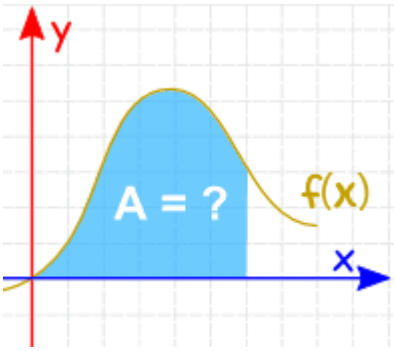
Integration Rules

Integration

Integration can be used to find areas, volumes, central points and many useful things. But it is often used to find the **area underneath the graph of a function** like this:



The integral of many functions are well known, and there are useful rules to work out the integral of more complicated functions, many of which are shown here.

There are examples below to help you.

Common Functions	Function	Integral
Constant	$\int a\,dx$	$ax + C$
Variable	$\int x\,dx$	$x^2/2 + C$
Square	$\int x^2\,dx$	$x^3/3 + C$
Reciprocal	$\int (1/x)\,dx$	$\ln\|x\| + C$
Exponential	$\int e^x\,dx$	$e^x + C$
	$\int a^x\,dx$	$a^x/\ln(a) + C$
	$\int \ln(x)\,dx$	$x\ln(x) - x + C$
Trigonometry (x in radians)	$\int \cos(x)\,dx$	$\sin(x) + C$
	$\int \sin(x)\,dx$	$-\cos(x) + C$
	$\int \sec^2(x)\,dx$	$\tan(x) + C$

Rules	Function	Integral

Multiplication by constant	$\int cf(x)\,dx$	$c\int f(x)\,dx$
Power Rule (n≠-1)	$\int x^n\,dx$	$\frac{x^{n+1}}{n+1} + C$
Sum Rule	$\int (f + g)\,dx$	$\int f\,dx + \int g\,dx$
Difference Rule	$\int (f - g)\,dx$	$\int f\,dx - \int g\,dx$
Integration by Parts	See Integration by Parts	
Substitution Rule	See Integration by Substitution	

Examples

Example: what is the integral of sin(x) ?

From the table above it is listed as being **−cos(x) + C**

It is written as:

$$\int \sin(x)\,dx = -\cos(x) + C$$

Example: what is the integral of 1/x ?

From the table above it is listed as being **ln|x| + C**

It is written as:

$$\int (1/x)\,dx = \ln|x| + C$$

The vertical bars **||** either side of **x** mean absolute value, because we don't want to give negative values to the natural logarithm function **ln**.

Power Rule

Example: What is $\int x^3\,dx$?

The question is asking "what is the integral of x^3 ?"

We can use the Power Rule, where n=3:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^3 \, dx = \frac{x^4}{4} + C$$

Example: What is $\int \sqrt{x} \, dx$?

$\sqrt{x}$ is also $\mathbf{x^{0.5}}$

We can use the Power Rule, where n=½:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^{0.5} \, dx = \frac{x^{1.5}}{1.5} + C$$

Multiplication by constant

Example: What is $\int 6x^2 \, dx$?

We can move the 6 outside the integral:

$$\int 6x^2 \, dx = 6\int x^2 \, dx$$

And now use the Power Rule on x^2:

$$= 6\,\frac{x^3}{3} + C$$

Simplify:

$$= 2x^3 + C$$

Sum Rule

Example: What is $\int \cos x + x \, dx$?

Use the Sum Rule:

$$\int \cos x + x \, dx = \int \cos x \, dx + \int x \, dx$$

Work out the integral of each (using table above):

$$= \sin x + x^2/2 + C$$

Difference Rule

Example: What is $\int e^w - 3 \, dw$?

Use the Difference Rule:

$$\int e^w - 3 \, dw = \int e^w \, dw - \int 3 \, dw$$

Then work out the integral of each (using table above):

$$= e^w - 3w + C$$

Sum, Difference, Constant Multiplication And Power Rules

Example: What is $\int 8z + 4z^3 - 6z^2 \, dz$?

Use the Sum and Difference Rule:

$$\int 8z + 4z^3 - 6z^2 \, dz = \int 8z \, dz + \int 4z^3 \, dz - \int 6z^2 \, dz$$

Constant Multiplication:

$$= 8\int z \, dz + 4\int z^3 \, dz - 6\int z^2 \, dz$$

Power Rule:

$$= 8z^2/2 + 4z^4/4 - 6z^3/3 + C$$

Simplify:

$$= 4z^2 + z^4 - 2z^3 + C$$

Integration by Parts

See Integration by Parts

Substitution Rule

See Integration by Substitution

Final Advice

- Get plenty of practice
- Don't forget the **dx** (or dz, etc)
- Don't forget the **+ C**

Question 1 Question 2 Question 3 Question 4 Question 5
Question 6 Question 7 Question 8 Question 9 Question 10