation to make each term true for its inputs.
  - Give a Sum-of-Products (SOP) expression.
- ... generate a a logic circuit.
  - Directly implementation of the SOP expression.

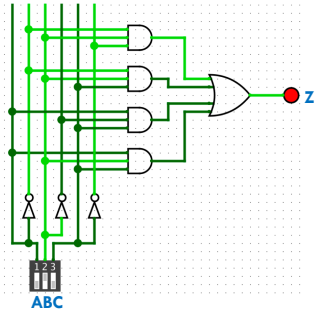
GIVEN A TRUTH TABLE …

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- … generate a logic expression.

$$Z = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

- … generate a a logic circuit.



LOGIC CIRCUIT DESIGN

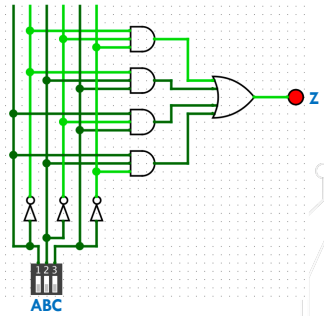
- Challenge: Design a circuit that computes an *odd parity check* on three inputs.
- Logical circuit design process:
  - Given a problem statement generate a truth table.
  - Convert the truth table to a logic expression in SOP form
  - Implement the SOP logic expression as a logic circuit.

LOGIC CIRCUIT DESIGN

- Challenge: Challenge: Design a circuit that computes an *odd parity check* on three inputs.

A	B	C	Z
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$Z = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$$



ABSTRACTING CIRCUITS

- Logic circuits can be packaged into integrated circuits.
  - 9 Bit even/odd parity generator/checker
    - https://www.ti.com/lit/ds/sdls152/sdls152.pdf (for the curious)

Image from: http://www.yd-tech.com.tw/product_info.php/products_id/63503