COMP314 Midterm Exam 2, Spring 2023
Time allowed: 75 minutes
Total points on exam: 75

Question 1. (15 points) Let THREEHC be a decision problem whose input is an undirected graph G. The solution is "yes" if all vertices of G can be visited by exactly three disjoint cycles. (This is the same idea as the usual Hamilton cycle problem, except we use three cycles to visit the vertices instead of only one cycle.) Give an explicit polyreduction from the decision version of UHC to THREEHC, and prove that your construction is a polyreduction.

Question 2. (10 points) Let THREEHC be as in the previous question, and let C be an NP-complete problem. Prove that there is there a polyreduction from THREEHC to C.

Question 3. Define the decision problem ISSQUARE as follows. The input is an ASCII string containing a positive integer M in decimal notation. (Example: "81".) The solution is "yes" if the input is a square number (i.e $1, 4, 9, 16, 25, \ldots$) and "no" otherwise.

Consider the following Python program. In answering the questions below, you may assume that all parameters are correctly formatted—so I and H are both strings containing a positive integer.

```python
def verifyIsSquare(I, S, H):
    if len(S)>3 or len(H)>len(I) or S!='yes':
        return 'unsure'
    M = int(I)
    H = int(H)
    total = 0
    for i in range(H):
        total += H
    if total == M:
        return 'correct'
    else:
        return 'unsure'
```

(a) (10 points) Is this program a verifier for ISSQUARE? Explain your answer, but rigorous mathematical proof is not required.

(b) (10 points) Is this program a *polytime* verifier for ISSQUARE? Explain your answer, but rigorous mathematical proof is not required.

Question 4. (20 points) The computational problem CIRCUIT6 is defined as follows. Informally, we are given a circuit that can output the numbers from 0 to 7 in binary, and we ask the question, "Can this circuit output the number 6?" Formally, the input is a description of a circuit similar to CIRCUITSAT except that there are three output wires instead of one. Because each of the three output wires can output a 0 or 1, we can interpret the output as a three-bit binary number. There are exactly 8 possibilities: $000, 001, 010, 011, 100, 101, 110, 111$. We interpret these outputs as representing the integers $0, 1, 2, 3, 4, 5, 6, 7$. The solution to a CIRCUIT6 instance is "yes" if there exists some input for which the given circuit will produce a 6, and "no" otherwise.

Is CIRCUIT6 NP-complete? Give a rigorous justification of your answer.

Question 5. (10 points) The decision problem ISINCREASINGSTRING is defined as follows. The input I is a single ASCII string. The string I is a positive instance if and only if it is a strictly increasing sequence of alphabetic characters, where we define the ordering of the characters as $a < b < c < \ldots < z < A < B < \ldots < Z$. If I contains any characters other than upper and lowercase letters, it is a negative instance. For example, the strings "befyAW", "abmnXYZ", and "PRTV" are positive instances; the strings "BdXz", "abc5ABC", "aa", and "ba" are negative instances.

Assuming we are using the default computational model of Python programs as in our textbook, list all of the following complexity classes that ISINCREASINGSTRING belongs to: Const, Lin, LogLin, Quad. Briefly justify your answer.

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